1 Concepts

Definition 1. (Extreme consequentialism)
$\succeq$ is said to be extremely consequential if, for all $(x_1, x_2; A_1, A_2), (x_1, x_2; B_1, B_2) \in \Omega$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.

Definition 2. (First opportunity set ranking strong consequentialism)
$\succeq$ is said to be first opportunity set ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\ (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| \geq |B_1|\ ]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 3. (Second opportunity set ranking strong consequentialism)
$\succeq$ is said to be second opportunity set ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\ (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_2| \geq |B_2|\ ]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 4. (Sum-ranking strong consequentialism)
$\succeq$ is said to be sum-ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\ (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| + |A_2| \geq |B_1| + |B_2|\ ]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 7. (First opportunity set ranking extreme nonconsequentialism)
$\succeq$ is said to be first opportunity set ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \geq |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Definition 8. (Second opportunity set ranking extreme nonconsequentialism)
$\succeq$ is said to be second opportunity set ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(x_1, x_2; B_1, B_1) \in \Omega$, $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Definition 9. (Sum-ranking extreme nonconsequentialism)
$\succeq$ is said to be sum-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Definition 10. (Weighted sum-ranking extremely nonconsequentialism)
$\succeq$ is said to be Weighted sum-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $\alpha|A_1| + \beta|A_2| \geq \alpha|B_1| + \beta|B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Definition 11. (Lexicographic extreme nonconsequentialism for first opportunity set)
$\succeq$ is said to be lexicographic extremely nonconsequential for first opportunity set if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| = |B_1| \Rightarrow [\ |A_2| \geq |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 12. (First opportunity set ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| = |B_1| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 13. (Second opportunity set ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_2| = |B_2| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 14. (Sum-ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| > |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| + |A_2| = |B_1| + |B_2| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 15. (Lexicographic strong nonconsequentialism for first opportunity set)
$\succeq$ is said to be lexicographic strongly nonconsequential for first opportunity set if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, $|A_1| = |B_1| \Rightarrow [\ |A_2| > |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)]$, and $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow [\ (x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 16. (Multiplicative-ranking strong consequentialism)
$\succeq$ is said to be Multiplicative-ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\ (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2) \Leftrightarrow |A_1| \times |A_2| \geq |B_1| \times |B_2|\ ]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 17. (Multiplicative-ranking extreme nonconsequentialism)
$\succeq$ is said to be multiplicative-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \times |A_2| \geq |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Definition 18. (Multiplicative-ranking strong nonconsequentialism)
$\succeq$ is said to be multiplicative-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \times |A_2| > |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, and $|A_1| \times |A_2| = |B_1| \times |B_2|$, then $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

2 Axioms

Axiom 1. Independence for Addition(IND)
For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$ and all $z_1 \in X_1 \backslash \{A_1 \cup B_1\}$ and all $z_2 \in X_2 \backslash \{A_2 \cup B_2\}$, $(x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow (x_1, x_2; A_1 \cup \{z_1\}, A_2) \succeq (y_1, y_2; B_1 \cup \{z_1\}, B_2)$ and $(x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow (x_1, x_2; A_1, A_2 \cup \{z_2\}) \succeq (y_1, y_2; B_1, B_2 \cup \{z_2\})$.

Axiom 2. Simple Indifference(SI)
For all $x_1 \in X_1$ and all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $x_2 \in X_2$ and all $y_2, z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2\})$ and $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, z_2\})$.

Axiom 3. Indifference(BI)
For all $x_1 \in X_1$ and all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $x_2 \in X_2$ and all $y_2, z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, y_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2, z_2\})$.

Axiom 4. Local Indifference 1(LI1)
For all $x_1 \in X_1$ and $x_2 \in X_2$, there exist $(x_1, x_2; A_1, \{x_2\}) \in \Omega$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; A_1, \{x_2\})$ where $A_1 \neq \{x_1\}$.

Axiom 5. Local Indifference 2(LI2)
For all $x_1 \in X_1$ and $x_2 \in X_2$, there exist $(x_1, x_2; \{x_1\}, A_2) \in \Omega$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, A_2)$ where $A_2 \neq \{x_2\}$.

Axiom 6. Local Strict Monotonicity 1 (LSM1)
For all $x_1 \in X_1$ and $x_2 \in X_2$,there exists $(x_1, x_2; A_1, \{x_2\}) \in \Omega \setminus \{(x_1, x_2; \{x_1\}, \{x_2\})\}$ such that $(x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$.

Axiom 7. Local Strict Monotonicity 2 (LSM2)
For all $x_1 \in X_1$ and $x_2 \in X_2$,there exists $(x_1, x_2; \{x_1\}, A_2) \in \Omega \setminus \{(x_1, x_2; \{x_1\}, \{x_2\})\}$ such that $(x_1, x_2; \{x_1\}, A_2) \succ (x_1, x_2; \{x_1\}, \{x_2\})$.

Axiom 8. Robustness(ROB)
For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$ and all $z_1 \in X_1$,$z_2 \in X_2$, if $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$ and $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1 \cup \{z_1\}, B_2)$ and $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2 \cup \{z_2\})$.

Axiom 9. Trinary Indifference(TI)
For all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2\})$

Axiom 10. Indifference of No-Choice Situations(INS)
For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$.

Axiom 11. Proportional Inddiference(PI)
For all $x_1 \in X_1$ and all $x_2 \in X_2$, there exists A_1, A_2, B_1 and B_2 such that $\alpha|A_1| + \beta|A_2| = \alpha|B_1| + \beta|B_2|$, $|A_1| \neq |B_1|$ and $|A_2| \neq |B_2|$, and $(x_1, x_2; A_1, A_1) \sim (x_1, x_2; B_1, B_2)$

Axiom 12. Weakly Robustness for first opportunity set (WROB1)
For all $(x_1, x_2; A_1, A_2), (x_1, x_2; B_1, B_2) \in \Omega$ and all $z_2 \in X_2$, $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2 \cup \{z_2\})$.

Axiom 13. Simple Preference for First Opportunities(SPO1)
For all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$

Axiom 14. Simple Preference for Second Opportunities(SPO2)
For all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$

Axiom 15. Stronly Robustness for first opportunity set (SROB1)
For all $(x_1, x_2; A_1, A_2), (y_1, y_2; B_1, B_2) \in \Omega$ and all $z_1 \in X_1$,$z_2 \in X_2$, $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2 \cup \{z_2\})$.

Axiom 16. Indifference for Multiplication (INDM)
$n \in \mathbb{N}$ and $i, j \in \{1, 2\}$. For all $(x_1, x_2; A_1, A_2)$,$(x_1, x_2; B_1, B_2) \in \Omega$ and all C_i,D_j such that $A_1 \cap C_1 = \emptyset$, $B_1 \cap D_1 = \emptyset$, $n \times |A_i| = |A_i \cup C_i|$ and $n \times |B_j| = |B_j \cup D_j|$, $(x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2) \Leftrightarrow (x_i, x_{3-i}; A_i \cup C_i, A_{3-i}) \succeq (x_j, x_{3-j}; B_j \cup D_j, B_{3-j})$.

Axiom 17. Semi-Local Indifference(SLI)
For all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1 \in K_1$, $(x_1, x_2; A_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\})$ or, for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_2 \in K_2$, $(x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_2\})$

Axiom 18. Semi-Local Strict Monotonicity(SLSM)
For all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\})$ or, for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_2, B_2 \in K_2$, $A_2 \supset B_2 \Rightarrow (x_1, x_2; \{x_1\}, A_2) \succ (x_1, x_2; \{x_1\}, B_2)$

Axiom 19. Semi-Strict Preference for Opportunity(SSPO)
For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ where $x_1 \neq y_1$ and $x_2 \neq y_2$, and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (y_1, y_2; B_1, \{y_2\})$ or, for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ where $x_1 \neq y_1$ and $x_2 \neq y_2$, and all $A_2, B_2 \in K_2$, $A_2 \supset B_2 \Rightarrow (x_1, x_2; \{x_1\}, A_2) \succ (y_1, y_2; \{y_1\}, B_2)$

3 Summary of Results

- $(IND) \oplus (BI)$
  - $\oplus(LI1) \oplus (LI2) =$ extreme consequentialism
  - $\oplus(LI1) \oplus (LSM2)$
    - $\oplus(ROB) =$ 2nd opp. set ranking strong conseq.
    - $\oplus(INS) =$ 2nd opp. set ranking extreme nonconseq.
    - $\oplus(SPO2) =$ 2nd opp. set ranking strong nonconseq.
  - $\oplus(LI2) \oplus (LSM1)$
    - $\oplus(ROB) =$ 1st opp. set ranking strong conseq.
    - $\oplus(INS) =$ 1st opp. set ranking extreme nonconseq.
    - $\oplus(SPO1) =$ 1st opp. set ranking strong nonconseq.
  - $\oplus(LSM1) \oplus (LSM2)$
    - $\oplus(ROB)$
      - $\oplus(TI) =$ sum-ranking strong conseq.
      - $\oplus(PI) =$ wighted-sum ranking strong conseq.
    - $\oplus(INS)$
      - $\oplus(TI) =$ sum-ranking extreme nonconseq.
      - $\oplus(PI) =$ weghted sum-ranking extreme nonconseq.
      - $\oplus(WROB1) =$ lexioco. extreme nonconseq.
    - $\oplus(SPO1) \oplus (SPO2)$
      - $\oplus(TI) =$ sum-ranking strong nonconseq.
      - $\oplus(PI) =$ weghted sum-ranking strong nonconseq.
      - $\oplus(SROB1) =$ lexioco. strong nonconseq.
- $(INDM)$
  - $\oplus(SLI) =$ extreme consequentialism
  - $\oplus(SLSM)$
    - $\oplus(ROB) =$ multiplicative-ranking strong consequentialism
    - $\oplus(INS) =$ multiplicative-ranking extreme nonconsequentialism
    - $\oplus(SSPO) =$ multiplicative-ranking strong nonconsequentialism

IND: Independence for addition
INDM: Indifference for Multiplication
BI: Baseline Indifference
LIi: Local Indifference for ith opportunity set
LSMi: Local Strict Monotonicity for ith opportunity set
SPO: Strong Preference for Opportunities
INS: Indifferece of No-choice Situations
ROB: Robustness
TI: Trinary Indifference
PI: Proportional Indifference
WROB1: Weakly Robustness for first opportunity set
*note; $(A) \oplus (B)$ indicates the logical combination of the two axioms A and B.