Implications of General and Specific Productivity Growth in a Matching Model

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Abstract

In this paper, I explore the implications of incorporating long-run productivity growth into a labor market matching model. I allow the productivity growth to be of general or specific nature, and consider an environment in which risk-averse workers are matched with firms and engage in long-term employment contracts, to which the former cannot commit. I claim that such an environment gives rise to two new channels through which growth may affect unemployment. A quantitative analysis of the model shows that these two channels are able to generate the negative effects of growth on unemployment comparable to empirical estimates, while the traditional "capitalization effect" is negligible. The analysis also finds that while a greater specificity of productivity growth tends to decrease unemployment rate and lengthen workers' tenure, it makes these variables more responsive to growth and job destruction rates. These results may potentially explain certain labor market differences among US, Europe and Japan.

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1 Introduction

In recent years, there has been active research on the cyclical properties of unemployment and vacancies in the Mortensen-Pissarides search and matching framework[1]. Much of the research was motivated by Shimer (2005), who argued that the textbook search and matching model cannot generate, in response to shocks of plausible magnitude to labor productivity, the observed cyclical behaviors of unemployment and vacancies. In contrast to this "Shimer puzzle", the implications of long-term productivity growth in the Mortensen-Pissarides model have drawn much less attention of economists, despite the strong negative relationship between long-run productivity growth and unemployment found by several empirical literatures.

In this paper, I examine the implications of incorporating long-run productivity growth into a labor market matching model. I allow the productivity growth to be of general or specific[2] nature, and consider an environment in which risk-averse workers are matched with firms and engage in long-term employment contracts, to which the former cannot commit. I then analyze the optimal contract as well as the stationary equilibrium, and claim that such an environment gives rise to two new channels through which growth may affect unemployment.

A quantitative analysis of the model shows that these two channels are able to generate the negative effects of growth on unemployment that are comparable to empirical estimates, while the traditional "capitalization effect" is negligible. The analysis also finds that, while a greater specificity of productivity growth tends to decrease unemployment rate and lengthen workers' tenure, it makes these variables more responsive to changes in growth and job destruction rates. These features of the model may potentially explain longer tenure and historically lower levels of unemployment rate in Europe and Japan compared to the U.S., as well as their labor market experiences, such as the surge of unemployment rate in Europe and changes to "lifetime employment" in Japan.

The main theoretical contributions of the paper lies in introducing the concept of specific productivity growth, as well as in shedding new light on the role played by workers' specific productivity. Specific productivity growth in the model resembles the widely used notion of specific human capital, in the sense that they both result in partial destruction of a worker's productivity following the termination of a match. It is novel, however, in that the amount of productivity subject to destruction depends on the rate of productivity

[1]Such work includes, for example, Hall (2005), Farmer (2005), and Hagedorn and Manovskii (2005).

[2]Throughout the paper, I use the term "specific" to imply match or firm specific, as opposed to industry or occupation specific.

growth. Now, such match-specific component of the workers' productivity turns out to have a greater role in this paper than in a more conventional setup, due to long-term contracts and limited commitment. More precisely, typical literatures impose a trade-off between general and specific human capital by assuming that the latter is more efficient within a given firm, while it is by definition less so outside of that firm. In my model, instead the key trade-off arises from the risk-sharing considerations between firms and workers. Since specific productivity growth relaxes the workers' commitment problem by reducing the workers' value of outside option, it is not necessarily inferior to general productivity growth, even when both are equally efficient within a given match.

2 Related literature

Besides the Mortensen-Pissarides model (Mortensen and Pissarides (1994), Pissarides (2000)), this paper is related to three large strands of literature. The first is the literature on dynamic labor contract under limited commitment. In particular, part of this paper follows the analysis of Chari, Restuccia, and Urrutia (2005), who develop a model of dynamic labor contract with one-sided lack of commitment and explore how firing costs affect the firms' investments in the training of workers. However, they do not pursue the implications of productivity growth, and since human capital is general in their model it does not affect the workers' commitment problem as in mine.

The second is the literature on growth and unemployment. Aghion and Howitt (1994) discuss the two channels through which growth may affect unemployment in the search and matching framework. They call these channels "capitalization effect" and "creative destruction effect", and claim that a faster growth reduces unemployment through the first channel, but increases it through the second. Several empirical studies, however, find a strong negative impact of long-run productivity growth on unemployment. Using the data for 20 developed countries, Blanchard and Wolfers (2000) estimate that a 1% drop in TFP growth rate leads to an increase in unemployment rate of $0.25 \sim 0.75\%$. Pissarides and Vallanti (2005) use the data for a similar group of countries and estimate that a 1% drop in TFP growth rate increases the unemployment rate by roughly 1.5% in the U.S., and 1.3% in Europe. Further, they develop a matching model with TFP growth under embodied and disembodied technology, and evaluate the quantitative effects of capitalization and creative destruction effects. Their analysis finds that the creative destruction effect dominates the capitalization effect under plausible parameters, so that the technology has to be totally disembodied for the model to yield a negative effect of growth on unemployment that they find in data. Now, in an economy with embodied technology, the technology used in an existing match gets more and more obsolete each period compared to that in new matches. In a sense my model pushes their argument one step further, because under specific productivity growth, a worker's productivity in the current match becomes increasingly more efficient compared to that outside of the current match.

The third is the literature on general and specific human capital, which dates back to Becker (1964). Wasmer (2006) incorporates the workers' choice of general and specific human capital in a matching model and examines under what condition one is preferred to the other, but in his model human capital is not related to growth. Ljungqvist and Sargent (1998, 2004, 2005a,b, 2006) emphasize the role of increased "turbulence", or the probability of skill depreciation when workers get unemployed, in the rise of unemployment rate in Europe. Den Haan, Haefke, and Ramey (2001) also explore the implications of this "turbulence", and find that the degree of turbulence largely influences the effects of growth on unemployment rate. The notion of "turbulence" in these papers is similar to the loss of match specific productivity in my model, but again the size of the "turbulence" is unrelated to growth. Regarding the cross-country differences in the degree of specificity of human capital, Hashimoto and Raisian (1985) find that the returns to firm-specific tenure is higher in Japan than in the U.S., which is consistent with the larger importance of specific human capital in the former. Wasmer (2006) presumes in his discussions that skills are more job-specific in Europe than in the U.S.

3 Model

3.1 General Environment

Time is discrete and goes to infinity, and there is a single perishable good. The economy is populated by continuum of risk-averse workers, whose mass in the labor force equals one, and continuum of firms. A worker stays in the labor market for $\bar{T}$ periods[3]. I let t denote the period in the economy (calendar date) and τ the "age" of an individual worker. A worker is born and begins to be matched with firms at date $\tau = -1$, enters the labor force and starts consuming at $\tau = 0$, and retires at $\tau = \bar{T}$. After retirement, he receives retirement benefits B_t^R each period[4], whose total discounted utility at the date of

[3]Since T_1 and T_2 explained below affect the productivity of a match, they are part of the state variables in the recursive problem that I will set up. Thus, unless I impose an exogenous upper bound $\bar{T}$ the state spaces are unbounded, making the problem computationally infeasible.

[4]Since the amount of retirement benefits is independent of the employment history in this model, theoretically it does not affect any of the results. It does, however, alter the level of workers' expected

retirement is denoted as V_t^R. Each period, new workers with mass $1/\bar{T}$ enter the labor force, so that the mass of workers in the labor force is constant at one. A firm goes out of business each period with probability γ. Firms can borrow or lend at a constant interest rate r, and hence discount their future profits with this rate. In contrast, workers can neither borrow nor lend, and there are no other financial markets.

Workers' period utility is described by a CRRA utility function over consumption, and workers discount future utility with β, so that their lifetime utility is given by

$$E_{-1}\sum_{\tau=0}^{\infty}\beta^{\tau}u(C_{\tau}) = E_{-1}\sum_{\tau=0}^{\infty}\beta^{\tau}\frac{C_{\tau}^{1-\sigma}}{1-\sigma} \tag{1}$$

Production is done by matches of a worker and a firm. Each worker is characterized by his work experience, which consists of two elements; the number of periods from his entrance into the labor force T_1, and his tenure T_2, or equivalently the number periods he has been in the current match. By definition, $T_1 = \tau$ for $\tau \in \{0, ..., \bar{T}-1\}$, and $T_2 \in \{0, ..., T_1\}$ for each T_1. In period t, a match between a firm and a worker produces

$$Y_t(A, T_1, T_2) = A\Psi_t(T_1, T_2) \equiv A(1+g)^t(1+g)^{(1-\alpha_1)(\alpha_2 T_2 - T_1)} \tag{2}$$

, where $A \in [\underline{A}, \bar{A}]$ is the idiosyncratic productivity specific to the match, and $\Psi_t(T_1, T_2)$ is the worker's productivity. All new matches start with the highest idiosyncratic productivity $\bar{A}$, and subsequently A remains constant with probability λ, and a new A is drawn from an *i.i.d.* distribution with probability $1-\lambda$. I assume that this distribution has a CDF $G(A)$, which is absolutely continuous with corresponding PDF $g(A)$. $\Psi_t(T_1, T_2)$ is a deterministic function of T_1 and T_2, where α_1 and α_2 are parameters common to all matches in the economy that govern how one's work experience (T_1, T_2) affects his productivity Ψ_t. $\alpha_1 \in [0, 1]$ represents the generality of technology. $\alpha_1 = 1$ corresponds to entirely general productivity growth, in which (T_1, T_2) are irrelevant. $\alpha_1 = 0$ is the opposite case, in which workers' characteristics have the maximum effect on productivity. In contrast, α_2, which I assume to satisfy $\alpha_2 \geq 1$, represents the impact of tenure on productivity. These parameter assumptions imply that $\Psi_t(T_1, T_2)$ is increasing in the worker's tenure T_2 and is decreasing in his years in labor force T_1, the latter reflecting the idea that new workers enter the labor force with better education.

The mass of new matches formed each period is described by a matching function $m(u, s)$, where u is the mass of unemployed workers, and s is that of vacancies posted by firms. I assume this matching function exhibits CRS, is increasing in both arguments,

discounted utility, and may cause scaling problems in the numerical analysis depending on the values assigned.

and satisfies $m(0,\cdot) = m(\cdot,0) = 0$. Posting a vacancy incurs the firm costs Φ_t, and a vacancy lasts for one period.

3.2 Productivity Growth

I summarize here some of the relationships that result from the functional form of $\Psi_t(T_1, T_2)$, combined with the assumptions $\alpha_1 \in [0,1]$ and $\alpha_2 \geq 1$.

$$\frac{\Psi_t(T_1+1, T_2+1)}{\Psi_t(T_1, T_2)} = (1+g)^{(1-\alpha_1)(\alpha_2-1)} \geq 1 \tag{3}$$

$$\frac{\Psi_{t+1}(T_1, T_2)}{\Psi_t(T_1, T_2)} = 1+g \tag{4}$$

$$\frac{\Psi_{t+1}(T_1+1, T_2+1)}{\Psi_t(T_1, T_2)} = (1+g)^{1+(1-\alpha_1)(\alpha_2-1)} \geq 1+g \tag{5}$$

$$\frac{\Psi_{t+1}(T_1+1, 0)}{\Psi_t(T_1, 0)} = (1+g)^{\alpha_1} \leq 1+g \tag{6}$$

(3) suggests that among the workers with the same $T_1 - T_2$, for example the workers who have been matched with the same firm since their first period in the labor force, senior workers are at least as productive as junior ones. (4) tells us that the productivity of a worker with given (T_1, T_2) grows at $1+g$, which implies that the economy grows at $1+g$ in a stationary equilibrium, in which the distribution of workers over (A, T_1, T_2) is constant. (5) implies that the productivity of a worker staying in the same match grows at a speed weakly larger than $1+g$. (5) and (6) combined indicate that unless $\alpha_1 = 1$, the growth rate of a worker's productivity outside the current match is strictly smaller than that in the current match. Therefore, under specific productivity growth ($\alpha_1 < 1$), there is in general a wedge between a worker's productivity within and outside the current match.

3.3 Labor Contract

I now describe the labor contract between a firm and a worker. What I refer to below as the "firm's value" is its expected present value of profits, and the "worker's value" is his expected present value of utility. When a firm and a worker with work experience (T_1, T_2) are matched at period t, they write a contract that specifies the stream of wages[5] $\{C_s(\{A_z\}_{z=t+1}^{s}, T_{1,t})\}_{s=t+1}^{\bar{T}-1-T_1+t}$, contingent on the history of idiosyncratic productivity A

[5]The stream of wage starts from $t+1$, when the match starts producing, and continues until the worker's last period in the labor force $\bar{T}-1$, which corresponds to $\bar{T}-1-T_1+t$ in the calendar time.

and the value of T_1 at date t. Notice that the worker's productivity Ψ_t is a deterministic function of (T_1, T_2), and since the values of (T_1, T_2) increase by 1 unit each period with $T_2 = 0$ at the beginning of the match, the sequence of Ψ_t can be summarized by the value of T_1 at date t. Under the assumptions that the good is nonstorable and workers have no access to financial markets, the stream of wages also equals the stream of the worker's consumption.

Firms can commit to the contract. However, a firm has probability γ of going out of business each period, which I refer to as an exogenous separation. Also, a bad draw may lead to an endogenous separation, or equivalently dismissal of the worker, which incurs the firm firing costs F_t. I assume that an endogenous separation occurs if and only if (i) A has changed, and (ii) the following condition holds[6].

$$\Pi_t^{e=0}(A, V_t^{un}(T_1), T_1, T_2)) < -F_t \tag{7}$$

Here, the LHS denotes the firm's value of continuing ($e = 0$) the match at period t, when it is matched with a worker with work experience (T_1, T_2), the idiosyncratic productivity of the match is A, and has promised to provide the worker the value of unemployment $V_t^{un}(T_1)$, which equals the worker's outside option. So this condition implies that the match is unable to provide the firm its outside option value of $-F_t$, if the firm must provide the worker at least his outside option value $V_t^{un}(T_1)$. Thus the firm's value of the match $\Pi_t(A, V, T_1, T_2)$ is given by

$$\begin{aligned} \Pi_t(A, V, T_1, T_2) &= -F_t \qquad\qquad\qquad\quad, \quad \text{if } \Pi_t^{e=0}(A, V_t^{un}(T_1), T_1, T_2)) < -F_t \\ &= \Pi_t^{e=0}(A, V, T_1, T_2), \quad \text{if } \Pi_t^{e=0}(A, V_t^{un}(T_1), T_1, T_2)) \geq -F_t \end{aligned}$$

In contrast to firms, workers cannot commit to the contract. They can walk away from the current contract at any date, become unemployed, and search for a new job. When unemployed, workers receive unemployment benefits $B_t^{un}(T_1) = (1+g)^{(1-\alpha_1)(-T_1)} B_t^{un}(0)$, where $B_t^{un}(0)$ is the value of benefits for unemployed workers with $T_1 = 0$[7].

[6]As I claim in Proposition 3, $\pi^{e=0}(A, v^{un}(T_1), T_1, T_2)$ is increasing in A, and hence so is $\Pi^{e=0}(A, V_t^{un}(T_1), T_1, T_2)$. Then it is never optimal for the firm to choose an endogenous separation for the values of A that does not satisfy (7), because the firm can increase profit by sustaining the match and providing the worker $V_t^{un}(T_1)$. When the current promised value is high, however, from the ex-ante perspective it may be optimal for the firm to sustain the match even if (7) holds, so that in general the threshold productivity will also depend on the current promised value. Since such consideration largely complicates the analysis without altering the main message of the paper, I assume the separation rule in the text which leads to the threshold productivity that only depend on (T_1, T_2). One may consider this as a restriction on the kind of commitment the firms can make.

[7]This formulation, along with the assumption on growth of parameters below, imply that unemployment benefits are proportional to unemployed workers' (potential) productivity $\Psi_t(T_1, 0)$.

The optimal contract under this environment is the contract that maximizes the firm's value of a match, subject to the worker's participation constraint and the requirement of providing the worker the initial promised value $V_t^{new}(T_1)$, which depends on the value of T_1 at the beginning of the match[8]. $V_t^{new}(T_1)$ is determined by the Nash bargaining, so that it maximizes the Nash product of the firm's and worker's surplus from the match, subject to their participation constraints[9]. Formally,

$$V_t^{new}(T_1) = \arg\max_V \{\Pi_t(\bar{A}, V, T_1, 0)^\theta (V - V_t^{un}(T_1))^{1-\theta}\} \tag{8}$$
$$s.t.\ \Pi_t(\bar{A}, V, T_1, 0) \geq 0,\ V \geq V_t^{un}(T_1)$$

where θ is the firm's bargaining power.

In (8), $\Pi_t(A, V, T_1, T_2)$ is the firm's value at period t from a match whose idiosyncratic productivity is A, promised value to the worker is V, and the worker's work experience is (T_1, T_2). Note that $A = \bar{A}$ and $T_2 = 0$ for a new match. Since the firm and the worker's outside options are respectively 0 and $V_t^{un}(T_1)$, $\Pi_t(\bar{A}, V, T_1, 0)$ and $V - V_t^{un}(T_1)$ are respectively their surplus from the match. I focus on the case in which there always exists a value of V that satisfies (8), so that all new matches lead to employment relationships. Formally, this requires $\Pi_t(\bar{A}, V_t^{un}(T_1), T_1, 0) \geq 0$ for all t and T_1.

3.4 Recursive Optimal Contract

I assume that the cost of posting a vacancy Φ_t, unemployment benefits B_t^u, retirement benefits B_t^R, and firing costs F_t all grow at a constant factor $1+g$. I can then stationarize the firm's problem by detrending profit, output and wage by $(1+g)^t$, and the variables related to the worker's utility by $(1+g)^{(1-\sigma)t}$. This allows me to formulate a recursive optimal contract in detrended variables, which I denote by lower case letters.

In this recursive formulation, the state variables for a matched firm are idiosyncratic productivity A, promised value v, and the worker's work experience (T_1, T_2). The separation rule (7) becomes

$$\pi^{e=0}(A, v^{un}(T_1), T_1, T_2) < -f \tag{9}$$

Now, when the match continues, the firm chooses the current wage c and the worker's

[8] Suppose a firm and a worker are matched at date t, when the worker has spent T_1 periods in the labor market. Since the match starts at date $t+1$, worker's initial value is $V_{t+1}^{new}(T_1+1)$.

[9] Note that the firm's participation constraint need to be satisfied only at the beginning of the match, due to the assumption that the firm can commit to the contract.

state-contingent promised utility next period $v'(A')$, to solve the following problem.

$$\pi^{e=0}(A, v, T_1, T_2) = \max_{c, v'(A')} \{A\psi(T_1, T_2) - c + (1-\gamma)(1+g)/(1+r)\hat{\pi}(A, v'(A'), T_1+1, T_2+1)\} \tag{10}$$

$$s.t. \quad u(c) + \beta(1+g)^{1-\sigma}\hat{u}(A, v'(A'), T_1+1, T_2+1) = v \tag{11}$$

$$v'(A') \geq v^{un}(T_1+1), \quad \forall A' \in [\underline{A}, \bar{A}] \tag{12}$$

$$c \geq 0$$

(11) is the promise-keeping constraint, and (12) are the participation constraints. $\hat{\pi}$ and $\hat{u}$ are respectively the firm and the worker's continuation value of the match, described below.

First, let us define the function $g^e(A, v, T_1, T_2)$[10], which indicates whether or not (9), a necessary condition for an endogenous separation, is satisfied;

$$\begin{aligned} g^e(A, v, T_1, T_2) &= 0 \text{ if } \pi^{e=0}(A, v^{un}(T_1), T_1, T_2) \geq -f \\ &= 1 \text{ if } \pi^{e=0}(A, v^{un}(T_1), T_1, T_2) < -f \end{aligned} \tag{13}$$

Then, for $T_1 \in [0, \bar{T}-2]$,

$$\begin{aligned} &\hat{\pi}(A, v'(A'), T_1+1, T_2+1) \equiv \lambda\pi^{e=0}(A, v'(A), T_1+1, T_2+1) \\ &+ (1-\lambda)\Big(\int_{\underline{A}}^{\bar{A}} \big[(1 - g^e(A', v'(A'), T_1+1, T_2+1))\pi^{e=0}(A', v'(A'), T_1+1, T_2+1) \\ &+ g^e(A', v'(A'), T_1+1, T_2+1))(-f)\big] dG(A')\Big) \end{aligned} \tag{14}$$

and

$$\begin{aligned} &\hat{u}(A, v'(A'), T_1+1, T_2+1) \\ &\equiv (1-\gamma)\Big[\lambda v'(A) + (1-\lambda)\int_{\underline{A}}^{\bar{A}} (1 - g^e(A', v'(A'), T_1+1, T_2+1))v'(A')dG(A')\Big] \\ &+ \Big[(1-\gamma)(1-\lambda)\int_{\underline{A}}^{\bar{A}} g^e(A', v'(A'), T_1+1, T_2+1)dG(A') + \gamma\Big]v^{un}(T_1+1) \end{aligned} \tag{15}$$

For $T_1 \in \bar{T}-1$,

$$\hat{\pi}(A, v'(A'), T_1+1, T_2+1) = 0 \tag{16}$$

and

$$\hat{u}(A, v'(A'), T_1+1, T_2+1) = v^R \tag{17}$$

where v^R is the value of retirement.

[10] As is obvious from the definition, $g^e(A, v, T_1, T_2)$ does not depend on v. I keep v as an argument just for the sake of notational consistency with other functions.

3.5 Stationary Recursive Equilibrium

I now define a stationary recursive equilibrium of the model.

Definition 1 *A stationary recursive equilibrium is*

- *A list of functions*

$$\pi^{e=0}(A, v, T_1, T_2),\ \ g^e(A, v, T_1, T_2),\ \ g^c(A, v, T_1, T_2),\ \ g^{v'(A')}(A, v, T_1, T_2)$$

- $\bar{T}$ *vectors* v^{un}, v^{new} *and a scalar* v^{nb}

- *Probabilities* p *and* q*,*

- *Stationary distributions of unemployed workers* $\mu^{un}(T_1)$*, of employed workers* $\mu^{em}(A, v, T_1, T_2)$*, and of new born workers* μ^{nb}*,*
  for $T_1 \in \{0, ..., \bar{T} - 1\}$*,* $T_2 \in \{0, ..., T_1\}$ *such that:*

1. The value function $\pi^{e=0}(A, v, T_1, T_2)$ solves the Bellman equation (10), $g^e(A, v, T_1, T_2)$ is as defined in (13), and $g^c(A, v, T_1, T_2)$ and $g^{v'(A')}(A, v, T_1, T_2)$ are the optimal policy rules.

2. The value of an unemployed worker with T_1 is given by

$$v^{un}(T_1) = u(b^{un}(T_1)) + \beta(1+g)^{1-\sigma}[pv^{new}(T_1+1) + (1-p)v^{un}(T_1+1)] \qquad (18)$$
$$, T_1 \in \{0, ..., \bar{T} - 2\}$$
$$v^{un}(\bar{T} - 1) = u(b^{un}(T_1)) + \beta(1+g)^{1-\sigma}v^R$$

3. The value of a new born worker is given by

$$v^{nb} = \beta(1+g)^{1-\sigma}[pv^{new}(0) + (1-p)v^{un}(0)]$$

4. The value of a new worker with T_1 is determined by the following Nash bargaining problem[11]

$$v^{new}(T_1) = \arg\max_v \{\pi(\bar{A}, v, T_1, 0)^{\theta}(v - v^{un}(T_1))^{1-\theta}\} \qquad (19)$$
$$s.t.\ \pi(\bar{A}, v, T_1, 0) \geq 0,\ \ v \geq v^{un}(T_1)$$

[11] Under the setup of the paper, this condition can be shown to be equivalent to (8).

5. A zero profit condition for posting a vacancy holds:

$$\phi = q\frac{1+g}{1+r}\frac{\mu^{nb}\pi^{new}(0) + \sum_{T_1=0}^{\bar{T}-2}\mu^{un}(T_1)\pi^{new}(T_1+1)}{\mu^{nb} + \sum_{T_1=0}^{\bar{T}-2}\mu^{un}(T_1)} \tag{20}$$

where $\pi^{new}(T_1)$ is the firm's value of a new match with a worker with T_1, defined as

$$\pi^{new}(T_1) \equiv \pi^{e=0}(\bar{A}, v^{new}(T_1), T_1, 0), \; T_1 \in \{0, ..., \bar{T}-1\}$$

6. The probabilities of finding a job and filling a vacancy are consistent with the matching function,

$$p = \frac{m(u,s)}{u}, \qquad q = \frac{m(u,s)}{s}$$

7. The stationary distributions of workers μ^{nb}, $\mu^{un}(T_1)$ and $\mu^{em}(A, v, T_1, T_2)$ satisfy the following laws of motion.

(i) For $T_1 = 0$,

$$\mu^{un}(T_1) = (1-p)\mu^{nb}$$

and

$$\begin{aligned}\mu^{em}(A, v, T_1, T_2) &= p\mu^{nb} \;\text{ for } (A, v, T_1, T_2) = (\bar{A}, v^{new}(T_1), T_1, 0)\\ &= 0 \;\text{ otherwise}\end{aligned}$$

(ii) For $T_1 \in \{1, ..., \bar{T}-1\}$,

$$\mu^{un}(T_1) = (1-p)\mu^{un}(T_1-1) + \sum_{T_2=0}^{T_1}\Big\{\int_{\underline{A}}^{\bar{A}}\int_V [\gamma + (1-\gamma)(1-\lambda)\cdot \int_{\underline{A}}^{\bar{A}} g^e(A', v'(A'), T_1, T_2)dG(A')]\mu^{em}(A, v, T_1-1, T_2-1)dvdA\Big\}$$

Moreover, for $T_2 = 0$,

$$\begin{aligned}\mu^{em}(A, v, T_1, T_2) &= p\mu^{un}(T_1-1) \;\text{ for } (A, v) = (\bar{A}, v^{new}(T_1))\\ &= 0 \text{ otherwise}\end{aligned}$$

and for $T_2 \in \{1, ..., T_1\}$,

$$\begin{aligned}&\mu^{em}(A, v, T_1, T_2)\\ &= (1-\gamma)\Big\{\lambda\int_{\{\tilde{v}\in V | g^{v'(A)}(A, \tilde{v}, T_1-1, T_2-1) = v\}}\mu^{em}(A, \tilde{v}, T_1-1, T_2-1)d\tilde{v}\\ &+ (1-\lambda)g(A)(1-g^e(A, v, T_1, T_2))\int_{\underline{A}}^{\bar{A}}\int_{\{\tilde{v}\in V | g^{v'(A)}(\tilde{A}, \tilde{v}, T_1-1, T_2-1) = v\}}\mu^{em}(\tilde{A}, \tilde{v}, T_1-1, T_2-1)d\tilde{v}d\tilde{A}\Big\}\end{aligned}$$

4 Analytical Results from the Model

In this section and the next, I present the implications of the model. One drawback of moving away from the typical assumption in the search and matching framework of risk-neutral workers and period-by-period Nash bargaining on wages is that, it becomes much more difficult to obtain analytical results. Accordingly most of my results are based on a numerical analysis, but there are several implications of the model I am able to obtain analytically, which I present below.

4.1 Wage Rule

The first set of results concerns the path of wages in a match.

Proposition 2 *(1)The path of wage c is described as*

$$\begin{aligned} c' &= c[\beta(1+r)]^{1/\sigma}/(1+g) \quad \textit{if the participation constraint doesn't bind} \\ &\geq c[\beta(1+r)]^{1/\sigma}/(1+g) \quad \textit{if the participation constraint binds} \end{aligned}$$

(2) Conditional on the continuation of the match, the wage and the promised value next period is independent of the value of A next period.

Proof. See Appendix. ■

Proposition 2(1) implies that when the participation constraint doesn't bind, the non-detrended wage C drifts upwards or downwards with factor $[\beta(1+r)]^{1/\sigma}$; as a special case, C remains constant if $\beta(1+r) = 1$. The intuition is that, given a discount factor β and an interest rate r, such path of wages is least expensive for the firm to provide the worker a given promised value. But when the participation constraint binds, the firm must increase the wage and provide the worker the value of his outside option, which equals the value of unemployment $v^{un}(T_1)$.

4.2 Threshold Productivity

The separation rule (9) leads to a threshold property for the idiosyncratic productivity.

Proposition 3 *(1) For all (T_1, T_2), there exists a threshold for idiosyncratic productivity $A^*_{T_1,T_2}$, such that an endogenous separation occurs if and only if the newly drawn A satisfies $A < A^*_{T_1,T_2}$.*
(2) The threshold productivity $A^*_{T_1,T_2}$ is decreasing in f.

Proof. I first show that $\pi^{e=0}(A, v, T_1, T_2)$ is strictly increasing in A. To see this, first note that a larger A increases the current output and implies a larger probability of the same large A (and hence the large output) next period, due to the persistence term λ in the productivity process. On the other hand, the current value of A does not affect the worker's outside option, nor the probability of an endogenous separation next period since conditional on change, the value of A next period is independent of the current value of A. Thus a larger A only has positive effects on $\pi^{e=0}(A, v, T_1, T_2)$.

Therefore the threshold $A^*_{T_1,T_2}$ is the value that satisfy

$$\pi^{e=0}(A^*_{T_1,T_2}, v^{un}(T_1), T_1, T_2) = -f \tag{21}$$

, where $A^*_{T_1,T_2} = \underline{A}$ if $\pi^{e=0}(\underline{A}, v^{un}(T_1), T_1, T_2) > -f$ and $A^*_{T_1,T_2} = \bar{A}$ if $\pi^{e=0}(\bar{A}, v^{un}(T_1), T_1, T_2) < -f$, which proves (1). Moreover, (2) is immediate from the monotonicity of $\pi^{e=0}(A, v^{un}(T_1), T_1, T_2)$ in A. ■

Proposition 3(1) implies that the threshold property of the idiosyncratic productivity for an endogenous separation, which is standard in the literature, holds also in our model. Proposition 3(2) tells us that the role of firing costs in the model is to lower this threshold productivity and hence, all else equal, reduce endogenous separation.

Proposition 4 *$A^*_{T_1,T_2}$ is non-increasing in tenure T_2.*

Proof. Since T_2 is reset to 0 once a worker moves to a new match, his outside option is independent of T_2. Therefore, a larger value of T_2 weakly increases the output without affecting the worker's participation constraint. Thus $\pi^{e=0}(A, v^{un}(T_1), T_1, T_2)$ is weakly increasing in T_2 (strictly so if $\alpha_1 < 1$), and hence the proposition follows from the monotonicity of $\pi^{e=0}(A, v^{un}(T_1), T_1, T_2)$ in A. ■

Proposition 4 implies that for a given T_1, the hazard rate of separation falls in tenure. This negative relationship between the separation hazard and tenure is consistent with the empirical findings in the literature, for example Jovanovic (1979) and Pries (2004).

5 Numerical Analysis

5.1 Overview

I now move on to a numerical analysis of the model. After calibrating the model and solving it numerically[12], I first examine how the unemployment rate u and the workers'

[12] The computational procedure is described in Appendix B.

tenure, for different values of (α_1, α_2), vary with the growth rate g in the stationary equilibrium. I then perform a similar experiment by varying the exogenous separation rate γ.

5.2 Preliminary Calibration

In order to conduct the numerical analysis, I need to pick certain functional forms and parameter values. I follow the common practice in literature and set the firm's bargaining power θ to 0.5. I assume that when the value of A changes, a new A is drawn from a uniform distribution with support $[\underline{A}, \bar{A}]$, so that $G(A) = (A - \underline{A})/(\bar{A} - \underline{A})$ and $g(A) = 1/(\bar{A} - \underline{A})$ for $A \in [\underline{A}, \bar{A}]$. I then normalize $\underline{A}$ to 1 and set $\bar{A}$ to 3[13].

I let one model period correspond to one quarter. I follow Chari, Restuccia, and Urrutia (2005) and assume that a matching function is given by a Cobb-Douglas function $m(u, s) = Bu^{\varkappa}s^{1-\varkappa}$, with $\varkappa = 0.4$ and $B = 0.776$. I set the coefficient of risk aversion σ to 2, and the discount factor β to 0.99. I then choose r such that $\beta(1 + r) = 1$, which is a popular choice in a small open economy model. I let $\bar{T} = 160$, so that each worker stays in the labor force for 40 years. In the baseline model, as well as in all exercises in the current version of the paper, I set firing costs f to 0.

I calibrate the rest of the parameters by solving the model. I pick them so that in the baseline case of $g = 0.005$, $\alpha_1 = 1$ and $\gamma = 0.025$, the targets from the U.S. economy below are met. These values of g and α_1 correspond to 2% annual growth and completely general productivity growth. The choice of γ in this baseline case is based on the job destruction rate in the U.S. reported in Davis, Haltiwanger, and Schuh (1996)[14], but I later experiment with this parameter. The calibration targets 1. and 2. below are the same as in Chari, Restuccia, and Urrutia (2005), and 3. is the figure used in Shimer (2005).

1. u/s ratio of 1.3153, computed from the average duration of vacancy and unemployment

[13] These assumption on $\underline{A}$ and $\bar{A}$ imply a standard deviation for A of $2/\sqrt{12}$, which is within the range of values used in the literature. For example, this figure equals $1/\sqrt{12}$ in Ljungqvist and Sargent (2004), $1/\sqrt{12} \sim 7.07/\sqrt{12}$ in Den Haan, Haefke, and Ramey (2005), and $15/\sqrt{12}$ in Chari, Restuccia, and Urrutia (2005).

[14] To be precise, this is a figure computed from their annual job destruction rate. I use this figure, rather than the quarterly job destruction rate that the authors report, since they state that "the annual job flow measures provide a better indication of permanent job reallocation activity" (p18).

2. Unemployment rate of 5.4%, the average figure for 1990-2000 as documented in OECD (2005)

3. Value of unemployment benefits and home production of 40%

The procedure above leads to the following set of parameters.

Parameters Selected with no Empirical Counterpart		
θ	0.5	Firm's bargaining power
$\underline{A}$	1	Lowest level of match productivity
$\bar{A}$	3	Highest level of match productivity

Parameters Calibrated without Solving the Model		
$\varkappa$	0.4	Unemployment elasticity in matching function
B	0.7776	Scale in matching function
σ	2	Coefficient of risk aversion
β	0.99	Discount factor
r	$1/\beta - 1$	Interest rate
$\bar{T}$	160	Number of periods in labor force
f	0	Firing Costs

Parameters Selected by Solving the Model		
λ	0.81	Persistence in match productivity
ϕ	4.4484	Cost of posting a vacancy
b_0	0.865	Unemployment benefits for $T_1 = 0$

5.3 Results on Growth and Unemployment

5.3.1 Capitalization Effect and Our New Channels

I first discuss the most important part of my numerical analysis, which concerns the effect of growth on unemployment. The model predicts that a faster growth leads to a substantial fall in unemployment rate. Before presenting my own results, however, I first provide a theoretical discussion on how growth affects unemployment in the standard matching model, as well as in mine. As mentioned earlier, the traditional channel in a matching model that creates a negative relationship between growth and unemployment is the "capitalization effect", whose intuitive mechanism is as follows. In this literature, a firm first pays a fixed cost in order to post a vacancy. Then, once a match is created it generates surplus, part of which will be captured, or "capitalized" by the firm. Now, a faster productivity growth increases the joint surplus, and hence the firm's surplus, of a given match. On the other hand, the cost of vacancy is paid up-front, so it is unaffected by the changes in growth rates. Therefore, under a faster growth, posting a vacancy becomes more attractive, which leads to a rise in vacancy and a fall in unemployment.

However, there are two issues with this explanation based on the "capitalization effect". First, this effect turns out to be quantitatively small, if wages are determined by the Nash bargaining, which is a popular practice in the literature. Pissarides and Vallanti (2005) find that under the Nash bargaining and a plausible choice of parameters, a 1% drop in TFP growth rate only leads to an increase in unemployment rate of the order of 0.01%. This figure is nowhere close to the estimated effect of $0.25 \sim 0.75\%$ in Blanchard and Wolfers (2000), or let alone $1.3 \sim 1.5\%$ in Pissarides and Vallanti (2005) I mentioned earlier. The problem here is a bit similar to the one in Shimer puzzle, in the sense that under the Nash bargaining, wages respond too much to the changes in productivity growth, dampening the effect of growth on unemployment. Second, the capitalization effect hinges critically on the assumption that the interest rate is exogenously determined. As analyzed in Pissarides (2000), if the interest rate is endogenously determined this channel may have an opposite implication on the effect of growth on unemployment.

Now, in the model of this paper, there are two new channels through which a faster growth may reduce unemployment. The first channel is intertemporal consumption smoothing. In my model, workers have CRRA utility, and the wages are determined by long-term contracts. Then, a match between a firm and a worker has an additional potential margin of creating surplus, which is to smooth worker's consumption over time. It turns out that under reasonable parameters, a faster growth enlarges this margin, and hence the firm's value of the match[15]. Thus under a faster growth, posting a vacancy becomes more attractive, and so vacancy rises and unemployment falls. This channel is absent in the canonical Mortensen-Pissarides model, in which both agents have linear utility and wages are determined by the period-by-period Nash bargaining. The second channel is specific productivity growth, which makes a worker's productivity outside the current match grow slower than that in the current match. Under the period-by-period Nash bargaining on wages, this wedge limits the responsiveness of wages to changes in productivity growth rate and reinforces the capitalization effect. When wages are determined by long-term contracts as in this paper, this wedge has an analogous but even more interesting role, which is to relax the commitment problem of the worker. In either

[15]Under the assumption $\beta(1+r) = 1$, the result holds for any $g > 0$. In general there exists a threshold for g, above which a faster growth increases this margin.

case, a larger growth rate makes posting a vacancy more attractive for the firm, which results in an increase in vacancy and a fall in unemployment.

These channels, I believe, are interesting for two reasons. First, as I present below, they amplify the effect of growth on unemployment and generate results comparable to empirical estimates. Second, unlike the capitalization effect, the specific productivity growth channel is not directly dependent on the assumption of an exogenous interest rate. Thus, this channel is more likely to be robust to different assumptions on the determination of the interest rate, the functional forms of agents' preference, or the parameter values used.

5.3.2 Results on Growth and Unemployment

Figure 1 plots the unemployment rate in the model, against the (annualized) growth rate. The three series represent (1) $\alpha_1 = 1$, the baseline case of completely general productivity growth, and two cases with specific productivity growth, (2) $\alpha_1 = 0.8$, $\alpha_2 = 1$ and (3) $\alpha_1 = 0.6$, $\alpha_2 = 1$. Here I set the exogenous separation rate γ to 0.025. Notice that the unemployment rate takes the same value for all series when $g = 0$, because the production function (2) is independent of α_1 and α_2 in such case. As we observe in the figure, a higher growth rate reduces the unemployment rate in all three cases. Notice that when $\alpha_2 = 1$, specific productivity growth ($\alpha_1 < 1$) is an "inferior" technology compared to general productivity growth ($\alpha_1 = 1$), in the sense that the productivity of a worker with any (T_1, T_2) is smaller in the former. Nevertheless, such an inferior technology (cases (2) and (3)) may lower unemployment rate as we observe in Figure 1, by relaxing the commitment problem of the worker. Therefore, in my model, specific productivity growth may be preferred to general productivity growth, even when both technologies are equally productive within a given match.

Having said that, probably a more realistic case is $\alpha_2 > 1$, which implies that conditional on $T_1 - T_2$, older workers are more productive. This means, for example, workers who entered the labor force and started working last period have a higher productivity than those who join the labor force and starts working this period. In Figure 2, I again plot the unemployment rate against the growth rate. This time there are four series; the baseline case of $\alpha_1 = 1$, and 3 series with $\alpha_1 = 0.9$ and $\alpha_2 = \{1, 3, 5\}$. $(\alpha_1, \alpha_2) = (0.9, 5)$ corresponds to the case in which, under 2% annual growth, a worker who has spent his entire career in the same match is approximately 50% more productive than the worker with the same age and zero tenure. This appears to be a mild assumption on the effect of tenure on productivity. Figure 2 shows that adding in such positive tenure effect on productivity further amplifies the effect of growth on unemployment. When

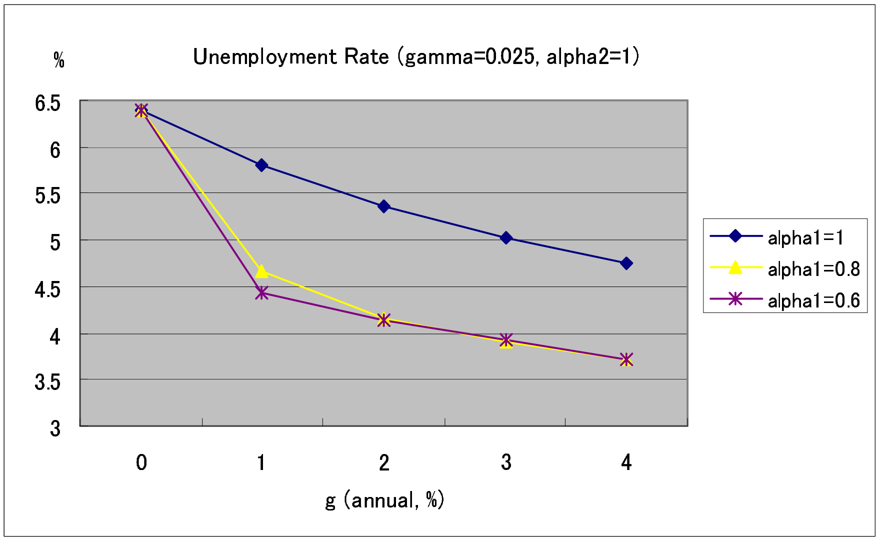

Figure 1: Growth and Unemployment under Different α_1

$(\alpha_1, \alpha_2) = (0.9, 5)$, a fall in growth rate from 4% to 0% increases the unemployment rate by 3.5%. Although a direct comparison requires caution due to differences in the basic environment of the model, this result is of a magnitude comparable to the estimates in Blanchard and Wolfers (2000) and Pissarides and Vallanti (2005) reported earlier.

5.4 Model's Implications on Tenure

Next I present the model's implications on workers' tenure. The main finding is that a higher growth rate tends to lengthen workers' tenure, especially under specific productivity growth.

Figure 3 and 4 respectively plot the tenure distribution in the U.S. and Japan, along with that from the model. The data for both countries are taken from OECD (1997), Chapter 5, and the parameters used for the model are $\alpha_1 = 1$, $g = 0.005$, $\gamma = 0.025$ for the U.S., and $\alpha_1 = 0.9$, $\alpha_2 = 5$, $g = 0.01$, $\gamma = 0.02$ for Japan[16]. These values are meant to capture the features of the economy of the U.S. and pre-1990s Japan. While the tenure distributions predicted by the model do not match the data for very short tenure, overall they fit the empirical tenure distributions fairly well. The plots from the data indicate that there are more workers with long tenure in Japan, which is replicated by the model.

[16] The annual job destruction rate for the U.S. and Japan, reported respectively in Davis, Haltiwanger, and Schuh (1996) and Higuchi (2001), leads to the quarterly job destruction rate of roughly 2.5% and 2%.

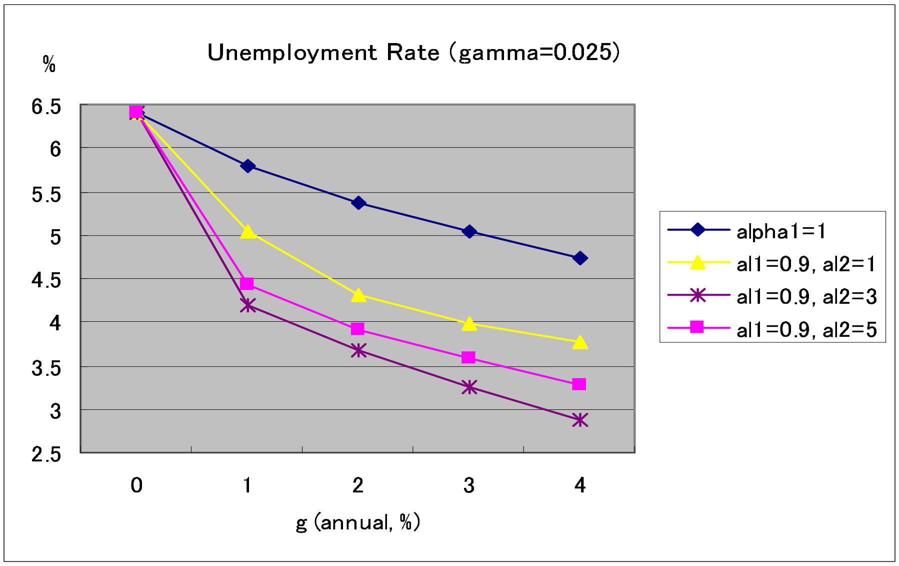

Figure 2: Growth and Unemployment with Positive Tenure Effects

Figure 5 and 6 compare the cumulative tenure distributions, again in the U.S. and Japan, with those computed from the model. The distributions from the model underestimate the fraction of workers with long tenure especially for Japan, but they reasonably replicate the empirical tenure distributions.

Next I discuss how the growth rate affects workers' tenure in the model. Figure 7 and 8 respectively plot, for $\gamma = 0.03$ and $\gamma = 0.015$, the median tenure of workers against the growth rate. In each figure, there are three series which correspond to $\alpha_1 = \{1, 0.9, 0.7\}$, with $\alpha_2 = 1$ for all cases.

The findings from these two figures are as follows. First, for all series, the median tenure rises as the growth rate rises. Second, this rise in tenure is larger for lower values of α_1, that is, when the productivity growth is more specific. Third, the median tenure is much more responsive to the growth rates when γ, the exogenous separation rate, is small. The results turn out to be quite similar when I plot the average tenure instead of the median tenure.

Figure 9 further examines the impact of γ on tenure. Here, I plot three series with the same values of (α_1, α_2) as in Figure 7 and 8. But this time, I fix g to 0.005, or equivalently to 2% annual growth, and plot the median tenure against γ (expressed in percentage). Figure 9 shows that greater the specificity of productivity growth (i.e. lower the α_1), more sensitive the median tenure is to the changes in exogenous separation rate.

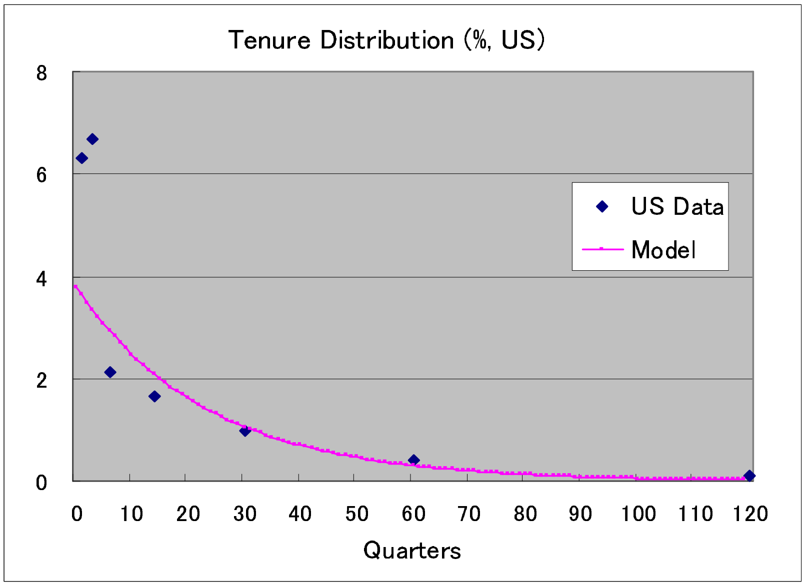

Figure 3: Tenure Distribution (U.S.)

The intuition behind these results is as follows. A higher growth rate increases the firm's value of the match through the intertemporal consumption smoothing channel. A greater specificity of productivity growth relaxes the commitment problem by increasing the amount of a worker's productivity that will be lost when the worker exits the current match, and further increases the firm's value of the match. Notice that the impact on the commitment problem is greater when the growth rate is higher, because the wedge between a worker's productivity within and outside the current match will be larger in such environment. As we observe from (9), the increased value of the firm reduces the threshold productivity $A^*_{T_1,T_2}$ and hence endogenous separations, which in turn result in a longer tenure. On the other hand, a larger value of γ directly increases the forced termination of matches and shortens the length of matches. This effect is amplified under specific productivity growth, because under this technology a long tenure results in a large wedge between a worker's productivity within and outside the match, which enlarges the surplus from the match. Conversely, a shorter average tenure dampens the benefits of specific productivity, and shortens the tenure even more.

5.5 Responsiveness to Growth and Exogenous Separation Rates

The results above indicated that the specific productivity growth tends to lower the unemployment rate. It is worth emphasizing, however, that a greater specificity of

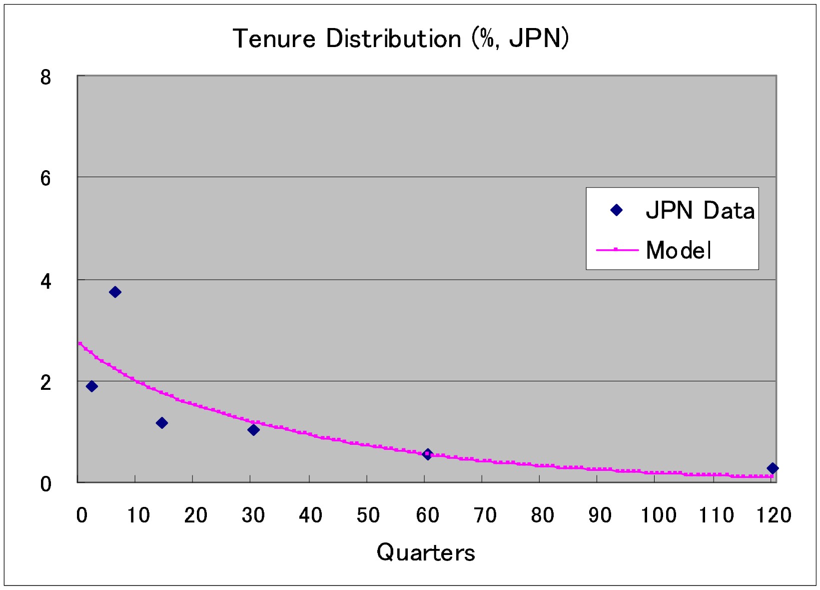

Figure 4: Tenure Distribution (Japan)

productivity growth and a larger tenure effect on productivity, characterized by a lower α_1 and a higher α_2, turn out to make the unemployment rate and workers' tenure more responsive to certain exogenous shocks, namely changes in the growth rate g and the exogenous separation rate γ.

That the unemployment rate and the median tenure are more responsive to changes in g under specific productivity growth can be observed from Figures 1, 2, 7 and 8. Since the values of unemployment rates and median tenure are common to all (α_1, α_2) at $g = 0$, smaller values of former and larger values of latter at $g > 0$ imply larger sensitivity to g under specific productivity growth. Therefore, an economy that enjoys a low unemployment rate, thanks to a low α_1 and a high α_2, may be hit by a large rise in unemployment following a growth slowdown.

The economy in which specific productivity is important also turns out to be more sensitive to changes in the exogenous separation rate γ. The analysis in the previous section already revealed this for the median tenure. I present below that in such economy, the unemployment rate and output are also more sensitive to changes in γ.

Figure 10 plots the unemployment rate against γ under $g = 0.005$, or 2% annual growth. The three series are (1) $\alpha_1 = 1$, plus two series with $\alpha_1 = 0.8$, which respectively use (2) $\alpha_2 = 1$, and (3) $\alpha_2 = 2.5$. As before, $(\alpha_1, \alpha_2) = (0.8, 2.5)$ implies that under 2% annual growth, a worker with 40 years of tenure is 50% more productive compared to a

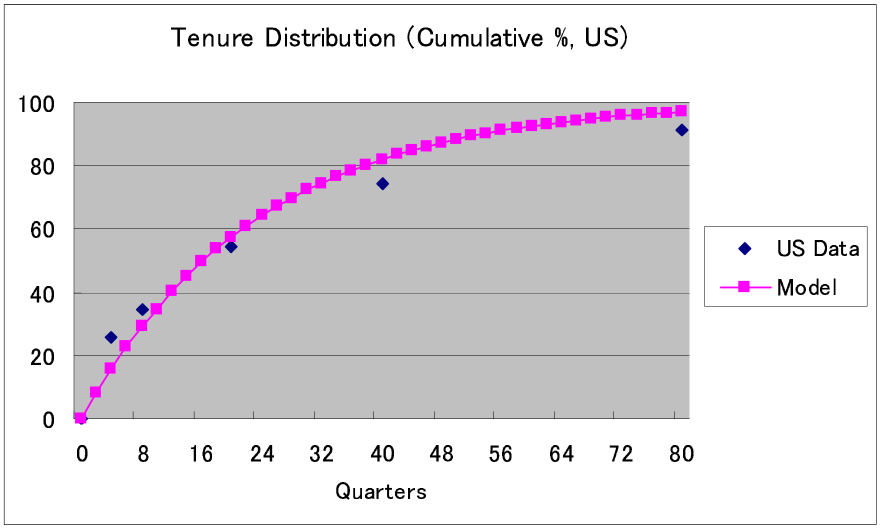

Figure 5: Cumulative Tenure Distribution (U.S.)

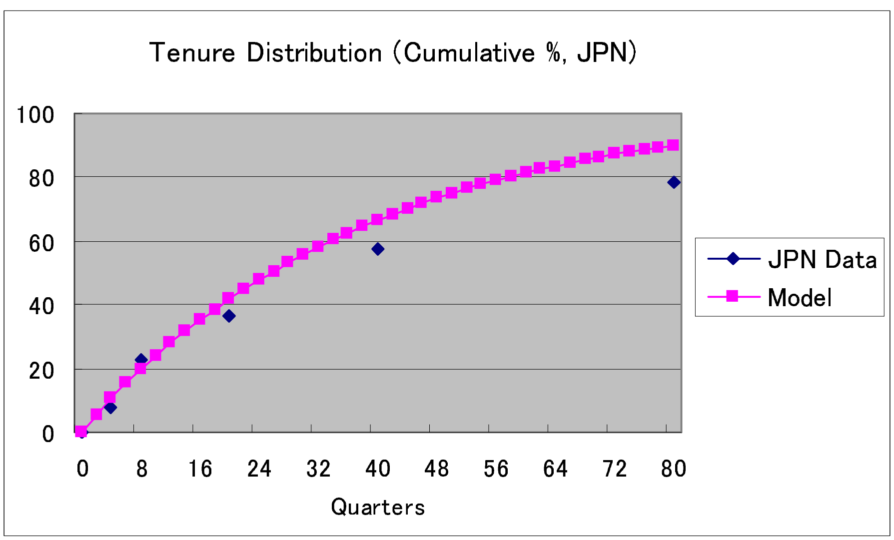

Figure 6: Cumulative Tenure Distribution (Japan)

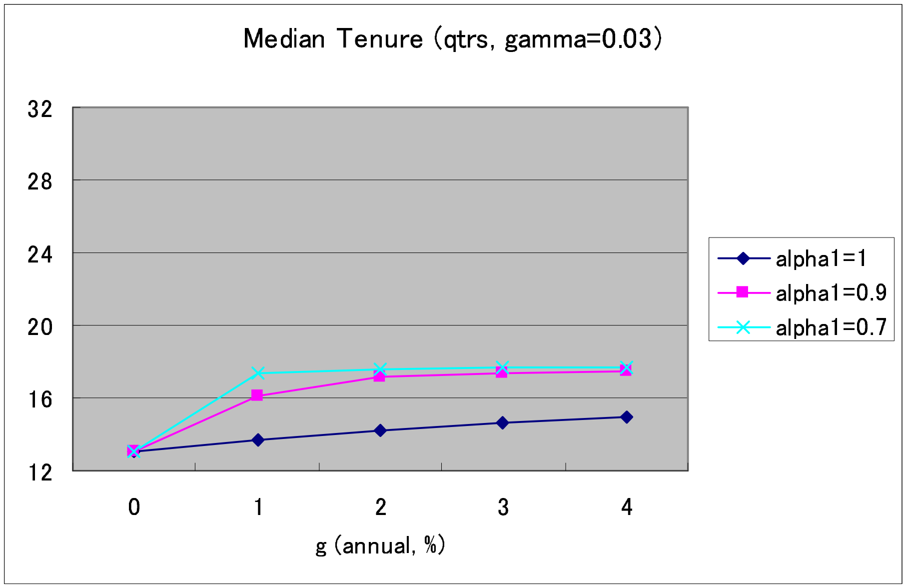

Figure 7: Growth and Median Tenure ($\gamma = 0.03$)

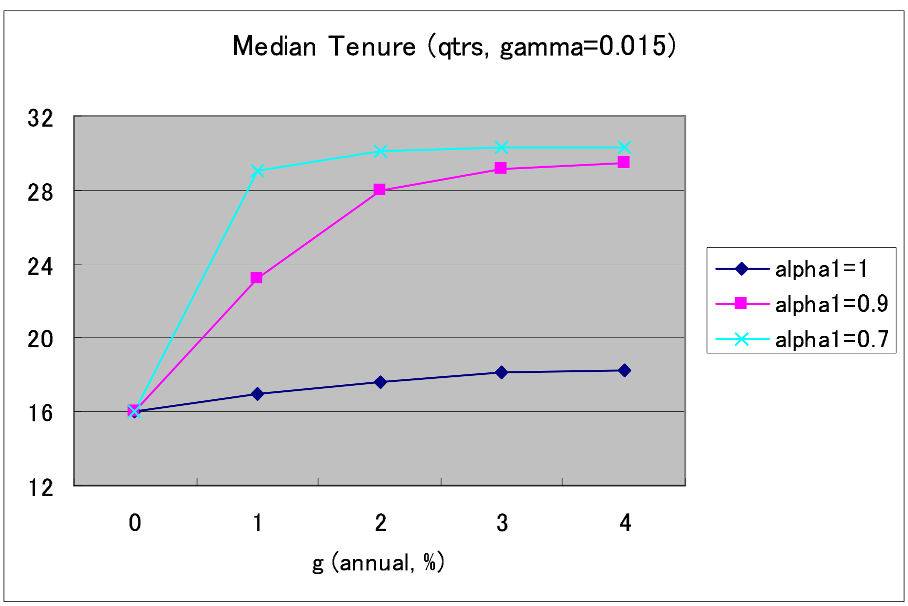

Figure 8: Growth and Median Tenure ($\gamma = 0.015$)

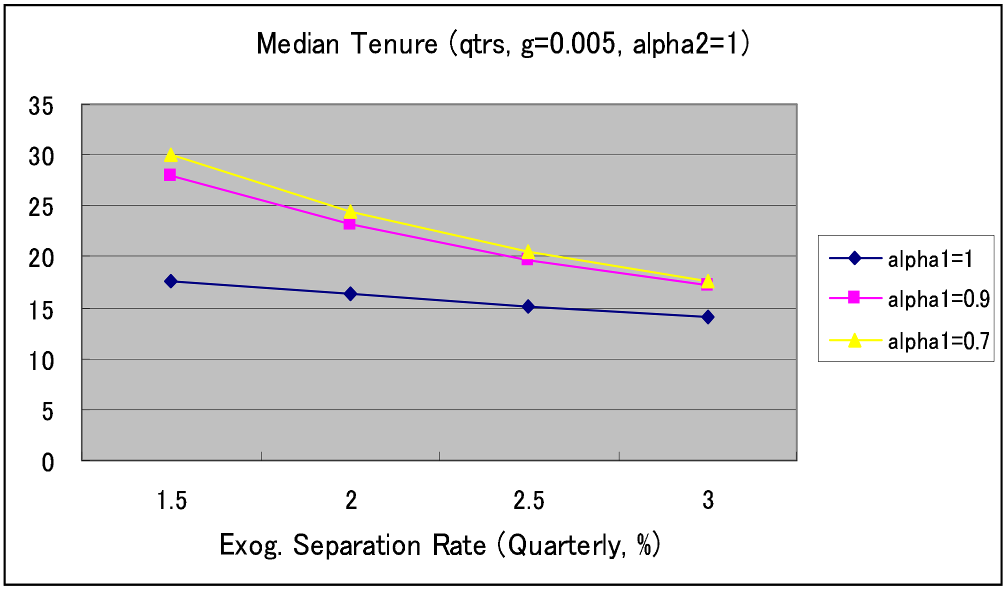

Figure 9: Exogenous Separation Rate and Tenure

worker with the same age and zero tenure. Figure 10 again shows that the unemployment rate is lower under specific productivity growth, and that the unemployment rates rise with the exogenous separation rate, which is a natural outcome. What is important here is that the two curves for $\alpha_1 = 0.8$ have steeper slopes than the one for $\alpha_1 = 1$; as the exogenous separation rate rises, the unemployment rate rises more quickly under specific productivity growth.

Figure 11 plots the aggregate output, net of unemployment benefits, against γ. To facilitate the comparison, I normalize to 1 the value of each series at $\gamma = 0.015$. The parameters used in the three series are the same as in Figure 10. The figure shows that smaller the α_1 and larger the α_2, the larger the percentage fall in output when γ rises. In other words, greater the specificity of productivity growth as well as the positive effect of tenure of productivity, the more sensitive is the output to the changes in the exogenous separation rate, which may, for example, result from increased bankruptcies of firms.

The intuition for the responsiveness of the unemployment rate and output, with respect to the exogenous separation rate, is as follows. A larger γ increases the inevitable destruction of workers' accumulated productivity under specific productivity growth, which reduces the average productivity of both employed and unemployed workers. These effects translate into lower output and profits, which lower the attractiveness of posting a vacancy, reducing vacancy and increasing unemployment. Another way to think about

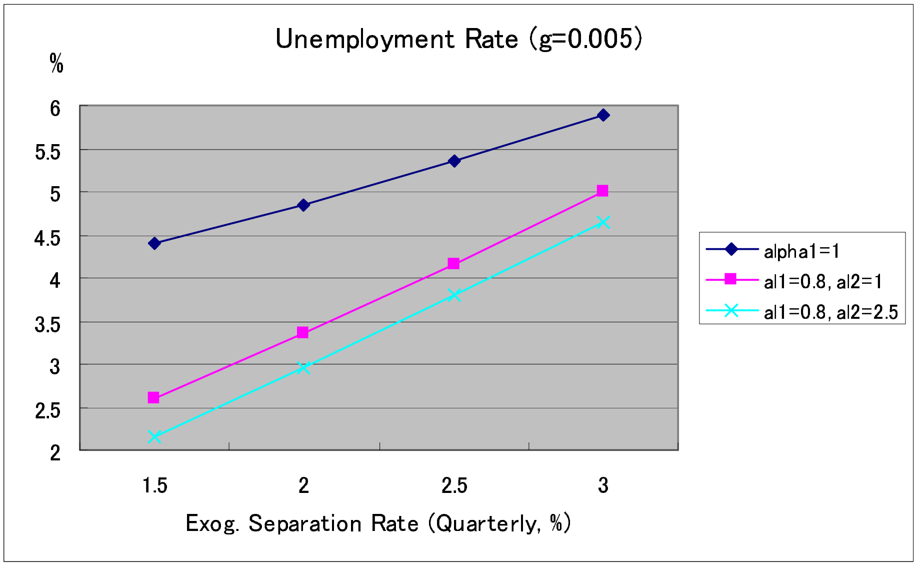

Figure 10: Exogenous Separation Rate and Unemployment

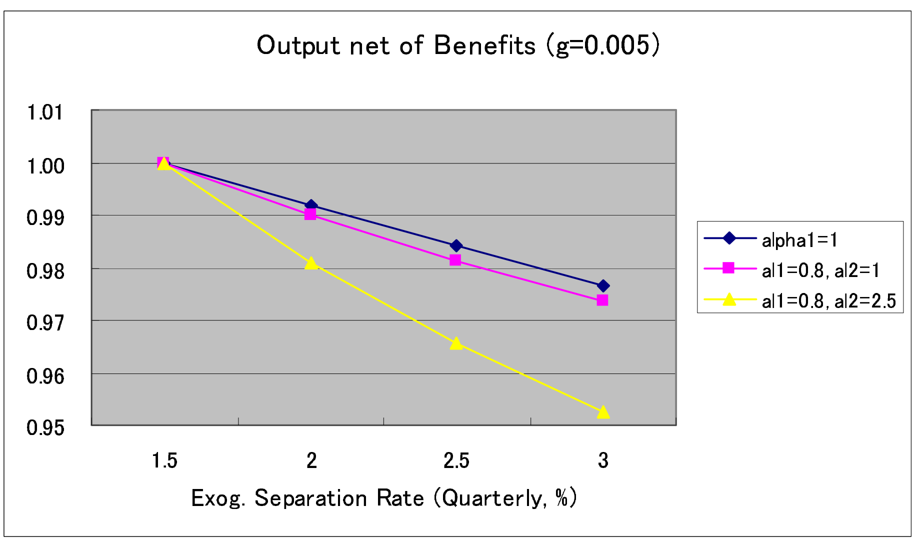

Figure 11: Exogenous Separation Rate and Output

these implications is that specific productivity growth is a costly commitment device, whose cost is increasing in the exogenous separation rate. My results above suggest that the level of exogenous separation rate, which I identify in this paper with the empirical job destruction rate, may be key in determining the desirability of specific productivity growth for the economy.

6 Conclusion

In this paper, I examined the implications of incorporating long-run productivity growth into a matching model. I proposed two new channels, namely intertemporal consumption smoothing and specific productivity growth, through which a faster growth may reduce unemployment. A quantitative analysis of the model indicated that under plausible parameters, these two new channels are able to generate the negative effects of growth on unemployment comparable to empirical estimates. Moreover, the analysis showed that specific productivity growth tends to (i) reduce the unemployment rate by relaxing the commitment problem, (ii) lengthen worker's tenure, and (iii) make these variables more responsive to changes in growth and exogenous separation rates.

I believe the framework in this model may be useful in explaining various labor market differences across countries, as well as certain labor market episodes. For example, the model implications appear to be consistent with the historically lower unemployment rates and longer tenure in Europe and Japan, where productivity growth was presumably relatively specific, in comparison to the U.S. Moreover, by incorporating the differences in the levels of unemployment benefits and firing costs[17], I believe our model may potentially account for the important labor market experiences in these two regions, such as the surge of unemployment rate in Europe, and changes to "lifetime employment" or even some aspects of the "lost decade" in Japan, as equilibrium responses to changes in TFP growth and job destruction rates. I leave such analyses for future research.

[17] According to OECD's summary measures of unemployment benefits and employment protection, (1)both measures are low in the U.S. and the U.K., (2)both are high in the Continental European countries, and (3)while the level of unemployment benefits is low, that of employment protection is high in Japan.

Appendix

Appendix A: Proof of Proposition 3

Recall the firm's problem given by equations (10)-(12) (I ignore the non-negativity constraint on c, which does not bind as $u'(0) = \infty$), and let η_1 and $\eta_2(A')$ respectively denote the Lagrange multipliers on the promise keeping and participation constraints. By setting up the Lagrangian adequately, many terms cancel out in the FOCs and the envelope conditions, leading to

$$1 = \eta_1 u'(c) \tag{22}$$

$$\frac{1+g}{1+r}\frac{\partial \pi^{e=0}(A', v'(A'), T_1+1, T_2+1)}{\partial v} + \beta(1+g)^{1-\sigma}\eta_1 + \eta_2(A') = 0 \tag{23}$$

$$\frac{\partial \pi^{e=0}(A, v, T_1, T_2)}{\partial v} = -\eta_1 \tag{24}$$

, $\forall A' \in [A^*_{T_1,T_2}, \bar{A}]$. These equations can be combined to yield

$$\eta_2(A') = \frac{1+g}{1+r}\left(-\frac{\partial \pi^{e=0}(A', v'(A'), T_1+1, T_2+1)}{\partial v}\right) - \beta(1+g)^{1-\sigma}\left(-\frac{\partial \pi^{e=0}(A, v, T_1, T_2)}{\partial v}\right) \tag{25}$$

$$= \frac{1+g}{1+r}\frac{1}{u'(c')} - \beta(1+g)^{1-\sigma}\frac{1}{u'(c)} \tag{26}$$

Therefore, whenever the participation constraint doesn't bind (i.e. $\eta_2(A') = 0$),

$$\frac{1}{1+r}\frac{1}{u'(c')} = \beta(1+g)^{-\sigma}\frac{1}{u'(c)}$$

But since $u'(c) = c^{-\sigma}$, it follows that

$$(c')^{\sigma} = \beta(1+r)(1+g)^{-\sigma}c^{\sigma}$$

or equivalently $c' = c[\beta(1+r)]^{\frac{1}{\sigma}}/(1+g)$. When the participation constraint binds, $\eta_2(A') \geq 0$ which yields $c' \geq c[\beta(1+r)]^{\frac{1}{\sigma}}/(1+g)$. This proves (1) of Proposition 3.

For the proof of (2), first recall that the RHS of (12) does not depend on A', because the match-specific productivity doesn't carry beyond the current match. Therefore, whether or not the participation constraint binds is independent of A'. Thus, when the participation constraint binds next period, it binds for all $A' \in [A^*_{T_1,T_2}, \bar{A}]$, and the wage and the promised value next period c', $v'(A')$ are given by

$$v'(A') = v^{un}(T_1+1)$$

$$c' = \left(-\frac{\partial \pi^{e=0}(A', v^{un}(T_1+1), T_1+1, T_2+1)}{\partial v}\right)^{\frac{1}{\sigma}}$$

When the participation constraint doesn't bind next period, c' and $v'(A')$ are the solutions to (25) and (26), where $\eta_2(A') = 0$. It immediately follows from the proof of (1) that c' is independent of A'. To prove that this is also true for $v'(A')$, it suffices to show that $\frac{\partial \pi^{e=0}(A,v,T_1,T_2)}{\partial v}$ is independent of A. This follows from the fact that in the firm's problem given by (10)-(12), A affects the current and future output but not the cost of providing a given promised value v, and hence the optimal value for c will be independent of A. Since (22) and (24) imply

$$\frac{\partial \pi^{e=0}(A,v,T_1,T_2)}{\partial v} = -\frac{1}{u'(c)}$$

, then, $\frac{\partial \pi^{e=0}(A,v,T_1,T_2)}{\partial v}$ is independent of A. Thus, in either case c' and $v'(A')$ are independent of A', which proves (2).■

Appendix B: Computational Procedure

I solve the model following the procedure below.

1. Choose an initial guess for u/s, and compute the job-finding rate p and vacancy-filling rate q from the matching function $m(s,u) = Bu^{\varkappa}s^{1-\varkappa}$.

2. For $T_1 = \bar{T} - 1$, solve the firm's problem and obtain the value and policy functions $\pi^{e=0}(A, v, T_1, T_2)$, $g^e(A, v, T_1, T_2)$, $g^c(A, v, T_1, T_2)$, $g^{v'(A')}(A, v, T_1, T_2)$, as well as the threshold productivity $A^*_{T_1,T_2}$, for $T_2 \in \{1, ..., T_1\}$. This is easy because the worker retires next period for sure, which means $v'(A') = v^R$. In other words, the firm just chooses the wage c to satisfy the promise keeping constraint, taking this promised value for worker next period as given. Also, use the Nash bargaining condition (19) to compute $\pi^{new}(T_1)$ and $v^{new}(T_1)$.

3. For $T_1 = \bar{T} - 2$, compute $v^{un}(T_1)$ from (18) and then solve the firm's problem for $T_2 \in \{1, ..., T_1\}$, using the results obtained in 2. Also, use the Nash bargaining condition (19) to compute $\pi^{new}(T_1)$ and $v^{new}(T_1)$. Iterate this process until you obtain all relevant functions and values for $T_1 \in \{0, ..., \bar{T} - 1\}$ and $T_2 \in \{0, ..., T_1\}$.

4. Using the values of threshold productivity $A^*_{T_1,T_2}$,recursively compute the stationary distributions of employed and unemployed workers, $\mu^{em}(A, v, T_1, T_2)$ and $\mu^{un}(T_1)$, for $T_1 \in \{0, ..., \bar{T} - 1\}$ and $T_2 \in \{0, ..., T_1\}$.

5. Using the values of $\pi^{new}(T_1)$ and $\mu^{un}(T_1)$ for $T_1 \in \{0, ..., \bar{T} - 1\}$ obtained above, check the zero-profit condition (20). If not satisfied, update the guess for u/s and go back to 1. Iterate 1.$\sim$ 5. until convergence.

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