

Measuring Dynamic Cost of Living Index from Consumption Data

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Abstract

In the U.S. the objective of CPI measurement is to measure the cost of living. However, the current CPI or in other words cost of living index (COLI) measures the cost of living in a static optimization problem. This paper proposes a new method to construct a dynamic cost of living index (DCOLI). Our method offers several advantages compared to other dynamic cost of living indices proposed in the literature. First, our measure is based on total wealth. Previous indices limited attention to financial wealth. Second, we consider an Epstein-Zin preference structure. Most previous literature has used log preferences. We derive formulas that relate our DCOLI to the COLI and derive conditions under which the two coincide. We also produce empirical measures of our DCOLI. We find that under standard assumptions on preferences, the volatility of our dynamic cost of living index is about the same as the COLI. In certain periods, e.g. 1977-1983, our measure differs sharply from the COLI.

Keywords: dynamic cost of living index; cost of life; CPI

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1 Introduction

The Consumer Price Index (CPI) is the most widely used measure of the general price level in the U.S. Taxes, welfare payments, retirement payments and labor contracts are all indexed to the CPI. Stabilizing CPI growth is a central objective of monetary policy.

In the U.S. it is generally accepted that the objective of CPI measurement is to measure changes in the purchasing power of money (for details, see e.g. Bureau of Labor Statistics (2007)). That is, the CPI is a cost of living index (COLI). Changes in a cost-of-living index are defined as the ratio of the expenditure function evaluated at different prices. Current COLI measurement implicitly assumes that the expenditure function is associated with a static expenditure minimization problem.

It has been known for a long time that there are some problems with this assumption. If a household is active for more than one period then the cost of living should reflect not just the price of today's goods but also future goods. Alchian and Klein (1973) point out this problem and propose a dynamic cost of life index (DCOLI) that recognizes that the money cost of goods includes future goods as well as current goods. Pollak (1975) provides a general theoretical treatment of intertemporal price indices.

A DCOLI has some very attractive properties. The DCOLI measures the money cost of yielding a reference level of lifetime utility. If a change in prices leads to an increase in the DCOLI, this implies that the money cost of goods has risen. In other words the lifetime utility delivered by the reference level of nominal wealth has fallen. In situations where households are active for many periods the properties of a DCOLI can in principle differ substantially from those of a COLI which just focuses on current period utility.

However, there are some major obstacles to measuring a DCOLI. One obstacle emphasized by Alchian and Klein (1973) is measuring nominal wealth. In principle one needs futures prices for each component of nominal wealth. Ab-

sent markets for these goods, one has to infer their prices indirectly by imposing restrictions on preferences and assuming complete markets.

Some of the first efforts to construct DCOLI occurred in Japan. The large swings in Japanese asset prices in the 1980s and 1990s precipitated a discussion about whether asset prices should be considered when setting monetary policy. Shibuya (1992) assumes that households have log utility, measures wealth as financial wealth and assumes that the real return on wealth is constant. He finds that the money price of goods using a DCOLI differs significantly from the COLI before the first oil price shock in 1973 and also between 1985 and 1990. Shiratsuka (1999) relaxes the assumption of constant real returns and takes up the question of whether a DCOLI should be used when setting monetary policy. He concludes that the answer is no. DCOLI is considerably more volatile than the GDP deflator, the reliability of the measurement of certain assets such as land and house prices that receive large weight in wealth is low and asset prices may respond to variations in spurious variables (e.g. sunspots). Reis (2006) constructs a DCOLI using U.S. data and also finds that it is much more volatile than the COLI. These problems have led Bryan et al. (2001) to adopt an empirical approach to measuring the dynamic cost of life that combines some restrictions from theory with an econometric approach for identifying good indicators of future prices. One common feature of all of this previous research is that human wealth is not used when constructing the measure of the DCOLI. Shiratsuka (1999) points out that the human wealth component is large but argues that it is hard to measure and only reports results for a DCOLI that uses financial wealth.

The measurement of wealth has received considerable attention in finance because wealth is important for asset pricing. Jagannathan and Wang (1996) emphasize the important role of accounting for human wealth for pricing the cross-section of returns. Campbell (1996) describes a methodology for deriving the dynamics of total wealth from a Vector Autoregression (VAR) and investigates the dynamics of asset pricing using Epstein-Zin preferences. Lustig et al.

(2007) estimate that 85 percent of total wealth is human wealth. They also propose a strategy for measuring human that is robust in the sense that they don't have to take an explicit position on the expected returns on human wealth or its growth rate. They find that the volatility of human wealth and thus total wealth is considerably lower than that of financial wealth. A common theme underlying this entire literature is that restrictions from preferences are not used to restrict the dynamics of human wealth.

One contribution of this paper is that we use restrictions from preferences to identify and estimate both human and total wealth. We adopt a specific preference structure, complete markets, and derive a stochastic pricing kernel. Then we use this pricing kernel to value dividends on human and financial wealth.

We also consider a class of preferences that is more general than that used in the previous literature on DCOLI measurement. Shibuya (1992) and Shiratsuka (1999) both assume log-preferences. Reis (2006) uses log preferences for most of his analysis but does consider a generalization to Epstein-Zin preferences. His analysis of this case imposes the assumption that equity prices follow a random walk and that goods prices follow an AR 1 in first differences. In addition, he doesn't produce an empirical measure for this preference structure.

We assume Epstein-Zin preferences throughout. Research by Bansal and Yaron (2004) finds that this preference structure in conjunction with the assumption that consumption growth has a small long run risk component can account for many key asset pricing anomalies. Using this preference structure we are able to derive a representation that decomposes the growth rate of DCOLI into two components: the growth rate of COLI in a static problem and the real dynamic cost of living index (RDCOLI). We find that when the EIS is very large the DCOLI coincides with the COLI. Our DCOLI also has the property that its long-run growth rate coincides with the COLI.

We summarize our empirical results. First, there are sharp differences between COLI and DCOLI during 1973-1976 and 1977-1983, i.e., around the first

and second oil crises. During the periods, RDCOLI, which is equal to DCOLI minus COLI, experienced the sharpest decline. This indicates that the prices of future goods sharply fell or in other words the expected future returns on total wealth increased. Second, the volatility of our DCOLI is about the same as the COLI. By experimentally calculating DCOLI only from financial wealth, we also find that the difference between our result and previous studies comes from the fact that we take into account human wealth. We also calculate the DCOLI where only dividends from financial wealth is taken into account and find that this the volatility of DCOLI is about four to eight times higher than our DCOLI which also takes into account dividends from human wealth at log utility case.

The rest of the paper is organized as follows. Section 2 lays out a household problem, defines DCOLI and RDCOLI, and derives the formula of DCOLI and RDCOLI that can be measured from data. Section 3 construct DCOLI and RDCOLI using these formula from consumption data. Finally, Section 4 concludes.

2 Model

2.1 DCOLI and RDCOLI

Consider a representative, infinitely-lived consumer who consumes a consumption good C_t at each period.¹ The consumer yields the utility $U(\{C_t\}_{t=0}^{\infty})$ from the (stochastic) consumption stream $\{C_t\}_{t=0}^{\infty}$. As in the standard financial models, we assume that all wealth, including human capital is tradable. She faces the following dynamic budget constraint:

$$\widetilde{W}_{t+1} = \widetilde{R}_{t+1}(\widetilde{W}_t - P_t C_t), \quad (1)$$

¹Although we assume one consumption good economy, it would not be difficult to extend the model to multi consumption goods economy.

where \widetilde{W}_t is the nominal total wealth, \widetilde{R}_{t+1} is the nominal gross return on the wealth, and P_t is the price of the consumption good in period t (these nominal terms are evaluated by dollar).

The indirect utility of the consumer can be written as $V(\widetilde{W}_t, s_t)$, where s_t represents state of period t , which contains information on current and future goods and asset prices.

Under the setting, we now introduce dynamic price index (DCOLI). We compare period t under state s_t , with period τ under state s'_τ . Hereafter, variables with subscript t and without prime, like \widetilde{W}_t , is the realized value of period t under state s , while those with subscript τ and with prime, like \widetilde{W}'_τ , is the realized value of period τ under state s' . Consider fictitious total nominal wealths $\widetilde{W}(s_t, \bar{V})$ and $\widetilde{W}(s'_\tau, \bar{V})$ which satisfy the following condition:

$$V(\widetilde{W}(s_t, \bar{V}), s_t) = V(\widetilde{W}(s'_\tau, \bar{V}), s'_\tau) = \bar{V}. \quad (2)$$

The $\widetilde{W}(s_t, \bar{V})$ and $\widetilde{W}(s'_\tau, \bar{V})$ are the total nominal wealth needed to attain a certain utility level, \bar{V} , under each states. Then, DCOLI $\pi(s_t|s'_\tau, \bar{V})$ is defined as follows:

$$\pi(s_t|s'_\tau, \bar{V}) = \frac{\widetilde{W}(s_t, \bar{V})}{\widetilde{W}(s'_\tau, \bar{V})}.$$

The DCOLI is the ratio of the nominal total wealths which are needed to attain a certain utility level. The definition is the same as DPI in Reis (2006), if \bar{V} is equal to the realized value of indirect utility under state s'_τ . In addition, if we assume homothetic preferences, as we do in the following sections, our DCOLI and Reis' DPI become exactly the same.

In a similar way, we can also define RDCOLI (real DCOLI instead of nominal one). RDCOLI $\pi_c(s|s', \bar{V})$ is defined as

$$\pi_c(s_t|s'_\tau, \bar{V}) = \frac{W(s_t, \bar{V})}{W(s'_\tau, \bar{V})},$$

where $W(s_t, \bar{V}) \equiv \widetilde{W}(s_t, \bar{V})/P_t$ and $W(s'_\tau, \bar{V}) \equiv \widetilde{W}(s'_\tau, \bar{V})/P'_\tau$ are real total

wealths (“real” means that it is evaluated in terms of the current consumption good). Thus RDCOLI is the ratio of real (instead of nominal) wealth needed to attain a certain utility level.

DCOLI and RDCOLI have the following properties.

First, obviously, DCOLI and RDCOLI have the following relation:

$$\ln \pi(s_t|s'_\tau, \bar{V}) = \{p_t - p'_\tau\} + \ln \pi_c(s_t|s'_\tau, \bar{V}), \quad (3)$$

where $p = \ln P$.

Second, related to the first one, DCOLI is a dynamic extension of COLI. We can check it as follows. Suppose a static problem, that is, the world consists only of one period. Then, since her consumption becomes equal to her total real wealth, $W(s_t, \bar{V}) = W(s'_\tau, \bar{V})$. Thus, RDCOLI becomes unity, and DCOLI becomes $p_t - p'_\tau$, which corresponds to traditional cost-of-living index (i.e., COLI).

Third, wealth levels can be ignored in the calculation of DCOLI and RDCOLI, if utility is homothetic (i.e., $U(\{\alpha C_t\}_{t=0}^\infty) = \alpha U(\{C_t\}_{t=0}^\infty)$ irrespective of α):

$$\pi(s_t|s'_\tau, \bar{V}) = \pi(s_t|s'_\tau) \text{ and } \pi_c(s_t|s'_\tau, \bar{V}) = \pi_c(s_t|s'_\tau).$$

In the following section, we assume the Epstein and Zin (1991) utility, which satisfies the homotheticity.

2.2 Epstein-Zin Utility

We use the recursive utility proposed by Epstein and Zin (1991):

$$U_t = \{(1 - \delta)C_t^{1-\frac{1}{\psi}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\}^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \quad (4)$$

where $U_t \equiv U(\{C_{t+j}\}_{j=0}^\infty)$, $\frac{1}{\delta} - 1$ is the rate of time preference, ψ is the elasticity of intertemporal substitution (EIS), and γ is the coefficient of the risk-aversion.²

²Needless to say, expectation E_t is the expectation conditional on state s_t .

If $\gamma = \frac{1}{\psi}$, then (4) collapses to the standard expected utility specification:

$$U_t^{1-\gamma} = (1-\delta)E_t \sum_{j=0}^{\infty} \delta^j C_{t+j}^{1-\frac{1}{\psi}}.$$

We derive DCOLI and RDCOLI under the utility specification. Epstein and Zin (1991) have shown that the optimal value of utility can be decomposed as

$$V(\widetilde{W}_t, s_t) = \phi(s_t)W_t \quad (5)$$

where $W_t \equiv \widetilde{W}_t/P_t$, and that

$$\phi(s_t) = \left[(1-\delta)^{-\psi} \frac{C_t}{W_t} \right]^{1/(1-\psi)}. \quad (6)$$

By substituting (5) and (6) into (2) and applying the definitions of RDCOLI and DCOLI, we obtain

$$\begin{aligned} \ln \pi_c(s_t|s'_\tau) &= -\{\ln \phi(s_t) - \ln \phi(s'_\tau)\} \\ &= \frac{1}{1-\psi} \{wc_t - wc'_\tau\}, \end{aligned} \quad (7)$$

and

$$\begin{aligned} \ln \pi(s_t|s'_\tau) &= \{p_t - p'_\tau\} - \{\ln \phi(s_t) - \ln \phi(s'_\tau)\} \\ &= \{p_t - p'_\tau\} + \frac{1}{1-\psi} \{wc_t - wc'_\tau\}, \end{aligned} \quad (8)$$

where $wc_t \equiv \ln(W_t/C_t)$.

2.3 Loglinear Approximations

In order to measure DCOLI and RDCOLI, we need to measure the difference of wc between state s_t and state s'_τ . In order to measure them, we apply Campbell (1993)'s loglinear approximation. We first decompose wc into weighted sum of expected future consumption growth rates and returns on total wealths. Because

the return on total wealth is unobservable in data, we rewrite it using the approximated Euler equations and by assuming conditionally homoscedasticity. Then, finally, we derive an expression of the difference of wc , expressed as a linear combination of expected consumption growth rates of states s_t and s'_τ .

2.3.1 Budget Constraint Approximation

The budget constraint (1) can be rewritten by real terms:

$$W_{t+1} = R_{t+1}(W_t - C_t),$$

where $R_{t+1} \equiv \tilde{R}_{t+1}(P_t/P_{t+1})$ is real gross return on the total wealth. The budget constraint can be rearranged as:

$$\frac{W_{t+1}}{C_{t+1}} \frac{C_{t+1}}{C_t} = R_{t+1} \left(\frac{W_t}{C_t} - 1 \right),$$

or in logs by letting $wc_t \equiv \ln \frac{W_t}{C_t}$, $\Delta c_{t+1} \equiv \ln \frac{C_{t+1}}{C_t}$, and $r_{t+1} \equiv \ln R_{t+1}$,

$$wc_t + \Delta c_{t+1} = r_{t+1} + \ln(\exp(wc_t) - 1).$$

Campbell (1993) approximates the second term around the long-run average log wealth-consumption ratio \overline{wc} :

$$\ln(\exp(wc_t) - 1) \simeq \ln(\exp(\overline{wc}) - 1) + \frac{\exp(\overline{wc})}{\exp(\overline{wc}) - 1} (wc_t - \overline{wc}).$$

Then,

$$wc_t \simeq \rho(\Delta c_{t+1} - r_{t+1}) - \rho\kappa + \rho wc_{t+1}$$

where

$$\rho \equiv \frac{\exp(\overline{wc}) - 1}{\exp(\overline{wc})} \text{ and } \kappa \equiv \ln(\exp(\overline{wc}) - 1) - \frac{1}{\rho} \overline{wc},$$

and hence, by assuming $\lim_{j \rightarrow \infty} \rho^j w c_j = 0$,

$$w c_t \simeq \sum_{j=1}^{\infty} \rho^j (\Delta c_{t+j} - r_{t+j}) - \frac{\rho \kappa}{1 - \rho}. \quad (9)$$

2.3.2 Euler Equation Approximation

Under the Epstein-Zin preferences, the Euler equation becomes

$$1 = E_t \left[\exp \left(\frac{1 - \gamma}{1 - \frac{1}{\psi}} \left(\ln \delta - \frac{1}{\psi} \Delta c_{t+1} + r_{t+1} \right) \right) \right].$$

Then, the second-order Taylor approximation around $E_t[-\frac{1}{\psi} \Delta c_{t+1} + r_{t+1}]$ as in Campbell (1993) yields:

$$0 \simeq \ln \delta - \frac{1}{\psi} E_t[\Delta c_{t+1}] + E_t[r_{t+1}] + \frac{1}{2} \frac{1 - \gamma}{1 - \frac{1}{\psi}} \text{var}_t \left(-\frac{1}{\psi} \Delta c_{t+1} + r_{t+1} \right). \quad (10)$$

2.3.3 Conditional Homoscedasticity

As in Campbell (1993), we assume that the consumption growth and asset returns are jointly conditionally homoscedastic. That is,

$$\text{var}_t \left(-\frac{1}{\psi} \Delta c_{t+1} + r_{t+1} \right) = \text{const.},$$

where var_t is the conditional variance at period t . Then, (10) implies:

$$E_t[\Delta c_{t+1} - r_{t+1}] \simeq \text{const.} + \left(1 - \frac{1}{\psi} \right) E_t[\Delta c_{t+1}]. \quad (11)$$

By substituting these equations into (9), we finally obtain

$$w c_t \simeq \text{const.} + \left(1 - \frac{1}{\psi} \right) \sum_{j=1}^{\infty} \rho^j E_t[\Delta c_{t+j}]. \quad (12)$$

Using (12), we finally obtain DCOLI and RDCOLI expressed as a linear combination of expected consumption growth rates. Since we assume conditional homoscedasticity, the constant term in (12) is equal between different states,

and it disappears when we take the difference of wcs . Therefore, we obtain

$$\ln \pi_c(s_t|s'_\tau) \simeq -\frac{1}{\psi} \sum_{j=1}^{\infty} \rho^j \{E_t[\Delta c_{t+j}] - E'_\tau[\Delta c'_{\tau+j}]\}, \quad (13)$$

$$\ln \pi(s_t|s'_\tau) \simeq \{p_t - p'_\tau\} - \frac{1}{\psi} \sum_{j=1}^{\infty} \rho^j \{E_t[\Delta c_{t+j}] - E'_\tau[\Delta c'_{\tau+j}]\}, \quad (14)$$

where expectation E'_τ is the expectation conditional on state s'_τ

2.4 Interpretation of DCOLI

In this section, we note properties obtained from (8) and (14).

First, from (14), the higher EIS is, the closer to COLI DCOLI is. Second, if as often assumed in financial literature, long-run wc is constant, which implies that long-run consumption growth rate is constant under our assumptions, long-run DCOLI coincides with long-run COLI.

Third, DCOLI in (14) becomes the ratio of a weighted average of prices in dynamic budget constraint, which correspond to $P_t, P_t/R_{t+1}, P_t/(R_{t+1}R_{t+2}), \dots$ ³ Let Q_{t+j}^t be the price of period $t+j$ consumption in the dynamic budget constraint (e.g., $Q_t^t = P_t, Q_{t+1}^t = P_t/R_{t+1}, Q_{t+2}^t = P_t/(R_{t+1}R_{t+2}), \dots$), and $q_u^t \equiv \ln Q_u^t$. Then, by using (11), (14) can be rewritten as

$$\begin{aligned} \ln \pi(s_t|s'_\tau) &\simeq \{p_t - p'_\tau\} - \sum_{j=1}^{\infty} \rho^j \{E_t[r_{t+j}] - E'_\tau[r'_{\tau+j}]\} \\ &= \sum_{j=0}^{\infty} \rho^j (1 - \rho) \{E_t[q_{t+j}^t] - E'_\tau[q'_{\tau+j}]\}, \end{aligned}$$

which is exactly the log of the geometric average of Q_{t+j}^t s.⁴ The above equations also indicate that if expected future returns decrease, the current prices of future goods increase, and cost of living increases (and vice versa).

³This is because by expanding (1), we obtain

$$P_t C_t + \sum_{j=1}^{\infty} \frac{P_t}{\prod_{i=1}^j R_{t+i}} C_{t+j} = \widetilde{W}_t.$$

⁴In the perfect foresight case, the above equation coincides with what Shibuya (1992) derives in his theoretical part, if ρ is equal to time preference δ .

2.5 RDCOLI with leisure

In many business cycle models, utility from leisure is taken into account, while in our baseline model it is not. Thus, in this section, we derive RDCOLI when leisure is considered. We assume the following utility function:

$$U_t = \left\{ (1 - \delta)(C_t^\eta L_t^{1-\eta})^{1-\frac{1}{\psi}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

where L_t is time spent for leisure. We also assume that consumer's budget constraint can be written as

$$W_{t+1} = R_{t+1}(W_t - C_t - X_t L_t),$$

where X_t is the opportunity cost of leisure, which is equal to wage rate in a standard setting.

In Appendix B, we show that RDCOLI can be expressed as follows:

$$\ln \pi_c(s_t | s'_\tau) = (1 - \eta)(x_t - x'_\tau) + \frac{1}{1 - \psi} \{w c_t - w c'_\tau\} \quad (15)$$

$$= (1 - \eta)\{(c_t - c'_\tau) - (l_t - l'_\tau)\} + \frac{1}{1 - \psi} \{\widehat{w} c_t - \widehat{w} c'_\tau\}, \quad (16)$$

where $x \equiv \ln X = l \equiv \ln L$ and

$$\begin{aligned} \widehat{w} c_t &\equiv \ln \left(\frac{W_t}{C_t + X_t L_t} \right) \\ &\simeq \text{const.} + \left(1 - \frac{1}{\psi} \right) \sum_{j=1}^{\infty} \rho^j E_t[\eta \Delta c_{t+j} + (1 - \eta) \Delta l_{t+j}]. \end{aligned}$$

3 Measuring DCOLI and RDCOLI from Data

In this section, we measure DCOLI and RDCOLI using the U.S. quarterly data from 1959:4 to 2003:1. We measure DCOLI growth rate, Δdcoli_t , and RDCOLI growth rates, Δrdcoli_t (we mainly focus on Δdcoli_t). Δdcoli_t is defined by $\ln \pi(s_t | s_{t-1})$, and Δrdcoli_t is defined by $\ln \pi_c(s_t | s_{t-1})$, where s_t is the realized

state variables at period t . Because the measurement of wc is crucial for our DCOLI measurement, we also report (demeaned) log RDCOLI, rdcoli_t , which is defined by (demeaned) $-\ln \phi(s_t)$ ($= wc_t/(1 - \psi)$).

We impose two different assumptions on household's expectation. In the first case, we assume that consumer's expectations for future consumption growth rates are the same as the realized values. In the second case, consumer forms her expectation for future consumption growth rates based on VAR model (we discuss it later). In the following two sections, we try these two cases.

In these two sections, leisure is not considered. As the third case, we also measure DCOLI where consumers also yield utility from leisure. In the third case, we follow the assumption on consumer expectations in the first case.

In order to measure DCOLI, we also need to specify the parameter of EIS, ψ , and long-run average log wealth-consumption ratio, \overline{wc} (or ρ). For the EIS, we try several values from 0.2 to 2.0. For \overline{wc} , we set the value of long-run average log price-dividend ratio on households' financial wealth, which is 4.627 in the U.S. quarterly data.⁵ Then, $\rho \approx 0.9902$.

3.1 Perfect foresight case

In this section, we measure DCOLI and RDCOLI based on the assumption that the expected values of future consumption growth rates coincide with the realized values (i.e., $E_t[\Delta c_{t+j}] = \Delta c_{t+j}$) before 2003, and that after 2003 they are equal to the average consumption growth rate over the sample periods. For consumption, we use per capita real consumption data (for details, see Appendix A). In this section, we do not consider leisure (for the leisure considered case, see Section 3.3).

We first look at the demeaned rdcoli . Figure 1 plots the demeaned rdcoli ⁶. The demeaned rdcoli captures economic boom from the latter half of 1960s to

⁵ We measure the households' financial wealth as in Lettau and Ludvigson (2001). For details, see Appendix A.5.

⁶Notice that although the shape of fluctuations in wc inverts at $\psi = 1.0$ (at $\psi = 1.0$, wc becomes constant), the shape of fluctuation in demeaned rdcoli (i.e., demeaned $wc/(1 - \psi)$) does not depend on ψ . We can confirm the property from (13).

the former half of 1970s, stagnation after the first and second energy crises, and boom around 2000.

Figures 3 and 2 plot Δcoli and Δdcolis . A property in (14) that as EIS becomes larger Δdcoli converges to Δcoli is confirmed in the figures. Since it might be difficult to see the differences between COLI and DCOLIs in these figures, in Figure 4, we also plot three-years moving averages of Δcoli , Δdcoli , and Δrdcoli (for EIS = 0.5), which is calculated as three-years average of inflation before and after the period, as in Reis (2006). During the first and second energy crises (i.e., 1973-1976 and 1977-1983), Δrdcoli , which is equal to the difference between Δdcoli and Δcoli , was the lowest. Along Section 2.4, it can be interpreted as the current prices of future consumption decreased, or in other words, future returns increased during the periods.⁷ On the other hand, Δrdcoli became the highest level around 1965 and 1985.

Table 1 reports the standard deviation and autocorrelation of Δcoli and Δdcoli and the correlation of Δcoli and Δrdcoli .⁸ Except for the case that EIS = 0.2, the standard deviations of Δdcolis are close to that of Δcoli .⁹ It is different from the results in the previous studies. The autocorrelation of Δdcoli is lower than Δcoli . Thus Δdcoli is less persistent. The correlation of Δcoli and Δrdcoli is negative. This means that when the price of current goods increases, the prices of future goods decrease, or in other words expected future returns increase.

In order to consider how the result is affected by human wealth, we also calculate two types of Δdcoli that use data on households' financial wealth instead of total wealth (we refer to them as financial Δdcolis). The one is Δdcoli which is calculated by (8) using the the price-dividend ratio data of the

⁷During the same periods, equity prices were relatively low.

⁸We also calculate the average and standard deviation of Δcoli and Δdcoli and the correlation of Δcoli and Δrdcoli for every ten years in Table Appendix 7.

⁹In the table, when EIS \geq 1.0 ($1/\text{EIS} = 1/\psi$ corresponds to risk-aversion parameter in CRRA utility and EIS = 1.0 corresponds to log utility case), the volatility of Δdcoli is less than that of Δcoli . This is because the covariance between Δcoli_t and Δrdcoli_t is negative. (notice that $\text{var}(\Delta\text{dcoli}_t) = \text{var}(\Delta\text{coli}_t) + \text{var}(\Delta\text{rdcoli}_t) + 2\text{cov}(\Delta\text{coli}_t, \Delta\text{rdcoli}_t)$ and that $\text{corr}(\Delta\text{coli}_t, \Delta\text{rdcoli}_t)$ is negative in data).

households' financial wealth instead of the wealth-consumption ratio.¹⁰ Note that because the data price-dividend ratio does not become constant at EIS = 1.0 (log utility case), it cannot be calculated at this EIS value. The other financial $\Delta dcoli$ is calculated in the way that dividend growth rates on financial wealth are used instead of consumption growth rates in (14).¹¹ Tables 2 and 3 report the standard deviations of the former and latter cases of financial $\Delta dcoli$. These are highly volatile compared with the $\Delta dcolis$ that take into account human wealth. Especially, the latter financial $\Delta dcoli$ is about eight times more volatile than $\Delta dcoli$ calculated from consumption data at EIS = 1.0.

3.2 VAR case

We measure DCOLI and RDCOLI where consumers (rationally) forecast future using a VAR model (and where leisure is not taken into account). We assume that expectation of household is formed by the following VAR:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \epsilon_t,$$

\mathbf{z}_t is state variables, and ϵ_t is i.i.d. with mean zero. Components of \mathbf{z}_t is basically the same as that of Lustig and Van Nieuwerburgh (2006), and $\mathbf{z}_t = (\Delta c_t, \Delta y_t, s_t, r_t^a, pd_t^a, YK_t, rtb_t, yspst_t)'$, where Δc_t is per capita real consumption growth, Δy_t is per capita real labor income growth, s_t is labor income share, r_t^a is real return on the households' financial wealth, pd_t^a is the log price-dividend ratio of the households' financial wealth, YK_t is output-(physical) capital ratio, rtb_t is relative T-bill return, and $ysps_t$ are yield spreads of several bonds. For details of these data, see Appendix A. We include real return r_t^a , because the return is related to consumption growth Δc_t through the Euler equation. We also include YK_t , because YK_t times capital intensity is the return on aggre-

¹⁰For the definition and construction of the households' financial wealth, see footnote 5 and Appendix A.5.

¹¹We assume that the expected values of future dividend growth rates coincide with the realized values before 2003, and that after 2003 they are equal to the average dividend growth rate over the sample periods.

gate capital under the Cobb-Douglas aggregate production function. Then, for example,

$$E_t[\Delta c_{t+j}] = E_t[\mathbf{e}_1 \mathbf{z}_{t+j}] = \mathbf{e}_1 \mathbf{A}^j \mathbf{z}_t,$$

where $\mathbf{e}_1 = (1, 0, \dots, 0)$. Matrix \mathbf{A} is estimated from data using OLS.

Figure 5 plots demeaned rdcoli. Compared with the perfect foresight case, a different point is after 2000. After 2000, demeaned rdcoli of the VAR case is higher than that of the perfect foresight case. The difference might be because in the perfect foresight case, we assume that after 2003 the expected consumption growth rate is equal to the average growth rate of consumption over the sample periods.

Next, we look at growth rates. Figure 6 plots Δd_{coli} in the VAR case. Since the basic tendencies are the same for different EISs, we only plot the EIS = 0.5 and 1.0 cases. As in the perfect foresight case, we also plot three-years moving average in Figure 7. Basic tendencies are similar to the perfect foresight case, except for after 2000. After 2000, three-years moving average Δd_{coli} of the VAR case is higher than that of the perfect foresight case. Table 4 reports the standard deviation and autocorrelation of Δcoli and Δd_{coli} and the correlation of Δcoli and Δrdcoli .¹² The volatilities of VAR Δd_{colis} are more volatile but still close to those in the perfect foresight case (the standard deviations are on average about 30% higher than those of the perfect foresight case). Properties on the low persistency of Δd_{coli} and negative correlation of Δcoli and Δrdcoli are the same as the perfect foresight case. As in the perfect foresight case, we compare these Δd_{colis} with financial wealth versions of Δd_{coli} . Financial Δd_{coli} consisting of the price-dividend ratio data of financial wealth is reported in Table 2, and another financial Δd_{coli} calculated from expected dividend growth rates is reported in Table 5.¹³ The latter financial Δd_{colis} are less

¹²As in the perfect foresight case, we calculate the average and standard deviation of Δcoli and Δd_{coli} and the correlation of Δcoli and Δrdcoli for every ten years in Table Appendix 7.

¹³The expected dividend growth rate is calculated by using the following relation:

$$E[\Delta d_{t+j}] = (\mathbf{e}_4 + \mathbf{e}_5) \mathbf{A}^j \mathbf{z}_t - \rho^{-1} \mathbf{e}_5 \mathbf{A}^{j-1} \mathbf{z}_t,$$

volatile than the perfect foresight version. Nonetheless, Δd_{colis} calculated from expected consumption growth rates are less volatile than the latter financial Δd_{colis} (the latter VAR version of financial Δd_{colis} is around four times volatile than our Δd_{colis} at $\text{EIS} = 1.0$).

3.3 Leisure considered case

Finally, we measure DCOLI and RDCOLI where leisure is taken into account. We assume perfect foresight, and the indices are calculated using formula in Section 2.5. We set $\eta = 1/3$ following Cooley (1995) and Heathcote et al. (2007). For consumption, as in the perfect foresight case, we use per capita real consumption data. For leisure, we assume that the max hours are \bar{H}/η , where H_t is per capita hours worked and \bar{H} is the mean of H_t . Then, following Heathcote et al., we calculate leisure time by $L_t \equiv (\bar{H}/\eta) - H_t$. For details of data, see Appendix A.

Figure 8 plots demeaned rdcolis . Compared with the perfect foresight case, basic tendencies are similar. However, in some periods especially around 2000, the movement of demeaned rdcolis with lower EIS differs from these of the perfect-foresight without leisure case: around 2000, the demeaned rdcolis with $\text{EIS} = 0.2$ is lower than zero (while the demeaned rdcolis with $\text{EIS} = 0.5$ or higher are higher than zero).

Next, Figure 9 plots Δd_{colis} in the leisure considered case. Figure 10 plots the three-years moving average of the leisure considered case. Δd_{colis} of the leisure considered case are higher than these of other cases. This is because of the upward trend in opportunity cost of leisure (i.e., wage rate), as confirmed in (21). Table 6 reports the standard deviation and autocorrelation of Δcolis and Δd_{colis} and the correlation of Δcolis and $\Delta \text{rdcolis}$.¹⁴ Δd_{colis} are less volatile than the perfect foresight without leisure case (the standard deviations are on average

where Δd_{t+j} is dividend growth rate and \mathbf{e}_i is a row vector with i -th element unity and other elements zero. This relation holds because $r_{t+j}^a = \Delta d_{t+j} + p d_{t+j}^a - \rho^{-1} p d_{t+j-1}^a$ holds (where variables are demeaned).

¹⁴See Table Appendix 7 for the decade-level comparison.

about 17% lower than those of the perfect foresight case). The autocorrelation is higher than the cases without leisure. The 8-lags autocorrelations with $EIS \geq 0.5$ are close to that of $\Delta coli$. On the other hand, negative correlation of $\Delta coli$ and $\Delta rdcoli$ is similar to other cases.

4 Concluding Remarks

This paper develops a practical method to construct DCOLI from consumption data, and measure DCOLI using the method. Compared with previous studies, there are three advantages: (1) our DCOLI can capture contribution from change in human wealth, (2) our DCOLI is less volatile, and (3) assumption on consumer preference is less restrictive.

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A Data Appendix

This appendix describes the data sources. We use quarterly data and the sample periods is 1959:4 to 2003:1.

A.1 Population and per capita hours worked

We take the working-age population (16-64 years old) and per capita hours worked data from Prescott et al. (2005).

A.2 Consumption

A.2.1 Consumption price indices

We construct the Fisher version of consumption price indices (CPI) using the formula:

$$\sqrt{\frac{\sum P_t Q_{t-1}}{\sum P_{t-1} Q_{t-1}}} \sqrt{\frac{\sum P_t Q_t}{\sum P_{t-1} Q_t}}.$$

We chain the indices to derive the price level of consumption. To construct the indices, we use the price data of “nondurable goods” (line 6) and “services” (line

13) in Table 2.3.4 and the quantity data of them in Table 2.3.3 in the National Income and Product Accounts (NIPA).

A.2.2 Per capita real consumption

In order to obtain real consumption data, we divide the nominal consumption by Fisher version of CPI explained above. We further divide the real consumption by population explained above.

Nominal consumption data are from Table 2.3.5 in the NIPA. Our nominal consumption data are the sum of nondurable goods (line 6) and services (line 13). These data are seasonally adjusted at annual rates. Thus, we divide the values by 4.

A.3 Labor income

A.3.1 Labor income share

Data on labor income share are taken from Table 2.1 in the NIPA. Labor income share are calculated by dividing the nominal labor income explained below by nominal “disposable personal income” (line 26).

We construct nominal labor income from “compensation of employees, received” (line 2)+ “government social benefits to persons” (line 17) - “Contributions for government social insurance” (line 24) - labor taxes. As in Lettau and Ludvigson (2001), labor taxes are imputed from a share of “personal current taxes” (line 25) to labor income, where the share is calculated as the ratio of “wage and salary disbursements” (line 3) to “wage and salary disbursements”, “proprietors’ income with inventory valuation and capital consumption adjustments” (line 9), “rental income of persons with capital consumption adjustment” (line 12), and “personal income receipts on assets” (line 13).

A.3.2 Per capita real labor income

Basically, data on per capita real labor income are taken from Table 2.1 in the NIPA. We obtain real labor income by multiplying labor income share defined above by real disposable personal income. The real disposable personal income is obtained by “disposable personal income” (line 26) \div CPI explained above.¹⁵ In order to obtain per capita real labor income, we divide it by population explained above. These data are seasonally adjusted at annual rates. Thus, we divide the values by 4.

A.4 Households’ financial wealth

A.5 Price-dividend ratio of households’ financial wealth

$$pd^a$$

In order to obtain price-dividend ratio of households’ financial wealth, pd^a , we divide the nominal financial wealth by nominal dividends minus savings, both explained below.

Nominal financial wealth data are obtained from the balance sheet of households and non-profit organizations, Flow of Funds Accounts Table B-100, provided by the Federal Reserve Board System. This wealth measure is on an end-of-period basis. Therefore, we use the $t - 1$ value of the data for period t wealth. Our measure of households’ financial wealth consists of: net worth (line 41) – consumer durable goods (line 7). Basically, our definition of nominal financial wealth is the same as that of Lettau and Ludvigson (2001) except that we exclude durable consumption from nominal financial wealth (because they are included in consumption in NIPA).

Nominal dividends minus savings are obtained from Table 2.1 in the NIPA. It is constructed from “proprietors’ income with inventory valuation and capital consumption adjustments” (line 9) + “rental income of persons with capital

¹⁵The reason that it is divided by CPI is that in our model economy, real terms are expressed in consumption good unit.

consumption adjustment” (line 12) + “personal income receipts on assets” (line 13) - “other current transfer receipts, from business (net)” (line 23) - capital taxes - personal saving (line 33). As of labor taxes in labor income share, capital taxes are imputed from a share of a share of “personal current taxes” (line 25) to capital income, where the share is calculated as the ratio of “proprietors’ income with inventory valuation and capital consumption adjustments”, “rental income of persons with capital consumption adjustment”, and “personal income receipts on assets” to “wage and salary disbursements” (line 3), “proprietors’ income with inventory valuation and capital consumption adjustments”, “rental income of persons with capital consumption adjustment”, and “personal income receipts on assets”.

A.5.1 Real return of the households’ financial wealth r^a

We obtain the real return on the households’ financial wealth, r^a from $r_{t+1}^a \equiv \ln R_{t+1} = \ln\left(\frac{P_{t+1}^a}{P_t^a - D_t^a}\right)$, where P^a is per capita real financial wealth, and D^a is per capita real dividends minus savings. P^a is calculated from nominal financial wealth explained above divided by CPI and population. D^a is calculated from nominal dividends minus savings divided by CPI and population.

A.6 Relative T-bill return rtb_t and yield spreads $ysps_t$

Relative T-bill return rtb_t and the yield spreads of several bonds $ysps_t$ used in the VAR case are taken from Van Nierwerburgh’s website. Precisely, rtb_t corresponds to relTbill and $ysps_t$ correspond to defsprBaaAAA, lefsprBaaTbond, and termspread in `quarterly_data_WSMS.xls` located in his website.

A.7 Output-(physical) capital ratio YK_t

We calculate Output-(physical) capital ratio YK_t of the U.S. from Braun et al. (2006) dataset. The dataset is available from Braun’s website. Notice that the YK_t is not taken log.

B Derivation of RDCOLI with leisure

We derive RDCOLI under the following consumer problem:

$$V_t = \max_{\{C_t, L_t\}} \left\{ (1 - \delta)(C_t^\eta L_t^{1-\eta})^{1-\frac{1}{\psi}} + \delta(E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}} \quad (17)$$

$$\text{s.t. } W_{t+1} = R_{t+1}(W_t - C_t - X_t L_t), \quad (18)$$

by the following steps.

Step 1. Intratemporal choice of consumption and leisure.

By solving intratemporal problem, we obtain the following equations:

$$C_t = \eta \widehat{C}_t, \quad (19)$$

$$(C_t^\eta L_t^{1-\eta}) = B_t \widehat{C}_t, \quad (20)$$

$$\frac{1}{X_t} = \frac{\eta}{1-\eta} \frac{L_t}{C_t}, \quad (21)$$

where $\widehat{C}_t \equiv C_t + X_t L_t$, and $B_t \equiv \eta^\eta \left(\frac{1-\eta}{X_t}\right)^{1-\eta} \widehat{C}_t$.

Step 2. Indirect utility and RDCOLI.

By the homotheticity, the indirect utility V_t can be written as

$$V_t = \phi_t W_t \quad (22)$$

In this step, we show that ϕ_t can be expressed as the function of W_t and \widehat{C}_t .

Then, as in the Section 2.2, expression on RDCOLI can also be obtained.

First, we obtain ϕ_t expressed by W_t and \widehat{C}_t . We define modified consumption wealth ratio as

$$\Psi_t = \frac{\widehat{C}_t}{W_t}. \quad (23)$$

Then, by substituting (18) and (20) into (4), and normalizing by (22) and (23),

we obtain

$$\phi_t = \max_{\Psi_t} \left\{ (1 - \delta) (B_t \Psi_t)^{1 - \frac{1}{\psi}} + \delta (E_t[(\phi_{t+1} R_{t+1} (1 - \Psi_t))^{1 - \gamma}])^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} \right\}^{\frac{1}{1 - \frac{1}{\psi}}} \quad (24)$$

FOC of (24) with respect to Ψ_t is

$$(1 - \delta) (B_t \Psi_t)^{1 - \frac{1}{\psi}} \Psi_t^{-1} = \delta (E_t[(\phi_{t+1} R_{t+1} (1 - \Psi_t))^{1 - \gamma}])^{\frac{1 - \frac{1}{\psi}}{1 - \gamma}} (1 - \Psi_t)^{-1}$$

Rearranging the FOC and applying (24), we obtain

$$\phi_t = \left[(1 - \delta) (B_t \Psi_t)^{1 - \frac{1}{\psi}} \Psi_t^{-1} \right]^{\frac{1}{1 - \frac{1}{\psi}}}. \quad (25)$$

Although (25) includes X_t , by using (21), X_t can be expressed by C_t and L_t . Then, as in (7), we can obtain RDCOLI between state s_t relative to state s'_τ as follows:

$$\begin{aligned} \ln \pi_c(s_t | s'_\tau) &= -\{\ln \phi_t - \ln \phi'_\tau\} \\ &= (1 - \eta)(x_t - x'_\tau) + \frac{1}{1 - \psi} \{\widehat{w}c_t - \widehat{w}c'_\tau\} \end{aligned} \quad (26)$$

$$= (1 - \eta)\{(c_t - c'_\tau) - (l_t - l'_\tau)\} + \frac{1}{1 - \psi} (\widehat{w}c_t - \widehat{w}c'_\tau), \quad (27)$$

where $\widehat{w}c_t$ is given by $-\ln \Psi_t$.

Step 3. Decomposition of $\widehat{w}c$.

In this step, we derive $\widehat{w}c_t \equiv -\ln \Psi_t$ as the sum of expected consumption growth.

First, by loglinear approximation, we obtain

$$\widehat{w}c_t \simeq \sum_{j=1}^{\infty} \rho^j (\Delta \widehat{c}_{t+j} - r_{t+j}) - \frac{\rho k}{1 - \rho}. \quad (28)$$

Next, we obtain an expression corresponding to (11) from the Euler equation.

By substituting (25) into (24), we obtain

$$(B_t \Psi_t)^{1-\frac{1}{\psi}} \Psi_t^{-1} = (B_t \Psi_t)^{1-\frac{1}{\psi}} + \delta (E_t [(((B_{t+1} \Psi_{t+1})^{1-\frac{1}{\psi}} \Psi_{t+1}^{-1})^{\frac{1}{1-\gamma}} R_{t+1} (1-\Psi_t))^{1-\gamma}]^{\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

By rearranging this equation using (18), (20), and (23), we obtain the Euler equation:

$$1 = \delta \left(E_t \left[\left(\left(\frac{X_{t+1}}{X_t} \right)^{-\frac{\theta}{1+\theta}} \left(\frac{\widehat{C}_{t+1}}{\widehat{C}_t} \right)^{1-\frac{1}{1-\frac{1}{\psi}}} R_{t+1}^{\frac{1}{1-\frac{1}{\psi}}} \right)^{1-\gamma} \right] \right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}.$$

By assuming conditional homoscedasticity, the Euler equation can be loglinearized as follows:

$$\begin{aligned} E_t [\Delta \widehat{c}_{t+1} - r_{t+1}] - \text{const.} &\simeq \left(1 - \frac{1}{\psi} \right) E_t [\Delta \widehat{c}_{t+1} - (1-\eta) \Delta x_{t+1}] \\ &= \left(1 - \frac{1}{\psi} \right) E_t [\eta \Delta c_{t+1} + (1-\eta) \Delta l_{t+1}], \end{aligned} \quad (29)$$

where we use (19) and (20).

By combining (28) and (29), we finally obtain the following equations:

$$\widehat{w}c_t \simeq \text{const.} + \left(1 - \frac{1}{\psi} \right) \sum_{j=1}^{\infty} \rho^j E_t [\eta \Delta c_{t+j} + (1-\eta) \Delta l_{t+j}]. \quad (30)$$

Although $\widehat{w}c_t$ can also be written as follows:

$$\widehat{w}c_t \simeq \text{const.} + \left(1 - \frac{1}{\psi} \right) \sum_{j=1}^{\infty} \rho^j E_t [\Delta c_{t+j} - (1-\eta) \Delta x_{t+j}],$$

we use (30) for the measurement.

	std[Δcoli]	AC(1)[Δcoli]	AC(4)[Δcoli]	AC(8)[Δcoli]	
	0.71%	0.82	0.63	0.37	
EIS	std[Δdcoli]	AC(1)[Δdcoli]	AC(4)[Δdcoli]	AC(8)[Δdcoli]	corr[$\Delta\text{coli}, \Delta\text{rdcoli}$]
0.2	2.35%	0.21	0.02	-0.16	-0.52
0.5	0.91%	0.27	0.09	0.01	-
1.0	0.62%	0.61	0.44	0.36	-
2.0	0.61%	0.81	0.64	0.47	-

Table 1: Standard deviation and autocorrelation of Δcoli and Δdcoli and correlation of Δcoli and Δrdcoli : the perfect foresight case. corr[$\Delta\text{coli}, \Delta\text{rdcoli}$] (where leisure is not considered) does not depend on the EIS value.

EIS	std[Δdcoli]
0.2	6.47%
0.5	10.34%
1.0	n.a.%
2.0	5.28%

Table 2: Standard deviation of Δdcoli : calculated using price-dividend ratio data of broad financial wealth instead of wealth-consumption ratio. For details of the the price-dividend ratio data, see appendix A.5.

EIS	std[Δdcoli]
0.2	24.01%
0.5	9.59%
1.0	4.81%
2.0	2.45%

Table 3: Standard deviation of Δdcoli : the perfect foresight case. Here Δdcoli is calculated assuming that households earn income only from financial wealth.

EIS	std[Δ dcoli]	AC(1)[Δ dcoli]	AC(4)[Δ dcoli]	AC(8)[Δ dcoli]	corr[Δ dcoli, Δ rdcoli]
0.2	3.16%	0.06	-0.05	-0.14	-0.29
0.5	1.30%	0.07	-0.02	-0.11	-
1.0	0.81%	0.31	0.21	0.06	-
2.0	0.69%	0.62	0.48	0.26	-

Table 4: Standard deviation and autocorrelation of Δ dcoli and correlation of Δ dcoli and Δ rdcoli: the VAR case. corr[Δ dcoli, Δ rdcoli] (where leisure is not considered) does not depend on the EIS value.

EIS	std[Δ dcoli]
0.2	15.85%
0.5	6.33%
1.0	3.19%
2.0	1.67%

Table 5: Standard deviation of Δ dcoli: the VAR case. Here Δ dcoli is calculated assuming that households earn income only from financial wealth.

EIS	std[Δ dcoli]	AC(1)[Δ dcoli]	AC(4)[Δ dcoli]	AC(8)[Δ dcoli]	corr[Δ coli, Δ rdcoli]
0.2	1.21%	0.14	0.01	-0.00	-0.40
0.5	0.69%	0.43	0.32	0.30	-0.49
1.0	0.63%	0.62	0.46	0.38	-0.51
2.0	0.64%	0.67	0.48	0.37	-0.48

Table 6: Standard deviation and autocorrelation of Δ dcoli and correlation of Δ coli and Δ rdcoli: the leisure considered case.

mean	1960s	1970s	1980s	1990s
mean[Δ coli]	0.62%	1.73%	1.10%	0.61%
mean[Δ dcoli] (PF EIS=0.5)	0.95%	1.39%	1.26%	0.65%
mean[Δ dcoli] (PF EIS=1.0)	0.78%	1.56%	1.18%	0.63%
mean[Δ dcoli] (VAR EIS=0.5)	0.90%	1.56%	1.12%	0.91%
mean[Δ dcoli] (VAR EIS=1.0)	0.76%	1.64%	1.11%	0.76%
mean[Δ dcoli] (PFL EIS=0.5)	1.17%	1.88%	1.52%	0.94%
mean[Δ dcoli] (PFL EIS=1.0)	1.12%	1.90%	1.55%	0.96%
std	1960s	1970s	1980s	1990s
std[Δ coli]	0.40%	0.77%	0.61%	0.33%
std[Δ dcoli] (PF EIS=0.5)	1.05%	0.80%	0.90%	0.73%
std[Δ dcoli] (PF EIS=1.0)	0.61%	0.49%	0.54%	0.38%
std[Δ dcoli] (VAR EIS=0.5)	1.17%	1.46%	1.54%	0.97%
std[Δ dcoli] (VAR EIS=1.0)	0.65%	0.85%	0.88%	0.57%
std[Δ dcoli] (PFL EIS=0.5)	0.76%	0.54%	0.58%	0.49%
std[Δ dcoli] (PFL EIS=1.0)	0.60%	0.50%	0.54%	0.38%
corr	1960s	1970s	1980s	1990s
corr[Δ coli, Δ rdcoli] (PF)	-0.14	-0.78	-0.58	-0.51
corr[Δ coli, Δ rdcoli] (VAR)	-0.19	-0.41	-0.21	0.03
corr[Δ coli, Δ rdcoli] (PFL EIS=0.5)	-0.17	-0.77	-0.56	-0.41
corr[Δ coli, Δ rdcoli] (PFL EIS=1.0)	-0.17	-0.77	-0.57	-0.51

Table 7: Appendix: Average and standard deviation of Δ coli and Δ dcoli, and correlation of Δ coli and Δ rdcoli for every ten years. PF, VAR and PFL denote the perfect foresight, VAR and perfect foresight with leisure cases. corr[Δ coli, Δ rdcoli] (where leisure is not considered) does not depend on the EIS value.

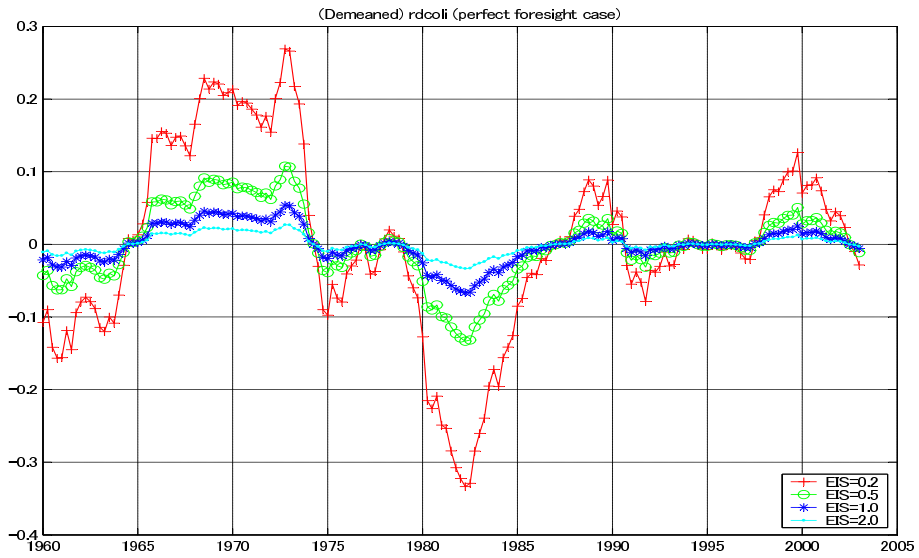


Figure 1: Demeaned rdcolis: the perfect foresight case.

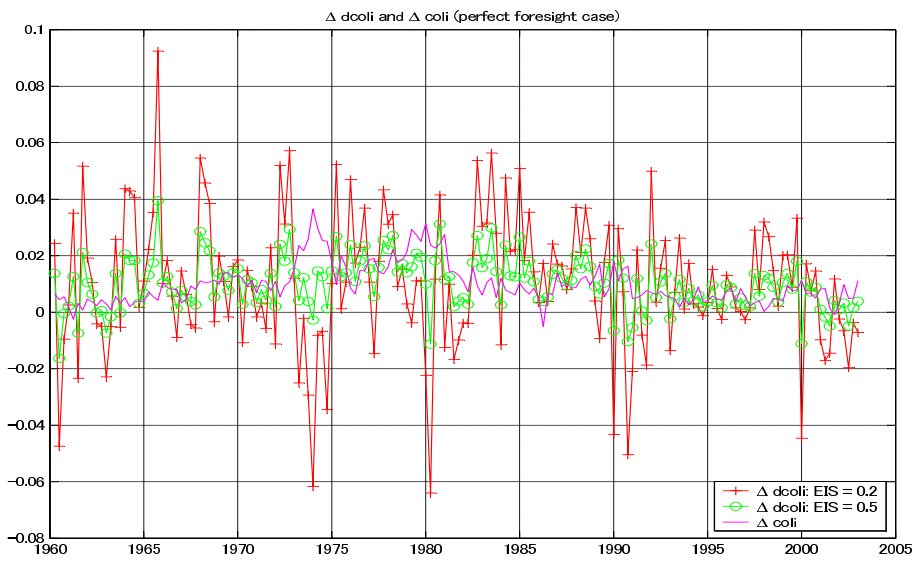


Figure 2: $\Delta colis$ and $\Delta dcolis$ (for DCOLI, EIS = 0.2 and 0.5): the perfect foresight case.

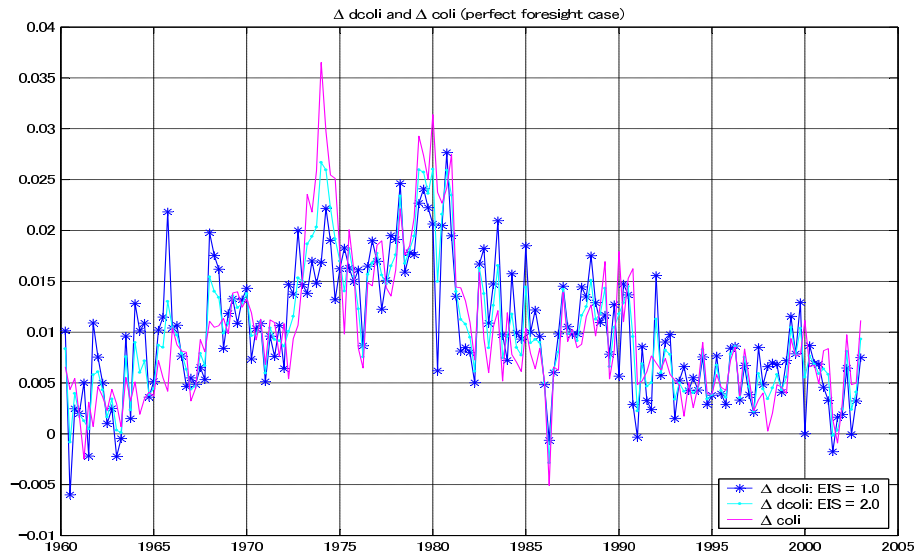


Figure 3: Δdcoli and Δdcolis (for DCOLI, EIS = 1.0 and 2.0): the perfect foresight case.

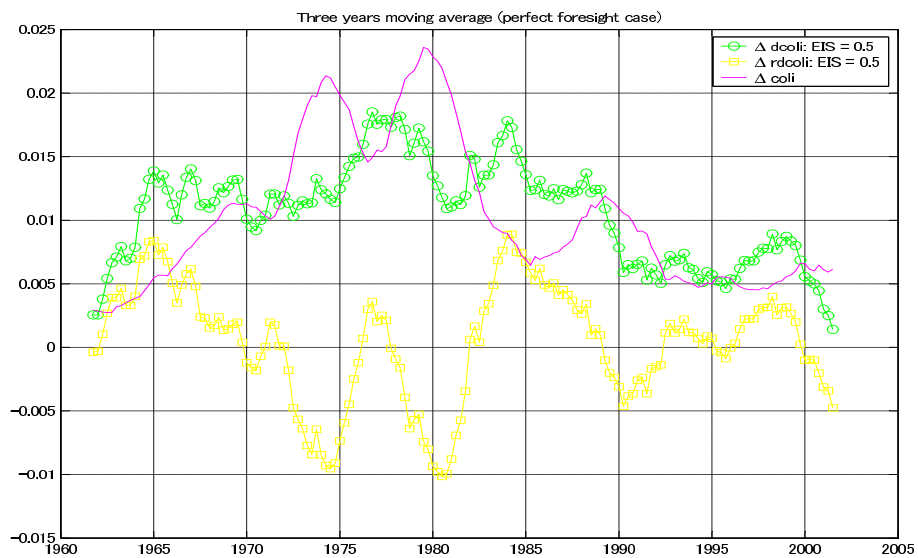


Figure 4: Three-years moving average of Δdcoli , Δdcolis , and Δrdcoli (for Δdcoli and Δrdcoli , EIS = 0.5): the perfect foresight case.

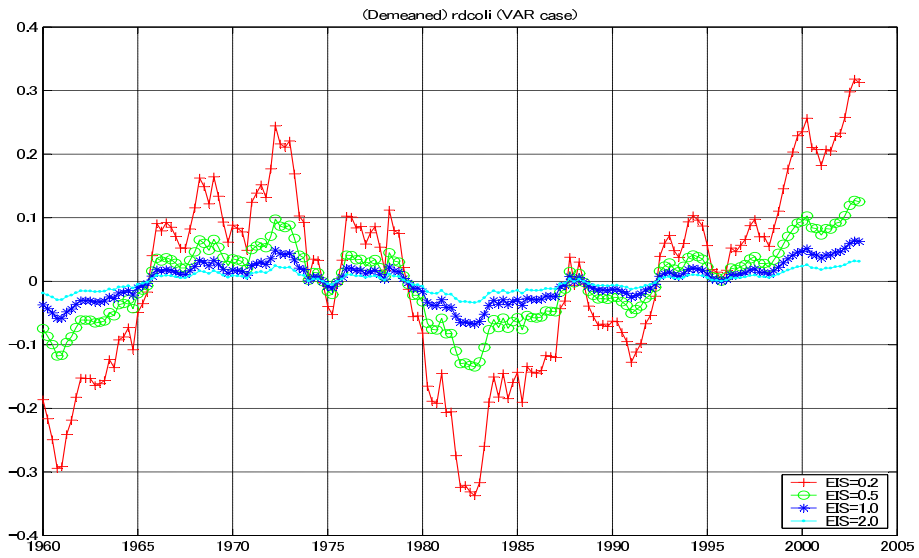


Figure 5: Demeaned rdcolis: the VAR case.

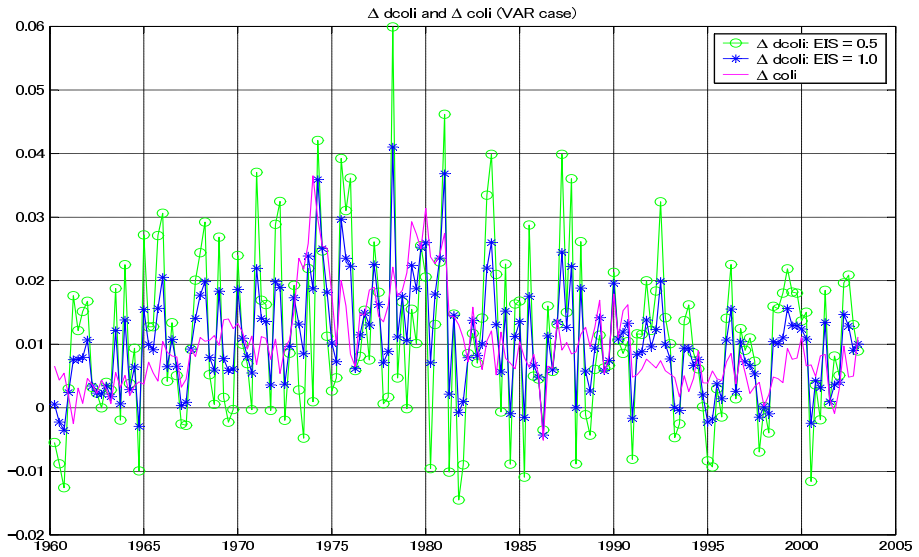


Figure 6: Δdcoli and Δcoli (for DCOLI, $\text{EIS} = 0.5$ and 1.0): the VAR case.

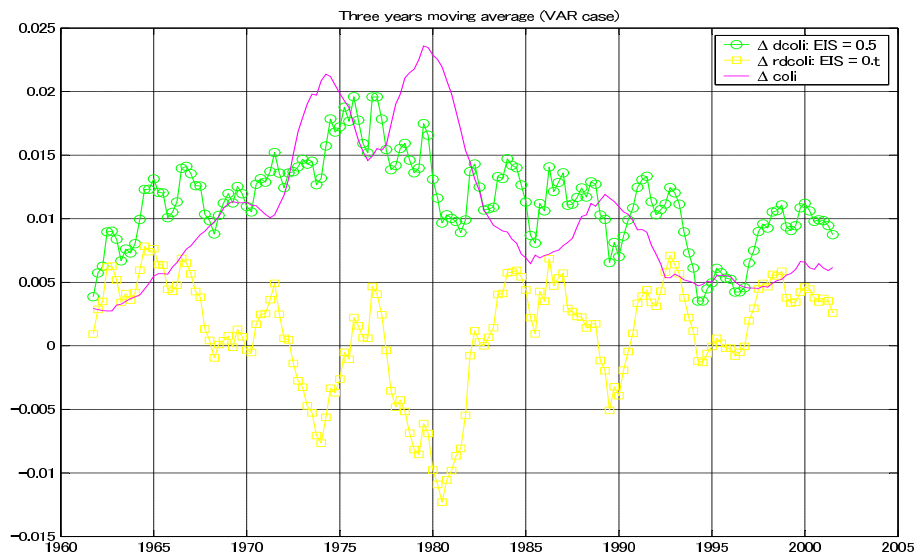


Figure 7: Three-years moving average of $\Delta colt$, $\Delta dcoli$, and $\Delta rdcoli$ (for $\Delta dcoli$ and $\Delta rdcoli$, $EIS = 0.5$): the VAR case.

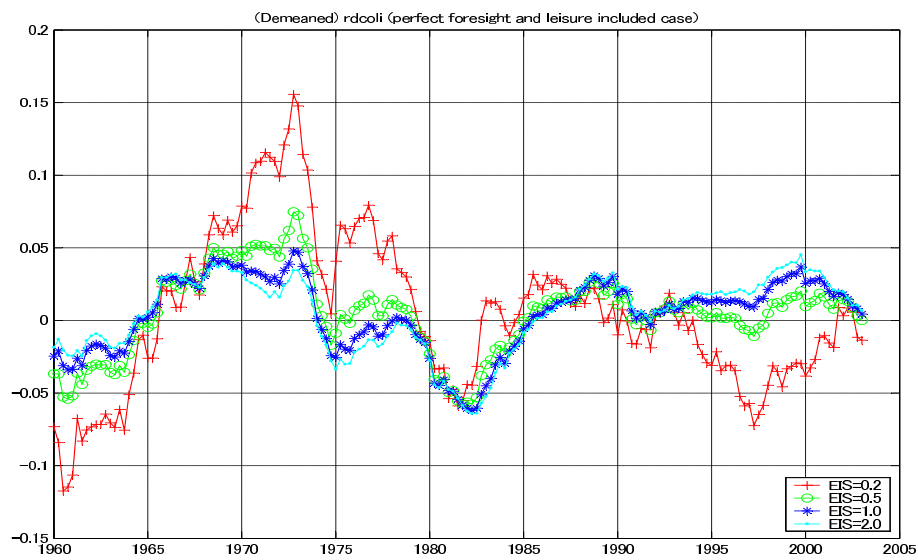


Figure 8: Demeaned rdcolis: the leisure considered case.

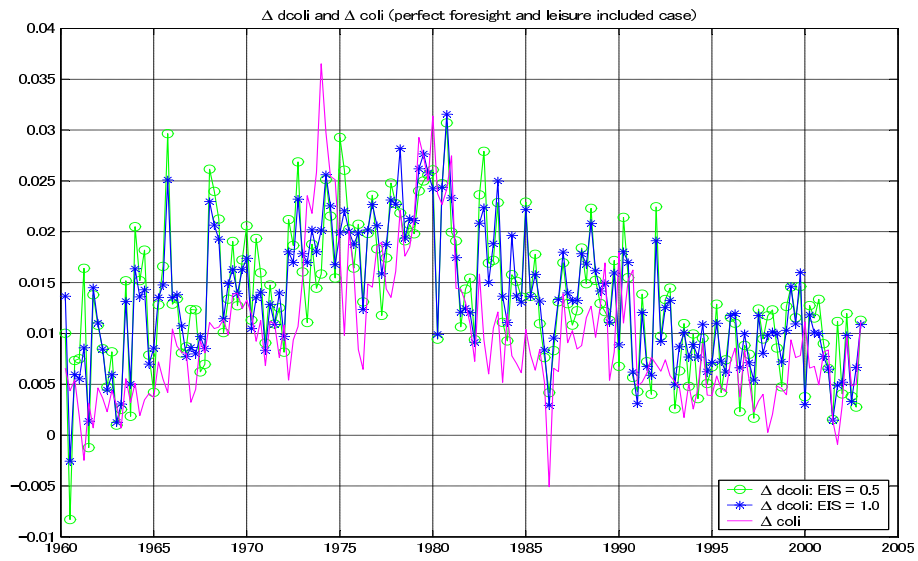


Figure 9: Δdcoli and Δdcolis (for DCOLI, EIS = 0.5 and 1.0): the leisure considered case.

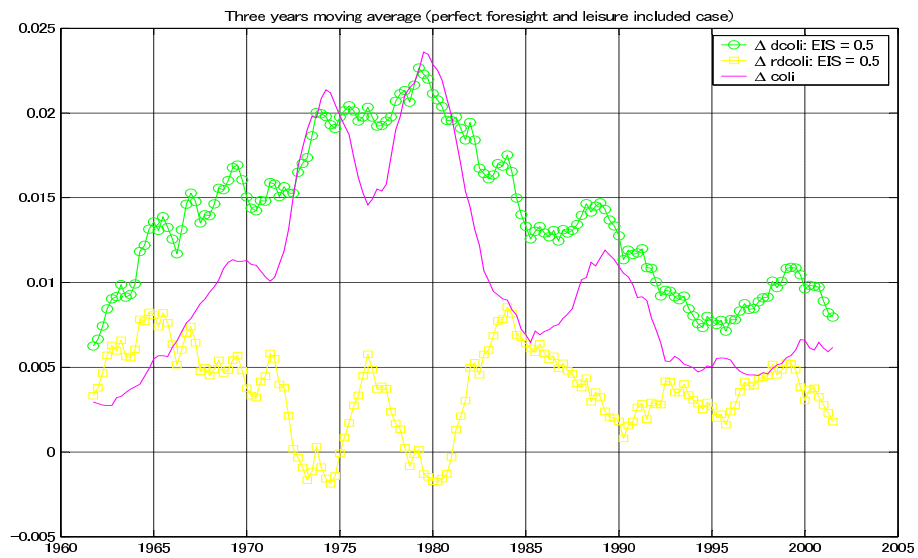


Figure 10: Three-years moving average of Δcoli , Δdcoli , and Δrdcoli (for Δdcoli and Δrdcoli , EIS = 0.5): the leisure considered case.