Heterogeneity and Aggregation in the Labor Market: Implications for Aggregate Preference Shifts*

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Abstract

The cyclical behavior of hours worked, wages, and consumption does not conform with the predictions of the representative agent model with standard preferences. The residual in the intra-temporal first-order condition for commodity consumption and leisure is often viewed as a failure of labor-market clearing. We demonstrate that a simple heterogeneous-agent economy with incomplete markets and indivisible labor generates an aggregation error that looks very much like the preference residual observed in the aggregate data. Our results caution against viewing the preference residual as a failure of labor-market clearing, or as a fundamental driving force of business cycles.

Keywords: Aggregation, Heterogeneity, Incomplete Market, Aggregate Preference Shifts

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1. Introduction

The equilibrium outcome of the representative-agent model with standard preferences is hard to reconcile with the joint behavior of aggregate hours worked, consumption, and wages over the business cycle (Barro and King, 1984; Mankiw, Rotemberg and Summers, 1989). A significant residual in preference is measured when an accounting method that uses the optimality condition for the choice between consumption and leisure is introduced (e.g., Hall, 1997). This is often viewed as a failure of the labor-market clearing.

In this paper we show that, when capital market is incomplete and labor supply is indivisible, an economy where agents have identical preferences, but heterogeneous productivity, can generate an aggregation error that looks like an aggregate preference shock. Although there are no inherent preference shifts, in the face of aggregate technology shocks, the model generates an aggregation error that is comparable to the preference residual in the data: that is highly correlated with hours worked, and as volatile as the cyclical component of hours. Our results caution against viewing the preference residual as a failure of labor-market clearing, or as a fundamental driving force of business cycles.

Our analysis quantitatively complements the theoretical work by Sheinkman and Weiss (1986), who show that capital-market incompleteness can lead to a stochastic term in aggregate preferences. It also complements Nakajima (2003) who derives aggregate preference shocks and TFP variation in a two-type household model with capacity utilization and government spending shocks. According to our model, however, an incomplete capital market alone cannot generate an aggregation error that is quantitatively comparable to the preference residual observed in the data. With divisible labor, hours are highly correlated across households in response to aggregate shocks, thus allowing for a fairly good aggregation.

On the other hand, with indivisible labor (Rogerson, 1988; Hansen, 1985), the intratemporal optimality condition for the choice between commodity consumption and leisure does not hold as equality at the individual level. While this is also true when the capital market is complete, at the *aggregate* level the consumption-leisure choice holds exactly in *efficiency units* under a complete market. An efficient allocation requires the marginal utility of aggregate consumption from the effective labor supplied by an additional worker being equal to the marginal dis-utility of his leisure. This implies that, under a complete market, the preference residual reflects a composition bias only in the measurement of aggregate wages and hours (e.g., Bils, 1985; Solon, Barsky and Parker, 1989; Hansen, 1993). This compositional bias has an impact mostly on the magnitudes, not the correlations—in other words, the compositional bias changes the *slope* of the conventionally measured Frisch labor-supply curve. However, in the data, the preference residual arises not only because the hours worked move little relative to wages, but also because there is no strong correlation with the measured price of leisure.

In addition to aggregation errors, a combination of an incomplete capital market and indivisible labor results in the magnification of the aggregate preference residual. The participation margin makes the aggregate labor supply more elastic, consumption fluctuates more over the length of the business cycle, as the household's expenditures depend on the employment status and current income, and the aggregate labor-supply curve itself is time-varying as the shape of reservation-wage distribution changes over time.

The paper is organized in the following fashion. Section 2 briefly discusses the preference residual in the aggregate data. Section 3 lays out a benchmark model economy in which the capital market is incomplete and labor supply is indivisible. In Section 4, we calibrate the model economy and compare the aggregate preference residual from the model to the one found in the data. In Section 5, we investigate the economies with and without the complete market and indivisible labor, in order to distinguish the role of market incompleteness and indivisible labor. Section 6 is the conclusion.

2. Preference Residual in Aggregate Data

One of the leading topics in macroeconomics is the identification of the fundamental driving forces behind economic fluctuations. Economists adopt accounting procedures that combine aggregate time series data with the equilibrium conditions of a prototype model. A prominent example of this is the Solow residual. Under certain assumptions about market competition and the production function, shifts in technology are identified (e.g., Kydland

and Prescott, 1982).[1] Likewise, the standard optimality condition on the choice between commodity consumption and leisure (along with the assumption that household efforts are paid for by their marginal product) identifies the preference residual (e.g., Hall, 1997). To illustrate, consider a representative agent who maximizes utility under a competitive labor market:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln C_t - B \frac{H_t^{1+1/\gamma}}{1+1/\gamma} \right\},$$

where β is the discount factor, C_t is consumption, and H_t is hours worked.[2] The parameter γ represents the compensated labor-supply elasticity and B is the utility parameter. Given the real wage rate W_t, hours worked and consumption satisfy the intra-temporal first-order condition (which is also referred to as the Frisch labor-supply function):

$$BH_t^{1/\gamma} = \frac{W_t}{C_t}. \tag{1}$$

Figure 1 shows the cyclical components of total hours worked (H) and two measures of marginal disutility (W/C) from working for U.S. for 1964:I-2003:II (detrended using the Hodrick-Prescott filter). The hours worked represent the total hours employed in the non-agricultural business sector. The two measures for marginal disutility differ in wage measures: these being the labor productivity of non-agricultural business sector (Y/H) and the real hourly earnings of the production and non-supervisory workers (W).[3] Consumption (C) reflects expenditures on nondurable goods and services. Hours worked often move in the opposite direction to its price, suggesting a serious departure from a stable labor-supply schedule. This is often viewed as a failure of labor-market clearing (more generally, as friction in the labor market) or as the existence of preference shifts.

[1] The Solow residual is subject to specification errors because of such factors as, imperfect competition, cyclical utilization, and aggregation across firms (e.g., Hall, 1987).

[2] While this form of utility is popular in both business cycle analysis and empirical labor supply literature, time non-separability does not account for the joint behavior of hours, wages, and consumption in the data (Mankiw et al., 1989).

[3] While the real hourly earning is a more relevant measure, the wage data at quarterly frequency is limited to those of production and non-supervisory workers only. Following Hall (1997), we use labor productivity for the rest of this paper. The conclusion and main results of this paper do not depend on the choice of wage measure.

Under the assumption that the aggregate labor-supply elasticity γ is 1, we compute the implied—or required—time series of preference residual B which accommodates the first-order condition (1). This elasticity is higher than a typical estimate in the micro data (e.g., MaCurdy, 1981), which is less than 0.5. If we assume an inelastic labor supply (a smaller value of γ), we obtain a bigger residual.[4] Conversely, an elastic labor supply produces a smaller residual. Nevertheless, the particular value of aggregate elasticity does not alter the conclusion of the paper. Figure 2 shows the time series of preference residual along with the hours worked. This is highly correlated with the labor supply, and its volatility is in the same order of magnitude as hours worked.[5] A strong interpretation of this residual would be shifts in the marginal rate of substitution between commodity consumption and leisure at the aggregate level, while a weak interpretation would be labor-market frictions, or the extent to which the labor market deviates from a competitive equilibrium. Finally, we note that two important premises in the calculation of the preference residual are: a competitive labor market and aggregation. While we focus solely on the latter, this does not necessarily reflect our views on the importance of a non-competitive labor market. Rather we simply maintain the competitive labor market to highlight the aggregation problem. In the next section, we present a model economy in which the preference residual arises as an aggregation error.

3. The Model

There is a continuum (measure one) of workers who have identical preferences but different productivity. Individual productivity varies exogenously according to a stochastic process with a transition probability distribution function $\pi_x(x'|x) = \Pr(x_{t+1} \leq x'|x_t = x)$. A worker maximizes his utility over consumption c_t and hours worked h_t:

$$\max_{\{c_t,h_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t - B \frac{h_t^{1+1/\gamma}}{1+1/\gamma} \right\} \tag{2}$$

[4] For example, Hall (1997) uses $\gamma = 1/1.7$.

[5] This feature identifies the preference residual as a potential driving force of economic fluctuations (e.g., Hall, 1997; Chari, Kehoe and McGrattan, 2002)

subject to

$$a_{t+1} = w_t x_t h_t + (1 + r_t)a_t - c_t. \tag{3}$$

Workers trade claims for physical capital, a_t, which yields the rate of return r_t and depreciates at the rate δ. The capital market is incomplete. Physical capital is the only asset available to workers who face a borrowing constraint: $a_t \geq \bar{a}$ for all t. Based on Hansen (1985) and Rogerson (1988), we abstract from the intensive margin, and assume that the labor supply is indivisible; i.e., h_t takes either zero or $\bar{h}(< 1)$. If employed, a worker supplies $\bar{h}$ units of labor and earns $w_t x_t \bar{h}$, where w_t is wage rate per effective unit of labor.[6]

The representative firm produces output according to a constant-returns-to-scale Cobb-Douglas technology in capital, K_t, and efficiency units of labor, L_t:[7]

$$Y_t = F(L_t, K_t, \lambda_t) = \lambda_t L_t^{\alpha} K_t^{1-\alpha},$$

where λ_t is the aggregate productivity shock with a transition probability distribution function $\pi_\lambda(\lambda'|\lambda) = \Pr(\lambda_{t+1} \leq \lambda'|\lambda_t = \lambda)$.[8]

The value function for an employed worker, denoted by V^E, is:

$$\begin{aligned} V^E(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}} \Big\{ &\ln c - B\frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} \\ &+ \beta E\Big[\max \big\{ V^E(a', x'; \lambda', \mu'), V^N(a', x'; \lambda', \mu') \big\} \big| x, \lambda \Big] \Big\} \end{aligned} \tag{4}$$

subject to

$$c = wx\bar{h} + (1 + r)a - a',$$

[6]In general, the labor supply decision operates on both the extensive and intensive margins. However, it is rare for workers to be allowed to choose completely flexible work schedules or to supply a small number of hours. Furthermore, it is well known that the variation in the number of employees is the dominant source of fluctuations in total hours worked (e.g., Heckman, 1984).

[7]This implicitly assumes that workers are perfect substitutes for each other. While this assumption abstracts from reality, it greatly simplifies the labor-market equilibrium.

[8]In this model economy, technology shock is the only aggregate shock. This does not necessarily reflect our view on the source of business cycles. As we would like to show that the preference residual contains a significant specification error, rather than true shifts in preferences, we intentionally exclude shocks that may shift the labor-supply schedule itself (e.g., shifts in government spending or changes in income tax rate) from the present paper.

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu),$$

where $\mathbf{T}$ denotes a transition operator that defines the law of motion for the distribution of workers $\mu(a, x)$.[9] The value function for a non-employed worker, denoted by $V^N(a, x; \lambda, \mu)$, is defined similarly with $h = 0$. Then, the labor-supply decision is characterized by:

$$V(a, x; \lambda, \mu) = \max_{h \in \{0, \bar{h}\}} \left\{ V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu) \right\}.$$

Equilibrium consists of a set of value functions, $\{V^E(a, x; \lambda, \mu), V^N(a, x; \lambda, \mu), V(a, x; \lambda, \mu)\}$, a set of decision rules for consumption, asset holdings, and labor supply, $\{c(a, x; \lambda, \mu), a'(a, x; \lambda, \mu), h(a, x; \lambda, \mu)\}$, aggregate inputs, $\{K(\lambda, \mu), L(\lambda, \mu)\}$, factor prices, $\{w(\lambda, \mu), r(\lambda, \mu)\}$, and a law of motion for the distribution $\mu' = \mathbf{T}(\lambda, \mu)$ such that:

1. Individuals optimize:

   Given $w(\lambda, \mu)$ and $r(\lambda, \mu)$, the individual decision rules $c(a, x; \lambda, \mu)$, $a'(a, x; \lambda, \mu)$, and $h(a, x; \lambda, \mu)$ solve $V^E(a, x; \lambda, \mu)$, $V^N(a, x; \lambda, \mu)$, and $V(a, x; \lambda, \mu)$

2. The representative firm maximizes profits:

$$w(\lambda, \mu) = F_1\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) \tag{5}$$

$$r(\lambda, \mu) = F_2\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) - \delta \tag{6}$$

   for all (λ, μ).

3. The goods market clears:

$$\int \big\{a'(a, x; \lambda, \mu) + c(a, x; \lambda, \mu)\big\} d\mu = F\big(L(\lambda, \mu), K(\lambda, \mu), \lambda\big) + (1 - \delta)K \tag{7}$$

   for all (λ, μ).

[9]Let $\mathcal{A}$ and $\mathcal{X}$ denote sets of all possible realizations of a and x, respectively. The measure $\mu(a, x)$ is defined over a σ-algebra of $\mathcal{A} \times \mathcal{X}$.

4. Factor markets clear:

$$L(\lambda, \mu) = \int xh(a, x; \lambda, \mu) d\mu \tag{8}$$

$$K(\lambda, \mu) = \int a d\mu \tag{9}$$

for all (λ, μ).

5. Individual and aggregate behaviors are consistent:

$$\mu'(A^0, X^0) = \int_{A^0, X^0} \left\{ \int_{\mathcal{A}, \mathcal{X}} \mathbf{1}_{a'=a'(a,x;\lambda,\mu)} \, d\pi_x(x'|x) d\mu \right\} da' dx' \tag{10}$$

for all $A^0 \subset \mathcal{A}$ and $X^0 \subset \mathcal{X}$.

4. Quantitative Analysis

4.1. Calibration

We briefly explain the choice of the model parameters. The unit of time is a business quarter. We assume that x follows an AR(1) process: $\ln x' = \rho_x \ln x + \varepsilon_x$, where $\varepsilon_x \sim N(0, \sigma_x^2)$. As we view x as reflecting a broad measure of earnings ability in the market, we estimate the stochastic process of x based on the wages from the PSID for 1979-1992. The values of $\rho_x = 0.939$ and $\sigma_x = 0.287$ reflects the persistence and standard deviation of innovation to individual wages.[10] The other parameters of the paper are in accordance with the business cycle analysis and empirical labor supply literature. A working individual spends one-third of discretionary time: $\bar{h} = 1/3$. The individual compensated labor-supply elasticity of hours, γ, is 0.4. The labor share of output, α, is 0.64, and the depreciation rate, δ, is 2.5 percent. We search for the weight parameter on leisure, B, such that the steady state employment rate is 60 percent, the CPS average for 1967:II-2000:IV. The discount factor β is chosen so that the quarterly rate of return to capital is one percent. Aggregate productivity shock, λ_t, follows an AR(1) process: $\ln \lambda' = \rho_\lambda \ln \lambda + \varepsilon_\lambda$, where $\varepsilon_\lambda \sim N(0, \sigma_\lambda^2)$. We set $\rho_\lambda = 0.95$ and $\sigma_\lambda = 0.007$ following Kydland and Prescott (1982). Table 1 summarizes the parameter values of the benchmark economy.

[10] These are maximum-likelihood estimates of Heckman (1979) correcting for a sample selection bias. See Chang and Kim (2004) for the details.

4.2. Cross-sectional Earnings and Wealth Distribution

As we investigate the aggregation issue, it is desirable for the model economy to possess a reasonable amount of heterogeneity. We compare cross-sectional earnings and wealth—two important observable dimensions of heterogeneity in the labor market—found in the model and in the data.

Table 2 summarizes both the PSID and the model's detailed information on wealth and earnings. Family wealth in the PSID (1984 survey) reflects the net worth of houses, other real estate, vehicles, farms and businesses owned, stocks, bonds, cash accounts, and other assets. For each quintile group of wealth distribution, we calculate the wealth share, ratio of group average to economy-wide average, and the earnings share.

In both the data and the model, the poorest 20 percent of families in terms of wealth distribution was found to own virtually nothing. In fact, households in the first quintile of wealth distribution were found to be in debt in both the model and the data. The PSID found that households in the 4th and 5th quintile own 18.74 and 76.22 percent of total wealth, respectively, while, according to the model, they own 23.88 and 65.49 percent, respectively. The average wealth of those in the 4th and 5th quintile is respectively, 0.93 and 3.81 times larger than that of a typical household, according to the PSID. These ratios are 1.19 and 3.27 according to our model. The 4th and 5th quintile groups of the wealth distribution earn, respectively, 24.21 and 38.23 percent of total earnings, according to the PSID. The corresponding groups earn 23.59 and 32.63 percent, respectively, in the model.

Overall, wealth distribution is found to be more skewed in the data. In particular, our model fails to match the highly concentrated wealth found in the right tail of the distribution. In the PSID, the top 5 percent of the population controls about half of total wealth (not shown in Table 2). Whereas, in our model, they possess only 20 percent of the total wealth. Our primary objective is not to explain the behavior of the top 1 or 5 percent of the population. Therefore, we argue that the model economy presented in this paper possesses a reasonable degree of heterogeneity, thus making it possible to study the effects of aggregation in the labor market.

4.3. Cyclical Properties of the Model

We numerically solve the equilibrium of the model using the "bounded rationality" method developed by Krusell and Smith (1998), and generate the aggregate time series through simulations.[11]

Table 3 shows the volatility of the key aggregate variables of our model economy. In the face of aggregate productivity shocks whose stochastic process resembles that of the U.S. TFP, the model-output exhibits a volatility of 1.18, slightly less than two thirds of actual-output volatility. This is not very different from the findings of the standard representative-agent models (e.g., Kydland and Prescott, 1982). Other statistics are also similar to those found in the standard models: consumption is about half as volatile as output and investment is about three to four times as volatile as output.

One distinguishing feature of our model is the labor market. The (relative) volatility of hours and productivity is very close to those found in the data, which is hard to achieve in a standard model unless unreasonably high labor-supply elasticity is assumed. The volatility of hours relative to output is 0.71 (0.85 in the data), and the volatility of labor productivity relative to output is 0.66 (0.50 in the data). This success is mostly due to the composition effect generated by the entry and exit of workers with heterogeneous productivity. On average, less-productive workers participate in the labor market during expansions and exit during contractions. The measured hours are more volatile than the hours in effective units and the aggregate wage is less volatile than individual wages. When we measure the hours worked in effective units, these are about half as volatile as output (0.41), similar to the result when using the representative-agent model with standard preferences.

Table 4 demonstrates the correlations. The correlations between output, consumption, and investment are higher than those in the data, a feature common in standard RBC models. However, the correlations of hours and labor productivity, respectively, with output are 0.84 and 0.73, close to those in the data. In particular, hours and labor productivity exhibit a fairly low correlation (0.23) in our model – it is 0.03 in the data. This is encouraging

[11] The model generated aggregated time series is based on the individual choices of 200,000 agents over 3,500 periods.

because it is well known that one of the most salient failures of the RBC models is its failure to generate a low correlation between hours and labor productivity in the data.

4.4. Preference Residual from the Model

If we were to use the competitive equilibrium of a representative agent to account for the model-generated aggregate time series of hours, wages, and consumption, we would face the same problem: the hours worked and the marginal disutility from working do not co-move. To illustrate we compute the preference residual from the model-generated aggregate time series.

Figure 3 shows total hours worked and the preference residual from the model (under the assumption that the labor supply elasticity is 1, which was also assumed for the actual data). In terms of the second moments of Hodrick-Prescott filtered data, the residual from the model exhibits stochastic properties similar to those found in the data. The standard deviation of the residual relative to output is 0.91 (1.23 in the data). In absolute terms, the residual in the model is about half as volatile as the one found in the data. The correlation between the residual and total hours worked is -0.90 (-0.92 in the data). The correlation between hours and measured marginal disutility of working (W/C) is -0.27 (-0.53 in the data).

Despite there being no inherent preference shifts (in the face of aggregate technology shocks) the model generates an aggregation error that looks very similar to the preference residual in the data. Our results caution against viewing the preference residual as the fundamental driving force of business cycles, as well as against the view that the residual reflects a failure of labor-market clearing.

5. Role of Incomplete Market and Indivisible Labor

The residual in our model stems from capital market incompleteness and indivisible labor supply. To investigate the marginal contributions of each, we consider three additional model economies. For comparison, we refer to the benchmark economy as HII which stands for "Heterogeneity-Incomplete Market-Indivisible Labor."

5.1. Alternative Model Specifications

Heterogeneity + Complete Market + Indivisible Labor (HCI) The second model we consider allows for complete capital markets but maintains indivisible labor: "Heterogeneity-Complete Market-Indivisible Labor" (HCI). Thanks to perfect risk sharing, agents enjoy the same level of consumption regardless of their employment status, productivity, or asset holdings.[12]

The equilibrium of this economy is identical to the allocation made by a social planner who maximizes the equally weighted utility of the population. The planner chooses the sequence of consumption $\{C_t\}_{t=0}^{\infty}$ and the cut-off productivity $\{x_t^*\}_{t=0}^{\infty}$ for labor-market participation. To assure an efficient allocation, the planner assigns workers who have a comparative advantage in the market (more productive workers) to work. If a worker's productivity is above x_t^*, he supplies $\bar{h}$ hours of labor.

The planner's value function in the complete market, $V^C(K, \lambda)$, and the decision rules for consumption, $C(K, \lambda)$, and cut-off productivity, $x^*(K, \lambda)$, satisfy the following Bellman equation:

$$V^C(K, \lambda) = \max_{C, x^*} \left\{ \ln C - B \frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} \int_{x_t^*}^{\infty} \phi(x) dx + \beta E\Big[V^C(K', \lambda') \big| \lambda \Big] \right\} \tag{11}$$

subject to

$$K' = F(K, L, \lambda) + (1 - \delta)K - C, \tag{12}$$

where $L = \bar{h} \int_{x^*}^{\infty} x\phi(x)dx$ is the aggregate efficiency unit of labor, and $\phi(x)$ is the productivity distribution of workers. The planner chooses the cut-off productivity x^* so that:

$$\frac{1}{C} F_L(K, L, \lambda) \bar{h} x^* \phi(x^*) = B \frac{\bar{h}^{1+1/\gamma}}{1 + 1/\gamma} \phi(x^*). \tag{13}$$

The left-hand side is the (society's) utility gain from assigning the marginal worker to production. There are $\phi(x^*)$ number of workers with productivity x^* in the economy. Each of

[12] The distribution of workers is no longer a state variable in the individual optimization problem. Moreover, due to the ergodicity of the stochastic process for idiosyncratic productivity, the cross-sectional distribution of workers is always stationary.

them supplies $\bar{h}x^*$ units of effective labor, and the marginal product of labor is F_L. The right-hand side represents the disutility incurred by these workers. The key point here is that, under complete markets, the first-order condition for the choice between hours and consumption is *exactly* defined in terms of the efficiency units of hours and wages at the aggregate level. Thus, the preference residual reflects the composition bias in wages and hours. In other words, the aggregate preference residual in this economy can always be eliminated by an appropriate choice of aggregate labor-supply elasticity. This is discussed in detail in section 5.2.

Heterogeneity + Incomplete Market + Divisible Labor (HID) The third model economy we consider allows for a divisible labor supply, but capital markets are incomplete: "Heterogeneity-Incomplete Market-Divisible Labor (HID)". The equilibrium of this economy can be defined similarly to that of HII in Section 3—equations (5) through (10)—with the worker's value function with divisible labor, $V^D(a, x; \lambda, \mu)$:

$$V^D(a, x; \lambda, \mu) = \max_{a' \in \mathcal{A}, h \in (0,1)} \left\{ \ln c - B \frac{h^{1+1/\gamma}}{1 + 1/\gamma} + \beta E \Big[V^D(a', x'; \lambda', \mu') \big| x, \lambda \Big] \right\} \tag{14}$$

subject to

$$c = wxh + (1 + r)a - a',$$

$$a' \geq \bar{a},$$

$$\mu' = \mathbf{T}(\lambda, \mu).$$

Representative-Agent Model (RA) The last model we consider is the "Representative-Agent (RA)" model. The value function of the representative agent, $V^R(K, \lambda)$, is:

$$V^R(K; \lambda) = \max_{C,H} \left\{ \ln C - B \frac{H^{1+1/\gamma}}{1 + 1/\gamma} + \beta E \Big[V^R(a', x'; \lambda', \mu') \big| x, \lambda \Big] \right\} \tag{15}$$

subject to

$$K' = F(K, H, z) + (1 - \delta)K - C. \tag{16}$$

5.2. Comparison of Four Model Economies

Except for β and B, the same parameter values are used across all models. In the RA model, β is 0.99 and B is chosen so that the steady-state hours worked is the same as the aggregate hours in the benchmark economy, which is 0.2 ($= \bar{h} \times 60\%$). For HCI, β is 0.99 and B is chosen to be consistent with 60% employment along with $\bar{h} = 1/3$.[13] For HID, β and B are jointly searched to be consistent with average hours of 0.2 and an interest rate of 1% in a steady state. The equilibrium of the HCI economy is solved by Marcet and Lorenzoni (1999)'s Parameterized Expectation Algorithm, while the equilibrium of HID is solved by Krusell and Smith (1998)'s "bounded rationality" method and the equilibrium of RA is solved by a value function iteration.

The preference residuals are computed using an aggregate labor-supply elasticity of 0.4 for the divisible-labor economies (RA and HID), which is the same as the individual elasticity. When the labor supply is indivisible the aggregate labor-supply elasticity can be very different from the individual elasticity. In the complete market economy (HCI), the aggregate labor supply is governed by (13); whereas, in the incomplete market economy (HII), it is dependent on the shape of the reservation wage distribution. We compute the residuals assuming that the aggregate elasticity of HII and HCI is 1 for two reasons. First, it allows for a direct comparison with the residual from the data in section 2. Moreover, according to Chang and Kim (2004), a model economy similar to HII exhibits an aggregate labor-supply elasticity of around 1 in the steady state. Nevertheless, the choice of a particular aggregate elasticity does not alter the main conclusions of this paper.

Figure 4 shows the sample paths (percentage deviations from the steady states) of the various models' preference residuals. These sample paths are comparable to each other because all model economies are subject to an identical sequence of aggregate productivity shocks. As expected, there is no preference residual in RA. The residual in HID represents an aggregation error that has been caused solely by market incompleteness. The residual

[13]Specifically, we find the steady-state cut-off productivity, x^*, from the 60th percentile of cross-sectional productivity distribution, $\phi(x)$: $\int_{x^*}^{\infty} x\phi(x)dx = 0.6$. Then, find B that satisfies the labor supply equation, (13).

in HCI reflects this model's compositional bias in aggregate hours and wages. Apparently, these residuals are not large enough to account for the preference residual in the data. The volatilities of the residual relative to output are 0.13 for both HID and HCI, which are, respectively, only one tenth of that found in the data (1.23). In this sense, our result contrasts with Mulligan (2001) in which the indivisible labor yields aggregate implications that are not very distinctive from those of representative-agent divisible-labor model economy.

Capital-market incompleteness alone does not generate a large aggregation error. With divisible labor, in response to aggregate productivity shocks, hours are highly correlated across households, allowing for a fairly precise aggregation. On the other hand, with indivisible labor (e.g., Rogerson, 1988; Hansen, 1985), the intra-temporal optimality condition for the choice between commodity consumption and leisure does not hold for most households, resulting in a serious aggregation errors. While this is true even when the capital market is complete, the consumption-hours choice holds exactly in *efficiency units* at the aggregate level under complete capital market—recall Equation (13). Thus, the preference residual reflects the measurement error due to composition effect in aggregate wages and hours.

It is well-known that low-wage and less-skilled workers enter the labor market during expansions and exit during recessions, making the aggregate hours more volatile than the effective unit of hours (Hansen, 1993), and making the aggregate wages less volatile than individual wages (Bils, 1985; Solon et. al 1989).[14] However, the compositional bias has an impact mostly on the volatilities, not on the correlations.[15] In other words, around the steady state (if the cutoff productivity is not far from steady-state x^*), the marginal rate of substitution between consumption and leisure holds with an appropriately chosen

[14]Bils (1985) and Solon et al. (1995), based on the individual panel data, find that aggregate wages are less cyclical than individual wages. Hansen (1993) computes the effective unit of hours based on the worker characteristics provided by the CPS. He finds that while the effective unit of hours adjusted for quality exhibits a bigger volatility, such an adjustment does not change the cyclical property of hours significantly. In practice, estimating quality-adjusted hours is not easy in practice as observed characteristics account for only a fraction of worker's productivity. For example, the R^2 of cross-sectional wage regression is usually well below 0.4.

[15]As a result, the hours of work are not strongly correlated with wages in HII at the aggregate level, whereas the correlation coefficients between aggregate hours and wages are, .97, .95, and .93 in the RA, HID, and HCI, respectively.

aggregate labor elasticity: the preference residual in HCI reflects the mis-specification error in aggregate labor-supply elasticity. In fact, if we assume aggregate labor-supply elasticity of 1.44 in HCI, there is virtually no preference residual. The preference residual in the data arises not only because wages (more exactly, wage-consumption ratio) move little relative to hours worked, but also because they are not strongly correlated with hours over the business cycle. Thus, from the data (as well as in HII), we find the preference residual regardless of aggregate labor-supply elasticity.

The preference residual arises when aggregate hours move more relative to the measured marginal disutility of working (with $\gamma = 1$, $B_t = \frac{W_t}{C_t H_t}$). We compare each component—hours, wages, and consumption—across the models. Figure 5 shows total hours worked (percentage deviations from the steady states). The hours worked in HID move similarly to those in RA, whereas the hours worked in indivisible labor economies (HII and HCI) fluctuate much more than those in RA. This suggests that with regards to the volatility of hours, indivisible labor is more important than incomplete markets. The hours worked fluctuate more in HII than they do in HCI. Under incomplete markets, high-skilled workers (as well as the low-skilled) perform intertemporal substitution in labor-market participation; whereas, under complete markets, only workers at the margin—whose productivity is around x^* in (13)—enter and exit the labor market in response to shifts in aggregate productivity. This can be seen more clearly when we measure the hours of work in effective units (Figure 6). The effective unit of hours is more volatile in HII than it is in HCI. Note also that, despite indivisible labor, the cyclical behavior of the effective unit of hours in HCI is almost identical to that of the RA. Figure 7 shows aggregate wages in each model. As the compositional bias works in a converse fashion here, wages fluctuate less in the indivisible labor economies (HII and HCI) than they do in divisible labor economies (HID and RA). Figure 8 shows aggregate consumption. The consumption of HCI is again almost identical to that of RA. Despite the incomplete capital market the consumption of HID is very close to that of RA. Aggregate consumption (as well as hours) fluctuates the most in HII as the household consumption depends on the current income—in particular, employment status—as well as on the wealth of the household. In sum, volatile consumption, combined with smooth wages, makes the measured marginal disutility of working (W/C) moves little relative to the hours worked in

HII model, creating a larger preference residual.

Finally, the aggregate labor-supply curve depends on the shape of the reservation-wage distribution, which, in turn, is dependent on the cross-sectional wealth distribution. As the wealth distribution of an economy changes, the response of the aggregate labor supply and of consumption varies over time.

Confronted with the inability of the standard equilibrium model to account for the joint behavior of aggregate hours worked, wages, and consumption, Mankiw et al. (1985) proposed four hypotheses: (1) measurement errors (2) aggregation (3) time-varying preference (4) failure of market clearing. While it is highly plausible that all of these have contributed to the measured preference residual, our analysis sheds some lights on these hypotheses: measurement errors (1) due to composition effect is not enough; (2) can be mis-interpreted as (3); the residual does not necessarily reflect (4).

6. Summary

The cyclical behavior of aggregate hours worked, wages, and consumption is hard to reconcile with the equilibrium outcome of the representative-agent model with standard preference. A significant preference residual can be measured when employing an accounting method that uses aggregate time series in conjunction with the optimality condition. We demonstrate that a heterogeneous-agent economy with incomplete capital markets and indivisible labor can generate an aggregation error that is quantitatively comparable the preference residual in the data. With indivisible labor, the intra-temporal optimality condition for the choice between commodity consumption and leisure does not hold at the household level. Under a situation of incomplete markets, this results in a serious aggregation error at the aggregate level. In addition, the aggregate labor-supply curve is not time-invariant, as the reservation-wage distribution changes over the business cycle. Our results caution against viewing this residual as a fundamental driving force of business cycles or as a failure of labor-market clearing.

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Table 1: Parameters of the Benchmark Model Economy

Parameter	Description
$\alpha = 0.64$	Labor share in production function
$\beta = 0.9785504$	Discount factor
$\gamma = 0.4$	Individual labor-supply elasticity with divisible labor
$B = 151.28$	Utility parameter
$\overline{h} = 1/3$	Labor supply if working
$\bar{a} = -2.0$	Borrowing constraint
$\rho_x = 0.939$	Persistence of idiosyncratic productivity shock
$\sigma_x = 0.287$	Standard deviation of innovation to idiosyncratic productivity
$\rho_\lambda = 0.95$	Persistence of aggregate productivity shock
$\sigma_\lambda = 0.007$	Standard deviation of innovation to aggregate productivity

Table 2: Characteristics of Wealth Distribution

	Quintile					
	1st	2nd	3rd	4th	5th	Total
PSID						
Share of wealth	-.52	.50	5.06	18.74	76.22	100
Group average/population average	-.02	.03	.25	.93	3.81	1
Share of earnings	7.51	11.31	18.72	24.21	38.23	100
Benchmark Model						
Share of wealth	-2.05	2.46	10.22	23.88	65.49	100
Group average/population average	-.10	.12	.51	1.19	3.27	1
Share of earnings	9.70	15.06	19.01	23.59	32.63	100

Note: The PSID statistics reflect the family wealth and earnings levels published in their 1984 survey.

Table 3: Volatilities of Aggregate Variables

Variable	Data	Model
σ_Y	2.04	1.18
σ_C/σ_Y	0.42	0.35
σ_I/σ_Y	3.59	3.06
σ_H/σ_Y	0.85	0.71
σ_L/σ_Y	–	0.41
σ_W/σ_Y	0.50	0.66
$\sigma_{W/C}/\sigma_Y$	0.55	0.38
σ_B/σ_Y	1.23	0.91

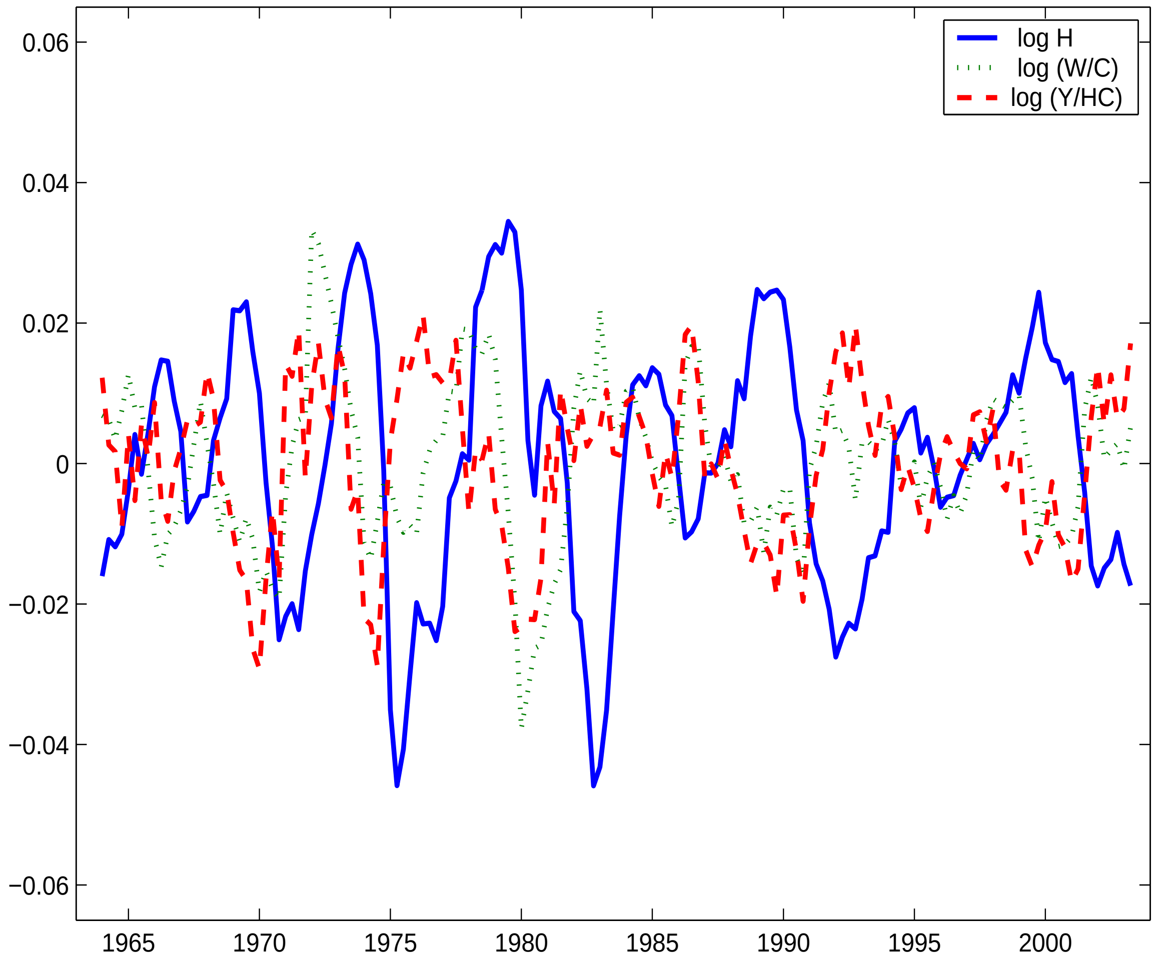
Note: All variables are logged and de-trended by the H-P filter. The volatility of output is measured by its standard deviation, and those of all other variables are by the relative standard deviations to output. The variable L denotes the effective unit of hours.

Table 4: Correlations of Aggregate Variables

Variable	Data	Model
$cor(Y, C)$	0.77	0.86
$cor(Y, I)$	0.88	0.99
$cor(Y, H)$	0.87	0.84
$cor(Y, L)$	–	0.90
$cor(Y, W)$	0.52	0.73
$cor(H, W)$	0.03	0.23
$cor(H, W/C)$	-0.53	-0.27
$cor(B, H)$	-0.92	-0.90
$cor(B, Y)$	-0.64	-0.54

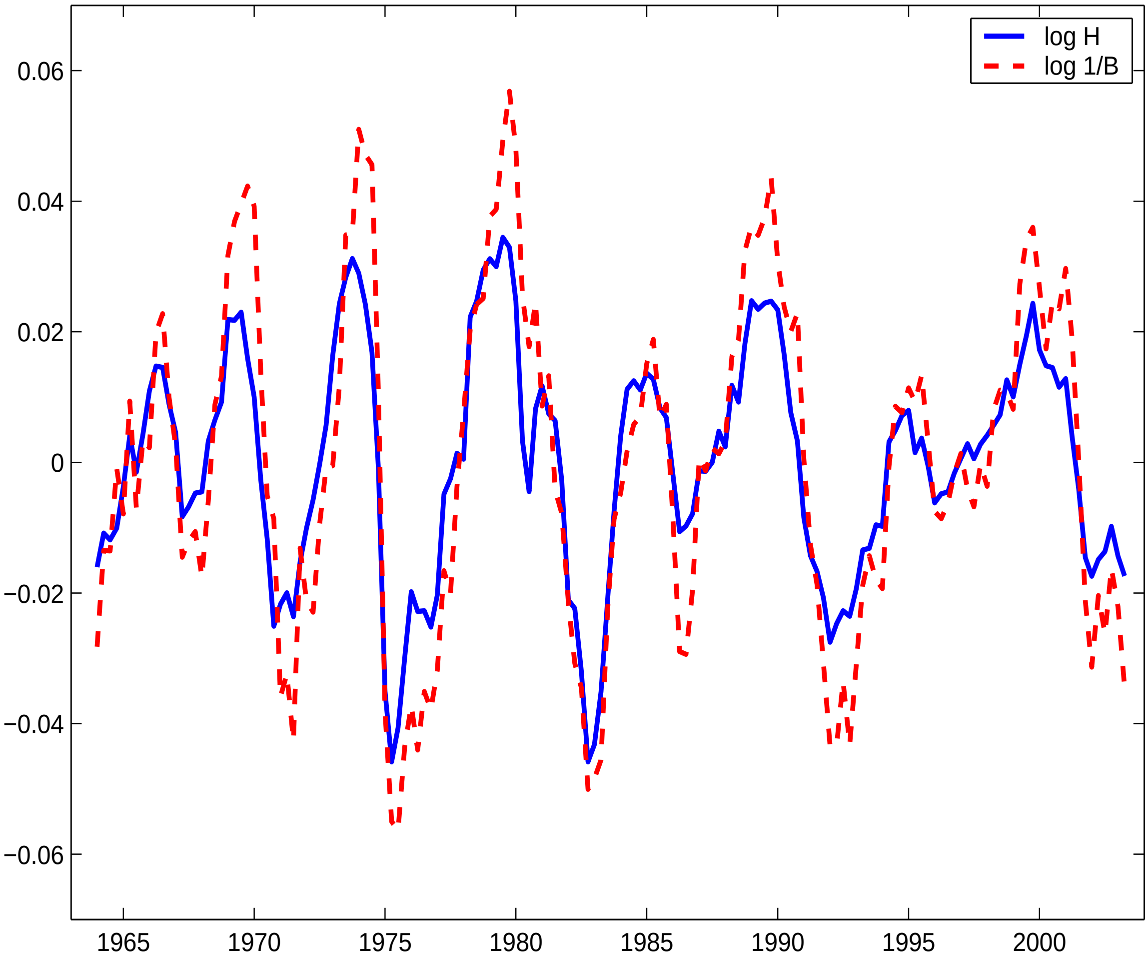
Note: See the note in Table 3 for variable descriptions.

Figure 1: Cyclical Components of Total Hours and Marginal Disutility from Working



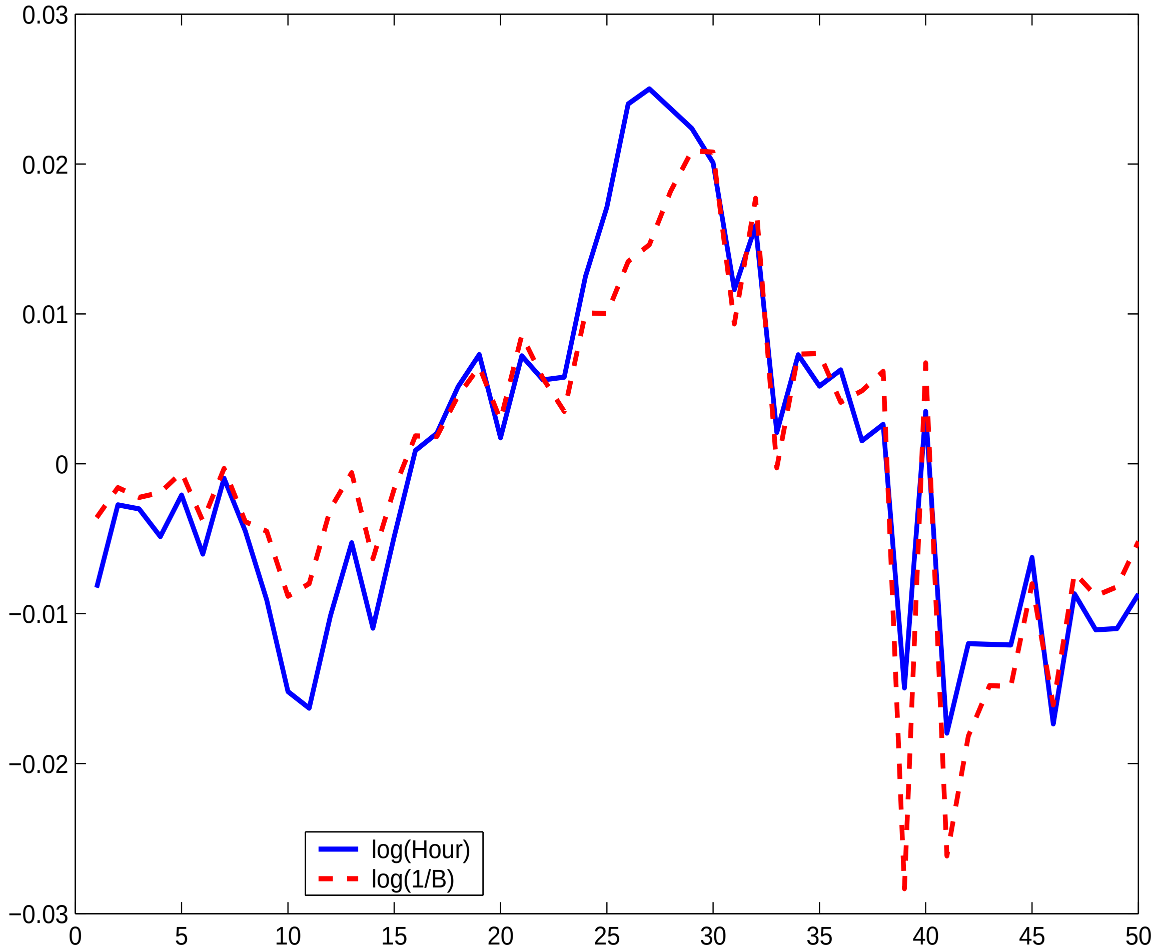
Note: Output and hours worked represent the non-agricultural private sector. Wage is real hourly earnings for non-supervisory and production workers. Consumption reflects expenditure on non-durable goods and services.

Figure 2: Cyclical Components of Hours and Preference Residuals for U.S.



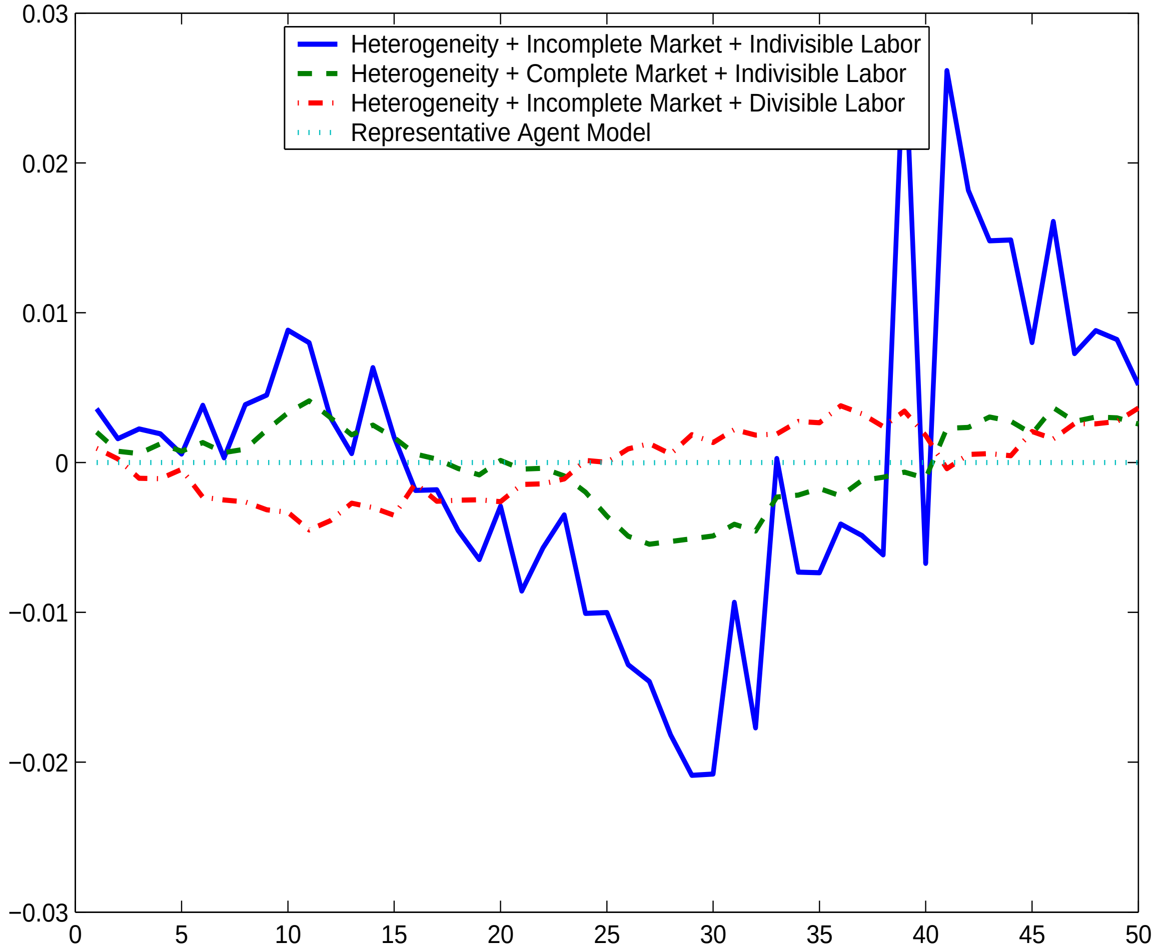
Note: The preference residual is computed from Equation (1) with the aggregate labor-supply elasticity (γ) of 1.

Figure 3: Total Hours and Preference Residual from the Benchmark Model



Note: The preference residual is computed from Equation (1) with the aggregate labor-supply elasticity (γ) of 1.

Figure 4: Preference Residuals from the Models



Note: The preference residual (B) is computed from Equation (1). We use the aggregate labor supply elasticity γ of 0.4 and 1, respectively, for divisible- and indivisible-labor models.

Figure 5: Total Hours Worked from the Models

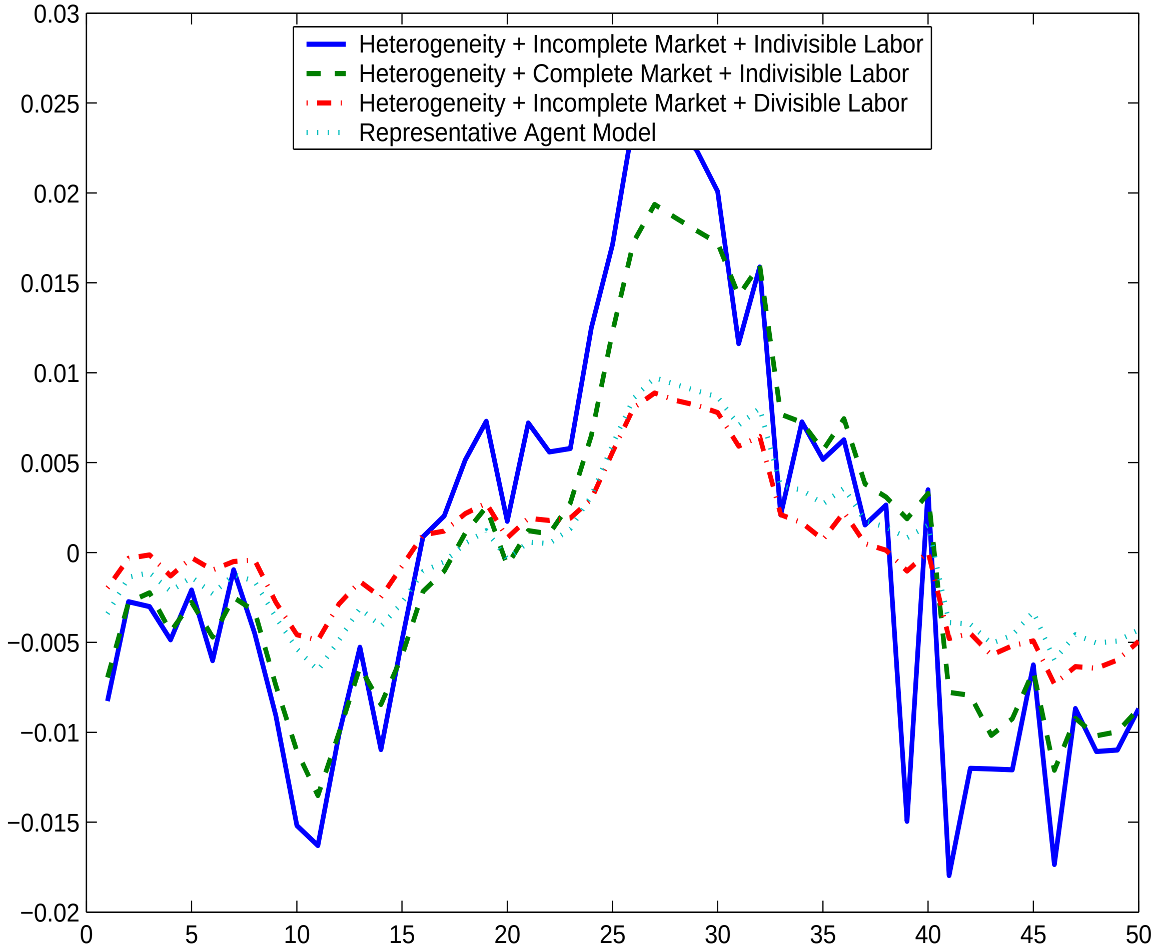


Figure 6: Effective Unit of Hours from the Models

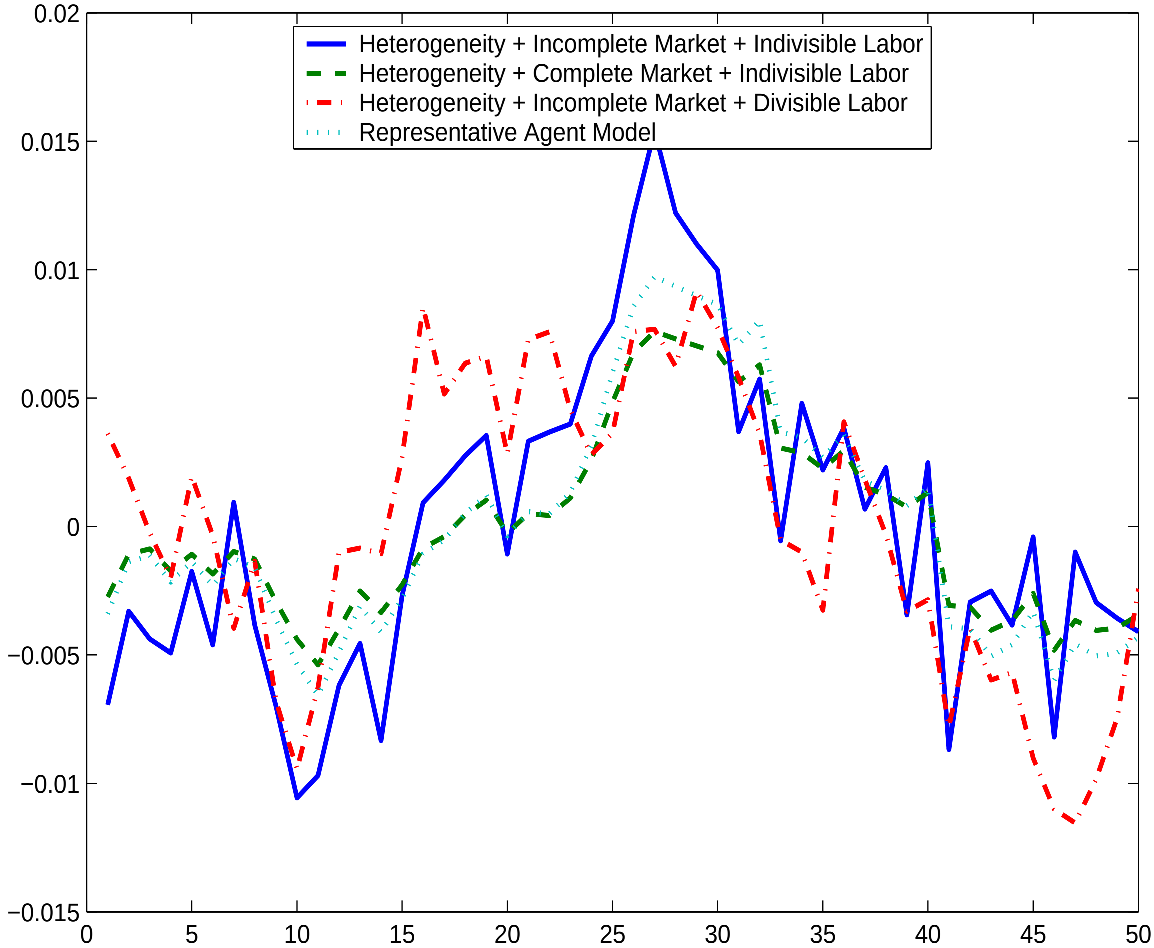


Figure 7: Aggregate Wages from the Models

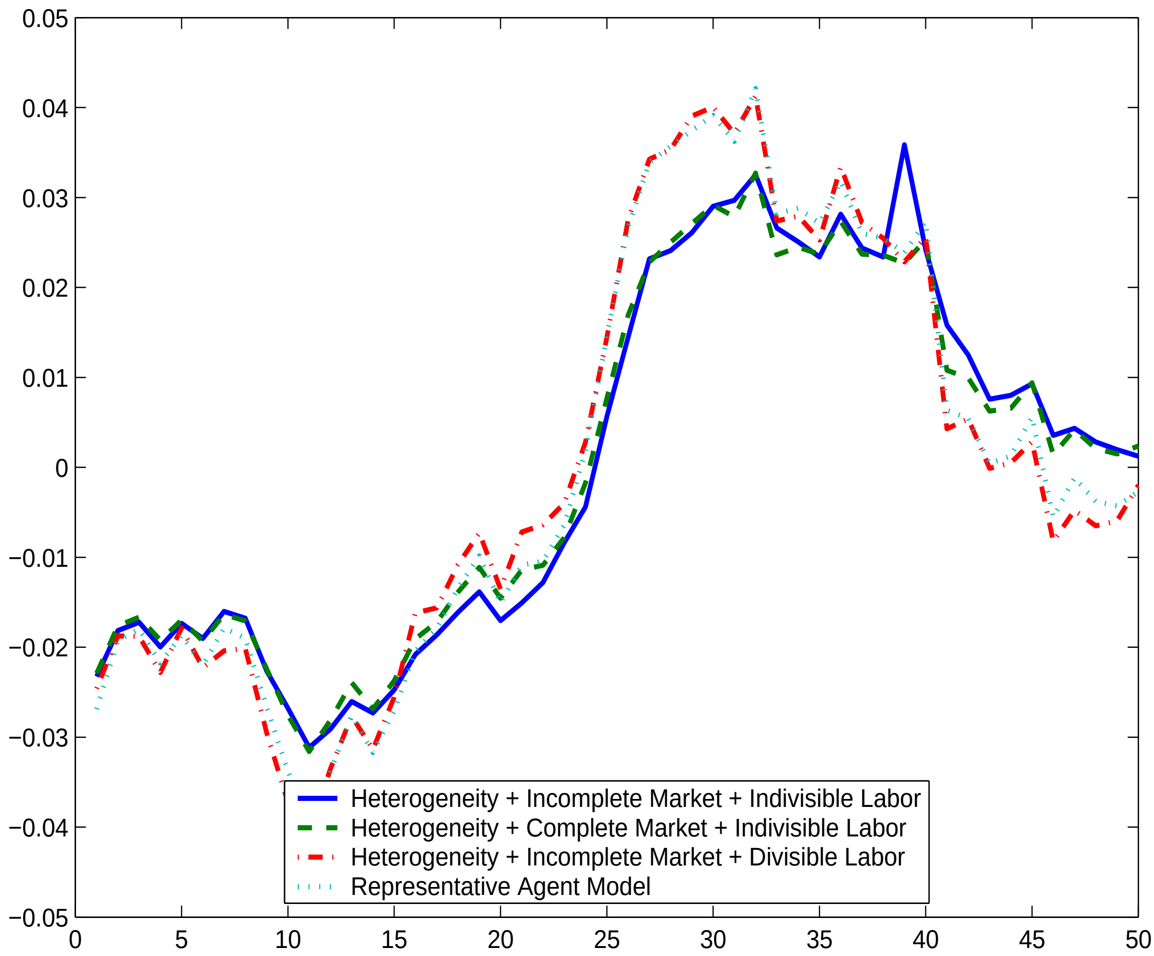


Figure 8: Aggregate Consumption from the Models