

Endogenous Productivity Differences and Patterns of International Capital Flows

By Kiminori Matsuyama

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1. Introduction.

- Capital often flows “upstream” from poor to rich countries, contrary to the neoclassical prediction. Why?
- Two often suggested mechanisms:
 - *Productivity Differences*: The North invests more productively than the South, so that the lenders would get higher return in the North.
 - *Institutional Differences*: The North has superior institution protecting the interest of lenders, so that they would get higher return in the North.
- A simple model could explain these mechanisms, as long as we treat these differences in productivity and institution *exogenously* (as done in Section 2).
- This paper aims to argue that this simplistic view of international capital flows needs to be substantially modified to the extent that *productivity differences are endogenous due to institutional differences*.

More specifically:

- Perhaps counter-intuitively, endogenous productivity differences have *opposite* implications from exogenous ones on capital flows. Better investment technologies (IT) due to a better institution quality (IQ), leads to
 - a higher capital stock, a higher wage rate, as in the exogenous case),
 - a *lower* investment, unlike the exogenous case
 - a current account *surplus* (i.e., capital *outflows*), unlike the exogenous case
 - *Ambiguous effects* of IQ on capital flows, due to the TWO effects working in the opposite directions.
 1. *Holding investment productivity constant*, a higher IQ causes to a current account **deficit** (i.e., capital inflows), because it makes the country a more attractive place to invest.
 2. Induced improvement in IT cause a current account **surplus** (i.e., capital outflows), because the country needs less resources to produce capital stock.
- Even if the North is found to have better IT and better IQ than the South, no reason to expect large capital flows in either direction between the two. Small capital flows do not necessarily imply significant barriers.

- *Non-monotonic effects* of IQ suggested by parametric examples:
 - Higher IQ, while monotonically increasing the capital stock and wages, leads to a **U-shaped** response of the investment and current account.
 - Initially, a *lower* investment & current account *surplus* (i.e., capital outflows)
 - Then, a *higher* investment & a current account *deficit*, (i.e., capital inflows).
 - If countries inherently differ only in their IQ,
 - countries in the middle run a current account surplus (i.e., capital outflows)
 - countries at high & low ends run a current account deficit (i.e., capital inflows)
 - By improving its IQ, a country can experience both *a growth miracle & a current account surplus* at the same time.

Exogenous versus Endogenous: An Intuition (First Attempt)

Higher investment productivity generally has two effects:

- 1) More capital stock can be produced with *less* investment.
- 2) Higher rate of returns makes the lender willing to finance *more* investment.

Exogenous case: Both effects operate. Under the “reasonable” assumption, 2nd effect dominates → *higher investment*.

Consider the *endogenous* case where,

- Different types of investment projects with **productivity-agency cost trade-off**; more productive ones come with bigger agency problems.
- Credit markets pick the projects that generate the highest return to the lenders, which are *not* generally the most productivity ones.
- IQ affects the trade-off, hence the types of projects financed, and hence productivity.
- Improving IQ shifts the credit toward more productive projects, which come with bigger agency problems.
- Productivity goes up, but not the rate of return to the lenders (inclusive of the agency cost).

Hence, 2nd effect is negligible; 1st effect dominates → *lower investment*.

Some related (and not so related) work:

Credit Market Imperfections and Reverse Capital Flows

- Net Worth Effect; Gertler-Rogoff (JME 1990); Boyd-Smith (JET 1997); Matsuyama (ECMA 2004); Matsuyama (JEEA 2005)
- Institutional Quality; Sakuragawa-Hamada (IER 2001)

Endogenous Productivity through Composition of Credit

- Matsuyama (AER 2007); closed economy business cycles with endogenous productivity

The model here borrows some features from my (JEEA 2005) and my (AER 2007)

U-shaped patterns of capital flows:

- Gourinchas-Jeanne (2007)'s "Allocation Puzzle"

Emphasis on investment productivity differences:

- DeLong-Summers (QJE 1991) and others
- Greenwood-Hercowitz-Krusell (AER 1997) and others

Plan of the Paper

1. Introduction.
2. Background: A Model with **Exogenous** Investment Technologies
 - Closed Economy Model, with exogenous institutional quality (IQ) & exogenous investment technologies (IT).
 - World Economy where countries differ inherently both in IT & IQ
3. **Endogenizing** Investment Technologies
Exogenous versus Endogenous Technologies
4. Patterns of International Capital Flows with **Endogenous** Technologies
 - A Two-Project Case
 - Two-Country World
 - Three-Country World
 - A Continuum of Projects Case
5. Concluding Remarks

2. Background: A Model with Exogenous Investment Technologies:

Two Periods: $t = 0$ & $t = 1$

In $t = 0$, the economy's endowment can be consumed or invested to projects, which generate capital, k , that will be made available in $t = 1$.

In $t = 1$, the consumption good is produced competitively with $y = F(k, 1) \equiv f(k)$

- $L = 1$ is the labor supply (introducing diminishing returns to capital)
- $f(k)$ satisfies $f' > 0 > f''$ and $f'(0) = \infty$.

Two Types of Agents: Savers/Workers & Borrower/Entrepreneurs

Savers/Workers;

- Endowed with ω units of the input at $t = 0$.
- Supply one unit of labor at $t = 1$, and earns the labor income, $w(k) \equiv f(k) - kf'(k)$.
- Quasi-linear preferences: $U^s = V(C^s_0) + C^s_1$

Maximize $U^s = V(C^s_0) + C^s_1$ subject to $C^s_1 = r(\omega - C^s_0) + w(k)$.

→ First-Order-Condition: $V'(C^s_0) \equiv r$

→ Saver's Saving: $S^s(r) \equiv \omega - C^s_0 \equiv \omega - (V')^{-1}(r)$.

A Unit Mass of Borrowers/Entrepreneurs:

- Each agent is endowed with $0 \leq \omega^b < 1$ units of the input at $t = 0$.
- They consume only at $t = 1$, hence save everything.
- Each agent can run (at most) one project, which converts one unit of the input to R units of “physical capital,” by borrowing $1 - \omega^b$ at the market rate of return, r .

Entrepreneur's Objective = Consumption in Period 1

$$U^b = \begin{cases} Rf'(k) - r(1 - \omega^b) & \text{if borrow and run the project} \\ r\omega^b & \text{otherwise} \end{cases}$$

Each borrower/entrepreneur is willing to borrow and run the project if and only if

Profitability Constraint (PC): $Rf'(k) \geq r$,

Credit Market Imperfections: The borrower can pledge only up to $\lambda Rf'(k)$ for the repayment ($0 < \lambda < 1$).

Borrowing Constraint (BC): $\lambda Rf'(k) \geq r(1 - \omega^b)$,

In equilibrium, investment takes place until one of (PC) and (BC) becomes binding:

$$(PC) + (BC): \quad \text{Max} \left\{ 1, \frac{1 - \omega^b}{\lambda} \right\} r = Rf'(k) = Rf'(RI),$$

where I is the aggregate investment, i.e., the total amount of the endowment that is left unconsumed and goes into the capital-generating projects.

Aggregate Investment Schedule:

$$\frac{k}{R} = I(r) \equiv \frac{1}{R} (f')^{-1} \left[\text{Max} \left\{ 1, \frac{1 - \omega^b}{\lambda} \right\} \frac{r}{R} \right],$$

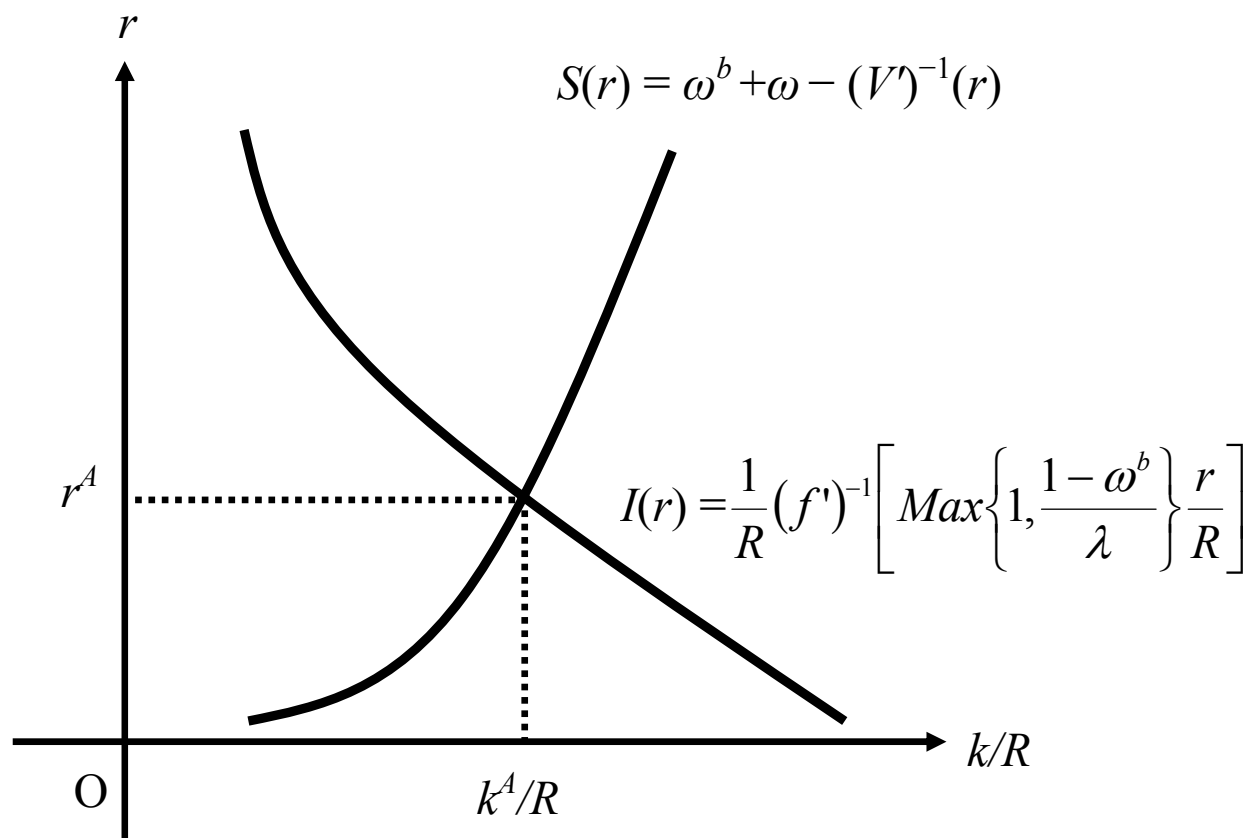
Aggregate Saving Schedule:

$$S(r) \equiv \omega^b + \omega - (V')^{-1}(r)$$

Current Account Schedule:

$$CA(r) \equiv S(r) - I(r)$$

Autarky Equilibrium: A Graphical Illustration



To keep it simple, let us assume $\omega^b = 0$ so that (BC) is always binding. Then,

Autarky Equilibrium: A Graphical Illustration

In autarky equilibrium,

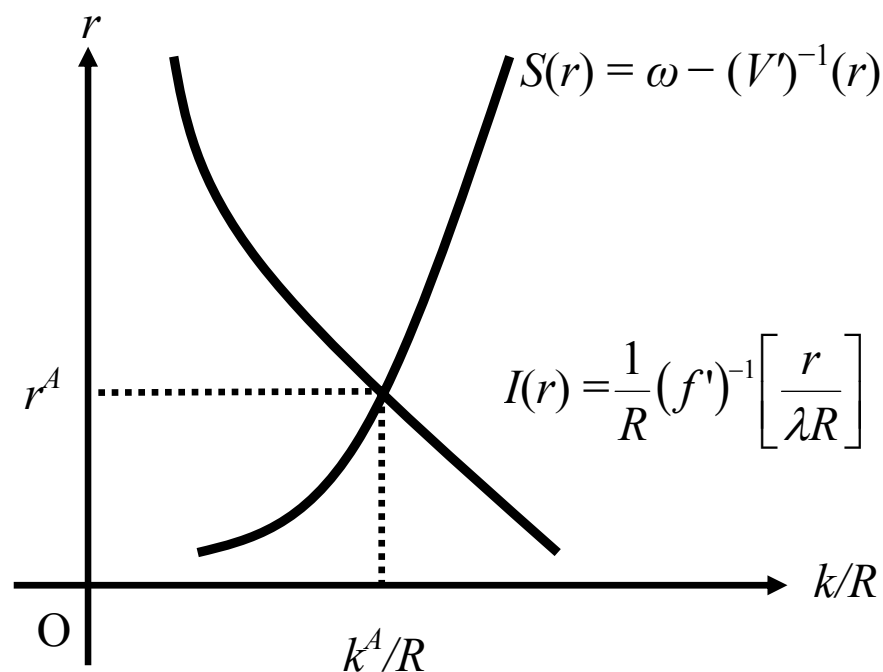
The capital stock, k^A , and the wage rate, $w^A = w(k^A) \equiv f(k^A) - k^A f'(k^A)$, are both

- increasing in ω ,
- increasing in λ ,
- increasing in R .

However,

The rate of return, r^A , is

- decreasing in ω .
- increasing in λ .
- increasing in R iff $\eta \equiv -kf''/f' < 1$ (e.g., Cobb-Douglas case).



Decomposing Credit Market Imperfections: $\lambda < 1$

$$\lambda = (\Lambda)^\theta < 1$$

$0 < \Lambda < 1$;

- ✓ Capturing the **Agency Problem** associated with the project:
- ✓ A larger Λ implies a smaller agency problem;

$\theta > 0$;

- ✓ Capturing the **Institutional Quality (IQ)**
- ✓ A smaller θ means a higher IQ, making λ larger.
- ✓ The benefit of a higher IQ is larger for a small Λ ; **strict log-supermodularity**

- At this point, it doesn't matter whether a change in λ comes from a change in Λ or in θ .
- However, this distinction will become important later.
- To make you prepare, I propose that you start thinking that variations in λ come primarily from θ , and hence a higher λ means a better IQ (a smaller θ).

A Digression: An Alternative Interpretation of R and k

No need to interpret R as the productivity in the I-goods sector. Instead of thinking that entrepreneurs run the project that produces “physical capital” to be rented out to the C-goods sector, imagine

- An entrepreneur invests one unit of the input in Period-0 to set up a firm.
- Each firm produces the consumption good in Period-1, using the labor input, n , with a concave function, $y = \varphi(n)$.
- Each firm hires labor in the competitive market, so that $\varphi'(n) = w$, and makes the profit equal to $\pi = y - wn = \varphi(n) - \varphi'(n)n$.
- Labor market equilibrium; $nI = 1$, where I is the aggregate investment, which is equal to the number of firms set up in Period-0.

Let $\varphi(n) \equiv F(R, n)$ and $k \equiv RI$. Here, R is the **productivity parameter in the C-Goods sector**; k is the aggregate supply of the **(intangible) assets** held by these firms. Then,

$$w = \varphi'(1/I) = F_n(R, 1/I) = F_n(k, 1) = f(k) - kf'(k).$$
$$\pi = F(R, 1/I) - w/I = [F(k, 1) - w]/I = R[f(k) - w]/k = Rf'(k).$$

The two interpretations are identical.

World Economy: C countries of the equal size for $c \in C$.

- Country Characteristics: ω^c , R^c , & $\theta^c = \log(\lambda^c)/\log(\Lambda)$
- Period-0 input & Period-1 consumption good are both intertemporally tradeable
- Capital stock and labor are not.

Under Autarky: The rate of return in $c \in C$ is determined by

$$S^c(r^{cA}) = \frac{k^{cA}}{R^c} = I^c(r^{cA}) \quad \Leftrightarrow \quad CA^c(r^{cA}) = 0$$

where $S^c(r) \equiv \omega^c - (V')^{-1}(r)$, $I^c(r) \equiv \frac{1}{R^c} (f')^{-1}\left(\frac{r}{\lambda^c R^c}\right)$ & $CA^c(r) \equiv S^c(r) - I^c(r)$.

Under Financial Integration: The rate of return is equalized across countries, and determined by:

$$\sum_{c \in C} S^c(r) = \sum_{c \in C} I^c(r) \quad \Leftrightarrow \quad \sum_{c \in C} CA^c(r) = 0.$$

Since $CA^c(r) \equiv S^c(r) - I^c(r)$ is strictly increasing in r ,

- If $r^{cA} < r$, $CA^c(r) \equiv S^c(r) - I^c(r) > 0$; $k^c = R^c I^c(r) < R^c I^c(r^{cA}) = k^{cA}$
- If $r^{cA} > r$, $CA^c(r) \equiv S^c(r) - I^c(r) < 0$; $k^c = R^c I^c(r) > R^c I^c(r^{cA}) = k^{cA}$

Thus,

Capital flows from countries with lower (autarky) rates of return to those with higher (autarky) rate of returns.

So, the key question is:

Do richer countries have higher or lower rates of returns than poor countries?

The answer depends on what we think are primary differences between rich and poor countries. Are they Endowments; Productivities, or Institutions?

Standard Neoclassical View: Capital flows from the Rich to the Poor

- **Endowment:** The rich has more saving, thus more resources to invest (a higher ω^c).
Recall that r^{cA} is strictly decreasing in ω^c .

Two Anti-Neoclassical Views: Capital flows from the Poor to the Rich

- **Productivity:** The rich has better investment technologies (a higher R^c)
Recall that r^{cA} is strictly increasing in R^c (as long as $\eta \equiv -kf'/f' < 1$, e.g., as in Cobb-Douglas case).
- **Institutional Quality:** The rich does better jobs protecting the interest of the lenders. (a smaller θ^c)
Recall that r^{cA} is strictly increasing in λ^c , hence strictly decreasing in θ^c .

These simple views of international capital flows need to be modified if productivity differences are caused by institutional differences.

3. Endogenizing Investment Technologies

- Each entrepreneur now has access to a set of projects, J . A type- j ($\in J$) project converts m_j units of the input to $R_j m_j$ units of “physical capital”. Only a fraction, λ_j , of the project revenue can be pledged to the lender.
- Each entrepreneur can run at most one project.

$$U^b = \begin{cases} R_j m_j f'(k) - r(m_j - \omega^b) = [R_j f'(k) - r] m_j + r \omega^b & \text{if borrow and run a project-}j \\ r \omega^b & \text{otherwise} \end{cases}$$

Each entrepreneur is willing to borrow and run a project- j if and only if

Profitability Constraint for a Type- j (PC- j): $R_j f'(k) \geq r,$

Borrowing Constraint for a Type- j (BC- j): $\lambda_j m_j R_j f'(k) \geq r(m_j - \omega^b),$

By combining (PC-j) and (BC-j), we can define

$$(PC-j) + (BC-j): \quad r_j \equiv \frac{R_j}{\text{Max}\{1, (m_j - \omega^b) / \lambda_j m_j\}} f'(k)$$

is the highest rate of return that an entrepreneur could credibly offer to the lender by running a type-j project.

In equilibrium, the credit goes only to the projects with the highest r_j :

$$\begin{aligned} r &= \text{Max}_{j \in J} \{r_j\} = \text{Max}_{j \in J} \left\{ \frac{R_j}{\text{Max}\{1, (m_j - \omega^b) / \lambda_j m_j\}} \right\} f'(k) \\ &= \left\{ \frac{R_{j^*}}{\text{Max}\{1, (m_{j^*} - \omega^b) / \lambda_{j^*} m_{j^*}\}} \right\} f'(R_{j^*} I), \quad \text{where } j^* \equiv \text{Arg max}_{j \in J} \{r_j\}. \end{aligned}$$

To keep it simple, let $\omega^b = 0$, so that

$$r = \underset{j \in J}{\text{Max}} \{r_j\} = \underset{j \in J}{\text{Max}} \{\lambda_j R_j\} f'(k) = \lambda_{j^*} R_{j^*} f'(R_{j^*} I)$$

or

$$I(r) = \frac{1}{R_{j^*}} (f')^{-1} \left(\frac{r}{\lambda_{j^*} R_{j^*}} \right), \quad \text{where } j^* \equiv \underset{j \in J}{\text{Arg max}} \{\lambda_j R_j\}.$$

Note: Crucial is that ω^b is small enough so that (BC) is binding and hence, the market picks the project that generates the highest return to the lender, which may or may not be the most productive project.

Decomposing the Credit Market Imperfections:

$$\lambda_j = (\Lambda_j)^\theta = [\Lambda(R_j)]^\theta$$

$$0 < \Lambda_j = \Lambda(R_j) \leq 1$$

- Project-specific component, capturing the **Agency Problem** of each project:
- A larger Λ_j implies a smaller agency problem.
- $\Lambda(\bullet)$ is strictly decreasing \rightarrow Trade-offs between productivity and agency problem

$$\theta > 0;$$

- Country-specific component, capturing the **Institutional Quality (IQ)** of each country
- A bigger θ means a lower IQ, making λ_j smaller, exacerbating the agency problem.
- The more productive projects with bigger agency problems benefit more from a lower θ (i.e., a higher IQ). Strict log-supermodularity.

How Institutional Quality affects Investment Technologies:

The market picks the project to solve $\max_{j \in J} \{\lambda_j R_j\} = \max_{j \in J} \{[\Lambda(R_j)]^\theta R_j\}$.

The solution is a function of θ , $R(\theta)$, with the properties:

- $[\Lambda(R(\theta))]^\theta R(\theta) \geq \lambda_j R_j$ for all $j \in J$;
- $R(\theta)$ is decreasing. (When an institutional quality improves, the credit switches towards more productive projects.)

Aggregate Investment Schedule:

$$I(r) = \frac{1}{R(\theta)} (f')^{-1} \left(\frac{r}{[\Lambda(R(\theta))]^\theta R(\theta)} \right)$$

Exogenous vs. Endogenous Productivity: An Intuition (Second Attempt)

Aggregate Investment with Exogenous Productivity:

$$I(r) = \frac{1}{R} (f')^{-1} \left(\frac{r}{(\Lambda)^\theta R} \right)$$

Increasing R exogenously affects $I(r)$ through two effects, working in the opposite directions:

- 1) Less investment is needed to produce more capital stock;
- 2) Higher return to the lenders, who become more willing to finance the investment.

Under the “reasonable” assumption on the elasticity of the marginal capital, $f'(k)$, satisfied by, e.g., Cobb-Douglas,

2nd effect dominates 1st effect. \rightarrow a higher R **increases** $I(r)$.

Aggregate Investment with Endogenous Productivity:

$$I(r) = \frac{1}{R(\theta)} (f')^{-1} \left(\frac{r}{[\Lambda(R(\theta))]^\theta R(\theta)} \right)$$

When $R(\theta)$ changes due to a change in θ ,

- 1st effect (less investment needed) is **of the first order**.
- 2nd effect (higher return to the lenders) is **of the second order**.

Intuition: The market always chooses the project to maximize the rate of return to the lenders, so that a change in $R(\theta)$ has no additional effect. **Envelope Theorem!!**

1st effect always dominates. \rightarrow a higher $R(\theta)$ **reduces** $I(r)$.

Notes:

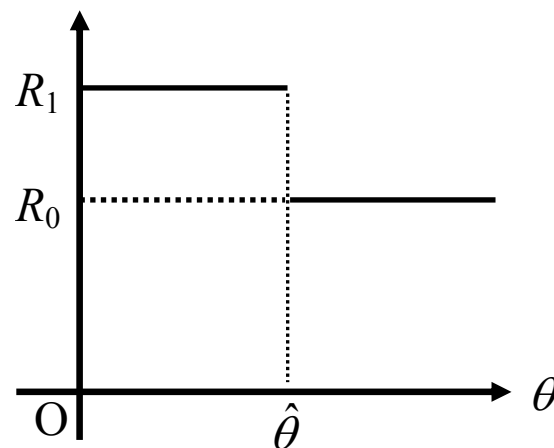
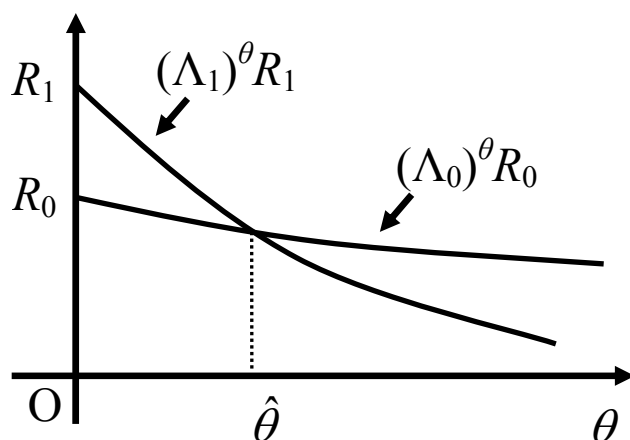
- 1) The argument does not depend on $f(k)$, unlike the case of exogenous changes.
- 2) The argument does not depend on the strict supermodularity.
- 3) θ also affects $I(r)$ directly. The combined effect of θ on $I(r)$ is generally ambiguous, so we need to look at some specific examples.

4. Patterns of International Capital Flows with Endogenous Technologies

A Two-Projects Case: $J = \{0,1\}$; Let $R_0 < R_1$; $1 \geq \Lambda_0 > \Lambda_1$.

Type-0 is less subject to the agency problem but less productive than Type-1.

- $\theta > \hat{\theta} \equiv \frac{\log(R_1/R_0)}{\log(\Lambda_0/\Lambda_1)} \rightarrow$ The credit goes to only Type-0.
- $\theta \leq \hat{\theta} \equiv \frac{\log(R_1/R_0)}{\log(\Lambda_0/\Lambda_1)} \rightarrow$ The credit goes to only Type-1.

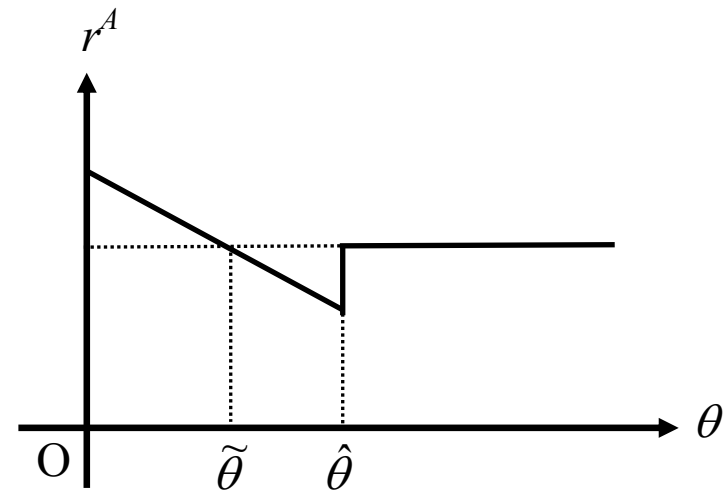


Note: At $\theta = \hat{\theta}$, productivity jumps, but the rate of return to the lender doesn't.

\rightarrow As IQ improves (a decline in θ), the aggregate investment drops discretely at $\theta = \hat{\theta}$.

$$I(r) = \frac{1}{R(\theta)} (f')^{-1} \left(\frac{r}{[\Lambda(R(\theta))]^\theta R(\theta)} \right) = \begin{cases} \frac{1}{R_0} (f')^{-1} \left[\frac{r}{(\Lambda_0)^\theta R_0} \right] & \text{if } \theta > \hat{\theta} \\ \frac{1}{R_1} (f')^{-1} \left[\frac{r}{(\Lambda_1)^\theta R_1} \right] & \text{if } \theta \leq \hat{\theta}. \end{cases}$$

This translates into a “U-shaped” response of r^A to a change in θ . (The graph assumes $\eta \equiv -kf''/f' < 1$ and $\Lambda_0 = 1$.)

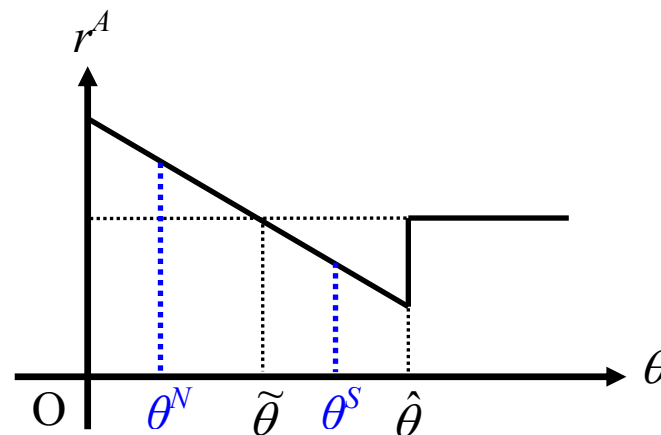


A Two-Country World: $C = \{N, S\}$; $\theta^N < \theta^S$. N for the rich North; S for the poor South. Assume also $\omega^N = \omega^S$ and $\Lambda^N(\cdot) = \Lambda^S(\cdot)$.

Case 1: $(\theta^N, \tilde{\theta} < \theta^S < \hat{\theta}) \rightarrow CA^N < 0 < CA^S$.

Capital flows from S to N .

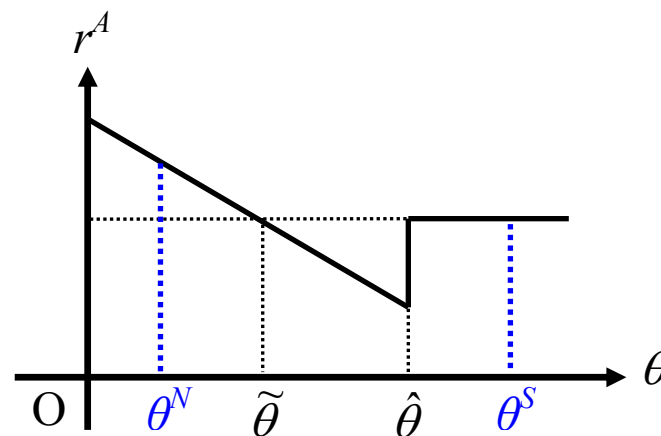
- Both countries use the same technologies.
- N 's superior institution causes the capital flows.



Case 2: $(\theta^N < \tilde{\theta} < \hat{\theta} < \theta^S) \rightarrow CA^N < 0 < CA^S$.

Capital flows from S to N .

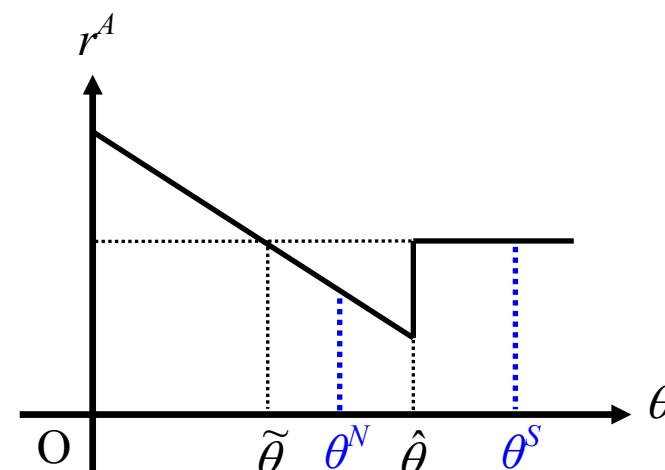
- Institutional difference is the real cause.
- Though N invests more productively, but it is *false* to attribute it to the capital flows. Productivity difference in fact partially offsets the effect of institutional difference on the capital flows.



Case 3: $(\tilde{\theta} < \theta^N < \hat{\theta} < \theta^S) \rightarrow CA^N > 0 > CA^S$.

Capital flows from N to S .

S is stuck with the less productive technology due to its inferior institution, and hence needs to invest more.

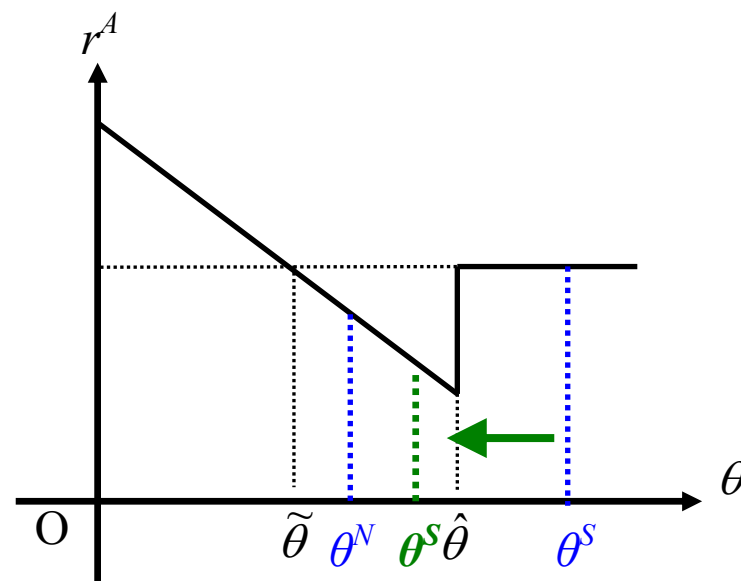


A Thought Experiment:

Imagine that, starting from Case 3, S improves its institution, as shown in the Figure. We now have Case 1.

Capital flows are reversed. S 's current account turns from a deficit to a surplus. (That is, capital starts flowing out, instead of flowing in.)

→ *Growth Miracle & Capital Outflows*



A Three-Country World: $C = \{N, M, S\}$ with $\theta^N < \theta^M < \theta^S$.

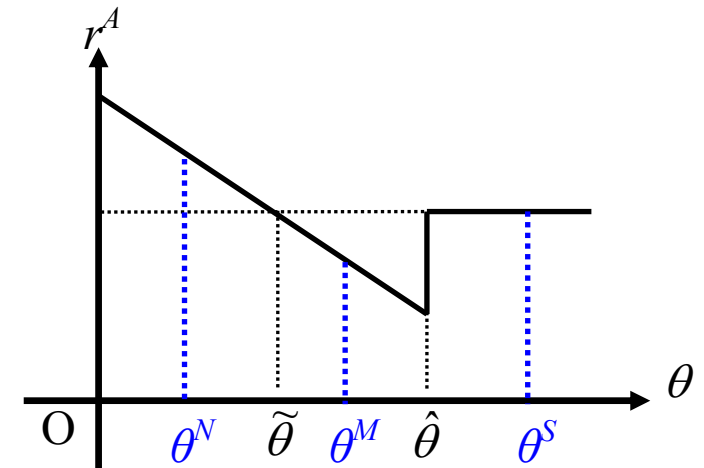
Assume they are identical in other dimensions.

Case 1: $\theta^N < \tilde{\theta} < \theta^M < \hat{\theta} < \theta^S \rightarrow CA^N < 0 < CA^M; CA^S?$

Capital flows into N . Capital flows out of M .

Among developing countries, capital flows from the more successful M to the less successful S .

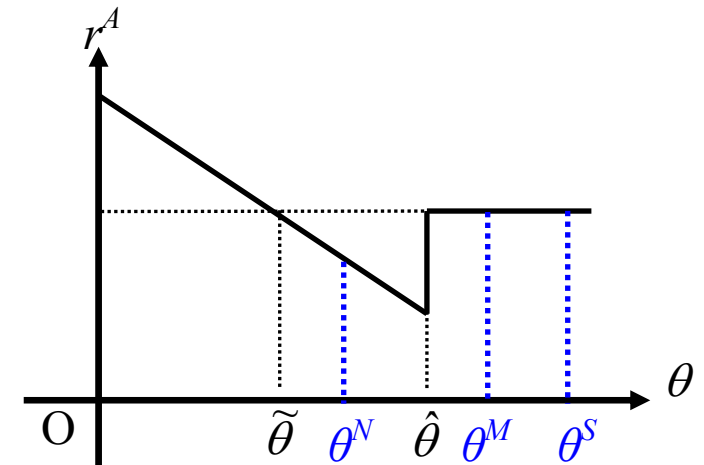
→ A solution to the “Allocation Puzzle”?



Case 2: $\tilde{\theta} < \theta^N < \hat{\theta} < \theta^M < \theta^S \rightarrow CA^N > 0 > CA^M, CA^S$

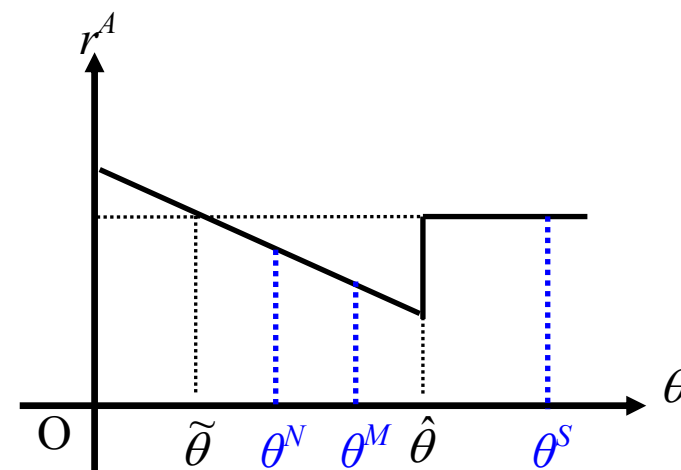
Capital flows from N to M and S .

This is *because* the most developed N are more productive in investment.



Case 3: $\tilde{\theta} < \theta^N < \theta^M < \hat{\theta} < \theta^S \rightarrow CA^N?; CA^M > 0 > CA^S$
 Capital flows into S . Capital flows out of M .

Again, among developing countries, capital flows from the more successful to the less successful \rightarrow A Solution to the “Allocation Puzzle”?

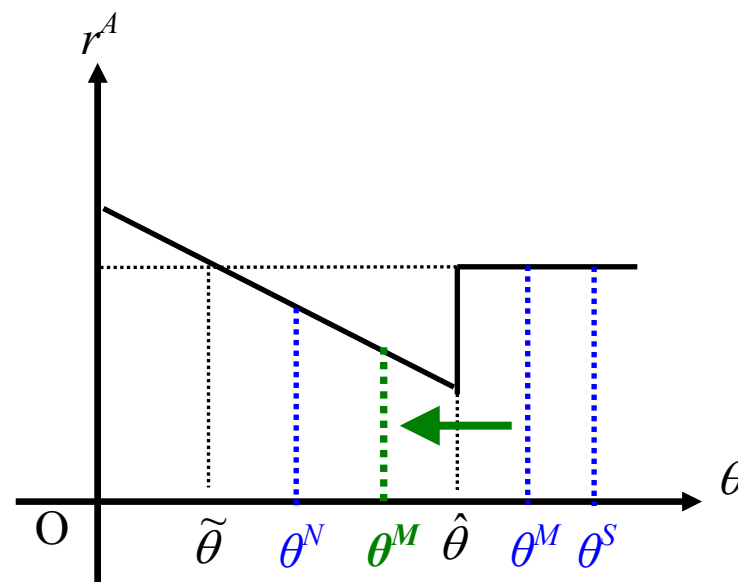


A Thought Experiment:

Imagine that M 's institution improves so that the situation changes from Case 2 to Case 3.

M 's current account turns from a deficit to a surplus.
 (Capital starts flowing out, instead of flowing in.)
 \rightarrow ***Growth Miracle & Capital Outflows***

These results are not driven by the discrete nature of the available technologies.



A Continuum of Projects Case: $j \in J = [0, \infty)$; $R_j \in [R_0, \infty)$ is increasing in j ;

$$\lambda_j = [\Lambda(R_j)]^\theta, \text{ where } \Lambda(R_j) = \exp\left[\frac{1}{\gamma} - \frac{1}{\gamma}\left(\frac{R_j}{R_0}\right)^\gamma\right] \text{ with } \gamma > 0.$$

- $\Lambda(R_0) = 1$; $0 < \Lambda(R_j) < 1$ for $R_j > R_0$; $\Lambda(R_j)$ is decreasing in R_j . \rightarrow Trade-off between productivity and the agency problem.
- For $0 < \theta < 1$, $\lambda_j R_j = [\Lambda(R_j)]^\theta R_j$ is maximized at $R(\theta) = R_0 / \theta^{1/\gamma} > R_0$ and attains $e^{(\theta-1)/\gamma} R(\theta) = R_0 (e^{(\theta-1)} / \theta)^{1/\gamma}$, which is decreasing in θ . As the institutional quality improves, the credit flows into more productive projects and the lenders are assured of higher returns.
- For $\theta > 1$, $\lambda_j R_j = [\Lambda(R_j)]^\theta R_j$ is maximized at R_0 and attains R_0 .

Aggregate Investment Demand:

$$e^{(\theta-1)/\gamma} R(\theta) f'(R(\theta)I) = r \quad \rightarrow \quad \eta \frac{d \log I}{d\theta} = \frac{1}{\gamma} \left(1 + \frac{\eta - 1}{\theta} \right), \text{ where } \eta \equiv -kf''/f'.$$

- If $\eta > 1$, I , and hence r^A are increasing in θ . Capital flows from the rich to the poor.
- If $\eta < 1$, I , and hence r^A are increasing in $\theta > 1 - \eta$, decreasing in $\theta < 1 - \eta$.

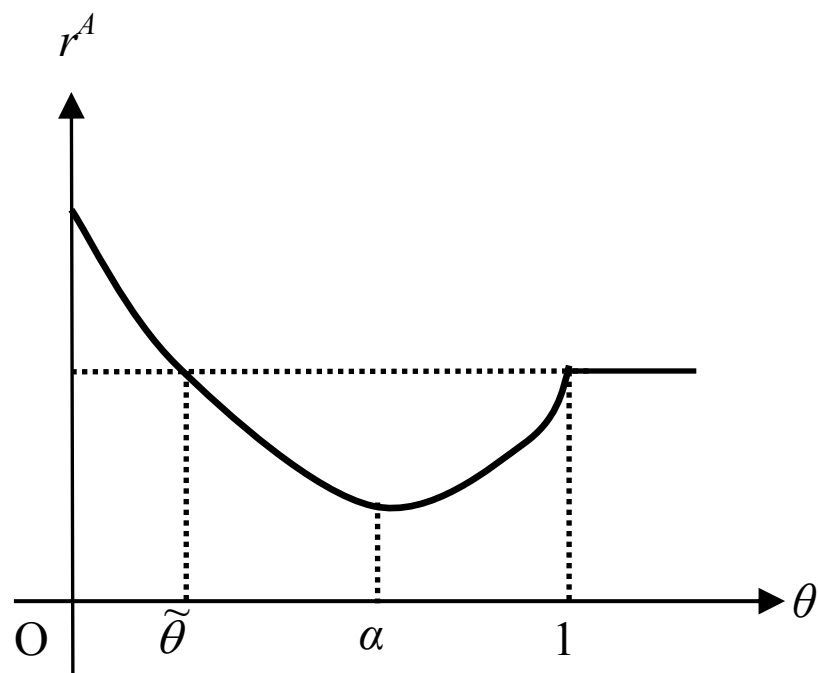
Cobb-Douglas Case: For $f(k) = k^\alpha$, $\eta = 1 - \alpha$.

$$\log I(r; \theta) = \begin{cases} \Omega(r) + \theta - 1 - \alpha \log \theta & \text{for } \theta < 1, \\ \Omega(r) & \text{for } \theta > 1, \end{cases}$$

where $\Omega(r)$ is independent of θ .

- $I(r; \theta)$ is decreasing in $\theta < \alpha$ and increasing in $\alpha < \theta < 1$.
- $I(r; \theta) > I(r; 1)$ if $\theta < \tilde{\theta}$ and $I(r; \theta) < I(r; 1)$ if $\tilde{\theta} < \theta < 1$, where $\tilde{\theta} \neq 1$ is the second solution to $h(\theta) \equiv \theta - 1 - \alpha \log \theta = 0$, and satisfies $0 < \tilde{\theta} < \alpha$.

This translates into *U-shaped* patterns of the (autarky) rate of return.



Note: $R(\theta)$ does not jump. Yet, the implications on the patterns of capital flows are similar to discrete cases.

5. Concluding Remarks

To sum up:

- Endogenous productivity differences have opposite implications on the aggregate investment and capital flows from exogenous ones.
- No reason to expect capital inflows when a country is more productive and has better institution protecting the interest of lenders.
- Improving (credit) institutions could lead to capital outflows.
- Not so puzzling that, among developing countries, capital flows out from the more successful and flows into the less successful, even when capital flows from developing countries to developed countries.
- Not so puzzling that some countries grow and run current account surpluses at the same time (or observing capital inflows into stagnant countries).

Future directions;

- Infinite Horizons; Showing the sustainable (a.k.a. steady state) patterns of capital flows by embedding into an OLG framework
- Saving Dynamics:
 1. Higher savings from a higher k and $w(k)$; again embedding into an OLG would do.
 2. Credit constraints on durable consumption goods; both tradeable & nontradeable (e.g., housing)
- Two Types of Institutions; Credit and Property Right
Two-Way Flows of Financial Capital (from the South to the North) and FDI (from the North to the South) by allowing entrepreneurs to start projects abroad.
- Interactions with exogenous sources of productivity differences
- Interactions with other mechanisms to endogenize productivity (such as externalities and agglomeration economies)
- Endogenous Institutional Quality; reverse causality from productivity to institution

To conclude:

- This model is roughly consistent with many “puzzles” in the patterns of capital flows.
- But, I don’t want to claim that it succeeds in “solving” these “puzzles.” After all, the model is highly stylized and omits many other factors affecting capital flows.
- However, the model suggests greater caution when interpreting empirical evidence and/or calibration exercises on capital flows.
- The following features of the model:
 - Poor IQ prevents productive technologies from being adopted
 - Quality changes in credit institutions causes productivity change;
 - Institution-driven productivity growth leads to a decline in the investment (and the rate of return);
 - Nonmonotone (U-shaped) effects of institution

might have wider applications besides capital flows.