How Non-Ricardian is the Life-Cycle Model?

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Abstract

This paper provides a quantitative analysis of the economic effects of public debt in a pure life-cycle economy. The effects on observable quantities are invariably small in cases where the change in the debt is accompanied by a change in lump-sum taxes and transfers. This is true of both the impact and long-run effects. A permanent increase in the debt can lead to a nontrivial long-run reduction in the welfare, however. A change in the debt accompanied by changes in either government purchases or distorting tax rates can significantly alter economic behavior, but even these effects can be small depending on the exact nature of the tax change. Taken together, the results suggest that if Ricardian Equivalence fails empirically, it does so not because of the finite horizons implied by the life-cycle model but because of more fundamental reasons.

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1 Introduction

The longstanding controversy over the consequences of public debt has centered on two related issues. The debates of the late 1950s and early 1960s dealt largely with the burden of the debt, and specifically with whether a larger public debt imposes a burden on future generations. Following Barro (1974), the more recent discussion has been framed in terms of the Ricardian Equivalence proposition, which states that under appropriate conditions the substitution of debt for tax finance does not affect economic behavior. The two issues are related because debt finance will have no effect on welfare if it leaves all allocations unchanged.

Abel (1987) defines Ricardian Equivalence as "the proposition that the method of financing any particular path of government expenditure is irrelevant. More precisely, the choice between levying lump-sum taxes and issuing government bonds to finance government spending does not affect the consumption of any household nor does it affect capital formation." Both Barro (1974) and the subsequent literature elaborate conditions that lead to Ricardian Equivalence or to its violation. A crucial issue is the effective planning horizon of households. A debt-financed tax reduction today implies a stream of future tax liabilities with a current market value equal to the debt issue. If households optimize over an infinite horizon, the debt issue may leave both their budget constraints and their behavior unchanged. Certain types of operative bequest motive can lead to long or infinite horizons. Conditions leading to shorter horizons include myopic behavior and binding constraints on borrowing, which cause even infinitely-lived households to solve a sequence of optimization problems over disconnected segments of time.

In a pure life-cycle model the planning horizon is finite by definition. Consequently, this model violates the requirements for Ricardian Equivalence, and it is generally regarded as the standard non-Ricardian alternative. The classic analysis of public debt in a life-cycle setting is by Diamond (1965), who studies the steady-state effects of a one-time, lump-sum transfer payment financed by a permanent increase in the debt. He finds that such a policy raises the return to capital and lowers both the capital stock and utility in a dynamically efficient, closed economy.

Strict Ricardian Equivalence implies that a debt issue which merely alters the timing of lump-sum taxes has no effect on observable economic aggregates or on economic welfare. How different from zero would these effects be in a plausibly calibrated, standard life-cycle model? In such a model, households have planning horizons that vary with age and possibly with other household characteristics. While some older households have short planning horizons, aggregate behavior depends on the distribution of horizons across the entire population. Diamond's results are qualitative rather than quantitative. His analysis thus does not indicate whether the finite planning horizons in a reasonably specified life-cycle model generate aggregate behavior that differs in a quantitatively significant way from that implied by Ricardian Equivalence. One purpose of this paper is to analyze this issue.

Much of the discussion of the quantitative effects of public debt has been based on the large body of empirical evidence produced in the wake of the U.S. budget deficits of the 1980s. This literature, which examined the relation between budget deficits and aggregate variables including consumption, interest rates, the trade balance, and foreign exchange rates, is too large to summarize here. Empirical analysis of aggregate time-series data has been complicated by several difficulties, including the fact that observed variation in the budget deficit is partly endogenous and rarely occurs in the Ricardian context of unchanging government purchase and tax rates. The various methods employed to deal with these complications have not been sufficiently convincing to yield a consensus. Thus, an exploration of the quantitative implications of the most standard non-Ricardian model is in order.

This paper is not the first to provide such an analysis. Miller and Upton (1974) use a simply calibrated, 80-period life-cycle model with inelastic labor supply to re-examine the debt-and-transfer policy described by Diamond (1965). They find that, on average, households perceive an increase in wealth equal to roughly 15 to 25 percent of the initial transfer, depending on the taxes chosen to finance the interest payments on the debt.[1] They do not report the steady-state effects on the capital stock, interest rates, or welfare.

Poterba and Summers (1987) use a 55-period life-cycle model and report calculations similar to those of Miller and Upton.[2] They also calculate the initial (year-1) response of consumption under standard assumptions about preferences. If the transfer program lasts only one year, consumption rises by about five to seven percent of the debt issue, depending on the parameter values used. If the transfer is kept in place for five years, consumption rises by about 19 to 25 percent of the annual transfer. Poterba and Summers do not calculate the change in consumption in subsequent years, nor do they compute the general-equilibrium effects on factor prices or welfare. Although they analyze a permanent increase in the debt-income ratio, they do not report steady-state effects. They note, however, that while "in any given year these saving effects may be small, they cumulate over time."

Auerbach and Kotlikoff (1987) . . .

Evans (1991) . . .

More recently, Gale and Orszag (2004) use a simple production technology to calculate the steady-state, general equilibrium effects of a permanent increase in the budget deficit under alternative assumptions about how consumption responds to the deficit. They find, for example, that if consumption increases

[1] Miller and Upton assume that the growth rates of population and per capita income and the probability of death before age 80 are all zero. A positive growth rate of aggregate income would imply lower debt-service payments at a given debt-income ratio, thus raising perceived wealth. A zero probability of premature death would tend to lower perceived wealth. Zero mortality and population growth rates together imply that all cohorts are of equal size, thus increasing the weight attached to cohorts with short horizons and increasing increasing perceived wealth.

[2] Poterba and Summers allow for positive population growth and use empirical cohort shares in computing weighted averages but maintain the assumption of an inelastic labor supply.

by 75 percent of the deficit, a permanent increase in the deficit equal to one percent of output lowers the capital-output ratio by about 4.4 percent and raises the marginal product of capital by 54 basis points.

This paper analyzes the effects of public debt using an 85-period overlapping generations model populated by pure life-cycle consumers. It considers the effects on both observable economic aggregates and welfare. It examines both the impact and steady-state effects of changes in the debt, as well as the transition path from one steady state to another, and deals with both transitory and permanent changes in the debt. It analyzes changes in the debt that occur under the assumptions of the Ricardian Equivalence proposition (i.e., accompanied by changes in lump-sum taxes or transfers) as well as more general alternatives in which government purchases or tax rates are allowed to vary. Finally, it examines both the general equilibrium effects in a closed economy and the effects in a small, open economy.

Changes in the debt accompanied by changes in lump-sum taxes and transfers are found to exert only small effects in observable variables. This is true of both transitory and permanent changes in the debt and of both the impact and steady-state effects. Thus, Ricardian Equivalence seems to be a reasonable approximation even in a pure life-cycle world. Despite the small effects on observables, permanent increases in the debt can substantially reduce the welfare of individuals born into a new, high-debt steady state. Variation in the debt accompanied by changes in either government purchases or distorting tax rates violate the assumptions of the Ricardian Equivalence proposition and can have substantial effects on variables including consumption, the capital stock, and interest rates.

The basic 85-period overlapping generations model is described in section 2 and its calibration is described in section 3. Section 4 compares economic behavior and welfare across steady states with different levels of debt. Section 5 deals with the impact effects of both transitory and permanent changes in the debt and with the transition from one steady state to another. It also includes an analysis of a temporary but long-lasting increase in the debt calibrated to the U.S. experience during and after World War II. Section 6 concludes.

2 The Model

The model is populated by overlapping generations of individuals who become economically active at a real-time age of 21 and who live for up to 85 additional periods, to a maximum age of 105. They face a positive probability of death at each age, and this is the only source of risk in the model. Individuals choose a level of work effort at each age until reaching mandatory retirement. They also choose non-negative holdings of physical capital and government debt, and these assets are perfect substitutes. Asset holdings are influenced by the existence of a mandatory, pay-as-you-go public pension system modeled after

U.S. Social Security. In the benchmark steady state, the constraint on negative asset holdings is binding both at the beginning of the life cycle and, in part because of the public pension, for more than a decade before the maximum possible age. This binding constraint makes the model less nearly Ricardian. Output is produced by competitive firms according to a constant-returns-to-scale technology, and factor prices are determined competitively by marginal productivity. Besides administering the public pension system, the government purchases output, imposes lump-sum taxes and transfers, and levies distorting taxes on consumption and income from labor and assets.

The remainder of this section describes the model in more detail.

2.1 Demographics

At each date t, a new cohort is born that is n percent larger than the previous cohort, and n is referred to as the "net fertility rate." Age is denoted by j, and $j = 1$ in the first period of life. Cohorts are indexed by i, which is equal to the calendar date corresponding to the first period of life. The relation between date, cohort, and age is given by $t = i + j - 1$. In general, a variable needs to be subscripted by at most two of these three indices. Variables that change over the life cycle but are stationary from one cohort to the next are indexed only by age. Aggregates describing the entire economy, as well as market-clearing factor prices, are indexed only by time.

Individuals face long but random lives and some live through age J, the maximum possible life span. Life-span uncertainty is described by ψ_j, the time-invariant conditional probability that an individual of age $j-1$ in period $t-1$ survives to age j in period t, with $\psi_1 \equiv 1$. The unconditional probability of surviving from birth in period t to age j in period $t+j-1$ is given by $\pi_j = \psi_j \pi_{j-1}$, where $\pi_1 \equiv 1$.

Let $\mu_{j.t}$ denote the number of individuals of age j in period t and μ_t be a $J \times 1$ vector whose elements are the $\mu_{j.t}$. The population evolves according to

$$\mu_t = \begin{bmatrix} (1+n)\psi_1 & 0 & 0 & \cdots & 0 \\ \psi_2 & 0 & 0 & \cdots & 0 \\ 0 & \psi_3 & 0 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & \psi_J & 0 \end{bmatrix} \mu_{t-1} = \Psi \mu_{t-1}.$$

The fraction of the population of age j at time t is $\widehat{\mu}_{j,t} = \mu_{j,t}/N_t$. The aggregate population in period t is given by $N_t = \sum_{j=1}^{J} \mu_{j,t}$, with $N_0 = 1$, and the population growth rate is given by $n_t = N_t/N_{t-1} - 1$. Given the assumption that both n and ψ_j are time-invariant, the population growth rate is also constant and is given by $n_t = n$, and the cohort shares are time-invariant and are given by

$$\widehat{\mu}_j = \frac{\psi_j \widehat{\mu}_{j-1}}{1+n}$$

where $\sum_{j=1}^{J} \widehat{\mu}_j = 1$.

2.2 Individual Preferences and Budget Constraints

The efficiency of a worker evolves exogenously with age, and the efficiency level at age j is denoted by $\varepsilon_{,j}$. This lifetime efficiency profile is assumed to remain constant from one cohort to the next, although the market wage of an efficiency unit may grow over time.

Preferences are standard. An individual of cohort i maximizes the utility derived from lifetime sequences of consumption $\{c_{i,j}\}$ and work effort $\{\ell_{i,j}\}$, and the lifetime utility function is given by

$$U_i = \sum_{j=1}^{J} \beta^{j-1} \pi_j \frac{[c_{i,j}^{\sigma}(1-\ell_{i,j})^{1-\sigma}]^{1-\gamma}}{1-\gamma},$$

where β is the subjective discount factor.

Individuals choose $c_{i,j}$, $\ell_{i,j}$, and $a_{i,j}$, their asset holdings at the end of age j, to maximize lifetime utility subject to a sequence of one-period budget constraints

$$c_{i,j} + a_{i,j} = (1 + r_{i+j-1})a_{i,j-1} + w_{i+j-1}\varepsilon_j \ell_{i,j} + b_{i,j} - \chi_{i,j} + \xi_{i+j-1}$$

as well as the constraints[3]

$$\begin{aligned} c_{i,j} &\geq 0 \qquad && j = 1, 2, \ldots, J \\ \ell_{i,j} &\geq 0 && j = 1, 2, \ldots, j^* - 1 \\ \ell_{i,j} &= 0 && j = j^*, j^* + 1, \ldots, J \\ a_{i,j} &\geq 0 && j = 1, 2, \ldots, J \\ a_{i,0} &= 0. \end{aligned}$$

Here the market-clearing wage rate and rate of return at date t are denoted by w_t and r_t, respectively. The other time-specific variable is ξ_t, a lump-sum transfer payment received by all individuals alive at date t. Members of cohort i at age j receive a social security benefit denoted $b_{i,j}$, which remains constant for ages $j \geq j^*$ and is zero before that. Taxes paid by a member of cohort i at age j are

$$\chi_{i,j} = \chi_{i,j}^{c} + \chi_{i,j}^{a} + \chi_{i,j}^{\ell} + \chi_{i,j}^{s}, \tag{1}$$

where the four items on the right-hand-side of equation (1) are revenues from the consumption tax, the tax on income from assets, the labor income tax, and

[3] Given the form of the utility function, the nonnegativity constraint on consumption is never binding. The zero constraint on labor is also redundant if the efficiency index ε_j is calibrated to equal zero for $j \geq j^*$.

the social security payroll tax. They are given by

$$\begin{aligned}
\chi^c_{i,j} &= \tau^c_{i+j-1} c_{i,j} \\
\chi^a_{i,j} &= \tau^a_{i+j-1} r_{i+j-1} a_{i,j-1} \\
\chi^\ell_{i,j} &= \tau^\ell_{i+j-1} w_{i+j-1} \varepsilon_{,j} \ell_{i,j} \\
\chi^s_{i,j} &= \tau^s_{i+j-1} w_{i+j-1} \varepsilon_{,j} \ell_{i,j},
\end{aligned}$$

where τ^c_t, τ^a_t, τ^ℓ_t, and τ^s_t adenote the respective flat tax rates. These tax rates may vary over time but are the same for all cohorts at any date.[4]

Social security benefits are based on an individual's earnings history according to an algorithm like that found in the U.S. system. Define the average indexed earnings of an individual of cohort i as

$$\overline{e}_i = \frac{1}{j^*-1} \sum_{j=1}^{j^*-1} w_{i+j^*-1} \varepsilon_{hj} \ell_{i,j}.$$

Use of the wage rate w_{i+j^*-1} rather than the current wage w_{i+j-1} indexes the individual's covered earnings at age j for aggregate wage growth between age j and the retirement age j^*. The social security benefit is zero for ages $j < j^*$, while for ages $j \geq j^*$ it is computed as a function of $\overline{e}_i$ according to the formula

$$b_{i,j} = b_i = \begin{cases} \theta_1 \overline{e}_i & \text{if } \overline{e}_i \leq \kappa_1 \\ \theta_1 \kappa_1 + \theta_2 (\overline{e}_i - \kappa_1) & \text{if } \kappa_1 < \overline{e}_i \leq \kappa_2 \\ \theta_1 \kappa_1 + \theta_2 (\kappa_2 - \kappa_1) + \theta_3 (\overline{e}_i - \kappa_2) & \text{if } \kappa_2 < \overline{e}_i \end{cases} \qquad (2)$$

where $\theta_1 > \theta_2 > \theta_3 > 0$ and $\kappa_2 > \kappa_1 > 0$. Note that the benefit remains constant throughout retirement.

For given values of the policy parameters and factor prices, the social security benefit can be regarded as a function of lifetime work effort, $b_i(\ell_{i,j}, ..., \ell_{i,j})$. Individuals are assumed to be fully aware of this dependence and to take it into account in determining their labor supply. The first-order condition for labor of an individual of cohort i and age j in period $i+j-1$ thus becomes

$$u_{c_j} \frac{w_{i+j-1} \varepsilon_j [1 - \chi'(\ell_{i,j})]}{1 + \tau^c_j} + \sum_{k=j^*}^{J} \left(\frac{\pi_k}{\pi_j} \right) \frac{\beta^{k-j} u_{c_k} b_k}{1 + \tau^c_k} + u_{\ell_j} \leq 0 \qquad (3)$$

where u_{c_j} is the marginal utility of consumption at age j, u_{ℓ_j} is the marginal utility of labor, $\chi'(\ell_{i,j})$ is the partial derivative of tax liability with respect

[4] Some public pension systems impose an upper limit on taxable earnings. A limit calibrated to that currently in force in the U.S. system is not binding in the model's initial steady state. This limit is binding in models with cross-sectional heterogeneity in the labor efficiency profile ε_j measured using U.S. data. Because such heterogeneity does not seem very important for the issues considered here, it is omitted from the model.

to work effort, and b_j is the partial derivative of the individual's future social security benefit with respect to work effort at age j.[5]

2.3 Production

Output is given by an aggregate Cobb-Douglas production function

$$Y_t = K_t^{\alpha}(A_t L_t)^{1-\alpha}$$

where K_t and L_t denote aggregate capital and labor, respectively, and $A_t^{1-\alpha}$ is total factor productivity. A_t, and thus per capita income, is assumed to grow at a constant, exogenously given rate g^*. Factor markets are assumed to be competitive, implying that factor prices are given by

$$\begin{aligned} w_t &= (1-\alpha)A_t^{1-\alpha}(K_t/L_t)^{\alpha} \\ r_t &= \alpha A_t^{1-\alpha}(K_t/L_t)^{1-\alpha} - \delta, \end{aligned}$$

where δ is the depreciation rate of capital.

2.4 Government

Government in this economy has three functions. First, it makes purchases of goods and services equal to G_t per period. Second, it makes a lump-sum transfer payment (which in principle could be negative) of ξ_t to each person alive at time t. Government finances these expenditures by imposing flat-rate taxes on consumption, labor income, and income from assets. In addition, the government is assumed to receive the assets of all individuals who die before the maximum age J. The aggregate amount of these receipts in period t is denoted φ_t.

The initial value of the transfer payment is determined as the residual amount needed to satisfy the government's budget constraint in the initial steady state. In some experiments, referred to as "Ricardian", the subsequent values

[5] If there are no binding liquidity constraints after age j,

$$\beta^{k-j}\left(\frac{\pi_k}{\pi_j}\right)u_{ck} = \frac{u_{c_j}}{\prod\limits_{v=j+1}^{k}(1+r_{i+v-1})}$$

and equation (3) becomes

$$u_{c_j}\left[\frac{w_{i+j-1}\varepsilon_j[1-\chi'(\ell_{i,j})]}{1+\tau_j^c} + \sum_{k=j^*}^{J}\left(\frac{b_k}{1+\tau_k^c}\right)\left(\prod_{v=j+1}^{k}\frac{1}{1+r_{i+v-1}}\right)\right] + u_{\ell_j} \leq 0$$

i.e., the marginal social security benefits, net of consumption taxes, are simply discounted at the interest rate. More generally, however, equation (3) applies.

are determined in a similar manner, with the transfer adjusting endogenously to satisfy the government budget constraint in response to changes in the public debt. In other experiments, called "non-Ricaridan", the transfer is fixed at the amount implied by the balanced growth path starting in the initial steady state and either government purchases or distorting taxes are adjusted to satisfy the government budget constraint.

The government's third function is to operate a pay-as-you-go social security system financed by a flat-rate payroll tax on earnings. In the steady state, aggregate social security contributions equal aggregate benefits in each period. The benefit at the retirement age j^* is a concave, piecewise linear function of average indexed covered earnings up through age $j^* - 1$, as shown in equation (2). The bend points (κ_1 and κ_2) and replacement rates (θ_1, θ_2, and θ_3) in the benefit formula are calibrated to match those recently applicable in the U.S. system. The size of the social security system is measured by the contribution rate τ_t^s, and the three replacement rates are adjusted upward or downward by a common proportionality factor so that the system's budget balances period-by-period in the initial and terminal steady states. During the transition from one steady state to another, the system is permitted to run a surplus or deficit, with the difference between contributions and benefits being made up from the general government budget.

The government's one-period budget constraint is given by

$$D_t = (1 + r_t)D_{t-1} + G_t + b_t + \xi_t - \varphi_t - \chi_t^a - \chi_t^\ell - \chi_t^s - \chi_t^c$$

where b_t denotes aggregate social security benefits, D_t is government debt outstanding at the end of period t, and the last four terms on the right-hand-side are the aggregate analogues of the individual taxes discussed above. Along a balanced growth path the debt grows at rate g, the rate of growth of aggregate output. In addition, aggregate social security contributions and benefits are equal so that, omitting time subscripts, the steady-state budget constraint becomes

$$\widehat{\chi}^a + \widehat{\chi}^\ell + \widehat{\chi}^c = \left(\frac{r - g}{1 + g}\right) \widehat{D} + G + \widehat{\xi} - \widehat{\varphi}. \tag{4}$$

Here the aggregate variables are defined by $\widehat{x}_t = x_t/N_t(A_t/A_0)$ and a similar definition for per capita variables involves deflation only for productivity growth. In the initial steady state, the government sets all tax rates and the social security benefit level. It also sets its initial level of purchases at G_0. The initial lump-sum transfer, ξ_0, is determined as a residual required to balance the budget in the initial steady state. In a subsequent steady state arising from a Ricardian experiment, purchases are set to the level $G_t = G_0 N_t(A_t/A_0)$ implied by the balanced growth path originating in the initial steady state, while tax rates are kept at their initial levels. In a terminal steady state arising from a non-Ricardian experiment, the transfer payment is set to $\xi_t = \xi_0 N_t(A_t/A_0)$, while purchases and distorting tax rates are permitted to differ from their initial values.

2.5 Equilibrium

The definition of equilibrium is standard. For a definition of the stationary recursive equilibrium characterizing the steady states, see İmrohoroğlu, İmrohoroğlu, and Joines (2003). The definition of the equilibrium transition paths follows Braun, Ikeda, and Joines (2004).

3 Calibration

The model is calibrated so that the initial steady state matches certain long-run features of the U.S. economy. The capital share parameter in the production function is set to 0.31, the depreciation rate to 0.044, and one model period is taken to be a year. The productivity parameter A is normalized so that output equals 1.0 in the initial steady state (period 0). The annual growth rate of per capita output is 1.65 percent, and the rate of growth of population is 1.20 percent. All of these parameter values are from İmrohoroğlu, İmrohoroğlu, and Joines (2003).

Age-specific survival probabilities are assumed to be time-invariant and are averages of the male and female rates for the cohort born in 1960 as reported in Bell and Miller (2002). The initial period of a model individual's life is assumed to correspond to a real-time age of 21, and the maximum possible life span J is 85, corresponding to a real-time age of 105. Mandatory retirement is assumed to occur at model age 45. All workers of a given age are equally efficient and the skill profile ε_j is taken from Hansen (1993). Social security benefits are given by a piecewise linear function of average indexed covered earnings with three segments. The replacement rates along the three segments are initially set to 0.90, 0.32, and 0.15 and are adjusted upward or downward by a common proportion to ensure that the social security budget balances in a given steady state. The bend points between the segments are set to equal roughly 0.17 and 0.99 times average annual earnings in the initial steady state and are indexed for productivity growth.

Fiscal parameters are calibrated to reflect values representative of the period since 1982 and in most cases are based on data from the National Income and Product Accounts. In the initial steady state, government purchases are set to 0.20, the average ratio of government purchases to GDP since 1982. The consumption tax rate is measured by the ratio of federal excise tax revenue plus state and local sales tax revenue to personal consumption expenditures net of these taxes. This ratio has averaged slightly more than 0.06 since 1982. In the model described above, the Social Security system is consolidated into the rest of the government budget, so that the relevant measure of the public debt is debt held by private investors. Such debt has averaged 35 percent of GDP since 1982.

The National Income and Product Accounts are less relevant for calibrating the remaining tax rates. The tax rate on labor income, τ^{ℓ}, is set to 0.20. This value is roughly equal to the average for the period after 1982 reported by Mendoza, Razin, and Tesar (1994), after deducting the payroll tax from their series. It is somewhat lower than the value of 0.26 implied by Prescott's (2004) calculations, after removing the payroll and consumption taxes from his overall tax rate of 0.40. The tax rate on income from assets is set to 0.40, the average value reported by Mendoza for 1982-1996.[6] The social security tax rate, τ^{s}, is set to 0.10, which is the current value of payroll tax payments to the Old-Age and Survivors (OASI) trust fund as a fraction of covered earnings (inclusive of the employer's share of the payroll tax for OASI and the other Social Security trust funds).

The baseline value of the inverse elasticity of intertemporal substitution γ is set to 2.0, and a lower elasticity is considered in the sensitivity analyses. Given $\gamma = 2$, the subjective discount factor β is set to 1.0016, resulting in a capital-output ratio of 2.52 in the initial steady state. The preference parameter σ is set to 0.35, resulting in an average labor input (whether measured cross-sectionally using population weights or over the life cycle using equal weights) slightly below 0.357, which in turn is 40 hours per week as a fraction of available hours (16 hours/day $\times$ 7 days/week).

Table 1 summarizes the calibration of the initial steady state.

4 Comparing Steady States

The analysis begins by comparing steady states with different levels of government debt since this task is simpler than analyzing the entire transition path between steady states. These quantitative results can be compared with the qualitative conclusions from Diamond's (1965) steady-state analysis.

The economy begins in an initial steady state with a debt-GDP ratio of 0.35, the characteristics of which are described below. It then moves to a new steady state with a different debt-GDP ratio. From the steady-state government budget constraint (4), it is apparent that a change in the debt generally requires a change in at least one category of revenue or outlays. Because behavior is different in the new steady-state equilibrium, government revenue from its three taxes and from confiscation of the assets of the prematurely deceased will generally also differ from that in the initial steady state, even at the original tax rates. Only by accident will this change in revenue match that required to satisfy the government budget constraint at the new debt-GDP ratio. The Ricardian Equivalence proposition deals with the case in which lump-sum taxes and transfers are altered to satisfy the government budget constraint, and experiments of this sort will therefore be referred to as Ricardian. Non-Ricardian experiments involve altering one of the three tax rates. Because government's

[6] The updated values are at http://www.bsos.umd.edu/econ/mendoza/pdfs/newtaxdata.pdf.

generally to not have recourse to lump-sum taxes, these non-Ricardian experiments are also reported. Comparing these results with those from the Ricardian experiments allows a decomposition of the total effects into those due to changes in distorting tax rates and those due only to changes in the debt.

This section begins with a description of the economy's initial steady state, after which it examines the effect of Ricardian changes in the public debt on asset holdings in a small, open economy with fixed factor prices. It then reports the general equilibrium effects on asset holdings, interest rates, and welfare in a closed economy for both the Ricardian and non-Ricardian cases.

4.1 Initial Steady State

Figure 1 shows consumption, assets, before-tax earnings, and labor over the life cycle in the initial steady state. The age-consumption profile has the humped shape seen in U.S. data, although the peak in consumption occurs later in the life cycle than is observed empirically. This discrepancy might be due to the fact that the current model does not allow the marginal utility of consumption to vary as a function of household size over the life cycle.[7] Consumption exhibits a discontinuous drop at retirement similar to that documented in a variety of studies of U.S. data and by Banks, Blundell, and Tanner (1998) for British data. The hours profile appears reasonable and corresponds to an average of slightly less than forty hours of work per week over the life cycle. The Social Security tax rate of 0.10 generates revenues that are about ten percent larger than aggregate benefit payments. Consequently, each segment of the benefit formula is adjusted upward by that amount to ensure that total benefit payments equal total revenues. Although the U.S. system is not currenty in a steady state, it has generated revenues that have averaged between 114 and 119 percent of benefits in recent years.

The capital-output ratio of 2.52 implies a before-tax real interest rate, r^*, of 7.9 percent. Given the tax rate on physical capital of 0.29, this in turn implies that r, the interest rate net of such taxes, is 5.6 percent.

[7] See, for example, the empirical work of Attanasio and Weber (1995), and Attanasio, Banks, Meghir, and Weber (1999) and the simulation model of Ríos-Rull (2001).

Table 1
Calibration of Baseline Initial Steady State

	Preferences	
β	subjective discount factor	1.0016
γ	inverse elasticity of intertemporal substitution	2.0
σ	weight on consumption	0.35
	Demographics	
n	population growth rate	0.012
J	maximum life span	85
j^*	mandatory retirement age	45
ψ_j	conditional survival probabilities	Bell and Miller (2002)
ϵ_j	labor efficiency profile	Hansen (1993)
	Technology	
α	capital share	0.31
δ	depreciation rate	0.044
A	normalized productivity parameter	1.852
g^*	growth rate of per capita output	0.0165
	Fiscal Policy	
D	public debt	0.35
G	government purchases	0.20
τ^c	consumption tax rate	0.06
τ^ℓ	labor income tax rate	0.20
τ^a	tax rate on personal income from ordinary assets	0.40
τ^s	social security tax rate	0.10
κ_1	first bend point in social security benefit formula	0.17
κ_2	second bend point in social security benefit formula	0.99
θ_1	first social security marginal replacement rate	0.90
θ_2	second social security marginal replacement rate	0.32
θ_3	third social security marginal replacement rate	0.15

4.2 Small, Open Economy

The open-economy analysis begins with a steady state in which foreign asset holdings are zero at the initial debt-GDP ratio of 0.35. In the new steady state, lump-sum taxes and transfers are altered to satisfy the government budget constraint. Figure 2 shows the response of the domestic capital stock and total asset holdings of domestic residents as the debt-GDP ratio varies. It also shows the breakdown of total assets into domestic and foreign components.

The responses of all four variable are nearly linear. Total private asset holdings are almost invariant to changes in the public debt, a finding strongly at odds with Ricardian Equivalence. In effect, the entire public debt is held externally. Because of the required interest payments to the rest of the world, an increase in the debt makes domestic residents poorer. Consequently, they supply more labor at higher levels of public debt. Because factor prices are determined externally, an increase in the domestic labor supply attracts capital from abroad. A one-dollar increase in the public debt results in an increase of 12.2 cents in the steady-state capital stock, implying that net foreign asset holdings decline more than one-for-one with an increase in the debt.

4.3 Closed Economy

Figure 3 shows the steady-state effects of variations in the public debt accompanied by changes in lump-sum taxes or transfers. The top panel displays the behavior of total private asset holdings and the capital stock, each of which responds almost linearly to changes in the debt. A one-dollar increase in the debt raises private assets by slightly more than 80 cents, thus reducing the capital stock by slightly less than 20 cents. The reaction of total asset holdings differs from that seen in the small, open economy because an increase in the debt in a closed economy tends to reduce the capital stock, thus raising the interest rate. The response of asset holdings depends on the parameters of preferences and technology. If the model is re-calibrated with a lower elasticity of intertemporal substitution (setting $\gamma = 4$ and adjusting the subjective discount parameter β so that the model yields the same capital-output ratio in the initial steady state), asset holdings are less responsive to changes in the debt. With this alternative calibration, a one-dollar increase in the debt raises private assets by about 68 cents and lowers the capital stock by about 32 cents.

The bottom panel of Figure 3 shows the interest rate for each of the two intertemporal substitution parameters. The interest rate responds almost linearly to variations in the debt. Under the benchmark calibration ($\gamma = 2$), a one-percentage-point increase in the debt-GDP ratio increases the interest rate by 0.81 basis points on average, and complete elimination of the debt would lower the interest rate by 29 basis points. With a lower elasticity of intertemporal substitution ($\gamma = 4$), a one-percentage-point increase in the debt-GDP ratio increases the interest rate by 1.30 basis points, and complete elimination of the debt would lower the interest rate by 41 basis points. These interest rate effects can also be expressed in terms of a flow variable, the budget deficit, rather than the stock of debt. From equation (4), the primary budget surplus in a steady state is equal to $(r-g)\widehat{D}/(1+g)$ and the conventional deficit is equal to $g\widehat{D}/(1+g)$. In the initial steady state with a debt-DGP ratio of 0.35, the conventional budget deficit is equal to 0.98 percent of GDP. Thus, complete elimination of the debt corresponds to a reduction in the deficit of about one percent of GDP.

Gale and Orszag (2004) calculate the change in interest rates associated with a change in the steady-state deficit. They use the Solow growth model with a production technology similar to that used here and make alternative assumptions about how private saving reacts to the budget deficit. They report that an increase in the deficit equal to one percent of GDP raises interest rates by 73 basis points if there is no change in private saving and by 35 basis points if saving increases by 50 percent of the deficit. Their assumed saving responses are smaller than those calculated here using either of the alternative values of the elasticity of intertemporal substitution, and their implied interest rate effects are correspondingly larger.

If governments do not have access to lump-sum taxes and transfers, then a change in the debt-GDP ratio requires modifying one of the distorting tax rates. Figure 4 shows the steady-state effect of public debt in three non-Ricardian cases, each corresponding to variation in one of the three distorting taxes under the benchmark parameterization. For purposes of comparison, the figure also displays the benchmark Ricardian experiment.

The top panel of Figure 4 shows total asset holdings as a function of the debt-GDP ratio. If the consumption tax is varied to satisfy the government budget constraint, assets react very much as in the Ricardian case. A one-dollar increase in the debt increases assets by about 77 cents and reduces capital by about 23 cents. Increases in the debt have a greater effect on the capital stock if the labor income tax rate is varied. This is because labor income taxes are paid earlier in the life cycle than consumption taxes, so that increases in labor income taxes cause greater reductions in asset holdings. If the labor income tax rate is adjusted, quadrupling the debt-GDP ratio from 0.35 to 1.40 reduces the capital stock by 13.5 percent, compared with 9.8 percent if the consumption tax rate is adjusted and 8.8 percent in the Ricardian experiment.

The economy behaves quite differently if the capital income tax rate is altered to balance the government budget constraint. Varying the debt-GDP ratio between zero and 1.0 results in very little change in total asset holdings. Assets begin to decline noticeably as the debt-GDP ratio is increased above 1.0. This is because large increases in the capital income tax rate are required to service the debt. Quadrupling the debt-GDP ratio to 1.4 is not feasible, since the government cannot generate enough revenue from the capital income tax to service the debt. The required tax rate would be in excess of 70 percent, which is near the peak of the Laffer Curve.

The bottom panel of Figure 4 shows interest rates, which respond almost linearly to changes in the debt-GDP ratio except in the case where the capital income tax rate is varied. Perhaps surprisingly, the interest rate effects are smaller when the consumption tax rate is varied than in the Ricardian experiment. A one-percentage-point increase in the debt-GDP ratio increases interest rates by 0.77 basis points if the consumption tax rate is adjusted and by 1.31 basis points if the labor income tax rate is adjusted, compared with 0.81 basis points in the Ricardian experiment. Completely eliminating the debt reduces the steady-state interest rate by 28 basis points if the consumption tax rate is reduced, by 34 basis points if the labor income tax rate is reduced, by 81 basis points if the capital income tax rate is reduced, compared with 29 basis points in the Ricardian experiment. The incremental interest rate effects of varying distorting tax rates other than that on capital income are small compared with the effects of departures from Ricardian Equivalence.

4.4 Welfare

As noted in the introduction, there has been considerable debate over the question of whether the public debt imposes a burden on future generations. One way of quantifying such a burden is to compare the expected lifetime utility of individuals born into steady states with different levels of debt. Here, these welfare effects are measured by the equivalent variation, expressed as the proportionate change in consumption at all ages in the initial steady state with a debt-GDP ratio of 0.35 required to make individuals indifferent between that debt level and an alternative one. The equivalent variation is positive if the alternative debt-GDP ratio results in higher expected lifetime utility and negative if it results in lower expected utility.

Figure 5 shows the steady-state welfare effects of public debt. The top panel considers three variants of the Ricardian experiment, two for the closed-economy cases with benchmark ($\gamma = 2$) and low ($\gamma = 4$) elasticities of intertemporal substitution and the third for the open-economy case with the benchmark calibration. A higher public debt entails noticeable welfare costs in all three cases. As emphasized by Modigliani (1961) the welfare losses in a closed economy are related to the reduction in the steady-state capital stock. In a closed economy, a lower elasticity of intertemporal substitution implies greater reductions in both the capital stock and welfare. The welfare effects are still larger in the benchmark, open economy model, a finding that appears to be due to the fact that public debt causes a greater reduction in net domestic assets in an open economy with fixed factor prices than in a closed economy.

The bottom panel of Figure 5 displays the results of the various non-Ricardian experiment. In each of these cases, debt entails a higher steady-state welfare cost than in the benchmark Ricardian experiment. The welfare losses are slightly higher if the consumption tax is varied and substantially higher if either of the income taxes is varied. Steady-state utility would increase by an amount worth 1.23 percent of lifetime consumption if the entire debt were eliminated and the consumption tax rate were lowered accordingly. The welfare gains would be 1.53 percent and 2.38 percent of lifetime consumption, respectively, if the labor or capital income tax rate were reduced. The gain from eliminating the debt in the Ricardian experiment is 1.03 percent of consumption.

5 Impact and Transition Effects

Much of the discussion of the consequences of public debt has been in terms of its impact effects. For instance, Barro's (1974) initial demonstration of Ricardian Equivalence was in terms of a one-time, temporary increase in the debt. Poterba and Summers (1987) analyze the impact effect of a one-time, permanent increase in the debt, and such an experiment also forms the basis for discussion of the debt going back to Ricardo (1821). This section traces out the impulse responses of variables including consumption, asset holdings, capital, and interest rates in response to both temporary and permanent changes in the debt. It also describes the subsequent evolution of these variables until the economy either returns to its initial steady state (if the debt increase is temporary) or settles at a new steady state with a permanently higher debt-GDP ratio. These impulse response functions can be compared with those implied by the many empirical studies that report estimates of the effect of deficits on consumption and interest rates. This section also traces out the response of the economy to a tripling of the debt-GDP ratio, as during World War II, followed by a reduction similar to that which followed the war.

5.1 Temporary Debt Increase

The analysis of a temporary increase in the debt begins in year 0 with the economy in an initial steady state with a debt-GDP ratio of 0.35. In period 1 the government unexpectedly makes a lump-sum transfer equal to one-tenth of GDP, distributed uniformly to everyone alive at that time and financed entirely by public debt. It also announces that it will levy a lump-sum tax in period 2 sufficient to return the debt-GDP ratio to 0.35 forever. This policy results in a temporary increase in the debt of about one third.

Figure 6 shows the responses of consumption, work effort, the capital stock and the interest rate to this shock. The first three variables are normalized so that the value on the balanced growth path originating in the initial steady state is 1.0.

Given the temporary nature of the debt increase, it might be expected to have small effects. The responses are indeed small, possibly even smaller than might be expected. Their signs are easily explained, however. Given the finite horizons of individuals in the model, it is not surprising that consumption increases in year 1. Individuals of model age 85 in year 1 will not be alive in year 2 and so consume the entire transfer. Because the population subject to increased taxes in year 2 will be larger than that receiving the transfer in year 1, the transfer causes a slight increase in wealth even for individuals younger than 85. As a result of this wealth increase, both consumption and leisure rise in year 1. Consumption rises 0.12 percent above trend, while labor input falls 0.14 percent below trend. The capital stock is unchanged relative to trend, having been determined by saving decisions in year 0, before the transfer was announced. The reduction in the capital-labor ratio means that the debt issue results in a small contemporaneous decline in the interest rate of 1.2 basis points. Higher consumption and lower earnings in year 1 imply that the capital stock is 0.07 percent below trend in year 2. The year-2 effects on consumption and work effort are opposite in sign to those for year 1, in part because the youngest cohort of workers entering the economy in year 2 faces both a wage rate that is below trend and its portion of the lump-sum tax levied to retire the temporary debt. Lower consumption and greater work effort in year 2 imply that the capital stock is above trend in year 3, after which it and the other variables gradually return to their trend values. While small, the impact effects of the deficit on both consumption and interest rates (allowing for the one-period lag due to the timing conventions of the model) are of the signs predicted by the traditional, non-Ricardian view. Note, however, that a temporary increase in the debt exerts a persistent effect on the capital stock that is opposite in sign to that predicted by the conventional view.

5.2 Permanent Debt Increase

Ricardo's (1821) discussion of the effects of public debt dealt with a one-time, permanent increase. As with the temporary change considered above, this increase in the debt is assumed to come about because of an unexpected lump-sum transfer in year 1. The only difference is that the government is assumed to maintain the higher debt-GDP ratio forever, financing the interest payments with lump-sum taxes. The economy eventually settles into a new steady state with a debt-GDP ratio of 0.45 and asset holdings, capital, and interest rate as shown in Figure 2 above.

Figure 7 shows the impact effects and transition to the new steady state. As with the temporary debt increase, consumption and leisure both initially rise. Consumption in year 1 is 0.50 percent above trend and labor input is 0.34 percent below trend. Lower earnings and higher consumption in year 1 imply that the capital stock falls 0.21 percent below trend in year 2. The capital stock continues declining until it settles at a value 0.94 percent below the original balanced growth path. Labor input recovers after its initial decline, eventually settling 0.05 percent above its original trend. Output is lower from year 1 onward, eventually stabilizing 0.24 percent below its original trend. Changes in the capital-labor ratio determine changes in the interest rate. Because labor input adjusts more quickly than the capital stock, the impact effect of a permanent increase in the debt is a decline in the interest rate of three basis points. The interest rate is higher from period 2 onward, with the effect being larger in the long-run than in the short run. The interest rate eventually settles eight basis points above its original value.\

Strict Ricardian Equivalence implies that all of these effects should be zero rather than the small numbers reported here. Poterba and Summers (1987) argue that Ricardian Equivalence provides a reasonable approximation of the short-run effects of permanent changes in the debt even in a pure life-cycle world. The calculations reported here confirm their contention and suggest that Ricardian Equivalence might also be a good approximation of the long-run effects. If Ricardian Equivalence fails empirically, it must do so not because of the finite horizons implied by the life-cycle model but because of more fundamental reasons.

5.3 World War II

Wars often lead to large increases in the public debt, and the highest debt-GDP in US history occurred at then end of World War II. The net Federal debt held by the public stood at 37 percent of GDP at the end of 1942 and increased to slightly more than 100 percent of GDP by mid-1945. Apart from slight increases associated with recessions in 1949, 1954, and 1958, the debt-GDP ratio declined steadily for thirty years after the war, reaching 0.36 in 1963. This episode is approximated by an experiment in which the economy begins in a steady state with a debt-GDP ratio of 0.35 in year 0. During years 1 to 3 the government makes a uniform, annual, lump-sum transfer that raises the debt-GDP ratio to 1.00, after which it levies a uniform lump-sum tax that returns the debt-GDP ratio to its starting value of 0.35 over the next 18 years. In another variant of this experiment, the proceeds of the initial increase in the debt are assumed to be used to purchase goods and services that do not enter directly into either private production or utility functions. This alternative, non-Ricardian experiment should more accurately represent the effects of deficit-financed war spending. Comparison of the two variants permits separating the effects of the government's appropriation of resources from those of the debt issue itself.

Figure 8 shows some details of the economy's response to these two experiments. Consider first the case in which the debt is used to finance transfers. As in the previously reported experiments, the immediate effect is to increase both consumption and leisure. Consumption remains above trend for five years, peaking at 1.16 percent above trend in year 1. Labor input remains about 0.4 percent below trend in years 2 through 4. This policy exerts a persistent effect on the capital stock, which remains below trend for roughly 60 years, i.e., for about 40 years after the debt-GDP ratio returns to its original value. The largest effects are in years 6 through 11, when the capital stock is about 1.35

percent below trend. After the debt-GDP ratio returns to its initial value, the capital stock remains with 0.5 percent of trend. The decline in the capital stock exerts a wealth effect causing labor input to remain trend from year 6 through the remainder of the transition back to the steady state. The lower capital-labor ratio implies that the interest rate remains above its initial value from year 2 until the steady state is reached. The peak effect occurs in years 7 through 12, when the interest rate increases by about 13 basis points. As with the experiments reported above, the effect of a temporary tripling of the debt appear quantitatively small.

The effects do not remain small if the proceeds of the debt are used to finance government purchases. The government's appropriation of output crowds out private spending on consumption and investment. An immediate increase in the interest rate is required to bring about this reallocation of resources, and the interest rate rises by 163 basis points by year 4. The combination of interest-rate and wealth effects lowers both consumption and leisure. Consumption remains depressed throughout the transition and is about six percent below trend in years 1 through 4. The interest rate peaks in year 4 at 163 basis points above its initial value. Work effort is above trend throughout the transition. The capital stock falls more than 15 percent below trend by year 4 and remains at least five percent below trend through year 19. The lower capital-labor ratio implies that the interest rate remains high for a protracted period. It is at least 100 basis points higher than its steady-state value through year 10 and 50 basis points higher through year 19. Thus an event like World War II can have pronounced and sustained economic consequences, but these consequences arise largely from the government's extraction of resources from the private sector rather than from the accompanying debt issue.

6 Conclusions

This paper provides a quantitative analysis of the economic effects of public debt in a pure life-cycle economy. The effects on observable quantities like consumption, the capital stock, and interest rates are invariably small in cases where the change in the debt is accompanied by a change in lump-sum taxes and transfers. This is true of both the impact and long-run effects. These findings suggest that Ricardian Equivalence is a reasonable approximation in such an economy. Even though the effects on observables are small, however, a permanent increase in the debt results in a nontrivial reduction in the welfare of individuals born into the steady state with higher debt.

The paper also conducts several non-Ricardian experiments in which the change in the debt is accompanied by changes in either government purchases or distorting tax rates. In certain of these cases, the effects remain fairly small. This is particularly true when the interest payments on a higher debt are financed with a consumption tax. The effects can be much larger when the interest payments are financed with a tax on income from capital, but the larger effects are due primarily to the distorting tax rate rather than to the debt itself. A similar conclusion applies when an increase in the debt is used to finance government purchases.

Taken together, these results suggest that if Ricardian Equivalence appears to fail empirically, it must do so not because of the finite horizons implied by the life-cycle model but because of more fundamental reasons. It is possible that individuals do not behave as assumed in the model. They might, for example, behave myopically or make their consumption and saving decisions by rules of thumb rather than by optimization. Alternatively, the difficulties might lie with the empirical analysis. For instance, it might not be possible to control adequately for the changes in government purchases and distorting tax rates (possibly future ones), that can exert large economic effects. Whatever the reasons for the discrepancies, however, it remains the case that a reasonably calibrated version of our most standard model of behavior over the life cycle implies that the effects of public debt are quantitatively small.

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Figure 1
Initial Steady State

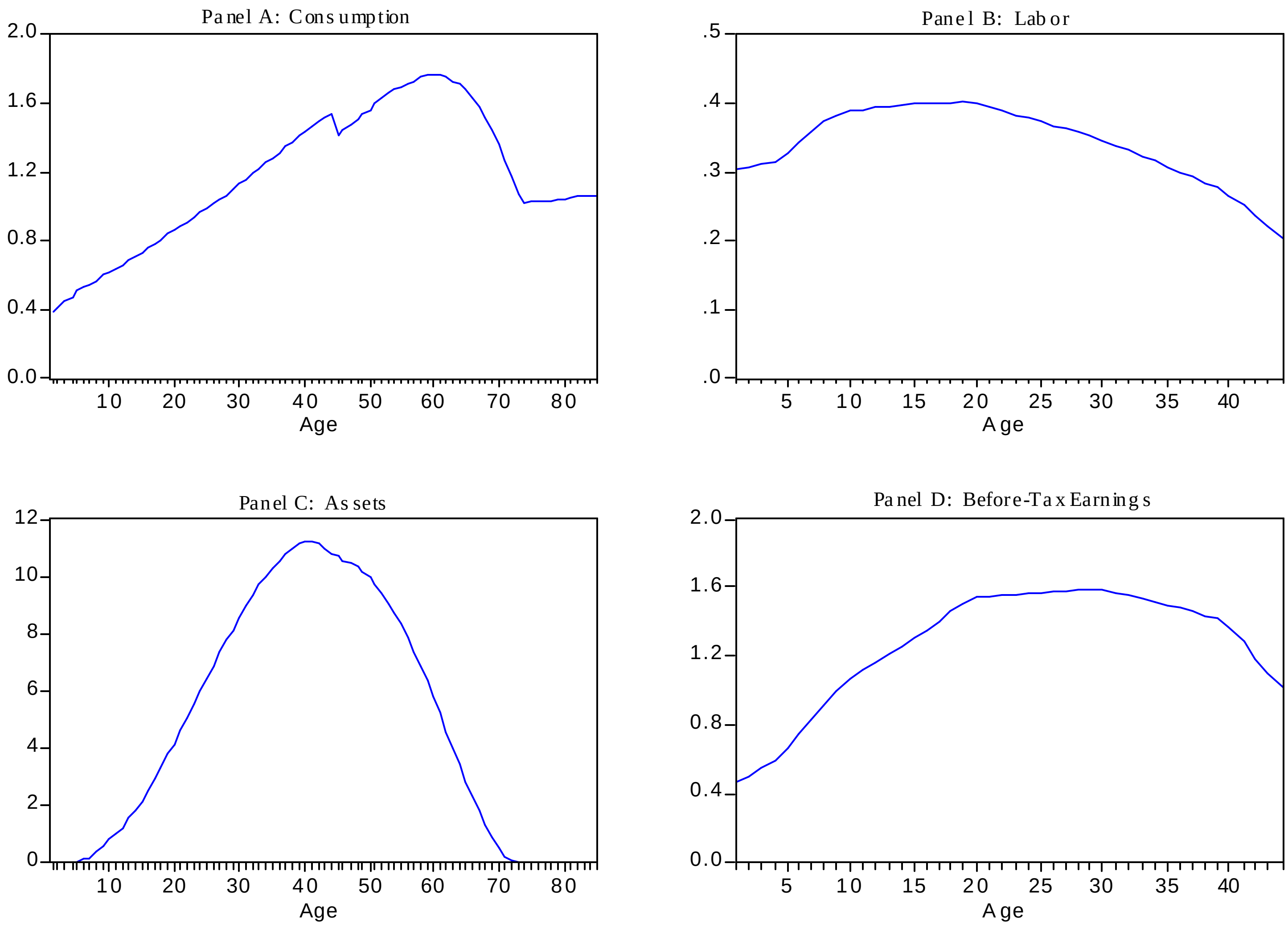


Figure 2
Steady-State, Open-Economy Assets

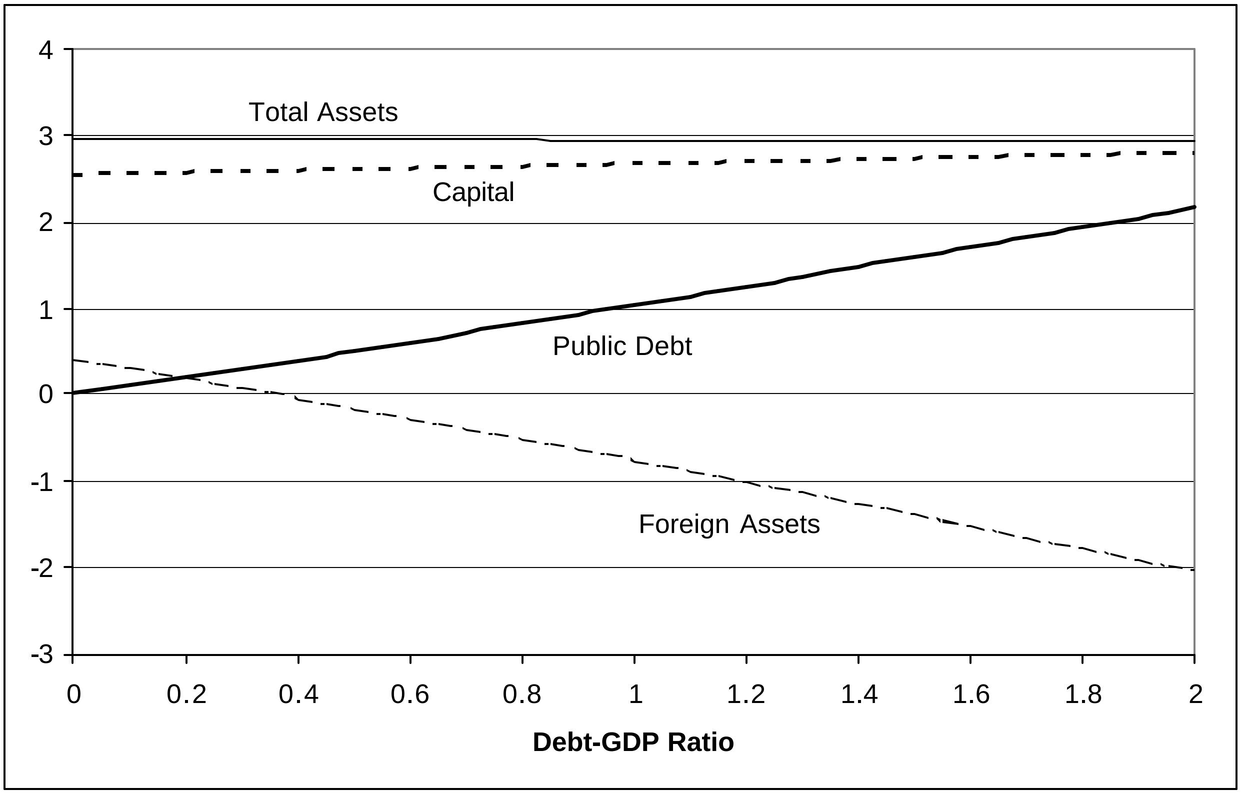
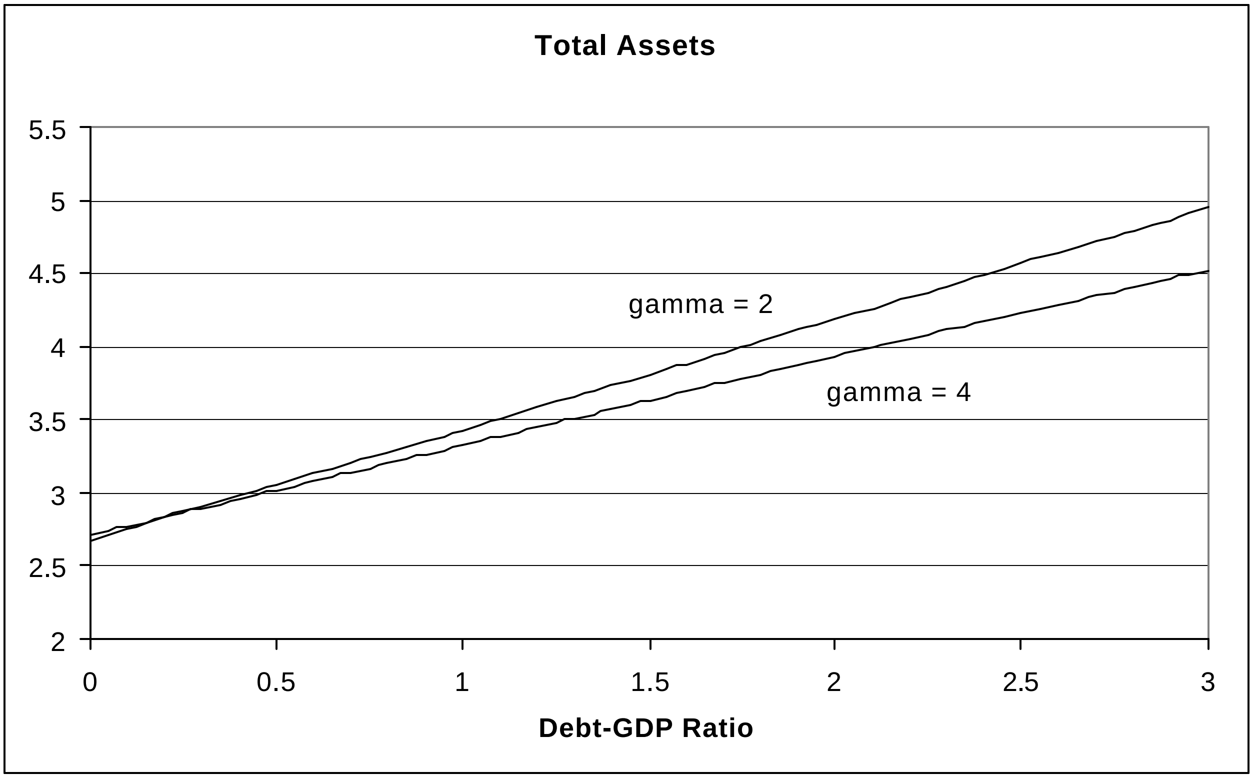


Figure 3
Steady-State Ricardian Experiments



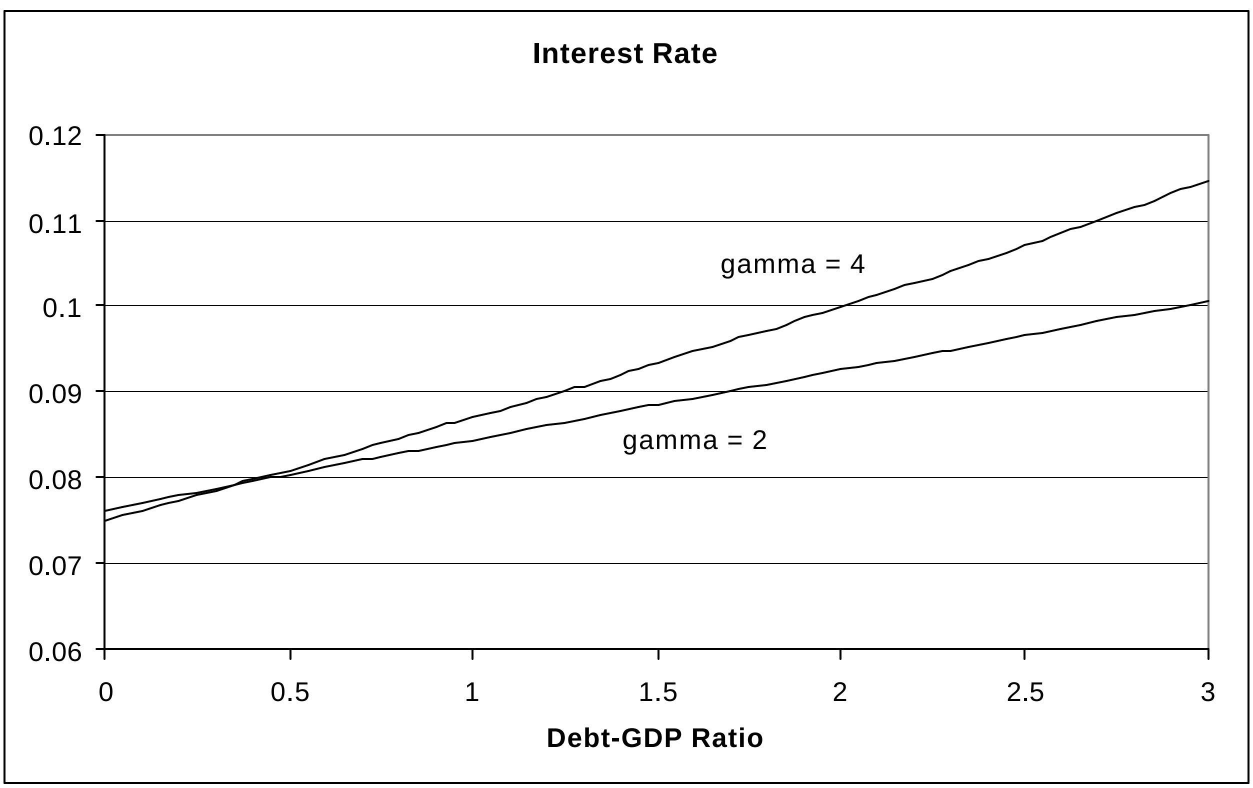
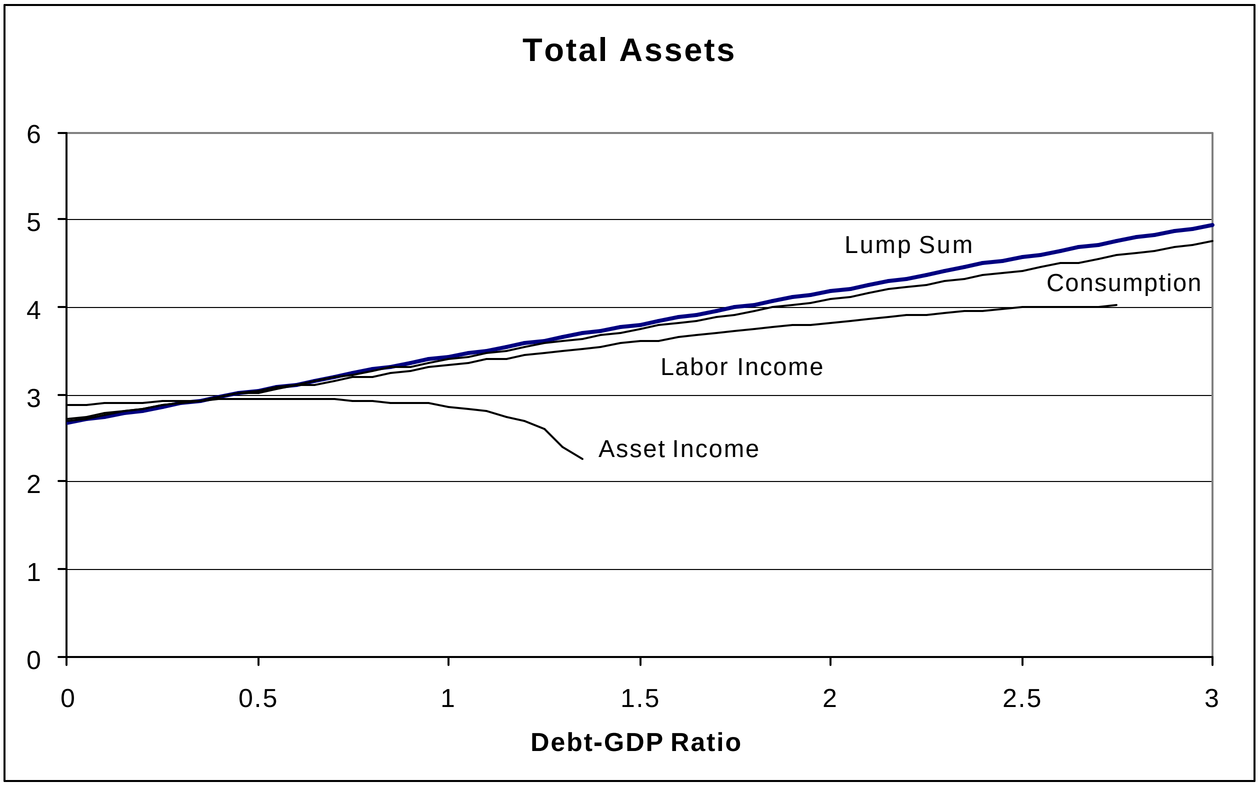


Figure 4
Non-Ricardian Experiments



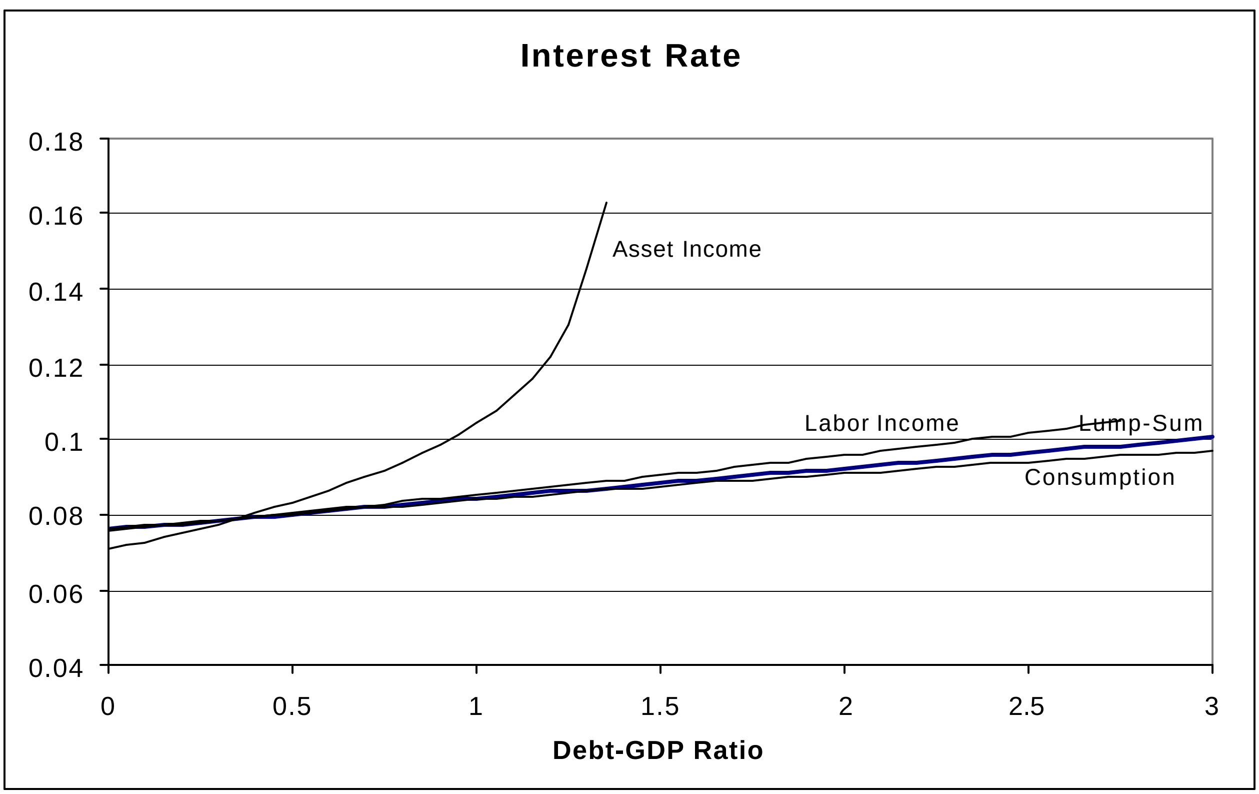
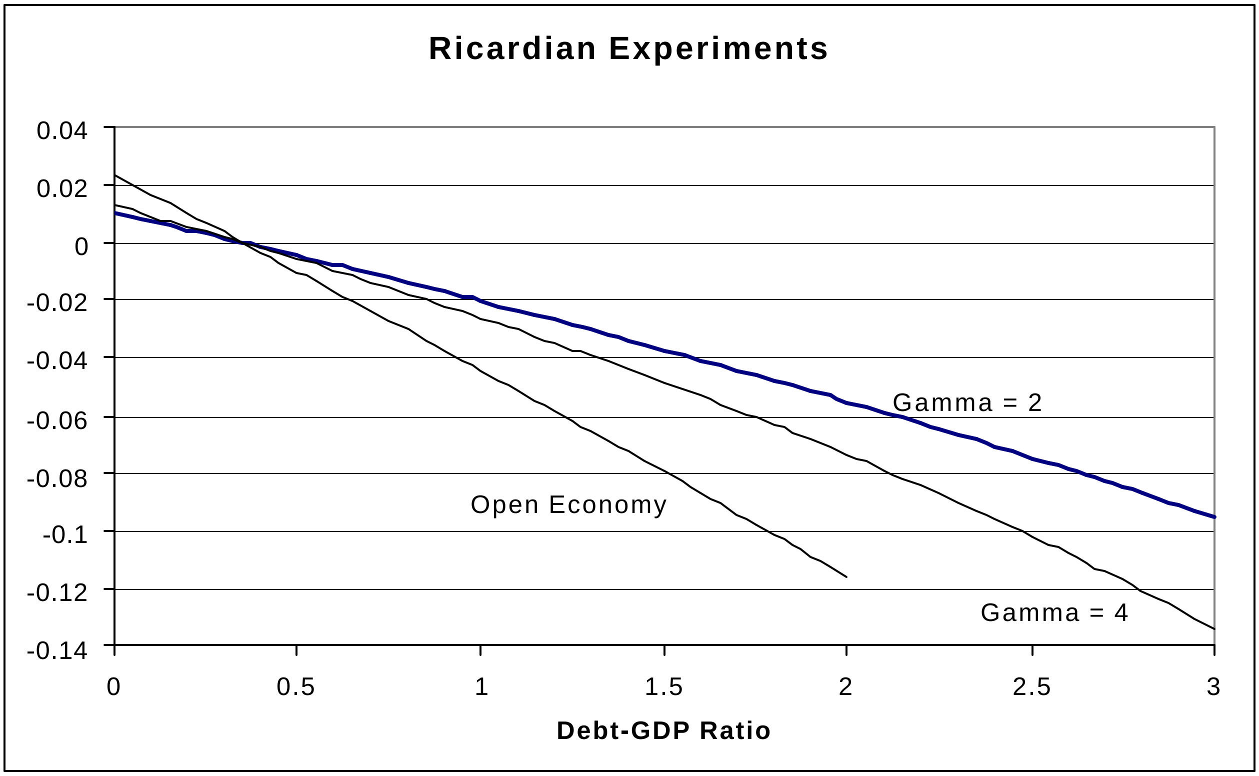


Figure 5
Steady-State Welfare
(Equivalent Variation as Fraction of Lifetime Consumption)



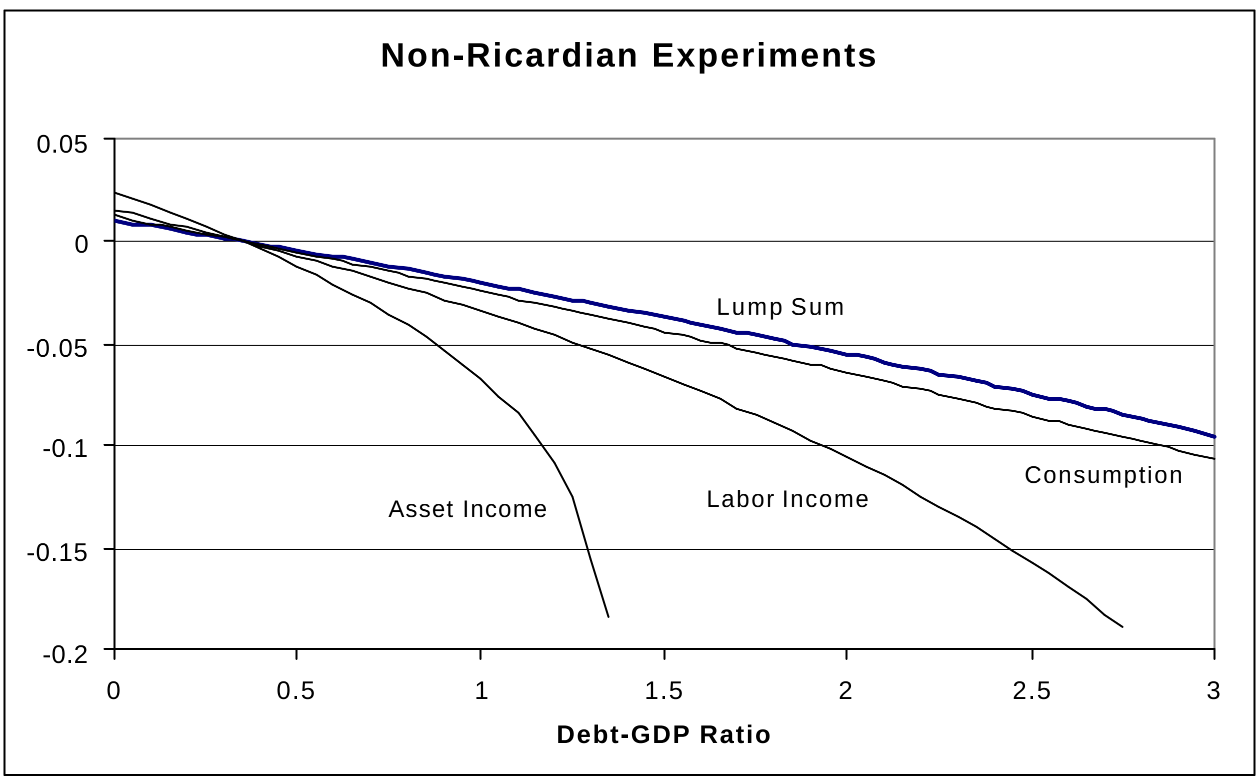


Figure 6
Temporary Debt Increase

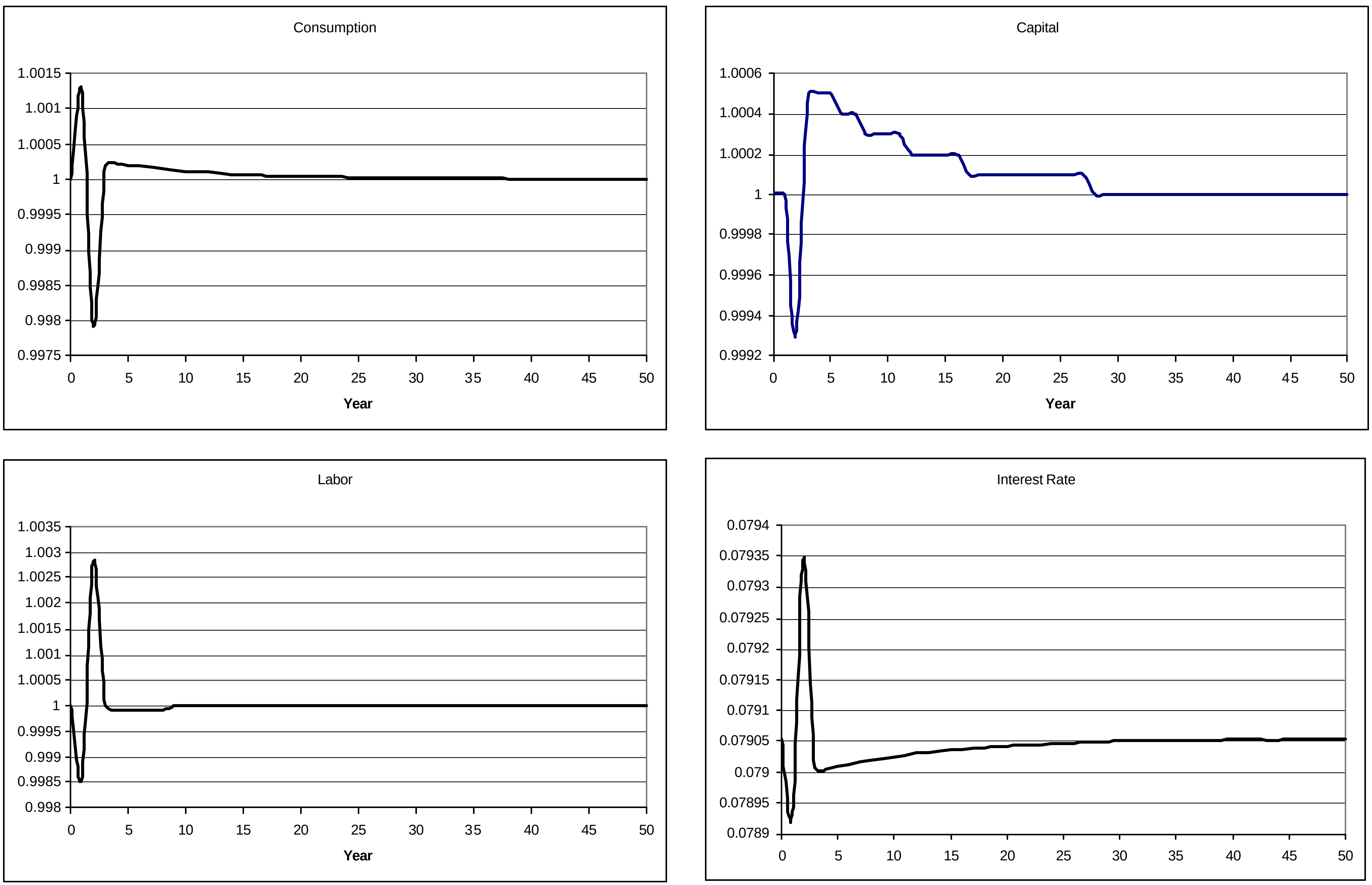


Figure 7
Permanent Debt Increase

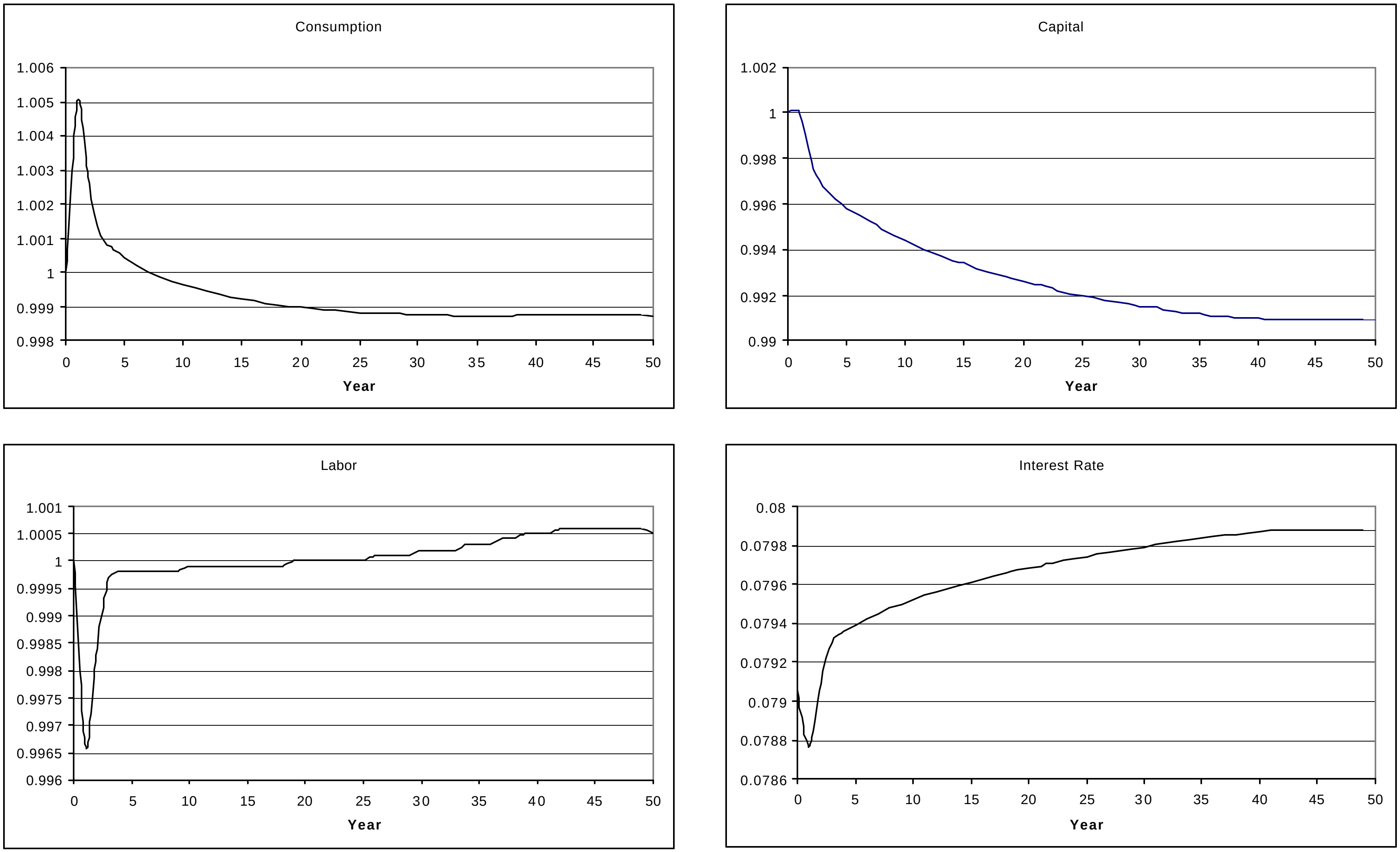


Figure 8
World War II