Aggregate Fluctuations of Discrete Investments

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Abstract

This paper demonstrates endogenous fluctuations of aggregate investments when firm-level investments follow an (S,s) policy and exhibit strategic complementarity. We present a method to characterize the aggregate fluctuations that arise from the interaction of the (S,s) policies. A closed-form distribution function of the output growth rate is derived in general environments. We show that the growth rate has a strictly positive variance even when the number of firms tends to infinity if the production exhibits constant returns to scale and the real wage and interest rate are fixed.

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1 Introduction

This paper presents a method to analyze the endogenous fluctuations of aggregate investments which arise from the interaction of the lumpy behavior of investments at the firm level. It demonstrates that the endogenous aggregate fluctuations can have a significant magnitude even when there are infinitely many agents if the micro-level discrete investments exhibit strategic complementarity. This result obtains when the strategic complementarity overwhelms the law of large numbers effect in which idiosyncratic shocks quickly cancel out with each other.

Recent developments in empirical studies on firm-level investments motivate this paper. Researchers have shown the importance of the discrete investment in the course of a firm's capital adjustment. Doms and Dunne (1998) found that the capital at the establishment level is adjusted only occasionally but by a jump. This finding led macroeconomists to investigate the aggregate consequence of the micro-level lumpy adjustments. For example, based on the similar empirical findings, Cooper, Haltiwanger, and Power (1999) and Caballero and Engel (1999) highlighted the effects of the lumpy investments in the aggregate fluctuation of investments. This paper presents a model in which the lumpy investment plays the central role to generate aggregate fluctuations.

The question of how to analyze the aggregate fluctuations that arise from micro-level discreteness, or more generally, micro-level nonlinearity, has been tackled by the literature on (S,s) economies and on interaction-based models independently. The (S,s) literature has developed an analytical method for the aggregate fluctuations without abstracting from the agent heterogeneity (Caplin (1985); Caplin and Leahy (1997); Danziger (1999); Fisher and Hornstein (2000); Thomas (2002); Khan and Thomas (2003)). Early development of the theory on (S,s) economies (Caplin and Spulber (1987); Caballero and Engel (1991)) revealed a robust tendency that the distribution of agents in the inaction band converges to a uniform distribution in one-sided (S,s) economies in which the micro adjustment occurs only in one direction. At the uniform distribution, the adjustment at the extensive margin works exactly like the adjustment at the intensive margin, and thus the aggregate behavior does not differ from the smoothly-adjusting case (the "neutrality" result). To the contrary, the models of interactions and nonlinear dynamics have focused on the possibility of endogenous fluctuations arising from the micro-level nonlinearity, such as in Brock and Hommes (1997), Glaeser and Scheinkman (2000), Brock and Durlauf (2001), and Topa (2001). This paper develops a method to analyze the aggregate fluctuations of an (S,s) economy by using the intuition of the interaction-based models.

We employ a standard multi-sectoral business cycles model by Long and Plosser (1983) to approach the question. Long and Plosser showed that the sectors can co-move without any common shocks when they are linked by input-output relations. This qualitative co-movement, however, has been questioned in terms of its quantitative significance. Dupor (1999) showed formally that the aggregate fluctuation in the Long-Plosser economy follows the law of large numbers, in which the aggregate variance shrinks linearly to the number of sectors. Horvath (2000) argued that the law of large numbers effect can be slowed down by a sparse input-output matrix. In this paper, we propose that a nonlinear response of firms leads to an aggregation mechanism that is different from the intuition of the law of large numbers.

We introduce lumpy capital adjustments in the multi-sectoral framework similar to Kiyotaki (1988) and Galí (1994). The model consists of many monopolistic firms that produce differentiated goods. Suppose that a capital adjustment is a discrete decision. An investment by a firm increases the aggregate capital and output in the next period. Because of the aggregate demand externality, the higher output induces the other firms to produce more in the next period and thus to invest more in this period. Then, there is a chance of a chain reaction of investments in which one firm's investment triggers another's. We formalize this chain reaction as a fictitious best response dynamics that converges to an equilibrium. The size of the chain reaction depends on the configuration of firms' positions in the inaction band. Even when the evolution of the configuration is solely driven by physical depreciation of capital and occasional capital adjustments, the evolution of the aggregate capital can be quite complex. By approximating the configuration by a vector of random variables, we obtain the analytical characterization of aggregate investments.

This paper delivers the following results. First, an asymptotic distribution function of the aggregate capital fluctuation is derived when the number of firms tends to infinity. The distribution has a heavier tail than the normal distribution. The fat tail indicates that the size of aggregate investment is sensitive to the detailed configuration of firms' positions in the inaction band. This sensitivity to the detailed configuration causes the aggregate investment to exhibit endogenous fluctuations in the course of evolution of the capital configuration driven by the depreciation and lumpy investments. Secondly, we show that the variance of the aggregate fluctuation does not vanish at the infinite limit of the number of firms when the technology exhibits constant returns to scale and when the wage and interest rate are fixed. Even though an economy consists of infinitely many firms, the nonlinear behavior at the firm level does not cancel out with each other in aggregation. This result forms a striking contrast to the sectoral models which lack a strong amplification mechanism of idiosyncratic shocks due to the law of large numbers. Thirdly, we compute the equilibrium path numerically. This confirms the emergence of endogenous aggregate fluctuations in a completely deterministic environment. The sensitivity analysis shows that constant-returns-to-scale is an important environment for the fluctuations. When the wage and interest rate are fixed and returns to scale are constant, the equilibrium of the product markets with monopolistic suppliers exhibits a "fragile" property. In this environment, the size of the chain reaction of investments depends crucially on the detailed configuration of the positions in the inaction band. The simulation also points to the presence of replacement echo effects in the time series.

Important contributions precede this research on endogenous macroeconomic fluctuations due to the synchronized timing of firms' discrete actions. Shleifer (1986) demonstrated that the event of synchronized actions can recur deterministically and endogenously through self-fulfilling expectations of periodic adjustments. Jovanovic (1987) highlighted the case where idiosyncratic shocks give rise to aggregate risks. Durlauf (1991, 1993) showed further that the aggregate size of synchronized actions depends on the detailed configuration of agents' states as well as can exhibit a long-run path-dependence. We extend this literature by presenting a sharper characterization of the synchronization in a standard multisectoral model. We obtain an analytical expression of the fluctuation magnitude with parameters which can be estimated from firm-level data. The model also identifies a mechanism of aggregate fluctuations which does not rely on a strong informational coordination or a strong nonlinearity such as the increasing returns to scale. The mechanism is best understood as a globally-coupled case of self-organized criticality (Bak, Chen, Scheinkman, and Woodford (1993); Scheinkman and Woodford (1994)). The dynamics of the capital profile organizes itself to a critical configuration, recurringly, at which the distribution of the number of synchronized firms exhibits a power-law tail.

The rest of this paper is organized as follows. Section 2 presents a simplified model and the main results of the paper. Section 3 shows that the analytical results hold in a dynamic general equilibrium setup. Numerical simulations of deterministic equilibrium paths confirm the analytical results. Section 4 concludes.

2 Simple Model

2.1 Model

This section provides a simplified presentation of our model and main results on endogenous investment fluctuations. Consider N firms that produce differentiated goods with a production function:

$$y_i = k_i^\theta \tag{1}$$

Each firm is endowed with an initial capital $k_{i,0}$ that is naturally depreciated at rate δ before it is available for current production. Each firm chooses an "investment" x_i to adjust the production level.[1]

$$k_i = (1-\delta)k_{i,0} + x_i \tag{2}$$

The investment x_i consists of a composite good produced by a symmetric CES function:

$$x_i = \left(\sum_{j=1}^{N} z_{i,j}^{(\eta-1)/\eta}\right)^{\eta/(\eta-1)} N^{1/(1-\eta)} \tag{3}$$

where $\eta > 1$ is the elasticity of substitution. The constant markup of firms is thus $\mu \equiv \eta/(\eta-1) > 1$. Firm i's profit is defined as $\pi_i = p_i y_i - \sum_{j=1}^{N} p_j z_{i,j}$. Households own the firms and collect all the profits. The representative household consumes a composite consumption good that is produced similarly as the capital good:

$$C = \left(\sum_{j=1}^{N} z_{c,j}^{(\eta-1)/\eta}/N\right)^{\eta/(\eta-1)}. \tag{4}$$

[1]Precisely speaking, x_i should be called intermediate inputs rather than investments in this static model, since x_i takes the resources produced by using (k_i). We will consider in Section 3 the case in which x_i is truly the investment that takes resources produced by using the capital in the previous period. We will see that the results obtained in this static model still hold in the dynamic model.

We define an aggregate output index $Y \equiv \left(\sum_{i=1}^{N} y_i^{(\eta-1)/\eta}/N\right)^{\eta/(\eta-1)}$. A price index is defined as $P \equiv \left(\sum_{i=1}^{N} p_i^{1-\eta}/N\right)^{1/(1-\eta)}$ and normalized to one. An aggregate capital index is defined as $K \equiv \left(\sum_{i=1}^{N} k_i^{\rho}/N\right)^{1/\rho}$ where $\rho \equiv \theta/\mu$. Note that $Y = K^\theta$ by construction. By following the procedure of Dixit and Stiglitz, we obtain the demand function for good i:

$$y_i = p_i^{-\eta} Y \tag{5}$$

Finally, we assume that the firm's capital choice is restricted by a binary set:

$$k_i \in \{(1-\delta)k_{i,0}, \lambda(1-\delta)k_{i,0}\} \tag{6}$$

where $\lambda(1-\delta) > 1$. The capital k_i has to be either at the depreciated level $(1-\delta)k_{i,0}$ or the depreciated level multiplied by the lumpiness parameter λ. This binary constraint is equivalent to assuming that the firm can choose the gross investment rate $x_i/k_{i,0}$ only either at $(\lambda-1)(1-\delta)$ or 0, namely, a lumpy investment or an inaction. This constraint is a shortcut for modeling the lumpy behavior which typically occurs as an optimal investment policy under fixed costs. This discreteness assumption is the only departure from the usual model of monopolistic product markets. Our main objective is to examine the aggregate consequence of the nonlinear behavior of firms induced by the discreteness constraint.

An equilibrium is defined as a pair of price vector (p_i) and allocation $(z_{i,j}, z_{c,j})$ such that the household maximizes its utility $U(C)$ subject to the budget constraint $\sum_i p_i z_{c,i} = \sum_i \pi_i/N$, that the firms maximize their profits π_i subject to the production function (1), the capital accumulation (2), the demand function for good i (5), and the discrete investment (6), and that satisfies the equilibrium conditions $y_i = \sum_j z_{j,i} + z_{c,i}$ for any good i. Note that the levels of y_i, k_i, x_i and their average variables are independent of N, whereas the derived demands $z_{c,j}, z_{i,j}$ are of order $1/N$, due to the normalization of the CES functions (3,4) by

N.

We assume $\mu > \theta$ so that the profit function is strictly concave with respect to k_i. The optimal strategy for a firm is to adjust the capital only if $(1-\delta)k_{i,0}$ is sufficiently away from the "desired" level of capital that would maximize the profit if the capital were chosen from a continuous set. Thanks to the discreteness assumption, we can easily derive the optimal inaction range for capital k_i. Let k^* denote the lower bound of the inaction band. The upper bound is λk^*. The optimal bounds must satisfy an indifference condition $\pi(k^*) = \pi(\lambda k^*)$. Then the bound is solved as:

$$k^* = a_0 K^{\phi} \tag{7}$$

$$\phi \equiv \frac{\mu - 1}{\mu/\theta - 1} \tag{8}$$

where a_0 is a constant $a_0 = ((\lambda^{\rho} - 1)/(\lambda - 1))^{1/(1-\rho)}$. The parameter ϕ represents the degree of strategic complementarity among firms. We have the following property immediately.

Lemma 1 $\phi \lesseqgtr 1$ *if and only if* $\theta \lesseqgtr 1$ *where the double signs correspond with each other.*

The spillover effect on the actual capital k_i is nonlinear because of the threshold policy. The average capital level K affects the threshold, but it may or may not induce an adjustment of k_i. The individual capital is insensitive to a small perturbation in the average capital, while it synchronizes with the average capital if the perturbation is large. The strength of the synchronization is determined by ϕ.

We restrict the support of the initial capital $k_{i,0}$ to an inaction band in order to capture the stationary behavior of the model economy. In this way, only the firms whose initial capital is near the lower bound will go below the inaction band due to the depreciation and increase the capital by a jump. We further assume for our first propositions that the initial capital $k_{i,0}$ is randomly drawn from the uniform distribution over the inaction band.

The randomness of the initial capital is interpreted as an idiosyncratic shock in this section, whereas in the next section it is reinterpreted as an unconditional distribution of the capital that evolves deterministically over time. Under the uniformity assumption, the equilibrium aggregate capital can be derived as $\bar{K} = (a_0((\lambda^\rho - 1)/(\rho \log \lambda))^{1/\rho})^{1/(1-\phi)}$ for $\phi < 1$ when there are a continuum of firms. The lower band of the inaction band is then $\bar{k} = a_0 \bar{K}^\phi$. Thus, the uniformity assumption is expressed as follows.

Assumption 1 $\log k_{i,0}$ *is a random variable following a uniform distribution over* $[\log \bar{k}, \log \lambda + \log \bar{k})$.

The uniformity assumption is not essential to our result, as we see later in Proposition 5. However, the assumption not only simplifies propositions greatly, but also corresponds to a robust feature of one-sided (S,s) economies. Let s_i denote a firm's position in the inaction band normalized by the band width: $s_i = (\log k_i - \log k^*)/\log \lambda$. The position s_i always takes a value between 0 and 1 at equilibrium. The uniform distribution of s_i is shifted to itself if s_i is decreased by depreciation and if the firms follow the one-sided (S,s) policy, as is the case in Caplin and Spulber (1987). Hence, the uniform distribution is an invariant measure of s_i in a typical one-sided (S,s) model. This property holds in our case if there are a continuum of firms: the equilibrium position s_i follows a uniform distribution if the initial position $s_{i,0} = (\log k_{i,0} - \log \bar{k})/\log \lambda$ follows the uniform distribution. Moreover, Caballero and Engel (1991) show that the position converges to the uniform distribution if its dynamics contain a random component whose distribution flattens over time. They also show that the heterogeneity of lumpiness λ_i (as well as the depreciation δ_i in our model) contributes to the convergence of a cross-section distribution of s_i to the uniform distribution. We will see in the dynamic model that the heterogeneity indeed drives $s_{i,t}$ to follow the uniform distribution.

The equilibrium condition is summarized by the inaction band: $k_i \in [k^*, \lambda k^*)$ for all

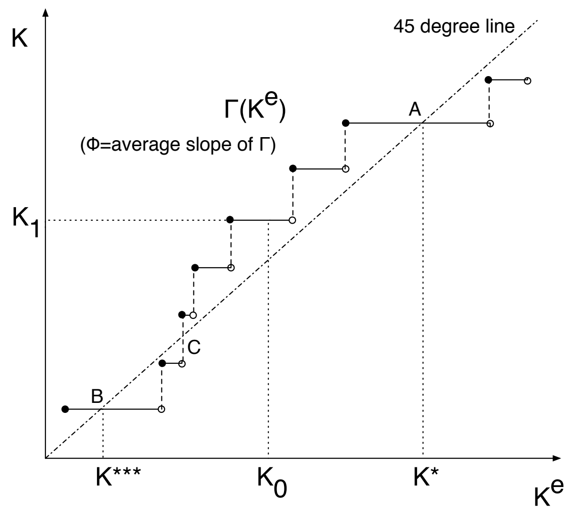

Figure 1: Aggregate reaction function Γ. K^* and K^{**} respectively show the equilibrium selected by Equilibrium Selection 1 and 2.

i. An equilibrium is a mapping from the initial capital vector $(k_{i,0})$ to the capital vector (k_i) that satisfies the inaction band. The probability measure for $k_{i,0}$ and the equilibrium mapping thus yield the probability measure for the equilibrium aggregate capital K. We will focus on the aggregate fluctuation that is represented by the distribution of the equilibrium K. The equilibrium aggregate capital is a fixed point of the aggregate reaction function Γ that is defined for each realization of $(k_{i,0})$ as follows:

$$K = \Gamma(K^e) \equiv \left(\sum_{\{i:(1-\delta)k_{i,0}<a_0(K^e)^\phi\}} \frac{(\lambda(1-\delta)k_{i,0})^\rho}{N} + \sum_{\{i:(1-\delta)k_{i,0}\geq a_0(K^e)^\phi\}} \frac{((1-\delta)k_{i,0})^\rho}{N} \right)^{1/\rho} \quad (9)$$

where K^e is an expected aggregate capital. Namely, $\Gamma(K^e)$ represents the aggregate capital when each firm optimally responds to the expected aggregate capital K^e. As depicted in Figure 1, Γ is a non-decreasing step function.

We note that multiple equilibria may exist as shown by points A and B in Figure 1. We need an equilibrium selection mechanism to pin down a unique solution for each draw of initial capital profile. We define two sets of the equilibrium selection mechanism. The first mechanism depends on the aggregate reaction function Γ:

Equilibrium Selection 1 *For each initial capital vector, pick the equilibrium aggregate capital K^* that has the minimum $|K - K_0|$ among equilibria K that satisfy* $\text{sign}(K - K_0) = \text{sign}(\Gamma(K_0) - K_0)$.

This mechanism selects the equilibrium aggregate capital that is closest to the initial aggregate capital in the direction toward which the firms are induced to adjust by the initial aggregate capital. In the case of Figure 1, this mechanism selects point A. Vives (1990) showed that the equilibrium selected by this mechanism can be reached as a convergent point of the best response dynamics $K_u = \Gamma(K_{u-1})$ starting at K_0. Cooper (1994) supported the use of this selection mechanism in macroeconomics on the grounds that the best response dynamics is a realistic tatonnement process in a situation where many agents interact with each other. The only information needed for an agent to make decisions in the tatonnement is the aggregate capital level. Because of this parsimony on public information, this selection mechanism excludes the possibilities of big jumps that arise from a purely informational coordination among agents.

The second equilibrium selection mechanism we use is simpler:

Equilibrium Selection 2 *For each initial capital vector, pick the equilibrium aggregate capital K^{**} that has the minimum $|K - K_0|$ among all equilibria K.*

By this mechanism, we construct the least volatile fluctuations of aggregate capital possible in equilibrium. In Figure 1, this mechanism selects whichever K^* or K^{***} is the closer to K_0.

2.2 Results

In this section, we derive the distribution of the aggregate capital fluctuations. We first analyze the fluctuation of the equilibrium selected by the first selection mechanism, and then proceed to analyze the one selected by the second mechanism.

Define $q \equiv \log \lambda / |\log(1-\delta)|$ as the natural frequency of the capital adjustment of a firm. In a dynamic context, q is the number of periods between a firm's two successive adjustments of capital when the aggregate capital is stationary. The inverse of the frequency, $1/q$, is the fraction of firms that engage in the lumpy investment in a period if there are a continuum of firms distributed uniformly.

Define $m_1 \equiv N(\log \Gamma(K_0) - \log K_0)/\log \lambda$, which represents the initial gap in aggregate capital measured in units of the number of firms. If $m_1 = 0$, then $K^* = K_0$ constitutes the equilibrium aggregate capital. Otherwise, $K^* \neq K_0$. Define the equilibrium aggregate capital growth rate $g^* \equiv \log K^* - \log K_0$. We obtain the asymptotic probability distribution of the capital growth rate as follows.

Proposition 1 *Under Assumption 1 and Equilibrium Selection 1, Ng^* converges in distribution to $(m_1 + M)\log \lambda$, where M conditional on m_1 follows a symmetric probability distribution function:*

$$\Pr(|M| = w \mid |m_1|) = |m_1| e^{-\phi(w+|m_1|)} \phi^w (w + |m_1|)^{w-1} / w! \tag{10}$$

The tail of the distribution function is approximated by:

$$\Pr(|M| = w \mid |m_1|) \sim (|m_1| e^{(1-\phi)|m_1|} / \sqrt{2\pi}) e^{-(\phi - 1 - \log \phi) w} w^{-1.5} \tag{11}$$

The initial gap $m_1/\sqrt{N}$ asymptotically follows a normal distribution with mean zero and *variance:*

$$\sigma_1^2 = \frac{1-\lambda^{-2\rho/q}}{2\rho\log\lambda} - \left(\frac{1-\lambda^{-\rho/q}}{\rho\log\lambda}\right)^2 \tag{12}$$

The proof is deferred to Appendix C. Here we outline the proof in order to elicit the mechanism behind the distribution. We utilize the best response dynamics of the capital profile as a workhorse for characterizing the aggregate fluctuations. The initial state and subsequent dynamics are defined as follows:

$$k_{i,1} = \begin{cases} \lambda(1-\delta)k_{i,0} & \text{if } (1-\delta)k_{i,0} < k_0^* \\ (1-\delta)k_{i,0} & \text{otherwise} \end{cases} \tag{13}$$

$$k_{i,u+1} = \begin{cases} \lambda k_{i,u} & \text{if } k_{i,u} < k_u^* \\ k_{i,u}/\lambda & \text{if } k_{i,u} \geq \lambda k_u^* \\ k_{i,u} & \text{otherwise} \end{cases} \tag{14}$$

where K_u and k_u^* are constructed by $(k_{i,u})$ as before: $K_u = \left(\sum_i k_{i,u}^{\rho}/N\right)^{1/\rho}$ and $k_u^* = a_0 K_u^{\phi}$. Note that the definition of K_u is consistent with the aggregate response dynamics we defined earlier: $K_u = \Gamma(K_{u-1})$. The mean number of firms that adjust in the first step is $N|\log(1-\delta)|/\log\lambda$. Their adjustments may or may not exactly balance with the aggregate capital depreciation, i.e., K_1 may not coincide with $\Gamma(K_0)$. If not, the optimal lower bound is updated and the adjustments in the second step take place. This procedure is iterated until there are no more firms that newly adjust.

Define m_u for $u = 2, 3, \ldots, T$ as the number of firms that adjust capital upward in step u. By convention, m_u is negative if the firms adjust downward. The number of adjusting firms are positive (negative) for all steps if $m_1 > 0$ ($m_1 < 0$). Define $M \equiv \sum_{u=2}^{T} m_u$ as the total number of firms that adjust capital subsequently after the initial deviation of K_1

from K_0. T is the stopping time of the best response dynamics, i.e., $T \equiv \min_{u:m_u=0} u$. An equilibrium capital vector is defined by the convergent point of the dynamics, $k_i = k_{i,T}$. $m_1 + M$ indicates the total deviation of the investment from the stationary level in units of the number of firms.

In the first step toward Proposition 1, we show that the capital growth rate is asymptotically proportional to the number of firms that adjust.

Lemma 2 *Under Assumption 1,* $N(\log K_{u+1} - \log K_u)$ *converges to* $m_{u+1} \log \lambda$ *as* $N \to \infty$ *almost surely for* $u = 1, 2, \ldots, T-1$.

Proof is in Appendix A. Lemma 2 implies that $N(\log K - \log K_0) \to (m_1 + M) \log \lambda$. Thus, the computation of the growth of aggregate capital reduces to counting the total number of adjusting firms. We then show that the number of adjusting firms in the best response dynamics asymptotically follows a Poisson branching process.

Lemma 3 *Under Assumption 1,* m_u *for* $u = 2, 3, \ldots, T$ *asymptotically follows a branching process, in which each firm in* m_u *bears firms in step* $u+1$ *whose number follows a Poisson distribution with mean* ϕ.

Proof is in Appendix B. A branching process is an integer stochastic process of a population. Each individual ("parent") in a generation bears a random number of "children" in the next generation. In a Poisson branching process, the number of children borne by a parent is a Poisson random variable. It is known that a branching process converges to 0 in a finite time with probability 1 if the mean number of children borne by a parent is less than or equal to 1 (see Feller (1957)). This fact confirms that the best response dynamics stops in a finite time T with probability 1 when $\phi \leq 1$. Thus, the best response dynamics is a valid algorithm of equilibrium selection even when $N \to \infty$.

The cumulative population size of the Poisson branching process is known to follow the Borel-Tanner distribution (Kingman (1993)). By combining the Borel-Tanner distribution with the Poisson distribution for m_2, we obtain the desired distribution (10).

We approximate the tail part of the distribution (10) by (11) by applying Stirling's formula. The distribution (11) shows that the normalized aggregate capital growth rate conditional on m_1 asymptotically follows a gamma-type distribution which combines a power function $w^{-1.5}$ and an exponential function $e^{-(\phi-1-\log\phi)w}$. Note that $\phi - 1 - \log\phi > 0$ for $\phi < 1$. Since the exponential function declines faster than the power function, the tail distribution is dominated by the exponential when $\phi < 1$. By Lemma 1, the condition $\phi < 1$ is equivalent to the decreasing returns to scale. As the returns-to-scale becomes constant, ϕ becomes one. In that case, the exponential part disappears, and thus the distribution converges to a pure power law.

In fact, the total population of any branching process is known to follow the gamma-type distribution with the power exponent 0.5 as in (11). The gamma-type often appears as the distribution for waiting time. The distribution of population in branching processes is closely related to the distribution of the first return time of a random walk, which has the same power-law exponent 0.5. In our case, the total number of adjusting firms is characterized by the waiting time for the best response dynamics to converge.

The gamma-type distribution implies a heavier tail than the normal distribution and an excess kurtosis. Thus our distribution is consistent with the finding of Caballero and Engel (1999) that the empirical investment rates at the sectoral level have excess kurtosis. They also found the skewness, which can be generated in our model when the initial distribution of capital is not uniform. Khan and Thomas (2003) obtained the excess kurtosis and the skewed response of the aggregate investment by numerically simulating a lumpy investment model under fixed prices. Our model shares the feature that the response of the aggregate investment to shocks depends on the cross-section distribution of capital.

Whether the tail obeys an exponential decay or a power decay has important implications for the moments of the distribution, since the existence of moments is determined by the tail behavior. If the tail decays exponentially, then any ξ-th moment exists, because $\int_0^\infty x^\xi e^{-x} dx$ is a gamma function and thus finite. To the contrary, if the tail decays in power with exponent ζ, then only the moments lower than the ζ-th exist, since $\int^\infty x^\xi x^{-\zeta-1} dx$ is finite only for $\xi < \zeta$. For our case where the exponent of the power law is 0.5, even the mean diverges if there is no exponential truncation.

The degree of strategic complementarity, ϕ, determines the speed of exponential truncation of the distribution. At $\phi = 1$, the exponential term disappears and the distribution becomes a pure power law without finite mean. This is because the mean number of children per parent, that is equal to ϕ, determines the trend growth of the population in the branching process. The population size of the n-th generation has mean ϕ^n if the population is originated by one individual. The population diverges to infinity with a positive probability when the process is supercritical, $\phi > 1$, whereas the population decreases to zero with probability 1 if subcritical, $\phi < 1$. Thus, $\phi = 1$ is the critical point at which the population size decreases to zero with probability 1 and yet the mean population size diverges.

The possibility of a power-law distribution of sectoral propagation was first pointed out by Bak et al. (1993) along the line of literature on self-organized criticality. The point of the literature is that the critical phenomena, which are broadly associated with the power-law distributions, can occur at the sink of a class of dynamical systems, whereas such criticality had been believed to require a fine tuning of parameters. The "self-organization" mechanism to arrive at a critical point can be interpreted as the convergence of s_i to the uniform distribution in a dynamic version of our model. The result differs in the exponent of the power-law distribution, which is 0.5 in our model and 1/3 in Bak et al. The difference arises from the topology of the network. Bak et al. assumed a two-dimensional lattice network in which two avalanches started from neighboring sites can overlap. This leads to the longer chain of reaction and thus the flatter power-law tail. Our model features a standard equilibrium market model that is essentially dimensionless in the firms' network and thus corresponds to an infinite-dimension case of the lattice models which yields the exponent 0.5 (Grimmett (1999)).

The distribution of M conditional on m_1 converges to a pure power-law distribution as ϕ approaches to 1. With the exponent 0.5, the power-law distribution does not have either mean or variance. The conditioning variable m_1, which is the initial gap in the best response dynamics, obeys the law of large numbers and its variance decreases linearly in N. It turns out that these two effects cancel out in the unconditional variance of $(m_1 + W)/N$, as we state in the following proposition.

Proposition 2 *Suppose that g^* follows the distribution in Proposition 1. Then, the standard deviation of g^* converges to a non-zero constant as $\phi \to 1$ and $N \to \infty$. The limit standard deviation is* $(\log \lambda)\sqrt{(2/\pi)(\sigma_1 + 1/3)\sigma_1}$.

Proof is deferred to Appendix D. The main idea is following. Proposition 1 showed that $m_1/\sqrt{N}$ asymptotically follows a normal distribution with finite variance. This implies that the absolute value $|m_1|$ has mean that scales as $\sqrt{N}$. Namely, the average initial gap of the best response dynamics in units of the number of firms increases as $\sqrt{N}$. Proposition 1 also showed that $Ng^*/\log\lambda - m_1$ conditional on $m_1 = 1$ follows the power-law distribution with exponent 0.5 if $\phi = 1$. Then, the variance of Ng^* conditional on $m_1 = 1$ diverges as $N^{1.5}$, because $\int^N x^2 x^{-1.5} dx \sim N^{1.5}$. Combining these two results, we obtain that Ng^* unconditional on m_1 has variance scaling as N^2, since Ng^* can be divided into $\sqrt{N}$ sets of sub-population each of which has variance that scales as $N^{1.5}$. Hence the variance of g^* scales as N^0.

The argument above shows that the power-law distribution is essential to obtain the non-trivial variance of g^*. The key environment that induces the emergence of the power law is $\phi = 1$, or equivalently, the constant returns to scale $\theta = 1$. It appears counterintuitive that the aggregate variance does not converge to zero when there are only idiosyncratic shocks to the initial capital. However, it is a natural consequence of the usual properties of equilibrium under the constant returns to scale. At the equilibrium factor prices, firms cannot determine the optimal size of production under the constant returns to scale, because firms are indifferent across production levels. Thus, any level of production can happen at the equilibrium factor price. The equilibrium is determinate in our model because of the frictions in firms' behavior. The indeterminacy of the constant returns to scale economy reappears, however, in the form of the power-law distribution. The power-law distribution is scale-free, i.e., the shape of the distribution does not depend on the measuring unit.[2] Due to the scale-free property, the economy experiences non-trivial fluctuations of the synchronized adjustments of firms regardless of the size of the economy.

The limit standard deviation of the growth rate in Proposition 2 is determined by the lumpiness parameter λ and the periodicity q of the capital oscillation at the firm level. Numerical examples for the standard deviation are shown in Table 1. First, we note that the fluctuation magnitude shows little dependence on the markup rate. In fact the standard deviation is not significantly changed even when the markup rate goes to infinity, at which σ_1^2 is simplified to $(1-1/q)/q$. This implies that the lumpiness parameter $\log \lambda$ has an almost proportional effect on the standard deviation when the periodicity q is held constant.

Secondly, Table 1 suggests that the empirically plausible range of lumpiness can generate the magnitude of fluctuations observed in the business cycles frequency. There exist some empirical estimates for the lumpiness of capital adjustments. Doms and Dunne (1998)

[2] Newman (2005) provides an illuminating survey of the scale-free distribution and the power law as well as the critical phenomena and the self-organized criticality.

markup $(\mu - 1)$		0.02				0.2			
lumpiness $(\log \lambda)$		0.02	0.05	0.1	0.2	0.02	0.05	0.1	0.2
	4	0.92	2.29	4.56	9.07	0.92	2.29	4.57	9.10
periodicity (q)	6	0.82	2.04	4.07	8.11	0.82	2.04	4.08	8.13
	8	0.75	1.87	3.73	7.43	0.75	1.87	3.73	7.44

Table 1: Limit standard deviations of aggregate capital growth rate (percent)

observe that over half of the plants in their data experience an annual capital adjustment of at least 37% of the capital. Cooper, Haltiwanger, and Power (1999) report that 20% of plants experience the gross investment-capital ratio more than 20% each year, which account for almost 50% of aggregate investment. Based on this observation, they choose 20% as a suitable threshold for defining a lumpy investment. Ellison and Glaeser (1997) report that the plant Herfindahl (the representative share of a plant's employment in an industry) is 2.8%. The plant Herfindahl may be interpreted as the lumpiness of investments if a capital adjustment is carried out by adjusting the number of plants. The aggregate fluctuation shown in Table 1 is sizable even for such small lumpiness.

Proposition 2 shows that the fluctuations of growth rates do not degenerate at the infinite limit of N if $\phi = 1$. The criticality condition $\phi = 1$ is equivalent to the constant returns to scale $\theta = 1$ by Lemma 1. We also note that the competitiveness of the market, μ, does not affect whether ϕ is at the critical level. In this sense, the returns to scale determines the "phase" of the spillover effects, whereas the markup rate only modifies the degree of complementarity without altering the phase. The criticality condition $\phi = 1$ is interpreted as the case of perfect strategic complementarity across firms. By perfect complementarity we mean that a proportional increase in capital of all the other firms induces the same proportional increase in capital of a firm, if the increment is larger than the lumpiness. A shock smaller than the lumpiness, however, does not cause a symmetric movement across firms. Thus, the firm's investment behavior at the criticality may be summarized as the local inertia combined with the global perfect strategic complementarity.

Proposition 2 demonstrates that, no matter how large the aggregative system is, the nonlinearity of the individual behaviors can add up to non-degenerate aggregate fluctuations. This is the theme pursued by Jovanovic (1987). His idea is that the multiplier effect, $1/(1-\phi)$ in our model, can be large enough to amplify an idiosyncratic shock to an aggregate fluctuation if ϕ approaches to 1 as $1-1/\sqrt{N}$. Our model shares the basic environment that the degree of strategic complementarity ϕ needs to approach to one. Our model differs in that the firm's response is nonlinear to the aggregate capital. The nonlinearity renders the multiplier effect quite sensitive to the detailed configuration of capital. It turns out that the multiplier effect follows a heavy tailed distribution, which causes the *average* multiplier effect to be the size of Jovanovic's. Thus, the idiosyncratic shocks are not simply magnified to the aggregate risks in our model. The essential factor for the aggregate fluctuations is the configuration of capital that determines the magnitude of the realized multiplier effect. In this sense, our result is analogous to Durlauf (1991).

Finally, we investigate the fluctuation magnitude of $g^{**} \equiv \log K^{**} - \log K_0$, the aggregate capital growth rate when the equilibrium is selected by Equilibrium Selection 2. We obtain the following result.

Proposition 3 *Under Assumption 1 and Equilibrium Selection 2, for a region of arbitrarily large N, there exists $\phi^* < 1$ such that the convergence of the variance of g^{**} to zero is not faster than $1/\sqrt{N}$ if $\phi \geq \phi^*$.*

Proof: For any large N, there exists ϕ^* close enough to 1 such that the exponential part in (11) has a negligible impact on the probability distribution for the region below N. Now consider the case $\Gamma(K_0) > K_0$ depicted in Figure 1. K^{***} is defined as the fixed point of Γ on the opposite side of K_0 from K^*. There exists a point between K^{***} and K_0 at which Γ

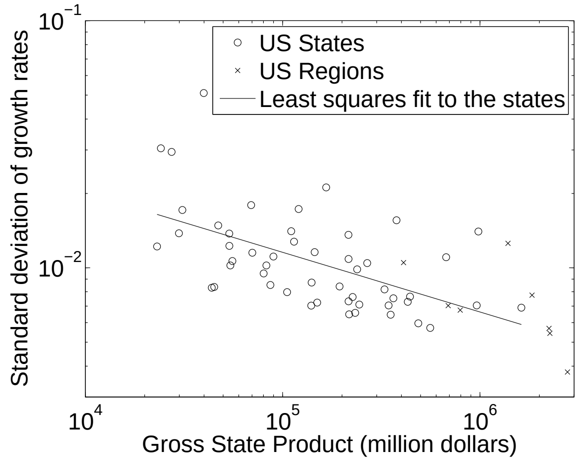

Figure 2: Dependence of the standard deviations of growth rates on the economy size

crosses the 45 degree line from below such as point C in Figure 1. By applying Proposition 1, the number of adjusting firms between B and C in Figure 1 follows the power law with exponent 0.5 if $\phi = 1$. Then, the tail distribution of $|g^{***}|$ cannot decay faster than the power function with exponent 0.5.

Suppose that $|g^{***}|$ follows the power-law tail with exponent 0.5. Since $|g^{*}|$ and $|g^{**}|$ are asymptotically independent conditional on m_1 and since $|g^{**}| = \min\{|g^{*}|, |g^{***}|\}$, we then have that $\Pr(\min\{|g^{**}| > g \mid m_1) = \Pr(|g^{*}| > g \mid m_1) \Pr(|g^{***}| > g \mid m_1) \propto g^{-0.5} g^{-0.5} = g^{-1}$. At the power exponent 1, the variance of g^{**} conditional on m_1 decreases as $\int^N x^2 x^{-2} dx / N^2 \sim 1/N$. We know that the mean of $|m_1|$ increases as $\sqrt{N}$. Then, proceeding as the proof of Proposition 2, we obtain that the variance of g^{**} decreases as $1/\sqrt{N}$. If the tail distribution of g^{***} decays more slowly than the power law with exponent 0.5, the variance of g^{**} also decreases more slowly than $1/\sqrt{N}$. □

Proposition 3 shows that, if we choose the least volatile equilibrium, the variance of the capital growth rate decreases to zero as N increases, but at the rate slower than the central limit theorem predicts. This again opens up the theoretical possibility that the lumpy investment at the micro level contributes to sizable aggregate fluctuations.

The slow decline of the variance corresponds to the empirical finding by Canning, Amaral, Lee, Meyer, and Stanley (1998) that the log standard deviation of GDP growth rates declines at the slope as flat as -0.15 when plotted against the log GDP across countries. Regional data also show this pattern. Figure 2 plots the standard deviation of the growth rates of the US states and BEA regions relative to the US growth rate. The gross state product (GSP) statistics were compiled by the Bureau of Economic Analysis. The state-level standard deviation has a slightly negative relation with the GSP size. If the state-specific fluctuations are entirely driven by the idiosyncratic technological shocks as in the Long-Plosser model, then the standard deviation should decline as a square root of the size of the economy as shown by Dupor (1999). Thus, the linear fit to the plot should have slope -0.5. However, the slope estimated by the linear regression is as flat as -0.24 (standard error 0.05). If we match the empirical slope by incorporating the aggregate shocks in the Long-Plosser model, then we need to assume that most of the state-specific fluctuations are due to the state-level aggregate shocks. Thus, the empirical pattern of the state fluctuations is hard to reconcile with the standard disaggregated model. The pattern is consistent with our model, however, in which the standard deviation of g^{**} declines at rate -0.25.

Let us finally examine the overall magnitude of the regional volatility in Figure 2. The standard deviation of the difference between the GSP growth rates and the US GDP growth rate is averaged as 1.19% across states. This magnitude falls in the range of the simulated standard deviations shown in Table 2 in Section 3.2 even though the number of operating manufacturing plants in the US amounts to be about 350000 (Cooper, Haltiwanger, and Power (1999)). It is thus quantitatively possible that the aggregate regional fluctuations with the magnitude of empirical business cycles endogenously arise from the complementarity of discrete investments.

Propositions 1 and 2 are robust in a more general setup where the initial capital is not distributed uniformly and the lumpiness and the depreciation rate are heterogeneous across firms. We defer these generalizations to Appendix F.

3 Dynamic Model

In this section, we construct a dynamic model in which the distribution of capital evolves over time deterministically due to the depreciation and lumpy investments. We use the standard business cycles model with monopolistic firms as in Galí (1994). First, we reestablish our analytical results in this framework. We observe that the wage and interest rate can dampen the investment amplification effects by lowering the degree of strategic complementarity. This corresponds to the general equilibrium effect identified by Thomas (2002). Second, we consider the case when the wage and interest rate are held constant, and compute the equilibrium path numerically. We observe that the endogenous deterministic fluctuation emerges. We also confirm that the uniform distribution provides a good approximate for an unconditional distribution of the firms' positions in the inaction band.

3.1 Model and the fluctuation result

Each firm i now uses labor $h_{i,t}$ as well as capital to produce good i:

$$y_{i,t} = Ak_{i,t}^{\alpha}h_{i,t}^{\gamma}. \tag{15}$$

The returns to scale is $\theta = \alpha + \gamma$. Capital is accumulated over time as in (2). The investment good $x_{i,t}$ is produced similarly as (3). The investment rate $x_{i,t}/k_{i,t}$ is chosen from a discrete set:

$$\frac{x_{i,t}}{k_{i,t}} \in \left\{(1-\delta)(\lambda_i^{\kappa_{i,t}}-1)\right\}_{\kappa_{i,t}=0,\pm1,\pm2,\ldots} \tag{16}$$

where $\lambda_i(1-\delta)>1$. Note that the choice space for $k_{i,t}$ is independent of the path: $k_{i,t} \in \{(1-\delta)^t k_{i,0}\lambda_i^{\tilde{\kappa}_t}\}_{\tilde{\kappa}_t=0,\pm1,\pm2,\ldots}$.

The representative household has preference over the sequence of consumption and hours worked: $\sum_{t=0}^{\infty}\beta^t(C_t^{1-\sigma}/(1-\sigma)-H_t)$. The representative household maximizes the utility subject to the sequence of budget constraints:

$$\sum_{i=1}^{N} p_{i,t}z_{c,i,t} = w_t H_t + \Pi_t, \quad \forall t \tag{17}$$

where w_t denotes the real wage and Π_t is the average dividend from firms: $\Pi_t \equiv \sum_{i=1}^{N}\pi_{i,t}/N$. Aggregate indices Y_t, K_t, and $P_t=1$ are defined similarly as before, by redefining $\rho \equiv (\theta-\gamma)/(\mu-\gamma)$. The new definition of ρ corresponds to the old one in the simple model when γ is set at 0. Aggregate investment is $X_t \equiv \sum_{i=1}^{N} x_{i,t}/N$. The labor market equilibrium condition is $H_t = \sum_{i=1}^{N} h_{i,t}/N$. The usual procedure yields a demand function for good i as (5).

The monopolist maximizes its discounted future profits as instructed by the representative household. The instructed discount rate, r_t^{-1}, is the marginal rate of intertemporal substitution of consumption. Then the monopolist's problem is defined as follows:

$$\max_{\{y_{i,t},k_{i,t+1},h_{i,t},i_{i,t},z^I_{l,i,t}\}} \sum_{t=0}^{\infty}(r_1\cdots r_t)^{-1}\pi_{i,t} = \sum_{t=0}^{\infty}(r_1\cdots r_t)^{-1}\left(p_{i,t}y_{i,t} - w_t h_{i,t} - \sum_{j=1}^{N}p_{j,t}z_{j,i,t}\right) \tag{18}$$

subject to the production function, the capital accumulation, the discreteness of the investment rate, and the demand function. By using the optimality condition for $h_{i,t}$, the profit in period t reduces to a function of $(k_{i,t}, k_{i,t+1})$ as:

$$\pi_{i,t} = a_0 w_t^{\frac{-\gamma}{1-\gamma}} K_t^{\frac{\rho(\mu-1)}{1-\gamma}} k_{i,t}^{\rho} - k_{i,t+1} + (1-\delta)k_{i,t} \tag{19}$$

where $a_0 \equiv (1-\gamma/\mu)(A(\gamma/\mu)^{\gamma})^{1/(1-\gamma)}$. The present discounted value of profits is concave in $k_{i,t}$ due to $\rho < 1$. Thus the optimal policy is characterized by an inaction band of $k_{i,t}$ with a lower bound $k^*_{i,t}$ and an upper bound $\lambda_i k^*_{i,t}$. Consider two sequences of $k_{i,\tau}$ which are identical except at $\tau = t$. Such sequences can be constructed by assigning a positive investment at $t-1$ and zero investment at t in one sequence and zero investment at $t-1$ and a positive investment at t in the other sequence. Then the lower bound is derived by solving for $k^*_{i,t}$ at which the two sequences yield the same discounted profit. Namely, if $k_{i,t}$ is strictly less than $k^*_{i,t}$, the firm is better off by adjusting it upward rather than waiting. Assign $k_{i,t} = k^*_{i,t}$ to the sequence that has zero investment at $t-1$, and $k_{i,t} = \lambda_i k^*_{i,t}$ to the other sequence. Then the both sequences have the same amount of capital at $t-1$ and $t+1$: $k_{i,t-1} = (1/(1-\delta))k^*_{i,t}$ and $\lambda_i(1-\delta)k^*_{i,t}$. Solving for $k^*_{i,t}$ which equates the discounted profits of the two sequences, we obtain:

$$k^*_{i,t} = a_i \left(r_t - 1 + \delta\right)^{\frac{-1}{1-\rho}} w_t^{\frac{-\gamma}{(1-\gamma)(1-\rho)}} K_t^{\bar{\phi}} \tag{20}$$

$$\bar{\phi} \equiv \frac{\rho(\mu-1)}{(1-\rho)(1-\gamma)} = \frac{\theta-\gamma}{1-\gamma}\frac{\mu-1}{\mu-\theta} \tag{21}$$

where $a_i \equiv (a_0(\lambda_i^{\rho}-1)/(\lambda_i-1))^{1/(1-\rho)}$. Equation (20) expresses the strategic complementarity between K_t and $k^*_{i,t}$, whereas the degree of complementarity is represented by $\bar{\phi}$. Note that $\bar{\phi}$ reduces to ϕ in the simple model when $\gamma = 0$. Lemma 1 continues to hold for $\bar{\phi}$, namely, $\bar{\phi} \leq 1$ if and only if $\theta \leq 1$.

The perfect foresight equilibrium path is determined by the following system of equations:

$$r_{t+1} = \frac{w_{t+1}}{w_t \beta} \tag{22}$$

$$w_t^{1/\sigma} = Y_t - X_t \tag{23}$$

$$Y_t = c_Y w_t^{-\gamma/(1-\gamma)} K_t^{(\theta-\gamma)/(1-\gamma)} \tag{24}$$

$$X_t = \sum_{\{i:s_{i,t}<z_{i,t}\}} (1-\delta)(\lambda_i - 1)\lambda_i^{s_{i,t}} k_{i,t}^*/N \tag{25}$$

$$K_{t+1} = \left(K_t^\rho + \sum_{\{i:s_{i,t}<z_{i,t}\}} (\lambda_i^\rho - 1)\lambda_i^{s_{i,t}\rho} k_{i,t}^{*\rho}/N \right)^{1/\rho} (1-\delta) \tag{26}$$

$$z_{i,t} \equiv 1/q_i + (\log k_{i,t+1}^* - \log k_{i,t}^*)/\log \lambda_i \tag{27}$$

where $c_Y \equiv (A(\gamma/\mu)^\gamma)^{1/(1-\gamma)}$. It is hard to compute the perfect foresight equilibrium path, since the future prices depend on the vector of individual capital rather than the aggregate capital. Thus we approximate the expectation formations of agents on the future prices. First, agents are assumed to expect that $s_{i,\tau}$ for $\tau > t$ follows a uniform random variable over the unit interval independently across i. Secondly, agents expect that w_τ for $\tau > t$ obeys a log-linearized dynamics around the steady state. Thus, when the firms decide the investment in t (and thus the capital in $t+1$), they rely on the forecasted factor price sequence that is the log-linearized equilibrium transition path to the steady state in the economy with a continuum of firms distributed uniformly over the inaction band in each period.

When $s_{i,t}$ follows a uniform distribution, (25) and (26) are simplified as follows:

$$X_t = c_X(K_{t+1} - (1-\delta)K_t) \tag{28}$$

$$K_{t+1}^{1-\bar{\phi}} = c_K w_{t+1}^{-\gamma/((1-\gamma)(1-\rho))} (r_{t+1} - 1 + \delta)^{-1/(1-\rho)} \tag{29}$$

where $c_K \equiv \mathrm{E}((\lambda_i^\rho - 1)a_i^\rho/(\rho \log \lambda_i))^{1/\rho}$ and $c_X = \mathrm{E}((\lambda_i - 1)a_i/\log \lambda_i)/c_K$ in which the expected value is taken across heterogeneous λ's. Equations (23,24,28) are summarized as:

$$w_t^{1/\sigma} = c_Y w_t^{-\gamma/(1-\gamma)} K_t^{(\theta-\gamma)/(1-\gamma)} - c_X(K_{t+1} - (1-\delta)K_t) \tag{30}$$

Then Equations (22,29,30) determine the equilibrium path of aggregate capital. There exists the steady state $(\bar{K}, \bar{w})$ of this dynamics.

We log-linearize the dynamics around the steady state. Let us define $v \equiv d\tilde{K}'/d\tilde{K} = d\tilde{w}'/d\tilde{w}$ where tilde denotes the log-difference of the variables to their steady state values. Then v is determined by:

$$(1-\bar{\phi})(1-\rho)\left(\frac{\gamma}{1-\gamma} + \frac{\bar{C}/\bar{Y}}{\sigma}\right) = -\left(\frac{\gamma}{1-\gamma} + \frac{1-1/v}{1-\beta(1-\delta)}\right)\left(\frac{\theta-\gamma}{1-\gamma} - \left(1-\frac{\bar{C}}{\bar{Y}}\right)\left(1-\frac{1-v}{\delta}\right)\right) \tag{31}$$

and we obtain a log-linearized equilibrium wage function with elasticity:

$$\eta_w \equiv \frac{d\tilde{w}}{d\tilde{K}} = \left(\frac{\theta-\gamma}{1-\gamma} - \left(1-\frac{\bar{C}}{\bar{Y}}\right)\left(1-\frac{1-v}{\delta}\right)\right) / \left(\frac{\gamma}{1-\gamma} + \frac{\bar{C}/\bar{Y}}{\sigma}\right) \tag{32}$$

The speed of convergence v can be solved explicitly when $\theta = 1$ (and thus $\bar{\phi} = 1$) as $v = (1-\gamma)/(1-\gamma\beta(1-\delta))$. Since this is strictly less than one, the dynamics is stable in the neighborhood of the steady state.

Let us focus on the constant returns to scale case $\theta = 1$. The firms expect w_{t+1} to be consistent with the forecasted equilibrium path: $\tilde{w}_{t+1} = \eta_w \tilde{K}_{t+1}$. Equation (32) indicates that the real wage is procyclical in the forecasted path. Thus, an upward deviation of K_{t+1} from the steady state raises the expected w_{t+1}. The contemporaneous wage w_t is determined by (23,24). An increased aggregate investment in t lowers the contemporaneous consumption, which raises the marginal rate of substitution between consumption and leisure and thus reduces the real wage. Both the increase in w_{t+1} and the decrease in w_t induces r_{t+1}

to rise in response to an increased K_{t+1}. Due to the increased w_{t+1} and r_{t+1}, the threshold $k^*_{i,t+1}$ decreases by (27) and dampens the incentive to invest. This is the general equilibrium effect that reduces the strategic complementarity across firms' investments. Let $\eta_{r,t+1}$ denote the elasticity of equilibrium interest rate to capital in $t+1$: $\eta_{r,t+1} \equiv d\tilde{r}_{t+1}/d\tilde{K}_{t+1}$. By using the log-linear approximation of the expected w_{t+1} and r_{t+1}, the threshold (20) is simplified as follows:

$$\tilde{k}^*_{i,t} = \hat{\phi}\tilde{K}_t \quad (33)$$

$$\hat{\phi} \equiv \bar{\phi} - \frac{\gamma\eta_w + (1-\gamma)\eta_{r,t+1}}{(1-\gamma)(1-\rho)} \quad (34)$$

The strategic complementarity is now lowered to $\hat{\phi}$ due to the general equilibrium effects if $\eta_w, \eta_{r,t+1} > 0$.

The equilibrium of goods markets, given the expected factor prices (w_{t+1}, r_{t+1}), is determined by a capital profile which satisfies the inaction band $k_{i,t+1} \in [k^*_{i,t+1}, \lambda_i k^*_{i,t+1})$. The capital configuration determines the forecasted equilibrium factor prices in turn. We redefine the equilibrium selection algorithm, in which the capital profile starts at $(k_{i,t})_i$ and converges to $(k_{i,t+1})_i$, by incorporating the adjustment of (w_{t+1}, r_{t+1}) according to the log-linearized pricing functions. Namely, the factor prices are updated in each step u during the best response dynamics as $\tilde{w}^u_t = \eta_w \tilde{K}^u_t$ and $\tilde{r}^u_t = \eta_{r,t+1}\tilde{K}^u_t$. The initial point of the equilibrium selection algorithm is set at $K^0_t = \bar{K}(K_t/\bar{K})^v$ which would be the aggregate capital if there were a continuum of firms distributed uniformly at t. Then we obtain the following proposition.

Proposition 4 *Suppose that λ and δ are common across i. Also suppose that $\log k_{i,t}$ is a random variable which follows a uniform distribution over the inaction band $[\log k^*_{i,t}, \log\lambda + \log k^*_{i,t})$. Then, the distributions in Proposition 1 hold with modified $\hat{\phi}$ and $\hat{\sigma}_1$ for $0 < \hat{\phi} < 1$.*

Also, $\log K_{t+1} - \log K_t$ *has a strictly positive variance when* $\hat{\phi} \to 1$ *and* $N \to \infty$*.*

Proof proceeds similarly to that of Proposition 1.

The aggregate capital fluctuates along with the evolution of configuration of the capital profile. To evaluate the magnitude of fluctuations analytically, we regarded the capital configuration as a vector of independent random variables that takes values within the inaction band. Then, we obtain that the parametric form of the fluctuation does not change from the static case, and that the aggregate fluctuation does not vanish as the number of firms tends to infinity if the key parameter $\hat{\phi}$ is one.

The crucial feature of the model is that the marginal product of capital is heterogeneous across firms due to the lumpy investment. The marginal product of capital would be equalized across firms if the capital adjustment were continuous. When the investment is lumpy, the cost of capital is equalized at the extensive margin of the marginal product of capital. The marginal product of capital for an individual firm is dependent on the level of aggregate capital, and thus the firms' investments exhibit strategic complementarity given the cost of capital. The extent of the amplification effects of the strategic complementarity depends on the distribution of firms around the extensive margin.

The complementarity parameter $\hat{\phi}$ is lowered from $\bar{\phi}$ by the general equilibrium effects η_w and $\eta_{r,t+1}$. If the factor prices co-move with aggregate investments, the exponential truncation of the distribution of aggregate fluctuations becomes faster, and thus the variance is smaller, than in the case of fixed factor prices. The static model shows that the aggregate fluctuations do not vanish even when there are an infinite number of firms if the returns to scale is constant. Whether this mechanism of endogenous fluctuations holds in the dynamic general equilibrium model depends on how elastically the wage and interest rate respond to the investments. Note that $\eta_{r,t+1} = \eta_w - \eta_{w,t}$ where $\eta_{w,t} \equiv d \log \tilde{w}_t / d \log \tilde{K}_{t+1}$. If we calibrate these elasticities by using the benchmark value $\sigma = 3$, the general equilibrium effect turns out to overwhelm the strategic complementarity effect $\bar{\phi}$. In this case, $\hat{\phi}$ becomes negative, and our equilibrium selection algorithm does not necessarily converge to the equilibrium. We can interpret this result as the case where the reluctance of households to substitute their consumption intertemporally dominates the firm's gain from synchronized investments. A small change in factor prices quickly dampens the firms' incentive to follow other firms' investments. On the contrary, if we calibrate η_w and $\eta_{w,t}$ by empirical correlations between $\tilde{Y}_{t+1}$ and $\tilde{w}_{t+1}$ or $\tilde{w}_t$ which tend to be low, then $\hat{\phi}$ becomes positive.

The dampening force of the general equilibrium effect is sensitive to how the agents form their expectations. Let us consider the following illustrative case. When $\theta = 1$, the forecasted relation of factor prices (29) reduces to a familiar condition:

$$w_{t+1}^{\gamma}(r_{t+1} - 1 + \delta)^{1-\gamma} = c_K^{(1-\rho)(1-\gamma)} \tag{35}$$

that must hold between the wage and the interest rate in any model with competitive factor markets with constant returns to scale. This relation holds regardless of the capital level. Note that the left hand side of this relation summarizes the effect of factor prices on the lower bound $k_{i,t+1}^{*}$ in (20). Thus, as long as the factor prices are constrained by this relation, their movement does not affect the incentive for a firm to invest. The firms expect this relation to hold only in $\tau > t + 1$ in the previous approximation of the future price path. Suppose now that the firms expect this relation to hold in $t + 1$ as well. Then, the firms' investment exhibits the perfect complementarity $\hat{\phi} = \bar{\phi} = 1$, which is the condition for our aggregate fluctuations in the limit of $N \to \infty$ to obtain. This alternative expectation formation has a particular relevance when the interest rate is exogenously determined as in a small open economy.

The perfect complementarity also emerges if $\theta = 1$ and if there are real rigidities in the wage and interest rate. In an economy where the factor employment contract is predetermined, for example, then $\hat{\phi} = 1$ holds for the frequency higher than the length of contract terms. Thus, the endogenous fluctuations of aggregate investments due to the strategic complementarity of lumpy investments may occur in a general equilibrium in the limit of $N \to \infty$ if there are real rigidities in expected factor prices.

3.2 Granular dynamics in regional economies with locally differentiated goods

We have so far assumed the randomness of the capital configuration in order to obtain the analytical solution for the distribution of aggregate fluctuations. We assumed a uniform distribution for Propositions 1, 2, and 4 and we generalized the proposition for an arbitrary distribution in Propositions 5 and 6. We showed that the aggregate investment is sensitive to the detailed configuration of capital in that environment.

In this section, we drop the assumption of randomized capital and show that the fluctuation results still hold in the equilibrium path where the capital evolves deterministically. We do so by numerically computing the equilibrium path when the wage and interest rate are fixed at the time-average level. We interpret the fixed wage and interest as a situation of regional economies in which the prices of locally differentiated products adjust flexibly while the wage and interest are exogenously determined by the national economy due to the perfect mobility of labor and capital across regions. We show that the aggregate output exhibits fluctuations along with the evolution of the capital configuration which is driven by the depreciation of capital and lumpy investments. Since the aggregate fluctuation is driven by discrete actions of finite firms, we may call the fluctuation as granular dynamics. Gabaix (2005) finds a granular effect in aggregate output fluctuations as the influence of big firms. In this paper, the aggregate fluctuation rather arises from interaction of small, but not atomless, granular firms.

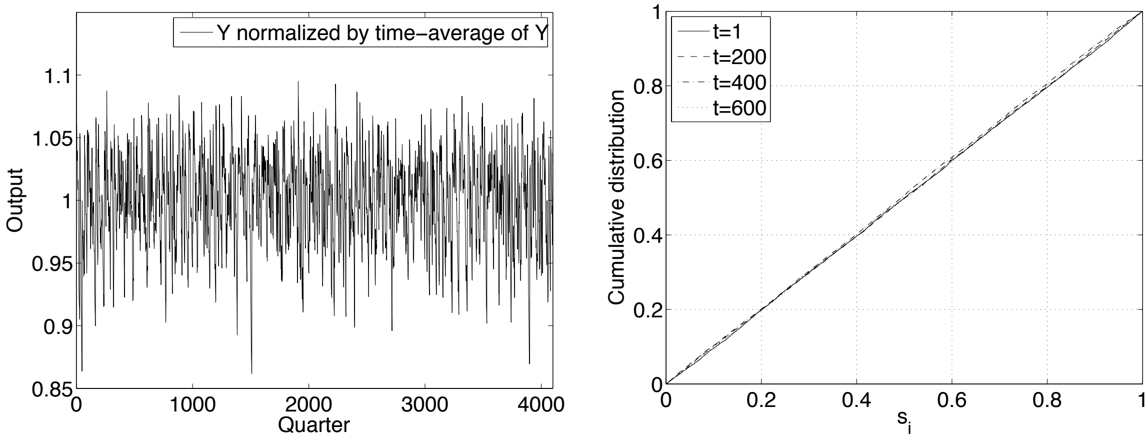

Figure 3: Left: Simulated path of output Y when $N = 10000$ and $\theta = 0.999$. Right: Cross-section distribution of simulated $s_{i,t}$

Parameter values are specified as follows. The labor's share of income γ/μ is equal to 0.58. The markup rate $\mu - 1$ is set at 0.2. The inverse of the elasticity of intertemporal substitution is set at $\sigma = 3$. The quarterly discount factor is set at $\beta = 0.99$. The returns-to-scale θ takes various values close to one. The quarterly depreciation rate of capital δ_i is drawn from a normal distribution with mean 0.02 and standard deviation 0.01, where the parameter values are obtained from the table of estimated depreciation rates of SIC 2-digit level industries shown in Horvath (2000). The distribution of lumpiness λ_i is similarly obtained by fitting an exponential distribution to the estimated investment-capital ratios shown in Cooper, Haltiwanger, and Power (1999). The mean λ_i is set at 0.2 and the distribution is truncated below at 0.1 so that the condition $\lambda_i(1 - \delta_i) > 1$ is satisfied. The equilibrium is computed sequentially for 500 quarters, and the first 100 quarters are discarded in order to focus on the stationary fluctuations. The left panel of Figure 3 plots a sample equilibrium path for the case $N = 10000$ and $\theta = 0.999$. We observe a considerable fluctuation of the output despite the large number of firms.

		Standard deviation of Y (%)			Autocorrelation of Y (%)		
	θ	0.9	0.99	0.999	0.9	0.99	0.999
	1,000	0.19	1.05	2.24	55.43	76.72	93.69
		(0.01)	(0.07)	(0.29)	(4.16)	(3.92)	(1.71)
N	10,000	0.06	0.42	1.68	55.05	72.81	88.35
		(0.00)	(0.02)	(0.15)	(3.62)	(3.78)	(2.38)
	100,000	0.02	0.15	0.99	54.23	69.67	83.03
		(0.00)	(0.05)	(0.19)	(3.96)	(4.00)	(2.93)

Table 2: Standard deviation and autocorrelation of output

We compute the standard deviation and the autocorrelation of Y for each path, and repeat the procedure for 100 times for each parameter set. Table 2 reports the average and standard error of the computed standard deviations and autocorrelations of output for various number of firms N and returns to scale θ. The standard errors of the estimates are reported in parentheses. Table 2 confirms our analytical result that the amplitude of the fluctuation increases as the returns-to-scale approaches to one. The amplitude decreases as the number of firms increases, but even for the case of 100,000 firms the model can generate sizable fluctuations. As we discussed along with Figure 2, the magnitudes of fluctuations in the US regions are consistent with the simulated statistics above.

We check whether the uniform distribution approximates the cross-section of $s_{i,t}$ well. The right panel of Figure 3 plots the cumulative distributions of $s_{i,t}$ for various periods t. The simulated distribution deviates little from the uniform distribution. This provides a ground for the uniformity assumption in our analysis in previous sections.

Even though the gap variable $s_{i,t}$ has a strong tendency to converge toward the uniform distribution, a slight deviation always occurs in a finite economy and the detailed change in the configuration of $s_{i,t}$ affects the aggregate investment significantly. Moreover, the fact that $s_{i,t}$ is serially correlated by construction leads to an interesting dynamic property of the aggregates. We observe in Table 2 that the model generates considerable autocorrelation in output. Since there is no autocorrelation built in the environment, we interpret this as the replacement echo effects, following Benhabib and Rustichini (1991) and Boucekkine, Germain, and Licandro (1997), in which the past clustering of capital adjustments brings out repercussions when the cluster comes back to the adjustment threshold. The simple principle of "phase-dynamics" in the weakly-coupled nonlinear systems might be useful to interpret this periodicity. The aggregate fluctuation occurs only if there is a certain degree of comovement across agents. Therefore, the frequency of aggregate fluctuations should center around the average frequency of the natural rate of adjustment of a firm, which in our model is determined by the lumpiness size divided by the depreciation rate: $q_i = \log \lambda_i / |\log(1-\delta_i)|$.

In sum, the synchronization of oscillating capital alone can generate significant amplitude and autocorrelation of the aggregate output fluctuations, while the significant heterogeneity in lumpiness and depreciation rates across firms prevents the capital from being completely synchronized.

4 Conclusion

This paper characterizes the aggregate fluctuations arising from spillover effects of discrete investments at the firm level. We evaluate the deterministic fluctuation of aggregate investment along the evolution of heterogeneous capital as if it is a stochastic fluctuation whose randomness arises from the stochastic configuration of capital. For each configuration, the equilibrium aggregate investment is determined as a convergent point of a fictitious best response dynamics of firms' investment decisions. The best response dynamics can be embedded in a branching process with the probability measure defined by the stochastic configuration. This enables us to derive the distribution function of the aggregate fluctuation in a closed form.

The fluctuation of the number of investing firms is shown to follow a power-law distribution with an exponential truncation at the tail. The truncation speed is determined by the degree of strategic complementarity among firms, which is ultimately determined by the returns to scale of production technology. Under the constant returns to scale, the distribution becomes a pure power law, and the standard deviation of the growth rate is shown to be strictly positive even when there are an infinite number of firms. The limiting standard deviation is shown to be almost proportional to the lumpiness of the firm-level investment.

The equilibrium path of the model is numerically computed without making the randomness assumption of the capital configuration. The simulation confirms the validity of the analysis above that utilizes the assumptions of randomness and uniformity of the capital configuration. The simulated output paths show strong persistence and mild periodicity. This expresses the echo effect in which a clustering of investments in a period reappears after several periods. The frequency of the echo effect is determined by the natural frequency of a firm's capital adjustment, which is equal to the lumpiness divided by the depreciation rate.

Appendix

A Proof of Lemma 2

Let H_u, $u = 2, 3, \ldots, T$, denote the set of firms that adjust capital in step u. Assume that H_u is finite with probability one when $N \to \infty$, which we verify later. We consider the case $m_1 > 0$ for the proofs of Lemma 2, 3, and Proposition 1 without loss of generality. Thus, $\log k_{i,u} = \log k_{i,u-1} + \log \lambda$ for $i \in H_u$.

We expand $N(\log K_{u+1} - \log K_u)$ around $(\log k_u)_{i \in H_{u+1}}$. The first derivative is $\partial N \log K_u / \partial \log k_{i,u} = (k_{i,u}/K_u)^\rho$. Thus $\partial K_u / \partial k_{i,u}$ is of order $1/N$. The second and higher derivatives with respect to the own $\log k_{i,u}$ are $\partial^n (k_{i,u}/K_u)^\rho / \partial \log k_{i,u}^n = \rho^n (k_{i,u}/K_u)^\rho + O(\partial K_u/\partial k_{i,u})$ for $n = 1, 2, \ldots$. The second cross derivatives, $\partial^2 \log K_u / (\partial \log k_{i,u} \partial \log k_{j,u})$, are of order $\partial K_u/\partial k_{j,u}$ and thus $O(1/N)$. Similarly, the cross derivative terms with respect to the capital of h distinct firms in H_{u+1} are of order $1/N^{h-1}$. Since H_{u+1} is finite, the n-th derivative of $N \log K_u$ has the finite number of the cross derivative terms for any finite n. Hence, the Taylor series expansion of $N(\log K_{u+1} - \log K_u)$ yields:

$$\sum_{n=1}^{\infty} \sum_{i \in H_{u+1}} \left(\frac{k_{i,u}}{K_u} \right)^\rho \frac{\rho^{n-1} (\log \lambda)^n}{n!} + O(1/N) = \frac{\lambda^\rho - 1}{\rho} \sum_{i \in H_{u+1}} \left(\frac{k_{i,u}}{K_u} \right)^\rho + O(1/N) \tag{36}$$

where we used $\lambda^\rho = \lambda^0 + \sum_{n=1}^{\infty} (d^n \lambda^\rho / d\rho^n)|_{\rho=0} (\rho^n/n!)$. Utilizing $k_{i,u} = k_u^* \lambda^{s_{i,u}}$, we obtain that $\sum_{i \in H_{u+1}} (k_{i,u}/K_u)^\rho = (\sum_{i \in H_{u+1}} \lambda^{s_{i,u}\rho}) / (\sum_{i=1}^N \lambda^{s_{i,u}\rho}/N)$. The denominator converges to $\mathrm{E}[\lambda^{s_{i,u}\rho}]$ as $N \to \infty$ almost surely by the law of large numbers, and we have $\mathrm{E}[\lambda^{s_{i,u}\rho}] = \int_0^1 \lambda^{s_{i,u}\rho} ds_{i,u} = (\lambda^\rho - 1)/(\rho \log \lambda)$. The numerator, $\sum_{i \in H_{u+1}} \lambda^{s_{i,u}\rho}$, converges to m_{u+1} for every event when H_{u+1} is finite, because $s_{i,u}$ is smaller than $\phi(\log K_u - \log K_{u-1})/\log \lambda$ for any $i \in H_{u+1}$ and thus of order $1/N$. Hence we obtain the lemma.

B Proof of Lemma 3

The conditional probability for firm i to invest in $u = 2, 3, \ldots, T$ is:

$$\Pr(i \in H_u \mid i \notin \cup_{v=2,3,\ldots,u-1} H_v) = \frac{\phi(\log K_u - \log K_{u-1})/\log \lambda}{1 - \phi(\log K_{u-1} - \log K_0)/\log \lambda}. \tag{37}$$

Thus m_u follows a binomial distribution with population $N - \sum_{v=2}^{u-1} m_v$ and probability (37). The mean of m_u converges to ϕm_{u-1} as $N \to \infty$, by using Lemma 2. Then, the binomial distribution of m_u converges to a Poisson distribution with mean ϕm_{u-1} for $u = 2, 3, \ldots, T$. Since a Poisson distribution is infinitely divisible, the Poisson variable with mean ϕm_{u-1} is equivalent to a m_{u-1}-times convolution of a Poisson variable with mean ϕ. Thus the process m_u for $u = 2, 3, \ldots, T$ is a branching process with a Poisson random variable with mean ϕ, where m_2 follows a Poisson distribution with mean ϕm_1. Note that m_1 is not included in the branching process because it is not necessarily an integer.

C Proof of Proposition 1

We first derive the asymptotic distribution of M conditional on m_1. It is known that the accumulated sum $M = \sum_{u=2}^{T} m_u$ of the Poisson branching process conditional on m_2 follows an infinitely divisible distribution called Borel-Tanner distribution (see Kingman (1993)):

$$\Pr(M = w \mid m_2) = (m_2/w)e^{-\phi w}(\phi w)^{w-m_2}/(w - m_2)! \tag{38}$$

for $w = m_2, m_2 + 1, \ldots$. Using that m_2 follows the Poisson distribution with mean ϕm_1, we obtain (10) as follows:

$$\begin{aligned}
\Pr(M = w \mid m_1) &= \sum_{m_2=0}^{w} ((m_2/w)e^{-\phi w}(\phi w)^{w-m_2}/(w - m_2)!)e^{-\phi m_1}(\phi m_1)^{m_2}/m_2! \\
&= m_1 e^{-\phi(w+m_1)}(\phi^w/w) \sum_{m_2=1}^{w} w^{w-m_2} m_1^{m_2-1}/((w - m_2)!(m_2 - 1)!) \\
&= m_1 e^{-\phi(w+m_1)}(\phi^w/w)(w + m_1)^{w-1}/(w - 1)! \\
&= m_1 e^{-\phi(w+m_1)}\phi^w(w + m_1)^{w-1}/w! \qquad (39)
\end{aligned}$$

where the third line utilized the binomial theorem. The approximation in (11) is obtained by applying the Stirling's formula $w! \sim \sqrt{2\pi}e^{-w}w^{w+0.5}$:

$$m_1 e^{-\phi(w+m_1)}\phi^w(w + m_1)^{w-1}/w! \sim (m_1 e^{-\phi m_1}/\sqrt{2\pi})(e^{1-\phi}\phi)^w w^{-1.5}(1 + m_1/w)^{w-1} \tag{40}$$

and using $(1+m_1/w)^{w-1} \to e^{m_1}$ as $w \to \infty$.

Next we derive the asymptotic normal distribution of $m_1/\sqrt{N}$. We split $m_1/\sqrt{N}$ into two terms as $m_1/\sqrt{N} = (\sqrt{N}/\log\lambda)(\log K_1 - \log(\sum_{i=1}^N((1-\delta)k_{i,0})^\rho/N)^{1/\rho}) + (\sqrt{N}/\log\lambda)(\log(\sum_{i=1}^N((1-\delta)k_{i,0})^\rho/N)^{1/\rho} - \log K_0)$. The second term represents the depreciation and is equal to $(\sqrt{N}/\log\lambda)\log(1-\delta) = -\sqrt{N}/q$. The first term represents the first-step adjustments induced directly by the depreciation. Define H_1 as the set of firms that adjust in the first step. Using $k_{i,0} = \bar{k}\lambda^{s_{i,0}}$, we obtain $K_1 = (1-\delta)\bar{k}((\lambda^\rho - 1)\sum_{i\in H_1}\lambda^{s_{i,0}\rho}/N + \sum_{i=1}^N \lambda^{s_{i,0}\rho}/N)^{1/\rho}$ and $(\sum_{i=1}^N((1-\delta)k_{i,0})^\rho/N)^{1/\rho} = (1-\delta)\bar{k}(\sum_{i=1}^N \lambda^{s_{i,0}\rho}/N)^{1/\rho}$. Thus the first term becomes:

$$\frac{\sqrt{N}}{\log\lambda}\left(\log K_1 - \frac{1}{\rho}\log\left(\sum_{i=1}^N((1-\delta)k_{i,0})^\rho\right)\right) = \frac{\sqrt{N}}{\rho\log\lambda}\log\left((\lambda^\rho - 1)\frac{\sum_{i\in H_1}\lambda^{s_{i,0}\rho}/N}{\sum_{i=1}^N \lambda^{s_{i,0}\rho}/N} + 1\right) \tag{41}$$

By Assumption 1, $s_{i,0}$ is distributed uniformly. Thus the denominator $\sum_{i=1}^N \lambda^{s_{i,0}\rho}/N$ in (41) converges to $\int_0^1 \lambda^{s_{i,0}\rho} ds_{i,0} = (\lambda^\rho - 1)/\rho\log\lambda$ with probability one by the law of large numbers. Let x denote that numerator: $x \equiv \sum_{i\in H_1}\lambda^{s_{i,0}\rho}/N$. Note that $i \in H_1$ is equivalent to $0 \le s_{i,0} < 1/q$. Then the asymptotic mean of x is $x_0 = \int_0^{1/q}\lambda^{s_{i,0}\rho} ds_{i,0} = (\lambda^{\rho/q} - 1)/(\rho\log\lambda)$ and, by the central limit theorem, $\sqrt{N}(x - x_0)$ converges in distribution to the normal distribution with mean zero and variance:

$$\int_0^{1/q}\left(\lambda^{s_{i,0}\rho}\right)^2 ds_{i,0} - \left(\frac{\lambda^{\rho/q} - 1}{\rho\log\lambda}\right)^2 = \frac{\lambda^{2\rho/q} - 1}{2\rho\log\lambda} - \left(\frac{\lambda^{\rho/q} - 1}{\rho\log\lambda}\right)^2 \tag{42}$$

We regard the right hand side of (41) as a function F of x. By the delta method, we obtain that $F(x)$ asymptotically follows the normal distribution with mean $F(x_0)$ and variance $F'(x_0)^2\mathrm{Avar}(x)$. $F(x_0)$ is calculated as:

$$\frac{\sqrt{N}}{\rho\log\lambda}\log\left(\frac{(\lambda^\rho - 1)(\lambda^{\rho/q} - 1)/(\rho\log\lambda)}{(\lambda^\rho - 1)/(\rho\log\lambda)} + 1\right) = \frac{\sqrt{N}}{q} \tag{43}$$

This cancels out with the second term of the split $m_1/\sqrt{N}$. $F'(x_0)^2\text{Avar}(x)$ is calculated as σ_1^2 in (12). Then, $m_1/\sqrt{N}$ asymptotically follows the normal distribution with mean zero and the variance σ_1^2. This completes the proof.

D Proof of Proposition 2

We focus on $(m_1+M)/N$, since Lemma 2 implies $(\log K - \log K_0)/\log\lambda \sim (m_1+M)/N$. Its unconditional variance is decomposed as follows:

$$\begin{aligned} \text{Var}\left(\frac{m_1+M}{N}\right) &= \text{E}\left(\text{Var}\left(\frac{M}{N} \mid m_1\right)\right) + \text{Var}\left(\frac{m_1}{N} + \text{E}\left(\frac{M}{N} \mid m_1\right)\right) \\ &= \text{E}\left[\text{E}\left(\text{Var}\left(\frac{M}{N} \mid m_1, m_2\right) \mid m_1\right) + \text{Var}\left(\text{E}\left(\frac{M}{N} \mid m_1, m_2\right) \mid m_1\right)\right] \\ &\quad + \text{Var}\left(\frac{m_1}{N} + \text{E}\left(\text{E}\left(\frac{M}{N} \mid m_1, m_2\right) \mid m_1\right)\right). \end{aligned} \tag{44}$$

m_u asymptotically follows a martingale branching process when $\phi \to 1$ and $N \to \infty$. By the nature of the branching process, $|M|$ conditional on $|m_2|$ is equivalent to the $|m_2|$-times convolution of M conditional on $m_2 = 1$. Using these facts, we obtain that:

$$\text{Var}(M/N \mid m_1, m_2) \sim |m_2|\text{Var}(M/N \mid m_2 = 1) \tag{45}$$

$$\text{E}(\text{E}(M/N \mid m_1, m_2) \mid m_1) \sim \text{E}(m_2 \mid m_1)\text{E}(M/N \mid m_2 = 1) \sim m_1\text{E}(M/N \mid m_2 = 1) \tag{46}$$

Also, $|m_2|$ conditional on m_1 asymptotically follows a Poisson distribution with mean $|m_1|$ and the unconditional distribution of m_2 is symmetric. Since $m_1/\sqrt{N}$ asymptotically follows $N(0, \sigma_1^2)$ by Proposition 1, we can use the formula $E(|m_1|/\sqrt{N}) \to \sigma_1\sqrt{2/\pi}$. Applying these, we obtain:

$$
\begin{aligned}
\mathrm{Var}\left(\frac{m_1+M}{N}\right) &\sim \mathrm{E}\left[\mathrm{E}\left(|m_2| \mid m_1\right)\mathrm{Var}\left(\frac{M}{N} \mid m_2=1\right)+\mathrm{Var}(m_2 \mid m_1)\left(\mathrm{E}\left(\frac{M}{N} \mid m_2=1\right)\right)^2\right] \\
&\quad + \mathrm{Var}\left(\frac{m_1}{N}+\mathrm{E}\left(\frac{M}{N} \mid m_2=1\right)\mathrm{E}(m_2 \mid m_1)\right) \\
&\sim (\sigma_1\sqrt{2/\pi})\mathrm{E}\left(\frac{M^2}{N^{1.5}} \mid m_2=1\right)+\sigma_1^2\left(\frac{1}{\sqrt{N}}+\mathrm{E}\left(\frac{M}{\sqrt{N}} \mid m_2=1\right)\right)^2 \qquad (47)
\end{aligned}
$$

Next we calculate $\lim_{N\to\infty}\mathrm{E}(M/\sqrt{N} \mid m_2=1)$, provided that the best response dynamics reaches an equilibrium before all the N firms adjust. Namely, we take the expectation conditional on $M \leq N$ for a fixed N by using the asymptotic probability function (38) when $\phi \to 1$:

$$\Pr(M=w \mid m_2=1, M\leq N)\Pr(M\leq N)=e^{-w}w^{w-1}/w!. \qquad (48)$$

By the property of a branching process with mean less than or equal to one, the probability of the event $M \leq N$ converges to one as $N \to \infty$. By using the following inequality (see Feller (1957)),

$$\sqrt{2\pi}w^{w+0.5}e^{-w+1/(12w+1)} < w! < \sqrt{2\pi}w^{w+0.5}e^{-w+1/(12w)} \qquad (49)$$

we can compute the upper and lower bounds of the asymptotic mean of $M/\sqrt{N}$ as follows.

$$
\begin{aligned}
\sum_{w=1}^{N} e^{-w}w^w/(w!\sqrt{N}) &< \int_0^N w^{-0.5}dw/\sqrt{2\pi N} \to \sqrt{2/\pi} \qquad (50)\\
\sum_{w=1}^{N} e^{-w}w^w/(w!\sqrt{N}) &> \int_1^{N+1} e^{-1/(12w)}w^{-0.5}dw/\sqrt{2\pi N} \to \sqrt{2/\pi} \qquad (51)
\end{aligned}
$$

Hence, $\mathrm{E}(M/\sqrt{N} \mid m_2 = 1, W \leq N) \to \sqrt{2/\pi}$. Similarly we obtain:

$$E(M^2/N^{1.5} \mid m_2 = 1) \to 1/(1.5\sqrt{2\pi}) \tag{52}$$

Collecting the results, we obtain $\mathrm{Var}((m+M)/N) \to (2/\pi)(\sigma_1 + 1/3)\sigma_1$. Hence, the capital growth rate has an asymptotic variance $(\log\lambda)^2(2/\pi)(\sigma_1 + 1/3)\sigma_1$.

E Derivation of Equations (50,51,52) in Appendix D

We use the inequality (51).

$$\begin{aligned}
E(M/\sqrt{N} \mid m_2 = 1, M \leq N)\,\mathrm{Pr}(M \leq N) &= \sum_{w=1}^{N} e^{-w} w^w/(w!\sqrt{N}) \\
< \quad & \sum_{w=1}^{N} e^{-w} w^w/(\sqrt{2\pi} w^{w+0.5} e^{-w+1/(12w+1)}\sqrt{N}) \\
= \quad & \sum_{w=1}^{N} e^{-1/(12w+1)} w^{-0.5}/\sqrt{2\pi N} \\
< \quad & \int_0^N w^{-0.5} dw/\sqrt{2\pi N} \\
\to_{N\to\infty} \quad & \sqrt{2/\pi}
\end{aligned} \tag{53}$$

The second to the last line holds because $e^{-1/(12w+1)}$ is bounded by one.

Similarly, the lower bound turns out to converge to the same value. Let us note that the function $e^{-1/(12w)} w^{-0.5}$ is decreasing for $w > 1/6$. Then we obtain:

$$\begin{aligned}
\sum_{w=1}^{N} e^{-w} w^w/(w!\sqrt{N}) &> \sum_{w=1}^{N} e^{-w} w^w/(\sqrt{2\pi} w^{w+0.5} e^{-w+1/(12w)}\sqrt{N}) \\
= \quad & \sum_{w=1}^{N} e^{-1/(12w)} w^{-0.5}/\sqrt{2\pi N}
\end{aligned}$$

$$
\begin{aligned}
&> \quad \int_1^{N+1} e^{-1/(12w)} w^{-0.5} dw / \sqrt{2\pi N} \\
&= \quad \left(e^{-1/(12(N+1))} \sqrt{N+1} - e^{-1/12} + \int_1^{N+1} w^{-1.5} e^{-1/(12w)} / 12 dw \right) / (0.5\sqrt{2\pi N}) \\
&\to_{N\to\infty} \quad \sqrt{2/\pi}
\end{aligned}
\tag{54}
$$

Hence, $\mathrm{E}(M/\sqrt{N} \mid m_2 = 1, M \leq N) \to \sqrt{2/\pi}$.

Similarly, $\mathrm{E}(M^2/N^{1.5} \mid m_2 = 1)$ is calculated as follows.

$$
\begin{aligned}
&E(M^2/N^{1.5} \mid m_2 = 1, M \leq N) = \sum_{w=1}^{N} e^{-w} w^{w+1} / (w! N^{1.5}) \\
&> \quad \sum_{w=1}^{N} e^{-w} w^{w+1} / (\sqrt{2\pi} w^{w+0.5} e^{-w+1/(12w)} N^{1.5}) \\
&= \quad \sum_{w=1}^{N} e^{-1/(12w)} \sqrt{w} / (\sqrt{2\pi} N^{1.5}) \\
&> \quad \left(\int_1^{N} e^{-1/(12w)} \sqrt{w} dw \right) / (\sqrt{2\pi} N^{1.5}) \\
&= \quad \left((e^{-1/(12N)} N^{1.5} - e^{-1/12}) / 1.5 + \int_1^{N} (w^{1.5}/1.5) e^{-1/(12w)} (1/(12w^2)) dw \right) / (\sqrt{2\pi} N^{1.5}) \\
&= \quad (e^{-1/(12N)} - e^{-1/12}/N^{1.5}) / (1.5\sqrt{2\pi}) + \int_1^{N} w^{-0.5} e^{-1/(12w)} dw / (18\sqrt{2\pi} N^{1.5}) \\
&\to \quad 1/(1.5\sqrt{2\pi})
\end{aligned}
\tag{55}
$$

where the inequality in the fourth line holds since the function $e^{-1/(12w)}\sqrt{w}$ is increasing in w. Similarly, the upper bound is obtained as follows.

$$
\begin{aligned}
&\sum_{w=1}^{N} e^{-w} w^{w+1} / (w! N^{1.5}) < \sum_{w=1}^{N} e^{-w} w^{w+1} / (\sqrt{2\pi} w^{w+0.5} e^{-w+1/(12w+1)} N^{1.5}) \\
&= \quad \sum_{w=1}^{N} e^{-1/(12w+1)} \sqrt{w} / (\sqrt{2\pi} N^{1.5})
\end{aligned}
$$

$$
\begin{aligned}
&< \int_1^{N+1} e^{-1/(12w+1)}\sqrt{w}dw/(\sqrt{2\pi}N^{1.5}) \\
&= \left(e^{-1/(12N+13)}(N+1)^{1.5} - e^{-1/13} + \int_1^{N+1} w^{-0.5}e^{-1/(12w+1)}(12/(12+1/w)^2)dw\right)/(1.5\sqrt{2\pi}N^{1.5}) \\
&\to 1/(1.5\sqrt{2\pi}) \qquad (56)
\end{aligned}
$$

Hence, we obtain that $E(M^2/N^{1.5} \mid m_2 = 1) \to 1/(1.5\sqrt{2\pi})$.

F Generalization

In this section, we extend our fluctuation results to the case where the distribution of initial capital is not uniform or where the lumpiness and depreciation rate are heterogeneous across firms.

First, we show the case of non-uniform distribution. Namely, we drop Assumption 1 and let the initial gap $s_{i,0}$ to follow any continuous density function $f_0(s_{i,0})$ defined over interval $[0, 1)$. A few notations are to be developed. $\bar{K}$ denoted the initial aggregate capital when there are a continuum of firms. The inaction band is $[\bar{k}, \lambda\bar{k})$ where $\bar{k} = a_0\bar{K}^{\phi}$. $\bar{K}$ is now redefined under the new density function f_0 as: $\bar{K} = (a_0 E_f[\lambda^{s_{i,0}\rho}]^{1/\rho})^{1/(1-\phi)}$. We construct an equilibrium aggregate capital $\hat{K}$ when there are a continuum of firms. On the one hand, $\hat{K}$ satisfies $\hat{K} = (a_0 E_g[\lambda^{s_i\rho}]^{1/\rho})^{1/(1-\phi)}$. On the other hand, $s_{i,0}$ is mapped as:

$$
s_i = \lfloor s_{i,0} - 1/q + \phi(\log \bar{K} - \log \hat{K})/\log \lambda \rfloor \qquad (57)
$$

where $\lfloor\cdot\rfloor$ takes the difference to the nearest integer that is less than the argument. This mapping and the density function f_0 determine the density function f_1 of s_i with $\hat{K}$ as a parameter. Then $\hat{K}$ and f_1 are determined simultaneously. Note that $\hat{K}$ might differ from $\bar{K}$, because the gap density f_1 may differ from f_0 if f_0 is not uniform.

Define ψ as the density evaluated at $s_i = 0$ at the equilibrium in the case of a continuum of firms. Namely, $\psi = f_0(1/q - \phi(\log \bar{K} - \log \hat{K})/\log \lambda)$. Also define $\log \tilde{\lambda} \equiv (\lambda^\rho - 1)/(\rho \mathrm{E}_{f_1}[\lambda^{s_{i,1}\rho}])$. We now obtain the following proposition.

Proposition 5 *Suppose that $k_{i,0}$ follows any continuous density function that has support $[\log \bar{k}, \log \lambda + \log \bar{k})$. Then, Proposition 1 holds with modified constants $\tilde{\phi} \equiv \phi\psi \log \tilde{\lambda}/\log \lambda$ and $\tilde{\sigma}_1$. Moreover, the limit variance of the capital growth rate is strictly positive when $\tilde{\phi} \to 1$ and $N \to \infty$.*

Proof: See Appendix G

Proposition 5 states that the deviation from the uniform distribution of $s_{i,0}$ does not change the parametric form of the distribution function of the aggregate growth rate. The key parameter that determines the speed of exponential truncation is a product of the degree of strategic complementarity ϕ and other two factors: ψ and $\log \tilde{\lambda}/\log \lambda$. The first factor represents the density at the threshold and the second factor indicates the aggregation effect of the non-uniformity. The both factors are 1 if $s_{i,0}$ follows the uniform distribution.

The first factor ψ has a particularly important implication. As the capital configuration evolves over time, the density at the threshold may fluctuate below or above 1. Thus, the fluctuation distribution may attain the critical level $\tilde{\phi} = 1$ due to the fluctuations of ψ over time, even if ϕ is less than 1 due to the decreasing aggregate returns to scale or the flexible wage and interest rate as we see in the dynamic version of the model. The time-varying ψ has another implication on the volatility of the aggregate growth rates over time. The aggregate growth rate exhibits higher volatility when ψ is high. This implies that the (S,s) economies can exhibit the echo effect not only in the level of production but also in the volatility.

Next generalization is to allow heterogeneity in the lumpiness and depreciation rate across firms. Suppose that there are finite L types of firms with parameter values $\delta_i = \delta(l)$ and

$\lambda_i = \lambda(l)$ for $l = 1, 2, \ldots, L$. Each type of a firm is drawn with probability $\sigma(l)$, where $\sum_{l=1}^{L} \sigma(l) = 1$. The lower bound of the inaction band becomes heterogeneous as $k_i^* = a_i K^\phi$ where $a_i \equiv ((\lambda_i^{\theta/\mu} - 1)/(\lambda_i - 1))^{\mu/(\mu-\theta)}$. Other variables such as $\bar{K}, \hat{K}, \bar{k}_i$ are defined similarly as before.

Let $\check{\psi}$ denote the density at $s_i = 0$ at the equilibrium when there are a continuum of firms. Also define $\log \check{\lambda}_i \equiv a_i^\rho (\lambda_i^\rho - 1)/(\rho \mathrm{E}[a_i^\rho \lambda_i^{s_{i,1}\rho}])$. Then we obtain the proposition.

Proposition 6 *Suppose that $k_{i,0}$ follows any continuous density function that has support $[\log \bar{k}_i, \log \lambda_i + \log \bar{k}_i)$. Moreover, λ_i and δ_i vary across firms, and they are randomly drawn from a finite set. Then, M conditional on $m_1 = 1$ follows the same tail distribution as (11):*

$$\Pr(|M| = w \mid m_1 = 1) = C_0 (e^{\check{\phi}-1}/\check{\phi})^{-w} w^{-1.5} \tag{58}$$

for a large integer w, where $\check{\phi} \equiv \phi \check{\psi} \mathrm{E}[\log \check{\lambda}_i / \log \lambda_i]$ and C_0 are constant. The asymptotic variance of the fraction of firms that adjust, $(m_1 + M)/N$, is strictly positive when $\check{\phi} \to 1$ and $N \to \infty$.

Proof: See Appendix H.

Proposition 6 shows that our fluctuation result is robust to the heterogeneity of firms. The tail distribution is derived explicitly and shown to coincide with our previous result. However, the exact distribution is obtained only implicitly as a form of the functional equation of the moment generating function of M (see Nirei (2003)).

Empirical studies attest enormous degree of heterogeneity of firms, which tends to render the collective dynamics of firms intractably complex. Nonetheless, the parametric form we derived for the distribution of the number of investing firms still stands. We also note that the heterogeneity accelerates the convergence of the firms' positions in the inaction band to the simple uniform distribution in one-sided (S,s) economies as Caballero and Engel

(1991) observed. The uniform distribution of firms' positions provides an important reference point in our model, even though we can derive the aggregate capital fluctuations for general distributions as shown by Proposition 5.

G Proof of Proposition 5

Randomly draw $s_{i,0}$ from density f_0 for N firms and construct the initial capital $\lambda^{s_{i,0}} a_0 \hat{K}^{\phi}$ in the inaction band. Set the initial aggregate capital K_0 at the aggregate of the $k_{i,0}$. The difference between K_0 and $\hat{K}$ can occur only due to the finiteness of the firms, and thus $\log K_0 - \log \hat{K}$ vanishes according to the law of large numbers.

The subsequent best response dynamics is defined similarly as before. The rest of the proof proceeds similarly with some modifications as follows. Define f_u as the density of $s_{i,u}$. Then $\sum_{i=1}^{N} \lambda^{s_{i,u}\rho}/N \to \mathrm{E}_{f_u}[\lambda^{s_{i,u}\rho}]$ as $N \to \infty$. Since $\log k_u^* - \log k_0^*$ is of order $1/N$, the density f_u converges to f_0 as $N \to \infty$. Then Lemma 2 is modified as $N(\log K_{u+1} - \log K_u) \to m_{u+1} \log \tilde{\lambda}$. Note that $\log \tilde{\lambda}$ is equal to $\log \lambda$ if f_1 is a uniform distribution.

Let us note that the conditional probability for firm i to invest in step u is the right hand side of Equation (37) times $f_u(0)$, because the density at the threshold $f_u(0)$ is not necessarily equal to one without Assumption 1. Since $\log k_u^* - \log k_0^*$ is of order $1/N$, $f_u(0) \to f_0(0) = \psi$ as $N \to \infty$. By combining this with the modified Lemma 2 as in the previous paragraph, Lemma 3 holds by modifying the Poisson mean ϕ to $\tilde{\phi}$. Then the proof of Proposition 1 holds by replacing ϕ with $\tilde{\phi}$.

The σ_1 is also modified to $\tilde{\sigma}_1$ so that the effect of ψ is taken into account. For i to be in H_1 is equivalent to $s_{i,0} < 1/q - \phi(\log \bar{K} - \log \hat{K})/\log \lambda$. Thus, the number of firms in H_1 follows the binomial distribution with population N and probability $1/\tilde{q} \equiv F_0(1/q - \phi(\log \bar{K} - \log \hat{K})/\log \lambda)$ where F_0 denotes the cumulative distribution of $s_{i,0}$. Then the modified $\tilde{\sigma}_1$ is obtained by following the computation of σ_1 with applying $\tilde{q}$ and $\log\tilde{\lambda}$.

H Proof of Proposition 6

Let $N(l)$ denote the total number of firms of type l and $m_u(l)$ denote the number of firms of type l that adjust capital in step u.

First, we show the counterpart of Lemma 2 as follows.

$$\begin{aligned} N(\log K_{u+1} - \log K_u) &= \sum_{n=1}^{\infty}\sum_{i\in H_{u+1}}\left(\frac{k_{i,u}}{K_u}\right)^{\rho}\frac{\rho^{n-1}(\log\lambda_i)^n}{n!} + O(1/N) \\ &= \frac{\sum_{i\in H_{u+1}} a_i^{\rho}\lambda_i^{s_{i,u}\rho}\sum_{n=1}^{\infty}\rho^{n-1}(\log\lambda_i)^n/n!}{\sum_i a_i^{\rho}\lambda_i^{s_{i,u}\rho}/N} + O(1/N) \\ &\to \frac{\sum_{i\in H_{u+1}} a_i^{\rho}(\lambda_i^{\rho}-1)/\rho}{\mathrm{E}(a_i^{\rho}\lambda_i^{s_{i,u}\rho})} \end{aligned} \tag{59}$$

Define Z_{u+1} as the right hand side of (59). It has mean $m_{u+1}\mathrm{E}[\log\check{\lambda}_i]$.

We then show that $(m_u)_u$ follows a branching process. Let F_1 denote the cumulative distribution function of $s_{i,1}$.

$$\Pr(i\in H_u, b_i = b(l) | i\notin \cup_{v=2,3,\dots,u-1}H_v) \tag{60}$$

$$= \sigma(l)\frac{F_1(\phi(\log K_u - \log K_0)/\log\lambda(l)) - F_1(\phi(\log K_{u-1} - \log K_0)/\log\lambda(l))}{1 - F_1(\phi(\log K_{u-1} - \log K_0)/\log\lambda(l))} \tag{61}$$

Thus $m_u(l)$ follows a binomial distribution with probability above and population $N(l) - \sum_{v=2}^{u-1} m_v(l)$. Considering that m_v, $v = 2,3,\dots,u-1$ are finite with probability one, we obtain the asymptotic mean of the binomial as $\sigma(l)\phi\psi Z_u/\log\lambda(l)$. Thus, $m_u(l)$ asymptotically follows a Poisson distribution with this mean. Hence, $m_u = \sum_{l=1}^{L} m_u(l)$ asymptotically follows a Poisson distribution with mean $\phi\psi\mathrm{E}[\log\check{\lambda}_i/\log\lambda_i]m_{u-1} = \check{\phi}m_{u-1}$.

The vector of Poisson random variables $(m_u(l))_l$ conditional on its sum m_u follows a multinomial distribution with probability vector $((\sigma(l)/\log\lambda(l))/\mathrm{E}[1/\log\lambda_i])_l$ and population m_u (Kingman, 1993, page 7). Z_u is a sum of the multinomial vector with weights $a(l)^\rho(\lambda(l)^\rho - 1)/(\rho\mathrm{E}(a_i^\rho\lambda_i^{s_{i,u}\rho}))$. Thus, Z_u conditional on m_u is equivalent to a m_u-times convolution of a random variable. Then, m_{u+1} conditional on m_u asymptotically follows a compound Poisson distribution. Since a compound Poisson distribution is infinitely divisible, $(m_u)_u$ follows a branching process in which each firm in step u bears children in step $u+1$ whose number follows the compound Poisson distribution that has mean $\check{\phi}$. By the theorem by Otter (see Harris (1989)), a cumulative sum of a branching process follows the distribution as in Proposition. Finally, the process (m_u) is finite with probability 1 if $\check{\phi} \leq 1$. This completes the proof.

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