Altruism, Environmental Externality and Fertility*

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Abstract

We investigate the interaction between environmental quality and fertility in an altruistic bequest model with pollution externalities created by the aggregate production. Despite the negative externality related to the endogenous childbearing decisions, the parents may choose to have fewer children in the competitive economy than in the social optimum. To achieve optimality, positive taxes on childbearing are required even with an insufficient number of children, if the social discount factor equals the parents' degree of altruism. On the other hand, child allowances may constitute the optimal policy if the social discount factor exceeds the parents' degree of altruism.

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1. Introduction

Falling fertility rates and falling environmental qualities are great problems among others for many modern societies, and, as often suggested, a close relationship is present between population growth and the magnitude of pollution. While there is no question that population growth contributes to environmental degradation,[1] a causal relationship in the opposite direction may also exist, i.e., environmental qualities may affect the fertility decisions in a family. If parents are altruistic toward their children and are concerned about their children's welfare (Barro, 1974), they may choose to have a smaller number of children in response to environmental degradation, which leave their children worse off. From this point of view, altruism could play a crucial role in the interaction between fertility and environmental qualities.

Jouvet, Michel and Vidal (2000) and Jouvet, Michel and Pestieau (2000) investigate environmental issues introducing altruistic bequests. (The latter authors consider the case where altruists and non-altruists coexist.) Assuming that individuals voluntarily contribute to pollution abatement, these studies show that a market economy results in under-contribution to pollution abatement and thus an under-provision of environmental quality due to the free rider problem. In these models, bequests also create environmental externalities via the production process, which lead to an over-accumulation of capital. To attain the social optimum, therefore, the government requires subsidies on contributions to pollution abatement and taxes on capital. These studies, however, assume exogenous fertility and the relationship between fertility and environmental qualities is outside their scope.

To the best of our knowledge, there is no study addressing the issues of fertility choices and environmental externalities in the presence of altruism, except Harford (1997, 1998). Harford (1998) considers a consumable capital

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[1] Cropper and Griffiths (1994) provide evidence for a significant effect of population density on deforestation.

good and a non-capital good, and consumption of the latter is assumed to create a pollution externality. While an increase in the number of children implies an increase in aggregate consumption of the polluting good, parents do not recognize such an impact of an extra child on pollution, and hence childbearing also has an external effect. He shows that Pareto efficiency requires taxes on childbearing as well as Pigovian pollution taxes. Taxes on capital are not called for in his model, since bequests of capital do not entail externalities.

In this paper we investigate the relationship between fertility and environmental qualities by assuming that production causes pollution and bequests embodied in productive capital create pollution externalities, as in Jouvet, Michel and Vidal (2000) and Jouvet, Michel and Pestieau (2000). Since aggregate production is increasing in population, pollution externalities of childbearing also prevail in our model. The co-existence of these two externalities leads to a result different from that obtained in the previous studies, namely, that the fertility rate determined in a market economy may be lower than the social optimum, although childbearing has a negative external effect on the environment. Parents choose the number of children so that the marginal benefit equals the marginal cost of having a child, and bequests toward each child constitute the marginal cost of a child. Thus, an increase in bequests raises the marginal cost of a child, and has a negative effect on the number of children. Therefore, if the level of bequests in the competitive equilibrium is higher than the social optimal level, this over-provision of bequests raises the private marginal cost of a child possibly to a level above its social marginal cost. In such a case, the number of children in the competitive equilibrium rather falls below the social optimum. On the other hand, we show that the level of pollution is unambiguously higher than the social optimum, whether the fertility rate (or per capita capital) is too high or too low.

We also examine what kind of policy is required to achieve social optimality. It is shown that, if the social discount factor equals the private degree of altruism, the government needs to tax both childbearing and inheritance so as to restore efficiency, even if the fertility rate is lower than its social optimal level. This is because an over-accumulation of capital is a necessary condition for under-production of children. Once capital is adjusted to its optimal level by inheritance taxes, the factor in the under-production of children disappears, and the fertility rate exceeds its social optimal level due to its environmental externalities. On the other hand, if the social discount factor is higher than the private degree of altruism, child allowances and/or subsidies on inheritance may be required to achieve optimality. However, the optimal policy never involves a combination of taxes on childbearing and subsidies on inheritance.

The rest of the paper is organized as follows. Section 2 presents a model, and characterizes the competitive equilibrium. Section 3 characterizes the social optimum, and compares it to competitive equilibrium in the benchmark case where the social discount factor equals to the private degree of altruism. Section 4 examines what kind of policy is required to decentralize the social optimum. Section 5, assuming that the social discount factor different from private degree of altruism, reexamines the result obtained in the previous sections. Section 6 provides the conclusions.

2. The Basic Model

Suppose that there are two periods and two generations. The parents' generation (generation 0) lives for period 0 and the children's generation (generation 1) lives for period 1, with no overlapping of the periods. Each member of the same generation is identical. The population of generation 0 is N, and each member of generation 0 produces n children.

As in Becker and Barro (1988), the parents decide to have n children because they are altruistic toward their children in that each child's welfare directly enters their utility functions. It is assumed that each child costs

$\beta\ (>0)$, so that $n\beta$ is the total cost of raising children. The parents allocate the remaining income after they have paid the cost of raising children between their own consumption and bequests toward their children. We also assume that the inheritance from the former generation determines the income of each generation.

The parents derive disutility from the level of pollution while deriving utility from consumption and their children's welfare. Their utility function is thus defined by

(1) $$U_0(c_0,\ \pi_0,\ n,\ U_1) = u_0[(1+r)b_0 - n(b_1+\beta)] - V_0(\pi_0) + n\delta(n)U_1,$$

where $c_0(=(1+r)b_0 - n(b_1+\beta))$ is the parent's consumption, b_0 is the inheritance they receive, b_1 is the bequests to each child, r is the interest rate, π_0 is the level of pollution in period 0, U_1 is the utility of each child, and $\delta(n)$ is the weight attached to each child's utility. We assume that $u_0' > 0$, $u_0'' < 0$, $V_0' > 0$, $V_0'' > 0$, $0 < \delta(n) < 1$, $\delta'(n) < 0$, $\delta(n) + \delta'(n)n > 0$ and $2\delta'(n) + \delta''(n)n < 0$.

The children consume the inheritance from their parents, and their utility function is defined by

(2) $$U_1(c_1,\ \pi_1) = u_1[(1+r)b_1] - V_1(\pi_1),$$

where π_1 is the level of pollution in period 1. We assume that $u_1' > 0$, $u_1'' < 0$, $V_1' > 0$ and $V_1'' > 0$.

Given $(1+r)b_0$, β and $\pi_i\ (i = 0,\ 1)$, the parents choose the number of children first and then choose the level of bequests to each child. In choosing the level of bequests, the parents maximize (1) with respect to b_1, given n. The first-order condition is

(3) $$-nu_0'[(1+r)b_0 - n(b_1+\beta)] + n\delta(n)(1+r)u_1'[(1+r)b_1] = 0,$$

from which we obtain $b_1 = b_1^*(n)$. In choosing the number of children, the parents substitute $b_1 = b_1^*(n)$ into (1) and maximize the utility function with respect to n. Using the envelope theorem, we have the following first-order condition:

$$(4) \qquad \begin{aligned} &-(b_1^*(n)+\beta)u_0'[(1+r)b_0 - n(b_1^*(n)+\beta)] \\ &\quad + (\delta(n)+n\delta'(n))\left\{u_1[(1+r)b_1^*(n)] - V_1(\pi_1)\right\} = 0. \end{aligned}$$

Denoting the solution of (4) as n^*, we obtain the parents' optimum $(b_1^*,\ n^*)$ from (3) and (4).

We assume that the level of pollution in each period is a linear function of current production Y_i, and that no pollutants survive the period. We thus have

$$(5) \qquad \pi_i = \alpha Y_i;\ \ \alpha > 0,\ \ i = 0,\ 1.$$

Assuming a linear technology, we define the production function as

$$(6) \qquad Y_i = AK_i;\ \ A > 0,\ \ i = 0,\ 1,$$

where K_i is the stock of capital in period i.

Equilibrium on the capital market implies

$$(7) \qquad b_i = k_i;\ \ i = 0,\ 1,$$

where $k_0 \equiv K_0 / N$ and $k_1 \equiv K_1 / nN$. At equilibrium the rate of interest is equal to the marginal productivity of capital net of depreciation:[2]

$$(8) \qquad 1 + r = A.$$

We hereafter denote k_1 as k for notational simplicity. Substituting (5), (6), (7) and (8) into (3) and (4) yields

$$(9) \qquad F(k,\ n) \equiv -nu_0'[Ak_0 - n(k+\beta)] + n\delta(n)Au_1'(Ak) = 0,$$

$$(10) \qquad \begin{aligned} G(k^*(n),\ n) \equiv &-(k^*(n)+\beta)u_0'[Ak_0 - n(k^*(n)+\beta)] \\ &+ [\delta(n)+n\delta'(n)]\left\{u_1[Ak^*(n)] - V_1[\alpha ANnk^*(n)]\right\} = 0, \end{aligned}$$

where $k^*(n) = b_1^*(n)$. The competitive equilibrium (k^*, n^*) is characterized by (9) and (10).

The first term in the RHS of (10) is the marginal disutility from the decrease in parental consumption by having an additional child, and can be defined as the private marginal cost of a child. The second term in the RHS of (10) is the increase in parental utility derived from altruism when adding an additional child, and can be defined as the private marginal benefit of a child. Thus, we can define $G(k^*(n),\ n)$ as the private marginal net benefit (PMNB) of a child.

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[2] We assume total depreciation after one period.

3. Social Optimum

In this section, we characterize the social optimum, and compare it to the competitive equilibrium obtained in the previous section. In particular, we show that the number of children chosen may be lower in the competitive equilibrium than in the social optimal allocation, albeit children create negative environmental externalities.

3.1 Characterizing the Social Optimum

We assume a central planner that adopts a utilitarian social welfare function consisting of the discount sum of individual's utilities. As a benchmark, we first assume that the social discount factor equals the parent's degree of altruism. The social welfare function is thus defined by

$$W = N\left\{u_0[Ak_0 - n(k+\beta)] - V_0(\pi_0(AK_0)) + n\delta(n)[u_1(Ak) - V_1(\alpha ANnk)]\right\}. \tag{11}$$

Given k_0, A, β and α, the central planner chooses n first and then chooses k so as to maximize (11). The first-order condition with respect to k is

$$F^S(k,\ n) \equiv -nu_0'[Ak_0 - n(k+\beta)] + n\delta(n)\left\{Au_1'(Ak) - \alpha ANnV_1'(\alpha ANnk)\right\} = 0, \tag{12}$$

from which we obtain $k = k^S(n)$. Substituting $k = k^S(n)$ into (11) and maximize that function with respect to n yields the following first-order condition:

$$\begin{aligned} G^S(k^S(n),\ n) \equiv & -(k^S(n)+\beta)u_0'[Ak_0 - n(k^S(n)+\beta)] \\ & + (\delta(n) + n\delta'(n))[u_1(Ak^S(n)) - V_1(\alpha ANnk^S(n))] \\ & - n\delta(n)(\alpha ANk^S(n))V_1'(\alpha ANnk^S(n)) = 0. \end{aligned} \tag{13}$$

Denoting the solution of (13) as n^S, we obtain the social optimum $(k^S,\ n^S)$ from (12) and (13). $G^S(k^S(n),\ n)$ can be defined as the social marginal net benefit (SMNB) of a child.

3.2 Comparing the Competitive Equilibrium to the Social Optimum

In our model, the parents do not take into account the effects of k and n

on pollution *via* the production process. This implies that both childbearing and bequeathing to children have pollution externalities. Comparing (10) to (13), it follows that the SMNB of n is smaller than its PMNB by $n\delta(n)(\alpha ANk)V_1'$ if $k^*(n) = k^S(n)$. Similarly, a comparison of (9) with (12) indicates that the SMNB of k is smaller than its PMNB by $n\delta(n)(\alpha ANn)V_1'$ given n. This does not imply, however, that k and n are determined higher in the competitive equilibrium than in the social optimum, because there exists an interaction between k and n. That is, an increase in bequests raises the marginal cost of having a child, and thus has a negative effect on the number of children. Therefore, if the level of capital accumulation in the competitive equilibrium is higher than the social optimal level, and this over-accumulation of capital raises the PMNB of a child to a level above its SMNB, the number of children in the competitive equilibrium rather falls below that in the social optimum.[3]

Paying attention to the interaction of k and n, we now derive conditions for $n^* < n^S$. For this purpose, we compare the relative magnitude of the SMNB and the PMNB of a child, with n fixed at n^S. Using the mean value theorem, (10) and (13) imply

$$\begin{aligned}(14)\qquad & G(k^*(n^S),\, n^S) - G^S(k^S,\, n^S) \\ & = [k^*(n^S) - k^S]G_k(\hat{k},\, n^S) + n^S\delta(n^S)\alpha ANk^S V_1'(\alpha ANn^S k^S),\end{aligned}$$

where $G_k(\hat{k},\, n^S) \equiv \partial G(\hat{k},\, n^S)/\partial k = n^S(\hat{k}+\beta)u_0'' + n^S\delta'(n^S)Au_1' - [\delta(n^S) + n^S\delta'(n^S)]\alpha ANnV_1' < 0$ and $\hat{k} \in (k^S,\, k^*(n^S))$. Since $k^*(n^S) > k^S$,[4] the sign of (14) is indeterminate in

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[3] Similarly, noting the impact of the number of children on the marginal cost of bequests, the relative magnitude of k^* and k^S is indeterminate.

[4] We evaluate (9) when $k = k^S$ and $n = n^S$. Substituting (12) into (9) yields $F(k^S,\, n^S) = n^S\delta(n^S)(\alpha ANn^S)V_1'(\alpha ANn^S k^S) > 0.$ From $F(k^*(n^S),\, n^S) = 0$ and $F_k \equiv \partial F(k,\, n)/\partial k = n^2 u_0'' + n\delta(n)A^2 u_1'' < 0$, we thus have $k^*(n^S) > k^S$.

general, and is negative if

(15) $$-[k^*(n^S)-k^S]G_k(\hat{k},\ n^S) > n^S\delta(n^S)(\alpha ANk^S)V_1'(\alpha ANn^S k^S)\ .$$

Since both $G(k^*(n),\ n)$ and $G^S(k^S(n),\ n)$ are decreasing in n,[5] we obtain the following proposition:

Proposition 1. $-[k^*(n^S)-k^S]G_k(\hat{k},\ n^S) > n^S\delta(n^S)(\alpha ANk^S)V_1'(\alpha ANn^S k^S) \Rightarrow n^* < n^S$

The implication of Proposition 1 is as follows. The LHS of (15) represents the decrease in the PMNB of a child, or the increase in the private marginal cost of a child, when capital (i.e., bequests) increases from k^S to $k^*(n^S)$. (Note that $k^*(n)$ unambiguously exceeds $k^S(n)$, given n, as a result of the pollution externality.) The RHS of (15) is the environmental effects of n which the parents do not take into account in calculating the PMNB of a child. If the former dominates the latter, then the SMNB of n exceeds the PMNB of n when $n = n^S$, and thus the number of children determined in the competitive equilibrium is lower than the social optimum.

We next compare the level of capital in the competitive equilibrium to its social optimal level. It is shown that capital is over-accumulated if the condition in Proposition 1 holds. As discussed earlier, this implies that $k^* > k^S$ is a necessary condition for $n^* \le n^S$.

Proposition 2. $n^* \le n^S \ \Rightarrow\ k^* > k^S$

Proof. From (9) we obtain

(16) $$\frac{\partial k^*(n)}{\partial n} = -\frac{F_n(k,\ n)}{F_k(k,\ n)} = -\frac{n(k+\beta)u_0'' + n\delta'(n)Au_1'}{n^2u_0'' + n\delta(n)A^2u_1''} < 0\ .$$

Since $k^*(n^S) > k^S$, this implis that $k^*(=k^*(n^*)) > k^*(n^S) > k^S$. □

[5] The second-order conditions for the social optimum ensure $dG^S(k^S(n),\ n)/dn < 0$. The proof for $dG(k^*(n),\ n)/dn < 0$ is available on request from the authors.

We next examine the relative magnitude of pollution between the competitive equilibrium and the social optimum. We show that the level of pollution is unambiguously higher in the competitive equilibrium than in the social optimum, irrespective of the relative magnitude between n^* and n^S and between k^* and k^S. We thus turn to the following proposition:

Proposition 3. $\pi^* > \pi^S$

Proof. We define the following function:

$$\begin{aligned}\hat{F}(k,\ n;\ \mu) \equiv & -nu_0'[Ak_0 - n(k+\beta)] + n\delta(n)Au_1'(Ak) \\ & - \mu\{n\delta(n)\alpha ANnV_1'(\alpha ANnk)\} = 0,\end{aligned} \tag{17}$$

$$\begin{aligned}\hat{G}(k,\ n;\ \mu) \equiv & -(k+\beta)u_0'[Ak_0 - n(k+\beta)] \\ & + (\delta(n) + n\delta'(n))\{u_1(Ak) - V_1(\alpha ANnk)\} \\ & - \mu[n\delta(n)\alpha ANkV_1'(\alpha ANnk)] = 0\end{aligned} \tag{18}$$

Note that the competitive equilibrium $k = k^*$ and $n = n^*$ satisfies (17) and (18) when $\mu = 0$, whereas the social optimum $k = k^S$ and $n = n^S$ satisfies them when $\mu = 1$.

Differentiating (17) and (18) yields[6]

$$\frac{dk}{d\mu} = \frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}F_n - G_n\right], \tag{19}$$

$$\frac{dn}{d\mu} = \frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}F_k + G_k\right], \tag{20}$$

where $G_n \equiv \partial G / \partial n = (k+\beta)^2 u_0'' + [2\delta'(n) + n\delta''(n)](u_1 - V_1) - (\delta(n) + n\delta'(n))\alpha ANkV_1' \ (<0)$, $\hat{F}_\mu \equiv \partial \hat{F} / \partial \mu = -n\delta(n)(\alpha ANn)V_1' \ (<0)$ and $D(\mu)$ (>0) is the determinant of the Jacobian.[7]

Differentiating $\pi_1 = \alpha ANnk$ with respect to μ and substituting (19) and (20) yields

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[6] See Appendix 1.

[7] The proof for $D(\mu) > 0$ is available on request from the authors.

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\begin{aligned}
\frac{d\pi}{d\mu} &= \alpha AN\left(n\frac{dk}{d\mu}+k\frac{dn}{d\mu}\right) \\
&= \frac{\alpha AN\hat{F}_{\mu}}{D(\mu)}\left[n\left(\frac{k}{n}F_n - G_n\right)+k\left(-\frac{k}{n}F_k + G_k\right)\right].
\end{aligned} \tag{21}
$$

Furthermore, subsituting $G_k = F_n - (\delta(n)+n\delta'(n))\alpha ANnV_1'$ and $G_n = U_{nn} - (\delta(n)+n\delta'(n))\alpha ANkV_1'$ (where $U_{nn} \equiv \partial^2 U_0/\partial n^2$) into (21) yields

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\begin{aligned}
\frac{d\pi}{d\mu} &= \frac{-\alpha ANn\hat{F}_{\mu}}{D(\mu)}\left[\left(\frac{k}{n}\right)^2 F_k - 2\frac{k}{n}F_n + U_{nn}\right] \\
&= \frac{-\alpha ANn\hat{F}_{\mu}}{D(\mu)F_k}\left[(\frac{k}{n}F_k - F_n)^2 + F_kU_{nn} - (F_n)^2\right].
\end{aligned} \tag{22}
$$

where we have $F_kU_{nn} - (F_n)^2 > 0$ from the second-order conditions for the parent's utility maximization, implying that $d\pi/d\mu < 0$. Thus, we have $\pi^* > \pi^S$. □

Propositions 1 and 3 imply the possibility that society may suffer from simultaneously insufficient fertility and excess pollution.

4. Optimal Policy

This section examines whether the social optimum can be decentralized. In our model, since the parents fail to take into account the effects of production on pollution in choosing the number of children and the amount of bequests to each child, *laissez faire* leads both the fertility rate and per capita capital to become suboptimal. To control two variables, decentralization requires two policy instruments. Among many instruments the government can use, we consider taxes (or subsidies) on inheritance and taxes on childbearing (or child allowances), which would directly affect the decisions on fertility and bequests in the family.

4.1 Decentralizing the Social Optimum

The government budget is balanced by lump-sum transfers to private individuals in each period. We thus have

(23) $$nT = \theta,$$

(24) $$(1+r)b_1\tau = \eta,$$

where T is a tax per child imposed on the parents, τ is the tax rate on bequests to each child, θ is a lump-sum transfer to each parent, and η is a lump-sum transfer to each child.

The parent's utility function (1) is rewritten as

(25) $$\begin{aligned} U_0 = u_0[(1+r)b_0 - n(b_1+\beta+T)+\theta] - V_0(\pi(Y_0)) \\ + n\delta(n)\left\{u_1[(1-\tau)(1+r)b_1+\eta] - V_1(\pi(Y_1))\right\}. \end{aligned}$$

The competitive equilibrium in this case, $k = k^*(\tau,\ T)$ and $n = n^*(\tau,\ T))$, satisfies the following conditions:

(26) $$\begin{aligned} \tilde{F}(k,\ n;\ \tau,\ T) \equiv -nu_0'[Ak_0 - n(k+\beta+T)+\theta] \\ + n\delta(n)(1-\tau)Au_1'[(1-\tau)Ak+\eta] = 0, \end{aligned}$$

(27) $$\begin{aligned} \tilde{G}(k,\ n;\ \tau,\ T) \equiv -(k+(\beta+T))u_0'[Ak_0 - n(k+\beta+T)+\theta] \\ + (\delta(n)+n\delta'(n))\left\{u_1[(1-\tau)Ak+\eta] - V_1(\alpha ANnk)\right\} = 0. \end{aligned}$$

If the government realizes the social optimum in a decentralized economy with $\tau = \tau^*$ and $T = T^*$, we have

(28) $$k^*(\tau^*,\ T^*) = k^S,$$

(29) $$n^*(\tau^*,\ T^*) = n^S.$$

Substituting (23), (24), (28) and (29) into (26) and (27) yields

(30) $$-n^S u_0'[Ak_0 - n^S(k^S+\beta)] + n^S\delta(n^S)(1-\tau^*)Au_1'(Ak^S) = 0,$$

(31) $$\begin{aligned} -(k^S+\beta+T^*)u_0'[Ak_0 - n^S(k^S+\beta)] \\ + (\delta(n^S)+n^S\delta'(n^S))[u_1(Ak^S) - V_1(\alpha ANn^S k^S)] = 0. \end{aligned}$$

Since (k^S, n^S) also satisfies (12) and (13), equations (30) and (12) with $k = k^S$ and $n = n^S$ imply

(32) $$-\tau^* n^S\delta(n^S)Au_1'(Ak^S) + n^S\delta(n^S)(\alpha ANn^S)V_1'(\alpha ANn^S k^S) = 0.$$

Hence we have

(33) $$\tau^* = \frac{\alpha Nn^S V_1'(\alpha ANn^S k^S)}{u_1'(Ak^S)} > 0.$$

Also, (31) and (13) with $k = k^S$ and $n = n^S$ imply

(34) $$-T^* u_0'[Ak_0 - n^S(k^S + \beta)] + n^S \delta(n^S)\alpha ANk^S V_1'(\alpha ANn^S k^S) = 0,$$

and thus

(35) $$T^* = \frac{n^S \delta(n^S)\alpha ANk^S V_1'(\alpha ANn^S k^S)}{u_0'[Ak_0 - n^S(k^S + \beta)]} > 0.$$

Hence we have the following proposition:

Proposition 4. If the social welfare function is given by (11), the social optimum can be decentralized with inheritance taxation and childbearing taxation that are defined in (33) and (35), respectively.

4.2 Implications of the Optimal Policy

Equation (35) implies that T^* is positive independent of the relative magnitude between n^* and n^S. (Similarly, (33) implies that τ^* is positive independent of the relative magnitude between k^* and k^S.) We now discuss why childbearing should be taxed to achieve social optimality, even when the number of children is too low in relation to the social optimum.[8]

In Figure 1, lines F and G respectively represent (26) and (27) with $\tau = T = 0$ in the (k, n) plane.[9] In this case, the number of children is too low and the level of bequests is too high at the equilibrium point E. Since an increase in τ shifts F to the left, and an increase in T shifts G downward,[10] these lines move to $\tilde{F}^*(=\tilde{F}(k, n;\ \tau^*,\ T^*))$ and

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8 A similar discussion could be applied to the reason τ should be positive.

9 Differentiating (26) and (27) with respect to k and n, it can be shown that both F and G slope downward, and F is steeper than G.

10 Differentiating (26) with respect to k, τ, T, θ and η, given n, and noting $Tdn + ndT = d\theta$ and $A(\tau dk + kd\tau) = d\eta$, which are derived from (23) and (24) respectively, yields $\partial k / \partial \tau = -(\tilde{F} / \tilde{F}_k) < 0$ and $\partial k / \partial T = 0$, where $\tilde{F}_k \equiv n^2 u_0'' + n\delta(n)(1-\tau)A^2 u_1'' < 0$ and $\tilde{F}_\tau \equiv -n\delta(n)Au_1' < 0$. Similarly, differentiating

$\tilde{G}^*(=\tilde{G}(k,\ n;\ \tau^*,\ T^*))$ if the government adopts $\tau=\tau^*(>0)$ and $T=T^*(>0)$. As a result, the social optimum S is achieved in the decentralized economy.

To explain the reason why the government should tax childbearing although the number of children is insufficient in the initial equilibrium, we first suppose that the government uses only an inheritance tax τ as a policy tool. We see that an inheritance tax suffices to attain the optimal level of capital k^S as shown in Figure 1, in which a bequest tax such that $\tau=\tau'$ alters the equilibrium to point D by shifting F to $\tilde{F}'$. However, the new equilibrium D is suboptimal because the number of children is too high relative to the social optimum ($n^*(\tau',\ 0)>n^S$). Once k is adjusted to its optimal level, the factor in the insufficiency of n disappears, and n exceeds its optimal level. In this stage, the government needs to tax childbearing to internalize a pollution externality children will create.

5. Alternative Social Discount Rate

Throughout the previous sections, we have maintained the assumption that the central planner counts the children's welfare only through the parent's welfare. In this section, we relax this assumption and consider that the social discount factor $\gamma(n)$ differs from the parents' degree of altruism $\delta(n)$. Since such an extension would not affect the results obtained above when $\gamma(n)<\delta(n)$, we concentrate on the case where $\gamma(n)>\delta(n)$. In particular, we suppose that the central planner counts all individuals' welfare equally, i.e., $\gamma(n)=1$, to simplify the analysis. In this case, the parents value the children's welfare less than the central planner, and the parents' behavior on fertility and inheritance creates other types of externalities.

The social welfare function is now defined as

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(27) with respect to n, τ, T, θ and η, given k, and noting $Tdn+ndT=d\theta$ and $A(\tau dk+kd\tau)=d\eta$ yields $\partial n/\partial\tau=0$ and $\partial n/\partial T=-(\tilde{G}_T/\tilde{G}_n)<0$, where $\tilde{G}_T\equiv -u_0'<0$ and $\tilde{G}_n\equiv(k+\beta+T)(k+\beta)u_0''+(2\delta'(n)+n\delta''(n))(u_1-V_1)-(\delta(n)+n\delta'(n))\alpha NAkV_1'<0$.

(36) $$\hat{W} = N\{u_0[Ak_0 - n(k+\beta)] - V_0(\pi_0(AK_0)) + n[u_1(Ak) - V_1(\alpha ANnk)]\}.$$

Accordingly, the social optimum is characterized by

(37) $$\hat{F}^S(k,\ n) \equiv -nu_0'[Ak_0 - n(k+\beta)] + nA[u_1'(Ak) - \alpha NnV_1'(\alpha ANnk)] = 0,$$

(38) $$\begin{aligned}\hat{G}^S(k^S(n),\ n) \equiv {} & -(k^S(n)+\beta)u_0'[Ak_0 - n(k^S(n)+\beta)] \\ & + [u_1(Ak^S(n)) - V_1(\alpha ANnk^S(n))] \\ & - n(\alpha ANk^S(n))V_1'(\alpha ANnk^S(n)) = 0.\end{aligned}$$

In this case, while Proposition 1 would be maintained by slight modification of the sufficient condition, Proposition 2 is no longer valid. Whether k^* is higher or lower than k^S, the number of children can be too low due to the positive externality of childbearing that stems from the difference between the private and social welfare weights. Thus, $k^* > k^S$ is not a necessary condition for $n^* \geq n^S$. This implies that we do not necessarily have $k^*(n^S) > k^S$ in this case.

The assumption that $\gamma(n) > \delta(n)$ has policy implications different from those in the previous section. Thus, Proposition 4 is also not fully maintained: the social optimum can be still decentralized, but the optimal policy does not necessarily imply taxing both on childbearing and inheritance.

Equations (33) and (35) are reduced to

(39) $$\hat{\tau}^* = \frac{\alpha Nn^S V_1'(\alpha ANn^S k^S) - [1-\delta(n^S)]u_1'(Ak^S)}{\delta(n^S)u_1'(Ak^S)},$$

(40) $$\hat{T}^* = \frac{\alpha ANn^S k^S V_1'(\alpha ANn^S k^S) - [1-\delta(n^S) - n^S\delta'(n^S)][u_1(Ak^S) - V_1(\alpha ANn^S k^S)]}{u_0'[Ak_0 - n(k^S+\beta)]}.$$

The sign of $\hat{\tau}^*$ and $\hat{T}^*$ may be positive or negative, depending on whether the pollution externalities, whose effects are captured by the first term in the numerator of (39) and (40), dominate or are dominated by the externalities arising from the parents' underestimation of the children's welfare, whose effects are captured by the second term in the numerator of (39) and (40). Thus, if the net external effect of children (bequests) is positive, child allowances (subsidies on bequests) are required to achieve optimality. It should be noted, however, that the sign of $\hat{\tau}^*$ and $\hat{T}^*$ is not to be determined independent of the sign of the other, as shown in the following proposition:

Proposition 5. $\hat{\tau}^* \leq 0 \Rightarrow \hat{T}^* < 0$

Proof. Using (37) and (38), we rewrite (39) and (40), respectively, as follows:

(41) $$\hat{\tau}^* = \frac{\alpha ANn^S k^S V_1' - [1-\delta(n^S)]k^S(u_0' + \alpha ANn^S k^S V_1')}{Ak^S \delta(n^S) u_1'(Ak^S)},$$

(42) $$\hat{T}^* = \frac{\alpha ANn^S k^S V_1' - [1-\delta(n^S) - n^S \delta'(n^S)][(k^S + \beta)u_0' + \alpha ANn^S k^S V_1']}{u_0'[Ak_0 - n(k^S + \beta)]}.$$

Subtracting the numerator of (42) from that of (41) yields

$$[1-\delta(n^S) - n^S \delta'(n^S)]\beta u_0' - n^S \delta'(n^S)(k^S u_0' + \alpha ANn^S k^S V_1') > 0.$$

Since the denominators of (42) and (41) are both positive, if $\hat{\tau}^* \leq 0$, then $\hat{T}^* < 0$. □

Proposition 5 implies that, if the social welfare function is given by (36), a combination of taxes on childbearing and subsidies on inheritance can not implement social optimum in a market economy.

6. Conclusion

Using an altruistic bequest model with endogenous fertility, in which both childbearing and bequests entail pollution externalities, we contrasted the fertility rate and the pollution level in the competitive equilibrium with those in the social optimum. It is shown that the fertility rate may be too low in the competitive equilibrium despite the negative externality created by childbearing, and, if this is the case, per capita capital over-accumulates. On the other hand, the level of pollution is unambiguously higher than the social optimum, whether the fertility rate (or per capita capital) is too high or too low.

Furthermore, we investigated what kind of policy is required to achieve social optimality. If the social discount factor equals the private degree of altruism, the government needs to tax both childbearing and inheritance so as to restore efficiency, even if fertility or capital accumulation falls short of the respective optimal level. On the other hand, if the social discount factor differs from the private degree of altruism and is assigned equally among all individuals, child allowances and/or subsidies to inheritance may be required to achieve optimality. However, the optimal policy never involves a combination of taxes on childbearing and subsidies on inheritance.

Appendix 1: Derivation of $dk/d\mu$ and $dn/d\mu$

Differentiating (25) and (26) with respect to k, n and μ yields

$$\begin{pmatrix} \hat{F}_k & \hat{F}_n \\ \hat{G}_k & \hat{G}_n \end{pmatrix} \begin{pmatrix} dk \\ dn \end{pmatrix} = -\begin{pmatrix} \hat{F}_\mu \\ \hat{G}_\mu \end{pmatrix} d\mu,$$

where

$$\hat{F}_k = n^2 u_0'' + n\delta(n) A^2 u_1'' - \mu n\delta(n)(\alpha ANn)^2 V_1'' < 0,$$

$$\begin{aligned}
\hat{F}_n &= n(k+\beta)u_0'' - u_0' + \left(\delta(n) + n\delta'(n)\right) Au_1' \\
&\quad - \mu\left\{\left(2\delta(n) + n\delta'(n)\right)(\alpha ANn)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''\right\} \\
&= n(k+\beta)u_0'' + n\delta'(n)Au_1' \\
&\quad - \mu\left\{\left(\delta(n) + n\delta'(n)\right)(\alpha ANn)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''\right\} < 0 \\
&\quad (\because u_0' = \delta(n)Au_1' - \mu\delta(n)(\alpha ANn)V_1'),
\end{aligned}$$

$$\begin{aligned}
\hat{G}_k &= n(k+\beta)u_0'' - u_0' + (\delta(n) + n\delta'(n))[Au_1' - (\alpha ANn)V_1'] \\
&\quad - \mu\left[n\delta(n)(\alpha AN)V_1' + n\delta(n)(\alpha AN)^2 nkV_1''\right] \\
&= n(k+\beta)u_0'' + n\delta'(n)Au_1' - (\delta(n) + n\delta'(n))(\alpha ANn)V_1'] \\
&\quad - \mu\left[n\delta(n)(\alpha AN)^2 nkV_1''\right] < 0 \\
&\quad (\because u_0' = \delta(n)Au_1' - \mu\delta(n)(\alpha ANn)V_1'),
\end{aligned}$$

$$\begin{aligned}
\hat{G}_n &= (k+\beta)^2 u_0'' + (2\delta'(n) + n\delta''(n))(u_1 - V_1) - (\delta(n) + n\delta'(n))(\alpha ANk)V_1' \\
&\quad - \mu\left[(\delta(n) + n\delta'(n))(\alpha ANk)V_1' + n\delta(n)(\alpha ANk)^2 V_1''\right] < 0,
\end{aligned}$$

$$\hat{F}_\mu = -n\delta(n)(\alpha ANn)V_1' < 0,$$

$$\hat{G}_\mu = -n\delta(n)(\alpha ANk)V_1' < 0.$$

Noting that $\hat{G}_\mu = \hat{F}_\mu k/n$, we have

$$\frac{dk}{d\mu} = \frac{1}{D(\mu)}\left[-\hat{F}_\mu \hat{G}_n + \hat{G}_\mu \hat{F}_n\right] = \frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}\hat{F}_n - \hat{G}_n\right] = \frac{\hat{F}_\mu}{D(\mu)}\left[\frac{k}{n}F_n - G_n\right],$$

$$\frac{dn}{d\mu} = \frac{1}{D(\mu)}\left[-\hat{G}_\mu \hat{F}_k + \hat{F}_\mu \hat{G}_k\right] = \frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}\hat{F}_k + \hat{G}_k\right] = \frac{\hat{F}_\mu}{D(\mu)}\left[-\frac{k}{n}F_k + G_k\right].$$

Appendix 2: Numerical Examples

Functional Specifications and the Baseline Values of Parameters

In this appendix, we consider numerical examples of our model to quantitatively assess the results obtained in Sections 3 and 4. For that purpose, we specify the utility functions from goods consumption, $u_0(c)$ and $u_1(c)$, and the disutility functions from pollution, $V_0(\pi)$ and $V_1(\pi)$, as follows:

$$u_0(c) = u_1(c) = c^{\sigma}, \quad 0 < \sigma < 1,$$

and

$$V_0(\pi) = V_1(\pi) = \frac{B\pi^{\nu}}{\nu}, \quad B > 0, \quad \nu > 1.$$

The degree of altruism toward children is assumed to take a form with a constant elasticity with respect to the number of children, $\delta(n) = \xi n^{-\varepsilon}$, where, $\xi > 0$, and $0 < \varepsilon < 1$, as employed in Becker and Barro (1988).

Let us now determine the baseline values of parameters. First, we normalize the population of parent's generation (N_0) to 1 and we assume initial endowment of each parent (b_0) equal to 1. Then, the total capital stock at first period (K_0) equals 1. The productivity parameter (A) is set to 3.8134. The value corresponds to the case where the annual rate of interest is equal to 0.055 when one period is considered to be 25 years (i.e., $3.8134 \approx (1+0.055)^{25}$). Emission coefficient (α) is set equal to 0.2 and parameters appearing in the functions specified above are set to $\sigma = 0.6$, $B = 1$, $\nu = 2$, $\xi = 0.25$, and $\varepsilon = 0.1$. Rearing cost per child (β) is set to 0.2, which implies that, in the case where each parameter takes the baseline value, the share of child rearing cost in income ($= n\beta/y_0$) in the competitive equilibrium is about 6.9 %, that the share of consumption in income for parents ($= c_0/y_0$) is about 72.3 %, and that the share of bequests to the children in income ($= nk/y_0$) is about 20.7 %. The baseline values of parameters are collected in Table 1.

When each parameter is set to its baseline value, the competitive equilibrium is given by k^* = 0.5990 and n^* = 1.3198, while the social optimum is calculated as k^S = 0.3907 and n^S = 1.2863. Notice that, in our baseline case where each parameter takes its baseline value, the number of children determined in the competitive equilibrium, n^*, is larger than the social optimum level n^S.

Comparing the Competitive Equilibrium to the Social Optimum

In the following, we perform a sensitivity analysis by changing the values of key parameters of the model. In particular, we focus on the following parameters: the level parameter of the degree of altruism toward children (ξ); the level parameter of the disutility functions from pollution (B); emission coefficient (α); and rearing cost per child (β). We change the value of each parameter and re-calculate the competitive equilibrium values of k, n, and π and the social optimum levels of these variables in each case, holding all other parameters constant at their baseline values. Henceforth, we call a value of a variable determined in competitive equilibrium as an *equilibrium value* of the variable, and a social optimum level of a variable as an *optimum value* of the variable. The results of our sensitivity analysis are reported in figures 2 – 5. The each figure presents the values of a given variable as a function of each parameter.

First we examine the sensitivity of the variables to changes in ξ. Figures 2(a) – 2(c) show, in order, the values of k, n, and π, corresponding to the values of ξ on the horizontal axis. As we can see from figure 2(a), when ξ moves from 0.2 to 0.4, the equilibrium value of k increases from about 0.44 to about 1.53, while the optimum value of k decreases from about 0.40 to about 0.37. The response of fertility rate is presented in figure 2(b). The figure shows that, over the same interval of ξ, the optimum value of n increases monotonically whereas the equilibrium value of n first increases and then decreases. The optimum value of n increases from about 0.77 to about 2.05. The equilibrium value of n increases for relatively small value of ξ ($0.20 \le \xi \le 0.27$) from about 0.89 to about 1.34, while, for relatively large values of ξ ($0.28 \le \xi \le 0.40$), n decreases from about 1.34 to about 1.02. In figure 2(c), the levels of pollution emission for different values of ξ are plotted. The social optimum level of π necessarily increases with ξ, since both k and n increase as ξ rises. Besides, the emission in the competitive equilibrium also increases monotonically in our example. When the value of ξ is relatively small, the emission increases with ξ since both k and n increase as ξ rises. As we saw above, when the value of ξ is relatively large, an increase in ξ raises k, but lowers n. However, in our example, the former effect dominates the latter effect, and hence, the emission increases with an increase in ξ, as shown in figure 2(c).

Summarizing, in our numerical example, as parents become more altruistic toward children, each parent tends to have less children and to leave larger bequest to each child than those of socially optimum levels. As for the level of pollution emission, it increases monotonically both in the competitive equilibrium and in the social optimum.

From Proposition 1, we know that the fertility rate determined in the competitive equilibrium may be higher or lower than the social optimum level, depending on the relative magnitude of LHS and RHS of (15). Now let us check the condition (15) for our parameter configurations. Remember that the LHS represents the decrease in PMNB of a child when capital increases from k^S to $k^*(n^S)$ and that the RHS is the environmental effects of n which the parents do not take into account in calculating the PMNB of a child. Figure 2(d) shows the LHS and RHS of (15) for each value of ξ. We can see from the figure that, for larger values of ξ (i.e., $\xi \ge 0.26$), the LHS is greater than the RHS and hence, the condition (15) is satisfied. In such cases, as shown in Proposition 1, the number of children chosen in the competitive equilibrium must be lower than that of the socially optimal level. And, figure 2(b) shows that, in our example, the fertility rate in the competitive equilibrium for relatively large value of ξ is indeed less than that of the socially optimal level.

Figures 3(a) – 3(c) show how the values of k, n, and π vary with the value of B. As we can see in figure 3(a), when B varies from 1.1 to 2.5, the equilibrium value of k increases from about 0.52 to about 0.73, while the optimum value of k is almost constant at about 0.39. In figure 3(b), changes in the value of n are plotted. From the figure, we can see that both the equilibrium and the optimum values of n decrease as B rises. The equilibrium value of n decreases from about 1.72 to about 0.85, while the optimum value of n decreases from about 1.60 to about 0.92. And, as shown in figure 3(c), the pollution emission becomes smaller when B rises in both competitive equilibrium and social optimum.

Let us here check the condition (15) for each value of B. The LHS and RHS of (15) for each value of B are presented in figure 3(d). The figure shows that, for larger values of B (i.e., $B \geq 1.3$), the LHS is greater than the RHS. Comparing figures 3(b) and 3(d), we can see that, for the relatively large values of B, the fertility rate in the competitive equilibrium is indeed lower than that of the socially optimal level.

Figures 4(a) – 4(c) present, respectively, the values of k, n, and π, corresponding to changes in the value of α. It is seen in figure 4(a) that, when α changes from 0.1 to 0.3, the equilibrium value of k increases from about 0.47 to about 0.72, while the optimum value of k decreases slightly from 0.40 to 0.39. Changes in the value of n are shown in figure 4(b). The figure shows that, both the equilibrium and the optimum values of n decreases as α rises. The equilibrium value of n decreases from about 2.08 to about 0.90, while the optimum value of n decreases from about 1.92 to about 0.96. And, in both cases, the pollution emission increases when α increases, as shown in figure 4(c). Figure 4(d) shows that the condition (15) is satisfied for relatively large values of α (i.e., $0.23 \leq \alpha$). For those values of α, the fertility rates chosen in the competitive equilibrium are lower than those of socially optimal levels, as shown in figure 4(b).

Finally, figures 5(a) – 5(c) report the values of k, n, and π, corresponding to the values of β on the horizontal axis. As shown in figure 5(a), when β increases from 0.1 to 0.3, the equilibrium value of k increases from about 0.51 to about 0.72 and the optimum value of k increases from about 0.19 to about 0.59. Both in the competitive equilibrium and in the social optimum solution, an increase in β reduces the fertility rate n. For the same rage of values of β, the equilibrium value of n decreases from about 2.00 to about 0.83, and the optimum value of n decreases from about 2.83 to about 0.73. In both cases, k increases and n decreases when β increases. In our numerical exercise, however, the former effect always dominates the latter effect, hence increasing the pollution emission π with an increase in β, as shown in figure 5(c). By comparing figures 5(b) and 5(d), we can see that, when the rearing cost per child is relatively small (i.e., $\beta \leq 0.18$), the condition (15) is satisfied and then the parents choose to have the smaller number of children than the optimum levels.

Optimal Policies

We reconsider by numerical examples the optimal policies examined in section 4.2. We change the value of each parameter and calculate the optimal policies, T and τ, in each case holding all other parameters constant at their baseline values. The results are reported in figures 6(1) – 6(4). Figure 6(1) presents the optimal policies for ξ from 0.2 to 0.4; figure 6(2) for B from 0.5 to 2.5; figure 6(3) for α from 0.1 to 0.3; and figure 6(4) for β from 0.1 to 0.3. Notice that, for ξ, B, and α, the optimal policies, T and τ, increase when values of the parameters rise, as shown in figures 6(1) – 6(3). The increase in τ can be understood from figures 2(a), 3(a), and 4(a). As can be seen in these figures, the difference between the competitive value of k and the optimum value of k increases as the value of each parameter rises. To reduce the differences, higher tax on bequests is required for higher values of the parameters, and hence τ must be increased. Next let us consider the tax on having a child, T. First note that the numerator of T is equal to RHS of (15), which represents the environmental effects of n which the parents do not take into account in calculating the PMNB of a child. The graphs of RHS of (15) were given in figures 2(d), 3(d), and 4(d). As shown in the figures, RHS is positively affected by ξ, B, and α, and it increases with the parameters. On the other hand, the changes in denominator of (35) are ambiguous. However, in our choice of parameter configurations, the former dominates the latter and hence T increases with the parameters.

Symmetrically, T and τ decrease when rearing cost per child increases. As for τ, we can understand the result in the same way above. As we can see in figure 5(a), the difference between the competitive value of k and the optimum value of k decreases as β rises. For higher values of β, a lower tax on bequests suffices to eliminate the difference, and hence τ decreases. As for the tax on having a child, the numerator of T decreases with β, as shown in figure 5(d). And, irrespective of the changes in denominator of (35), the optimal policy T decreases as β increases, as shown in figure 6(4).

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Figure 1: Optimal Policies

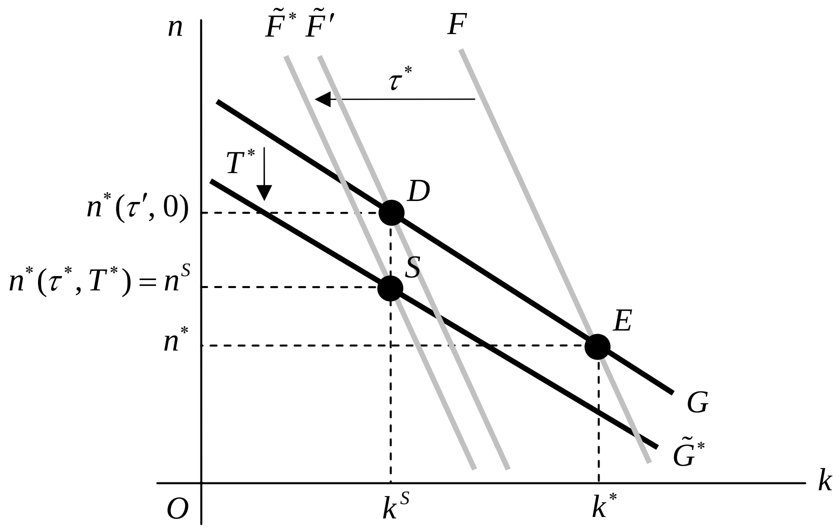

Figure 2: Sensitivity to ξ

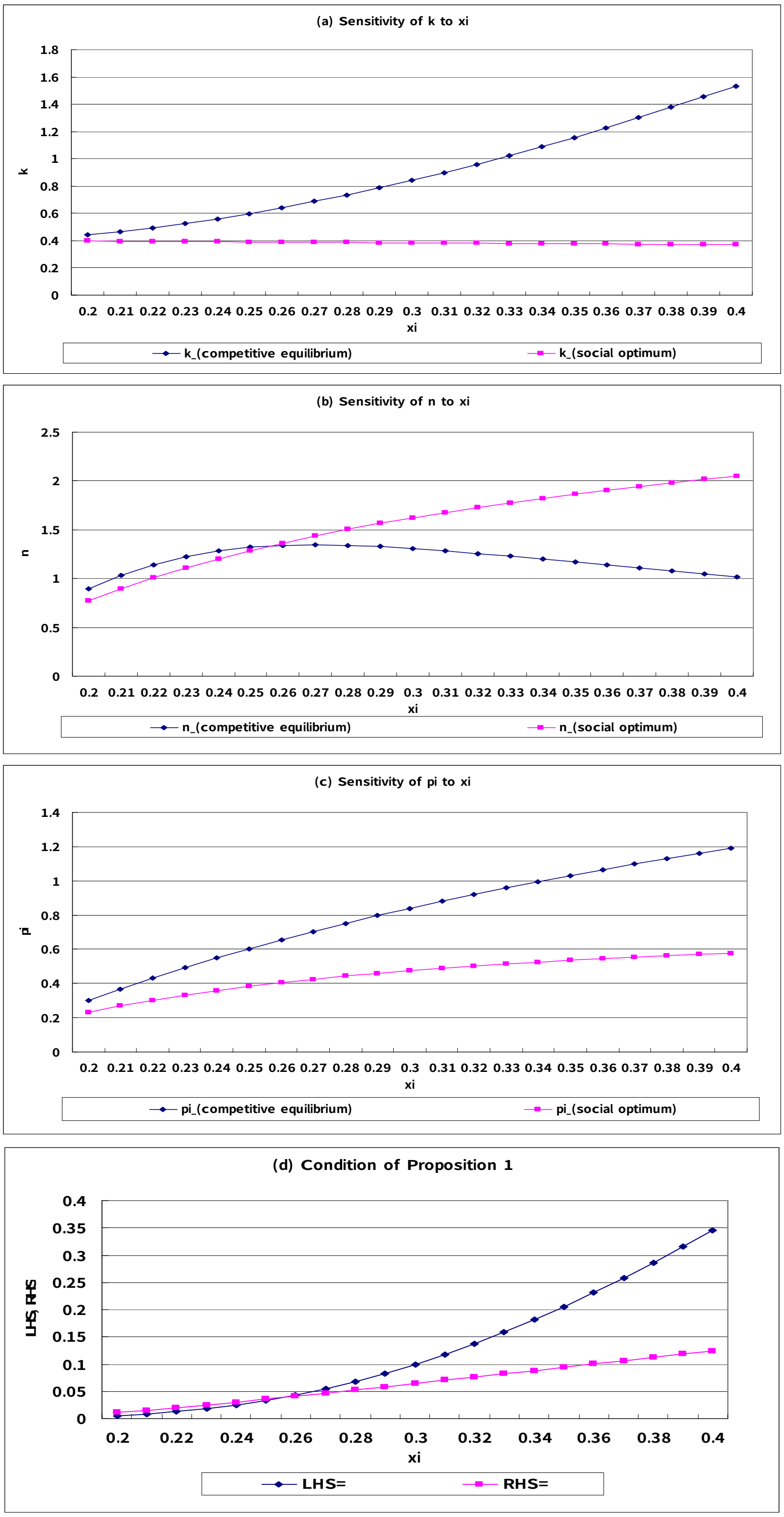

Figure 3: Sensitivity to B

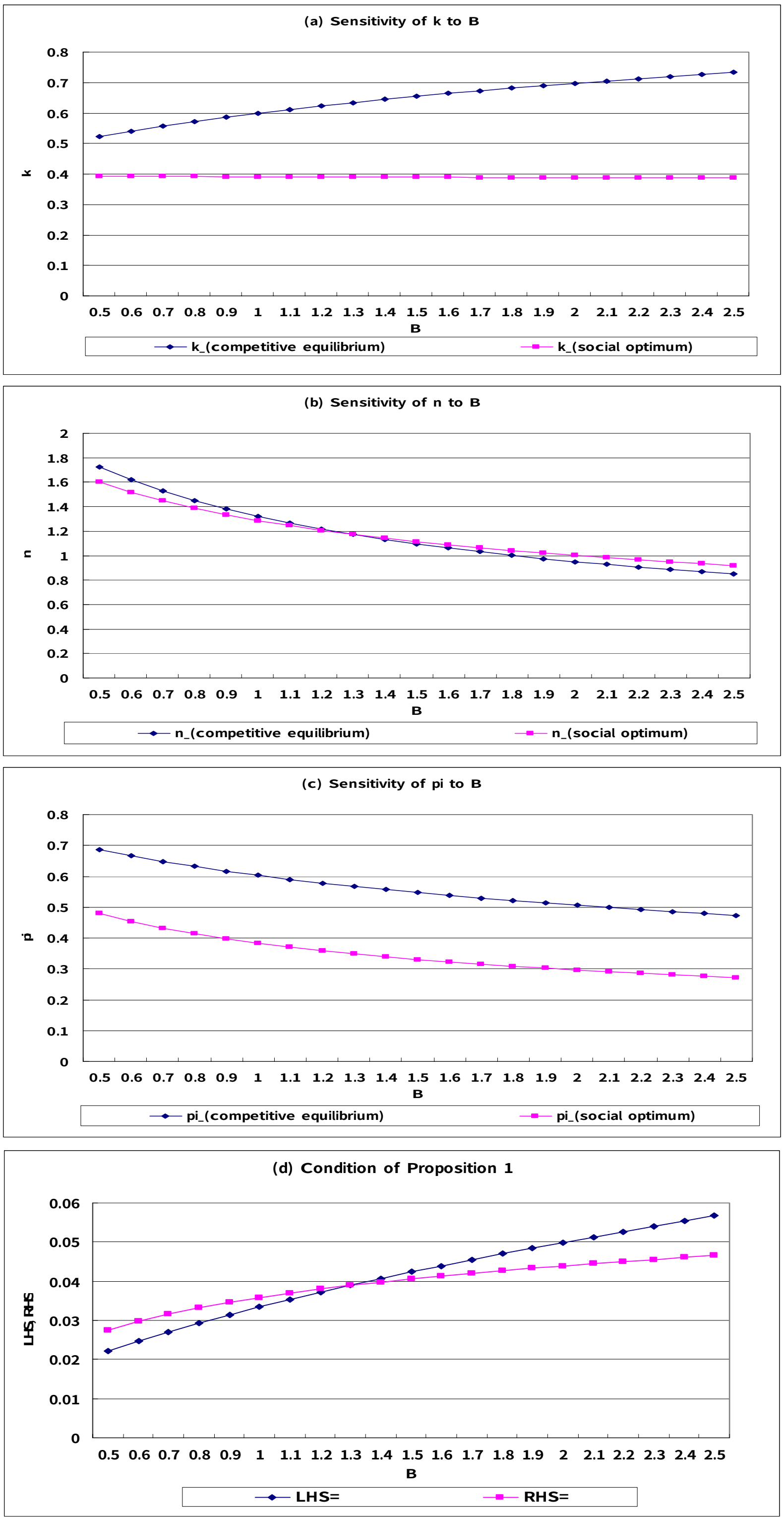

Figure 4: Sensitivity to α

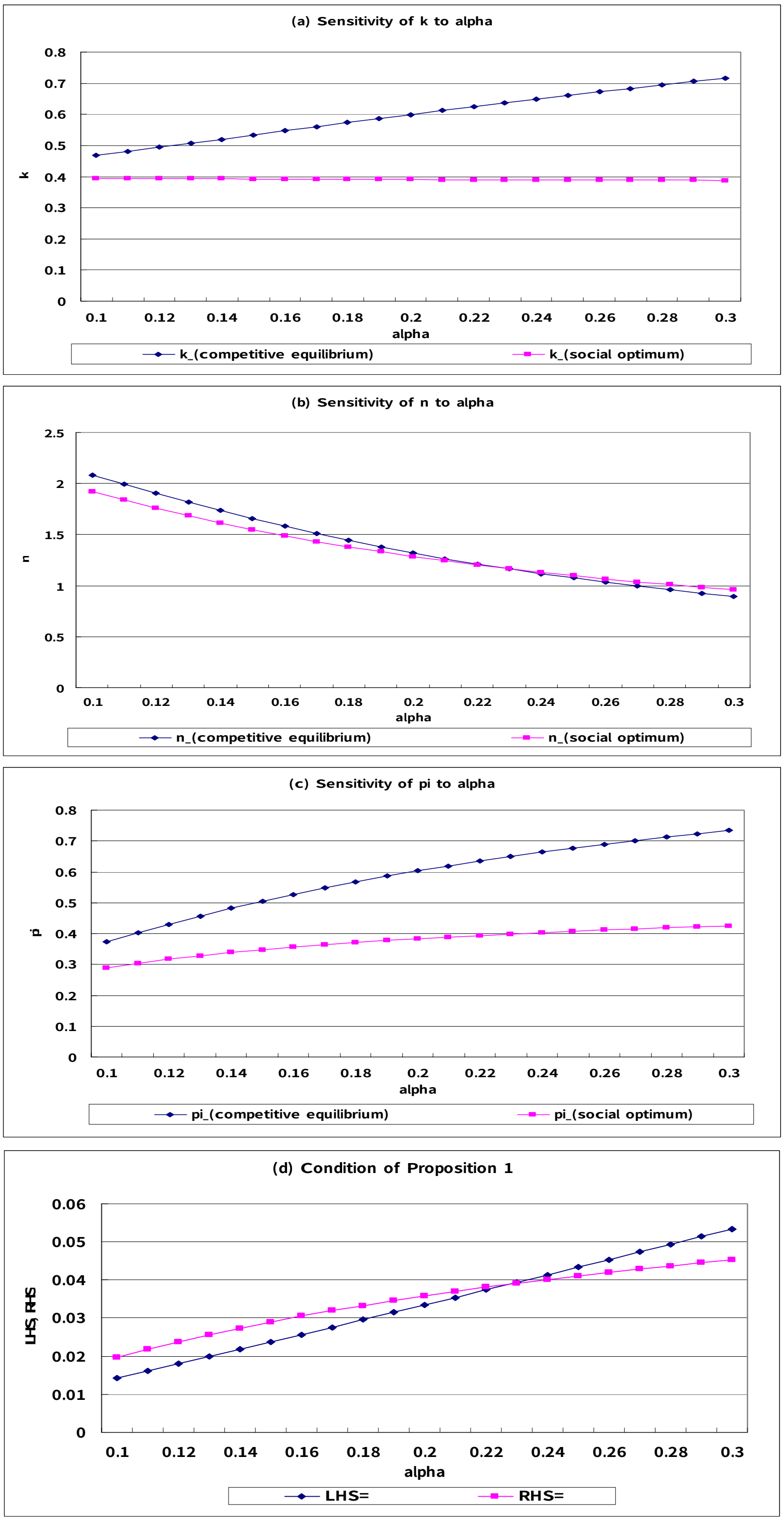

Figure 5: Sensitivity to β

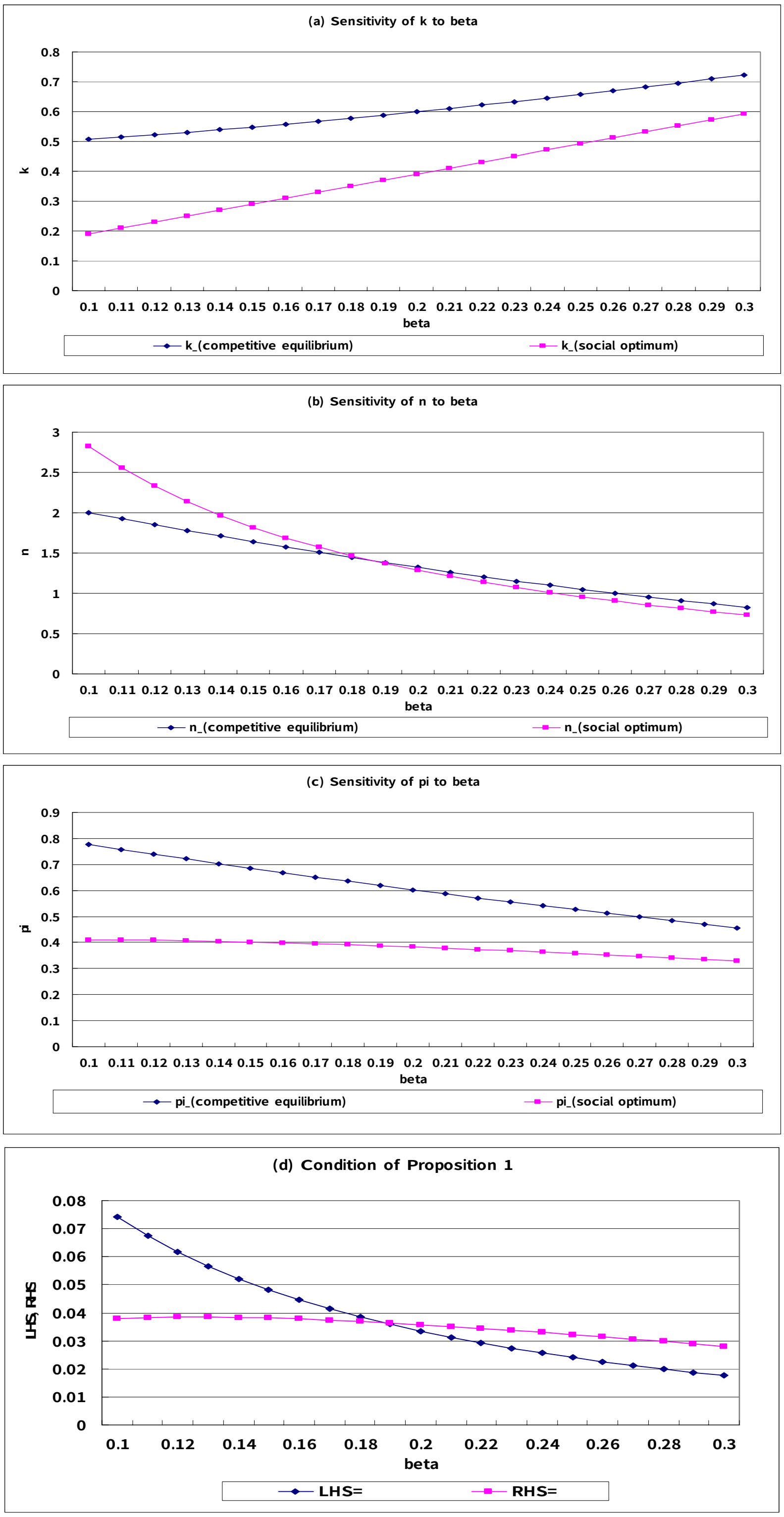

Figure 6: Optimal Policies

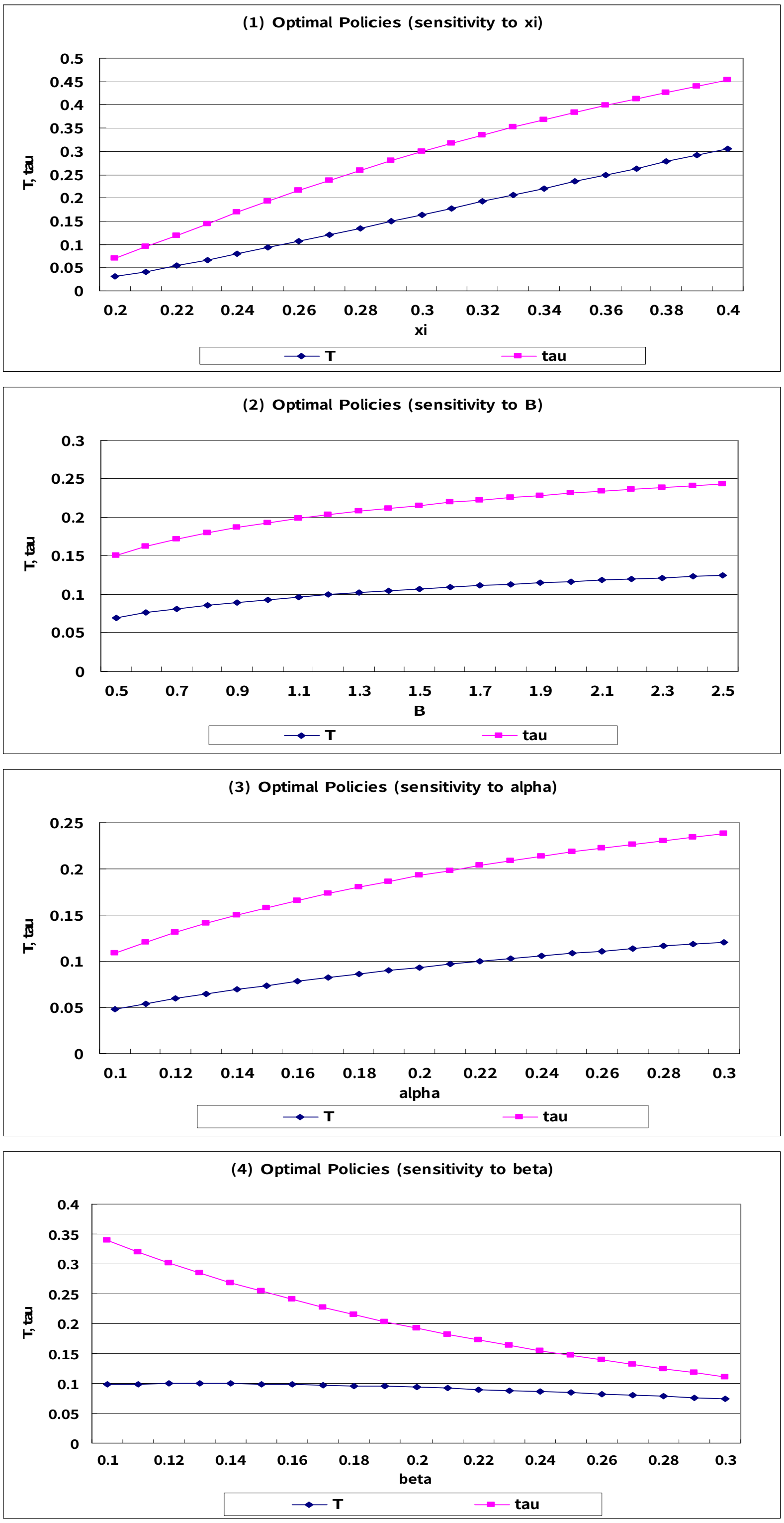

Table 1: The baseline values of parameters

Parameter	Values
β	0.2
σ	0.6
B	1
ν	2
ξ	0.25
ε	0.1
A	3.8134
α	0.2
b_0	1
N	1