Talking Freedom of Choice Seriously *

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Abstract

In actual life, we face the opportunity of many choices, from that opportunity set, we have to choose from different alternatives. So, Freedom of Choice is essential when you lead a life as a valuable human being. However, we may often consider that certain some opportunity sets are important, while other opportunity sets are not so important. Thus, in order to grasp the relation between two or more opportunity sets, I extended Suzumura and Xu(2001,2003) to multiple-opportunity sets, and axiomatized the concept of various consequentialism and nonconsequentialism.

JEL classification D00; D60; D63; D71

Keywords Freedom of Choice; Multiple opportunities; Extended alternatives; Consequentialism; Nonconsequentialism

Contents

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1 Introduction

Welfarist-Cosequentialism has long dominated the tradition of economic thought.[1] On the view of Welfarist-Cosequentialism, the intrinsic value of freedom of choice is ignored. Today, some people are pointing out the intrinsic value of freedom of choice, they claim that "choosing may itself be a value part of living"(Sen,1992).[2] Note that only Freedom of Choice which affects one's life can have the intrinsic value of opportunities. According to Sen, it is "life of genuine choice with serious options may be ... richer". For example, it would not have the intrinsic value, even if Freedom of Choice of choosing others' lunch were given. Freedoms that are guarantee by the constitution, such that political liberty, freedom of speech and freedom of choice of work, have the intrinsic value.[3]

The purpose of this paper is extending the issue on freedom of choice of the individual who faced multiple-choices and examine the above problems. In many papers, the representative individual who likes freedom of choice is formalized.[4] And evaluation of choice in the situation which consists of single-opportunity set is dealt with in almost all papers. However, there is an important aspect of freedom of choice which cannot be grasped exists in single-opportunity set approach. Then, treating multiple-opportunity sets simultaneously, it is possible to grasp the relation between two or more opportunity sets.

In actual life, we face various choice situations such as a life plan, choices in educational choice, books, and food ... In such situation, people enjoy Freedom of Choice and utilities by choice. When we are in the choice situation we faced various opportunity sets, how can we evaluate degree of our Freedom of Choice? Freedom of Choice of what is it? Can we ask which opportunity set is important, for example, Freedom of Choice of work or Freedom of Choice of food? It is known that in actual life, many people think that Freedom of Choice of work is more important than Freedom of Choice of food. Here, two persons, Eric, and Layla, exist and there are two choice situations, the choice of Eric's job and Layla's lunch. Suppose that Eric has the right for choosing of Layla's lunch. So, he has Freedom of choice, and can choose what she eats as he like. However, choice of Layla's lunch is not essential for the life of Eric. In the life of Eric, choice of his job is essential, and Freedom of Choice to work of Eric gives the intrinsic value of opportunities to him. According to this argument, although some Freedom of Choice has the intrinsic value of opportunities for an individual, some other Freedom of Choice does not have it. That is, the diversity about the intrinsic value of opportunities exists. In this paper, we examine the diversity of Freedom of Choice by extending some frameworks and using multiple-opportunity sets.

In order to achieve our purpose there are two possible ways; binary relation over opportunity sets and binary relation over extended alternatives. The former is traditional approach and developed by Kreps(1979), Jones and Sugden(1982), Bosert, Pattanaik and Xu (1994) et all.[5] By considering binary relation over the set of available alternatives, and axiomatized the relation. In formal, $A \succeq B$ where A, B

[1] See Sen(1979,1987) and Suzumura(2000).

[2] See also Arrow(1995) and Sen(1985a,1985b,1988,1991,1992,2002).

[3] For political philosophy, it is important problem whether the constitution guarantee essential freedoms for intrinsic values or for instrumental values.

In Rawls(1971), as a result of choice of persons in the original position, a list of liberties are guaranteed by the first principle of justice. However, we cannot conclude that such freedoms have the intrinsic value for them. Since he does not know a certain personal status, they may agree a list of liberties for instrumental values. This issue related with Hart(1973) and the discussion of the priority of liberty, and we does not dealt here.

Sen(1999) insists that political liberty have not only the instrumental value but also the intrinsic value.

[4] See Baharad and Nitzen(2000), Bossert, Pattanaik and Xu(1994), Pattanaik and Xu(1998,2000), Puppe(1995,1996) and Romeo-Medina(2001).

[5] Barbera, Bossert and Pattanaik(2004) provides an extemsive review for this.

are opportunity sets. The latter is dealt with in recently papers, Suzumura and Xu(2001,2003,2004), it is used in order to characterizing consequentialism and nonconsequentialism. [6] They consider the ordering over extended alternative (x, A), which means "Choosing alternative from opportunity set". [7] Since we also interest the feature of consequentialism and nonconsequentialism, then we adopt the approach of Suzumura and Xu(2001,2003).

Since our main purpose is consideration in the choice from multiple opportunity sets, our extended alternative $(x_1, x_2, \cdots, x_n; A_1, A_2, \cdots, A_n)$ means "choosing x_1 from A_1 and choosing x_2 from A_2 and $\cdots$ and choosing x_n from A_n". Vector of alternatives $(x_1, x_2, \cdots, x_n)$ represents consequence of the choice situation, and utility $u(x_1, x_2, \cdots, x_n)$ is given by numerical function $u(\cdot)$. According to Suzumura and Xu(2001), extreme consequentialist only consequence $(x_1, x_2, \cdots, x_n)$, and extreme nonconsequentialist mind only the cardinality of opportunity sets $(|A_1|, |A_2|, \cdots, |A_n|)$.

Consider extremely non-consequential person who faces the choice situation in books and jobs. Let denote A_1, B_1 opportunity sets of job and A_2, B_2 opportunity sets of book, and suppose consequence is same and cardinality of opportunity set is as follows; $(|A_1|, |A_2|) = (2, 1)$ and $(|B_1|, |B_2|) = (1, 2)$. which does the individual prefer $(x_1, x_2; A_1, A_2)$ or $(x_1, x_2; B_1, B_2)$? If the individual consider the freedom of choice of book is more important than one of job, then $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$. If the individual consider the freedom of choice of job is as important as one of book, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$. We must remark the relation between single opportunity set approach and multiple one. Suppose that $A_1 = \{$Lady Chatterley's Lover(L), My Fair Lady(M)$\}$ and $A_2 = \{$Teacher(T), Artist(A)$\}$. 4 consequences from this choice situation are possible; (Lady Chatterley's Lover, Teacher), (Lady Chatterley's Lover, Artist), (My Fair Lady, Teacher) and (My Fair Lady, Artist). Constructing the opportunity set $A = \{(L, T), (L, A), (M, T), (M, A)\}$, we can obtain single opportunity set. In general, if individual face n opportunity sets $A_1, \cdots, A_n$, we reconsider single opportunity set by taking $A = A_1 \times A_2 \times \cdots \times A_n$. Therefore, our approach is reinterpreted as decomposition A into categories $A_1, \cdots, A_n$. [8]

Finally the structure of this paper is described as follows. In section 2, we explain basic notation and basic axioms, and show those simple implications by it. The decision under two opportunity sets, which is the simplest case, is covered first. In section 3 we define consequentialism, and examine them axiomatically. In section 4, we discuss nonconsequentialism and characterize the concept of various nonconsequentialism axiomatically. Section5, we treat the relationship with Suzumura and Xu(2001), and present axioms which are essential and differ from ones in previous section. Furthermore, in section 6 we extend the model to general case, which is considering the decision-making under limited opportunity sets. Finally section 7 conclude this paper and give some remarks.

[6]The word of consequentialism originates from Anscombe(1958). On the detail definition of the concept, see Perfit(1984), Scheffler(1982). See also Nagel(1979) and Williams(1971,1981).

[7]Before Suzumura and Xu(2001,2003,2004), Grevel(1994.1998) analyzes the ordering over extended alternatives.

[8]How single opportunity set A is decomposed into multiple opportunity sets may be subjective. Consider 3 opportunities, choice of where you go, choice of how to go and choice of lunch. Suppose that opportunity set of where you go A_1 is $\{$London(L),Tokyo(T)$\}$, opportunity set of how you go A_2 is $\{$Airplane(A),Ship(S)$\}$, opportunity set of choice of lunch A_3 is $\{$Curry(C),Pasta(P)$\}$. Some people feel that they face 3 opportunities, but other recognize 2 opportunities, choice from $\{$(L,A),(L,S),(T,A),(T,S)$\}$ and choice from $\{$C,P$\}$.

2 The Basic Model

2.1 Notations

Let X_1 and X_2, where $3 \leq |X_1|, |X_2| < \infty$, be sets of all mutually exclusive and jointly exhaustive alternatives in category 1 and 2 respectively, and $X_1 \cap X_2 = \emptyset$. The elements of X_1 will be denoted by $x_1, y_1, z_1, \cdots$. K_1 denotes a collection of non-empty subset of X_1. Similarly, The elements of X_2 will be denoted by $x_2, y_2, z_2, \cdots$, and K_2 denotes a collection of non-empty subset of X_2. The elements in K_1 will be denoted by $A_1, B_1, C_1, \cdots$, and the elements in K_2 will be denoted by $A_2, B_2, C_2, \cdots$, and they are called opportunity sets. Let $X_1 \times X_2 \times K_1 \times K_2$ will be denoted by $(x_1, x_2; A_1, A_2), (y_1, y_2; B_1, B_2), (z_1, z_2; C_1, C_2), \cdots$, and they are called extended alternatives. Let $\Omega = \{(x_1, x_2; A_1, A_2) | A_1 \in K_1, A_2 \in K_2, x_1 \in A_1$ and $x_2 \in A_2\}$. That is, Ω contains all $(x_1, x_2; A_1, A_2)$ such that A_1 is an element in K_1, A_2 is an element in K_2, ,x_1 is an element in A_1 and x_2 is an element in A_2. It should be clear that $\Omega \subset X_1 \times X_2 \times K_1 \times K_2$, and for all $(x_1, x_2; A_1, A_2)$, $x_1 \in A_1$ and $x_2 \in A_2$ holds. For all $(x_1, x_2; A_1, A_2)$, the intended interpretation is that alternative x_1 is chosen from the opportunity set A_1 and x_2 is chosen from the opportunity set A_2. In this formulation, the consequence is $(x_1, x_2) \in X_1 \times X_2$. Note that opportunity sets are multiple, but consequence from opportunity sets is only one. Moreover, let $(x_2, x_1; A_2, A_1)$ means $(x_1, x_2; A_1, A_2)$.

Let $\succeq$ be a reflexive, complete and transitive binary relation over Ω. That is, $\succeq$ is an ordering over Ω. The asymmetric and symmetric part of $\succeq$ will be denoted by $\succ$ and $\sim$, respectively. For any $(x_1, x_2; A_1, A_2), (y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$ is interpreted as "choosing x_1 from A_1 and x_2 from A_2 is at least good as choosing y_1 from B_1 and y_2 from B_2".

Note that, in this framework, If we define $X = X_1 \times X_2$ and $K = K_1 \times K_2$, the extended alternatives corresponds to the framework of Suzumura and Xu(2001,2003). Moreover, in this framework, it is possible to deal difference of intrinsic value between opportunity sets.

2.2 Basic Axioms

In this subsection, we propose basic axioms and show the implication of them. We extend axioms proposed by Suzumura and Xu(2001,2003) to axioms with multiple opportunity sets.

Axiom 1. Independence for Addition(IND)
For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$ and all $z_1 \in X_1 \setminus \{A_1 \cup B_1\}$ and all $z_2 \in X_2 \setminus \{A_2 \cup B_2\}$, $(x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow (x_1, x_2; A_1 \cup \{z_1\}, A_2) \succeq (y_1, y_2; B_1 \cup \{z_1\}, B_2)$ and $(x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow (x_1, x_2; A_1, A_2 \cup \{z_2\}) \succeq (y_1, y_2; B_1, B_2 \cup \{z_2\})$.

Axiom 2. Simple Indifference(SI)
For all $x_1 \in X_1$ and all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $x_2 \in X_2$ and all $y_2, z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2\})$ and $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, z_2\})$.

Axiom 3. Baseline Indifference(BI)
For all $x_1 \in X_1$ and all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $x_2 \in X_2$ and all $y_2, z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, y_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2, z_2\})$.

(IND) is a similar concept to the independence axiom proposed by Suzumura and Xu(2001,2003,2004). Its implication is simple: if z_1 is not in A_1 and z_2 is not in A_2, the ordering over $(x_1, x_2; A_1 \cup \{z_1\}, A_2)$ and $(y_1, y_2; B_1 \cup \{z_1\}, B_2)$, and that over $(x_1, x_2; A_1, A_2 \cup \{z_2\})$ and $(y_1, y_2; B_1, B_2 \cup \{z_2\})$ are corresponding to the ordering over $(x_1, x_2; A_1, A_2)$ and $(y_1, y_2; B_1, B_2)$ regardless of the nature of z_1 and z_2.

(SI) and (BI) are very similar axioms. (SI) requires that choosing consequence (x_1, x_2) from opportunity sets such that $|A_1| = 2$ and $|A_2| = 1$ are indifferent, and choosing (x_1, x_2) from opportunity sets that $|A_1| = 1$ and $|A_2| = 2$ are indifferent. That is, alternative that do not choose is ignored. (BI) requires that if consequence (x_1, x_2) is same, choices from opportunity set that have two alternatives are indifferent. In the case of multiple opportunity sets, however, (IND) occur much problems.

The problem is which we need both (SI) and (BI) or do not. The following result summarizes the relation of above three axioms, that is (IND), (SI) and (BI) are not independent.

Lemma 1. *If $\succeq$ satisfies (BI) and (IND), then it satisfy (SI)*

Proof. Let $\succeq$ satisfy (IND) and (BI). From (BI), for all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $y_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, y_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2, y_2\})$. By using (IND), for all $x_1 \in X_1$ and all $y_1, z_1 \in X_1 \setminus \{x_1\}$ and all $x_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2\})$.

By similar argument, we can show for all $x_1 \in X_1$ and all $x_2 \in X_2$ and all $y_2, z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2\})$. Q.E.D.

The following implication of (IND) and (BI) prove very useful.

Theorem 1. *Suppose $\succeq$ satisfy (IND) and (BI). If $|A_1| = |B_1| and |A_2| = |B_2|$, then $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.*

Proof. First, we consider the case that $A_1 \cap B_1 = \{x_1\}$ and $A_2 \cap B_2 = \{x_2\}$. Suppose that $A_1 = \{x_1, a_{11}, a_{12}, \cdots, a_{1l}\}$ and $A_2 = \{x_2, a_{21}, a_{22}, \cdots, a_{2k}\}$ and $B_1 = \{x_1, b_{11}, b_{12}, \cdots, b_{1l}\}$ and $B_2 = \{x_2, b_{21}, b_{22}, \cdots, b_{2k}\}$. Let $l, k < \infty$. From (BI), $(x_1, x_2; \{x_1, a_{1g}\}, \{x_2, a_{2h}\}) \sim (x_1, x_2; \{x_1, b_{1i}\}, \{x_2, b_{2j}\})$ for all $g, i = 1, 2, \cdots, k$ and $h, j = 1, 2, \cdots, l$.

By using (IND), $(x_1, x_2; \{x_1, a_{11}, a_{12}\}, \{x_2, a_{2h}\}) \sim (x_1, x_2; \{x_1, a_{11}, b_{12}\}, \{x_2, b_{2j}\})$ and $(x_1, x_2; \{x_1, a_{11}, b_{12}\}, \{x_2, a_{2h}\}) \sim (x_1, x_2; \{x_1, b_{11}, b_{12}\}, \{x_2, b_{2j}\})$. From the transitivity, $(x_1, x_2; \{x_1, a_{11}, a_{12}\}, \{x_2, a_{2h}\}) \sim (x_1, x_2; \{x_1, b_{11}, b_{12}\}, \{x_2, b_{2j}\})$. Since opportunity sets are finite, we obtain that, for all $A_1, B_1 \in X_1$ and $a_{2h}, b_{2j} \in X_2$, $(x_1, x_2; A_1, \{x_2, a_{2h}\}) \sim (x_1, x_2; B_1, \{x_2, b_{2j}\})$ by using (IND) repeatedly.

Moreover, by using (IND),$(x_1, x_2; A_1, \{x_2, a_{21}, a_{22}\}) \sim (x_1, x_2; B_1, \{x_2, a_{21}, b_{22}\})$ and $(x_1, x_2; A_1, \{x_2, a_{21}, b_{22}\}) \sim (x_1, x_2; B_1, \{x_2, b_{21}, b_{22}\})$. By the transitivity of $\succeq$, $(x_1, x_2; A_1, \{x_2, a_{21}, a_{22}\}) \sim (x_1, x_2; B_1, \{x_2, b_{21}, b_{22}\})$. By using (IND) and the transitivity, we can obtain $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$

Next, we consider the case that $A1 \cap B1 = \{x_1\} \cup C_1$ and $A2 \cap B2 = \{x_2\} \cup C_2$ where C_1 or C_2 is nonempty. Note that $(x_1, x_2; A_1 \setminus C_1, A_2 \setminus C_2) \sim (x_1, x_2; B_1 \setminus C_1, B_2 \setminus C_2)$ because $|A_1 \setminus C_1| = |B_1 \setminus C_1|$ and $|A_2 \setminus C_2| = |B_2 \setminus C_2|$. Then, we can get $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$ by (IND). Q.E.D.

3 Consequentialism

In this section, we define and characterize extreme consequentialism and strong consequentialism.

3.1 Extreme consequentialism

To begin with, we define extreme consequentialism as follows.

Definition 1. (Extreme consequentialism)
$\succeq$ is said to be extremely consequential if, for all $(x_1, x_2; A_1, A_2), (x_1, x_2; B_1, B_2) \in \Omega$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.

Our consequentialism definition corresponds to one in Suzumura and Xu[**??**]. Extreme consequentialism implies that the ordering depend on only consequences $(x_1, x_2), (y_1, y_2)$, and any opportunity sets A_1, A_2, B_1 and B_2 don't affect the ordering. To characterize extreme consequentialism, we propose the following axioms.

Axiom 4. Local Indifference 1(LI1)
For all $x_1 \in X_1$ and $x_2 \in X_2$, there exist $(x_1, x_2; A_1, \{x_2\}) \in \Omega$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; A_1, \{x_2\})$ where $A_1 \neq \{x_1\}$.

Axiom 5. Local Indifference 2(LI2)
For all $x_1 \in X_1$ and $x_2 \in X_2$, there exist $(x_1, x_2; \{x_1\}, A_2) \in \Omega$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, A_2)$ where $A_2 \neq \{x_2\}$.

(LI1) require that there exists $A_1 \in K_1$, which is not $\{x_1\}$ such that choosing x_1 from A_1 and x_2 from $\{x_2\}$ is regarded as being indifferent as choosing x_1 from $\{x_1\}$ and x_2 from $\{x_2\}$. Similarly, (LI2) require that there exists $A_2 \in K_2$ which is not $\{x_2\}$, such that choosing x_1 from $\{x_1\}$ and x_2 from A_2 is regarded as being indifferent as choosing x_1 from $\{x_1\}$ and x_2 from $\{x_2\}$. Combining these axioms with (IND) and (BI), we can get the following result.

Theorem 2. *$\succeq$ satisfies (IND),(BI), (LI1) and (LI2) if and only if it is extremely consequential.*

Proof. If $\succeq$ is extremely consequential, it is satisfies (IND),(BI),(LI1) and (LI2). Therefore, we have only to show that if $\succeq$ satisfy (IND),(BI), (LI1) and (LI2), then it is extremely consequential.

Let $\succeq$ satisfy (IND),(BI), (LI1) and (LI2). Now, since $\succeq$ satisfy (IND) and (BI), from Theorem1, we have the following.

$$|A_1| = |B_1| \text{ and } |A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2) \tag{1}$$

Therefore, we have only to show the case where $|A_1| \neq |B_1|$ or $|A_2| \neq |B_2|$. To begin with, we show the following relation.

$$(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \text{ for all } y_1 \in X_1 \setminus \{x_1\} \tag{2}$$

Suppose that there exist $a_1 \in X_1 \setminus \{x_1\}$ such that $(x_1, x_2; \{x_1, a_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$. From (BI) and the transitivity, we can get the following.

$$(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\}) \text{ for all } y_1 \in X_1 \setminus \{x_1\} \tag{3}$$

By using (IND), $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1, z_1\}, \{x_2\})$ for all $z_1 \in X_1 \setminus \{x_1, y_1\}$. From Theorem1 and the transitivity of $\succeq$, $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$ for all $y_1, z_1 \in X_1 \setminus \{x_1\}$. By using similar argument, we can show the following.

$$\forall A_1 \in K_1 \text{ where } |A_1| \geq 2, \forall x_1 \in X_1, \forall x_2 \in X_2, (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\}) \tag{4}$$

Equation(4) is contradiction with (LI1). Similarly, if there exist $b_1 \in X_1 \setminus \{x_2\}$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1, b_1\}, \{x_2\})$, this cause another contradiction. So, equation(2) holds. From equation(1) and (2), since opportunity sets is finite, by using (IND) and the transitivity repeatedly,

$$\forall A_1 \in K_1, \forall x_1 \in X_1, \forall x_2 \in X_2, (x_1, x_2; A_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}). \tag{5}$$

By same argument, we can show that $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\})$ for all $y_1 \in X_1 \setminus \{x_1\}$ Hence, from (IND) and transitivity, we can get the following.

$$\forall A_2 \in K_2, \forall x_1 \in X_1, \forall x_2 \in X_2, (x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \tag{6}$$

From equation(5) and equation(6), For all $A_1 \in K_1$, all $A_2 \in K_2$, all $x_1 \in X_1$ and all $x_2 \in X_2$, $(x_1, x_2; A_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, A_2)$.

So,$(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2, z_3\})$. By simple application of (IND), $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2, a_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2, a_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, a_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2, a_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2, z_3, a_2\})$. These relationships and equation(1) lead us to have the following.

$$
\begin{gathered}
(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \sim \\
(x_1, x_2; \{x_1, z_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \sim \\
(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2, z_3\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2, z_3, w_2\}).
\end{gathered}
$$

By using this argument, since opportunity sets is finite, we can get the case where $|A_1| \neq |B_1|$ or $|A_2| \neq |B_2|$.

Q.E.D.

3.2 Strongly consequentialism

According to Suzumura and Xu(2001,2003,2004), we define strong consequentialism. Strong consequentialism requires that when the individual prefers $(x_1, x_2; \{x_1\}, \{x_2\})$ to $(y_1, y_2; \{y_1\}, \{y_2\})$, then he or she prefers $(x_1, x_2; A_1, A_2)$ to $(y_1, y_2; B_1, B_2)$, so opportunities do not matter. If alternatives are indifferent in choice from singleton opportunity sets, he ore she evaluates by cardinalities of each opportunity set. That is, opportunities matter only when the individual is indifferent between $(x_1, x_2; \{x_1\}, \{x_2\})$ and $(y_1, y_2; \{y_1\}, \{y_2\})$. In our framework, several version of strong consequentialism can be defined.

Before proposing strong consequentialism, we present the following axioms that will be use in our characterization results.

Axiom 6. Local Strict Monotonicity 1 (LSM1)
For all $x_1 \in X_1$ and $x_2 \in X_2$,there exists $(x_1, x_2; A_1, \{x_2\}) \in \Omega \backslash \{(x_1, x_2; \{x_1\}, \{x_2\})\}$ such that $(x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$.

Axiom 7. Local Strict Monotonicity 2 (LSM2)
For all $x_1 \in X_1$ and $x_2 \in X_2$,there exists $(x_1, x_2; \{x_1\}, A_2) \in \Omega \backslash \{(x_1, x_2; \{x_1\}, \{x_2\})|$ such that $(x_1, x_2; \{x_1\}, A_2) \succ (x_1, x_2; \{x_1\}, \{x_2\})$.

(LSM1) requires that there exist a first opportunity set A_1 such that choosing x_1 from first opportunity set A_1 and x_2 from second opportunity set $\{x_2\}$ is strictly better than choosing x_1 and x_2 from the singleton sets $\{x_1\}$ and $\{x_2\}$. Similarly, (LSM2) requires that there exist a first opportunity set A_2 such that choosing x_1 from first opportunity set $\{x_1\}$ and x_2 from second opportunity set A_2 is strictly better than choosing x_1 and x_2 from the singleton sets $\{x_1\}$ and $\{x_2\}$. These axioms are a minimal and local requirement for the intrinsic value of freedom of choice.

Combining (IND) and (BI), Local Strict Monotonicity axiom imply the following results that will prove useful in establishing the reminder of our results in this paper.

Lemma 2. *If $\succeq$ satisfy (IND), (BI) and (LSM1), then, for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$, $|A_1| > |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$.*

Proof. Let $\succeq$ satisfy (IND), (BI) and (LSM1).

To begin with, we show the following.

$$\forall x_1 \in X_1, \forall y_1 \in X_1 \setminus \{x_1\}, (x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\}) \tag{7}$$

Suppose that there exist $a_1 \in X_1 \backslash \{x_1\}$ such that $(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, a_1\}, \{x_2\})$. From (BI) and the transitivity, we can get the following.

$$(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, y_1\}, \{x_2\}) \text{ for all } y_1 \in X_1 \setminus \{x_1\} \tag{8}$$

By using (IND), $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\})$ for all $z_1 \in X_1 \setminus \{x_1, y_1\}$. From Theorem1 and the transitivity of $\succeq$, $(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\})$ for all $y_1, z_1 \in X_1 \setminus \{x_1\}$. By using similar argument, we can show the following.

$$\forall A_1 \in K_1 \text{ where } |A_1| \geq 2, \forall x_1 \in X_1, \forall x_2 \in X_2, (x_1, x_2; \{x_1\}, \{x_2\}) \succeq (x_1, x_2; A_1, \{x_2\}) \tag{9}$$

Equation(9) is contradiction with (LSM1). Hence, by completeness of $\succeq$, equation(7) holds. By using equation(7) and (IND) repeatedly, we get the following.

$$For all A_1, B_1 \in K_1, all x_2 \in X_2, |A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\})$$

From this equation and (IND) and Theorem1, for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$, $|A_1| > |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$. Q.E.D.

Lemma 3. *If $\succeq$ satisfies (IND), (BI) and (LSM2), then, for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$, $|A_1| = |B_1|$ and $|A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$.*

Proof. Exactly analogous to that of Lemma2 Q.E.D.

Corollary 1. *If $\succeq$ satisfies (IND), (BI) and (LSM1) and (LSM2), then, for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$, $|A_1| \geq |B_1|$ and $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2)$.*

Proof. Let $\succeq$ satisfy (IND),(BI), (LSM1) and (LSM2). There are 3 cases, (i)$|A_1| > |B_1|$ and $|A_2| = |B_2|$, (ii)$|A_1| = |B_1|$ and $|A_2| > |B_2|$, and (iii)$|A_1| > |B_1|$ and $|A_2| > |B_2|$. It is straightforward to show the first case and the second case by Lemma2 and Lemma3.

Therefore, we have only to show that, for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$, $|A_1| > |B_1|$ and $|A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$. There exist $C_1 \in K_1$ such that $C_1 \subset A_1$, $x_1 \notin C_1$ and $|C_1| = |B_1|$. Since $|C_1| = |B_1|$ and $|A_2| > |B_2|$, by Lemma3, $(x_1, x_2; C_1, A_2) \succ (x_1, x_2; B_1, B_2)$. Moreover, since $|A_1| > |C_1|$ and $|A_2| = |A_2|$, by Lemma2, $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; C_1, A_2)$. By the transitivity of $\succeq$, $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$. Q.E.D.

Now, we define several variations of strong consequentialism. These concepts is deferent in how to evaluate opportunities.

Definition 2. (First opportunity set ranking strong consequentialism)
$\succeq$ is said to be first opportunity set ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\, (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| \geq |B_1| \,]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 3. (Second opportunity set ranking strong consequentialism)
$\succeq$ is said to be second opportunity set ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\, (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_2| \geq |B_2| \,]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 4. (Sum-ranking strong consequentialism)
$\succeq$ is said to be sum-ranking strongly consequential if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\, (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| + |A_2| \geq |B_1| + |B_2| \,]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

To characterize strong consequentialism, we need additional axiom.

Axiom 8. Robustness(ROB)
For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$ and all $z_1 \in X_1$,$z_2 \in X_2$, if $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$ and $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1 \cup \{z_1\}, B_2)$ and $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2 \cup \{z_2\})$.

(ROB) require that for all $z_1 \in X_1$ and all $z_2 \in X_2$, if the individual ranking higher $(x_1, x_2; A_1, A_2)$ than $(y_1, y_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2)$ is still ranked higher than $(y_1, y_2; B_1 \cup \{z_1\}, B_2)$ and $(y_1, y_2; B_1, B_2 \cup \{z_2\})$.

Theorem 3. *$\succeq$ satisfy (IND), (BI), (LSM1), (LI2) and (ROB) if and only if it is first opportunity set ranking strongly consequential.*

Proof. If $\succeq$ is first opportunity set ranking strongly consequential, then it satisfy (IND), (BI), (LSM1), (LI2) and (ROB). Therefore, we have only to show that if $\succeq$ satisfy (IND), (BI), (LSM1), (LI2) and (ROB), then it is first opportunity set ranking strongly consequential.

Let $\succeq$ satisfy (IND), (BI), (LSM1), (LI2) and (ROB). By similar argument in Theorem2, we have the following.

$$\forall A_2 \in K_2, \forall x_1 \in X_1, \forall x_2 \in X_2, (x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_2\}). \tag{10}$$

By using (IND) and Theorem1, for all $A_2 \in K_2$ and for all $A_1, B_1 \in K_1$ such that $|A_1| = |B_1|$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, \{x_2\})$. From Lemma2, for all $(x_1, x_2; A_1, A_2), (x_1, x_2; B_1, B_2) \in \Omega$,

$$|A_1| \geq |B_1| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2). \tag{11}$$

First, consider that, for all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$,$(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. By using (IND) and Theorem1, $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2)$. From equation(11) and the transitivity, we obtain $|A_1| \geq |B_1| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Next, suppose that, for all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By using (ROB), $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1, z_1\}, \{y_2, z_2\})$. Moreover, by using (ROB) repeatedly, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; A_1, A_2)$. By simple application of equation(11) and the transitivity of $\succeq$, we have the following.

$$\text{for all } (x_1, x_2; A_1, A_2), (y_1, y_2; B_1, B_2) \in \Omega, (x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \\ \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$$

Q.E.D.

Theorem 4. *$\succeq$ satisfy (IND), (BI), (LI1), (LSM2) and (ROB) if and only if it is first opportunity set ranking strongly consequential.*

Proof. Exactly analogous to that of Theorem3 Q.E.D.

Since sum-ranking strong nonconsequentialism allows trade-off between the value of first opportunity set and one of second, to characterize it, we need more axiom.

Axiom 9. Trinary Indifference(CI)
For all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2\})$

According to (TI), if consequence (x_1, x_2) is same, choices from opportunity set such that the summation of the cardinality is three are indifferent, independent of the nature of the alternative that is not chosen.

We note the following results which will prove useful in establishing the reminder of our results in this paper.

Lemma 4. *If $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (TI), then $|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2)$.*

Proof. Let $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2) and (TI). First, by Theorem1, for all $(x_1, x_2; A_1, A_1), (x_1, x_2; B_1, B_1) \in \Omega$,

$$|A_1| = |B_1| \text{ and } |A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2) \tag{12}$$

Moreover, (TI) implies that, $|A_1| + |A_2| = |B_1| + |B_2| = 3$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.

From simple application of (TI) and (IND), for all distinct $x_1, y_1, z_1 \in X_1$ and all distinct $x_2, y_2, z_2 \in X_2$,

$$(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, y_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, y_1\}, \{x_2, y_2, z_2\}) \tag{13}$$

By using equation(12) and equation(13), for all $A_1, B_1 \in X_1$ and all $A_2, B_2 \in X_2$, $|A_1| + |A_2| = |B_1| + |B_2| \leq 4 \Rightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.

Since opportunity sets is finite, by similar argument, we obtain the following. For all $(x_1, x_2; A_1, A_2), (x_1, x_2; B_1, B_2) \in \Omega$,

$$|A_1| + |A_2| = |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2) \tag{14}$$

Now, suppose $|A_1| + |A_2| > |B_1| + |B_2|$. There exists $C_1 \in X_1$ and $C_2 \in X_2$ such that $|A_1| + |A_2| = |C_1| + |C_2|$ and $|C_1| > |B_1| \wedge |C_2| = |B_2|$. By Lemma2, $(x_1, x_2; C_1, C_2) \succ (x_1, x_2; B_1, B_2)$. And, by using equation(14), since $|A_1| + |A_2| = |C_1| + |C_2|$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; C_1, C_2)$. From transitivity of $\succeq$, $|A_1| + |A_2| > |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ .(x_1, x_2; B_1, B_2)$. Then, we obtain that, for all $(x_1, x_2; A_1, A_1), (x_1, x_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2)$. Q.E.D.

We are now ready to axiomatize completely sum-ranking strong consequentialism.

Theorem 5. *$\succeq$ satisfy (IND), (BI), (TI), (LSM1), (LSM2) and (ROB) if and only if it is sum-ranking strongly consequential.*

Proof. If $\succeq$ is sum-ranking strongly consequential, then it satisfy (IND), (BI), (TI), (LSM1), (LSM2) and (ROB). Therefore, we have only to show that if $\succeq$ satisfy (IND), (BI), (TI), (LSM1), (LSM2) and (ROB), then it is sum-ranking strongly consequential.

Let $\succeq$ satisfy (IND), (BI), (TI), (LSM1), (LSM2) and (ROB). By Lemma4, we have the following. for all $(x_1, x_2; A_1, A_2)$,$(x_1, x_2; B_1, B_2)$$\in \Omega$,

$$|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2) \tag{15}$$

First, suppose that, for all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$,$(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. By (IND), for all $z_1 \in X_1$ and all $z_2 \in X_2$, $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1, z_1\}, \{y_2\})$ and $(x_1, x_2; \{x_1\}, \{x_2, z_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2, z_2\})$. By using (TI) and Theorem1, we obtain that $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (y_1, y_2; \{x_1, y_1\}, \{y_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2, y_2\}) \sim (y_1, y_2; \{y_1\}, \{x_2, y_2\})$. From the finiteness of opportunity sets, by using (IND) repeatedly, we have the following.

$$\text{for all } (x_1, x_2; A_1, A_2),\ (y_1, y_2; B_1, B_2) \in \Omega, \text{ if } (x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}), \text{ then } (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| + |A_2| = |B_1| + |B_2|$$

By repeated application of equation(15) and the transitivity, we have the following.

$$\text{For all } (x_1, x_2; A_1, A_2),\ (y_1, y_2; B_1, B_2) \in \Omega, \text{ if } (x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}), \text{ then } (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \Leftrightarrow |A_1| + |A_2| \geq |B_1| + |B_2|$$

Next, suppose that, for all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By using (ROB), $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1, z_1\}, \{y_2, z_2\})$. Moreover, by using (ROB) repeatedly, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; A_1, A_2)$. By simple application of equation(10) and the transitivity of $\succeq$, we have the following.

$$\text{for all } (x_1, x_2; A_1, A_2),\ (y_1, y_2; B_1, B_2) \in \Omega,\ (x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \\ \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$$

Q.E.D.

4 Nonconsequentialism

4.1 Extreme nonconsequentialism

In this subsection, we define and characterize various type of extreme nonconsequentialism. First, we define the basic concept of extreme nonconsequentialism.

Definition 5. (Extreme nonconsequentialism)
$\succeq$ is said to be extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; A_1, A_2)$.

This formulation of extreme nonconsequentialism is different from Suzumura and Xu(2001). They present extreme nonconsequentialism as the concept that require increasing the cardinality of opportunity sets is good. However, essential requirement of extreme consequentialism is no regard for consequences. Our concept of extreme nonconsequentialism says that if first and second opportunity set have the same cardinality respectively, then extended alternatives are indifferent. Note that by this formulation we cannot completely judge two choice situation, $(x_1, x_2; A_1, A_2)$ and $(y_1, y_2; B_1, B_2)$.

To characterize extreme nonconsequentialism, we introduce the following axiom.

Axiom 10. Indifference of No-Choice Situations(INS)
For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$.

(INS) requires that two choice situation that both opportunity sets are singleton is indifferent. In many papers relating with freedom of choice, there are very similar concepts. [9]

Next, we characterize the extreme consequentialism.

Theorem 6. *$\succeq$ satisfy (IND), (BI) and (INS) if and only if it is extremely nonconsequential.*

Proof. If $\succeq$ is extreme nonconsequential, then it clearly satisfies (IND), (BI) and (INS). Hence, we have only to show that, if $\succeq$ satisfy (IND), (BI) and (INS), then it is extremely nonconsequential.

Let $\succeq$ satisfy (IND), (BI) and (INS). By (INS), for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. Moreover, by (IND) and the transitivity of $\succeq$, For all $x_1, y_1, z_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1, z_1\}, \{y_2\})$. From Theorem1, for all $x_1, y_1, z_1, w_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1, w_1\}, \{y_2\})$.

By using (IND) and Theorem1 repeatedly, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; A_1, A_2)$. Q.E.D.

[9](INS) corresponds to the axiom in Suzumura and Xu(2001). Similar concepts axiomatized in a form that Pattanaik and Xu(1990) call "indifference between no choice situation" and Jones and Sugden(1982) call "principle of no choice". Carter(2004) discuss for the axiom of "indifference between no choice situation".

By our definition of extreme nonconsequentialism, when he individual judge two choice situation by opportunities, the individual may not value the freedom of choice and consider harmful. The following definition guarantees that extreme nonconsequentialist values the freedom of choice.

Definition 6. (opportunity lover)
$\succeq$ is said to be opportunity lover if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \geq |B_1|$ and $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Note that the concept of opportunity lover implies the increasing freedom of choice is good. Between two choice situation $(x_1, x_2; A_1, A_1)$ and $(y_1, y_2; B_1, B_1)$such that $|A_1| \geq |B_1|$ and $|A_2| \geq |B_2|$, we judge which is better. But between two choice situation $(x_1, x_2; A_1, A_1)$ and $(y_1, y_2; B_1, B_1)$ such that $|A_1| \geq |B_1|$ and $|A_2| \leq |B_2|$, we cannot judge which is better.

Lemma 5. $\succeq$ *satisfy (IND), (BI), (LSM1), (LSM2) and (INS) if and only if it is opportunity lover.*

Proof. If $\succeq$ is opportunity lover, then it satisfy (IND), (BI), (LSM1), (LSM2) and (INS). Hence, we have only to show that if $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2) and (INS), then it is opportunity lover.

Let $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2) and (INS). To begin with, from Corollary1, we have the following. for all $(x_1, x_2; A_1, A_2)$,$(x_1, x_2; C_1, C_2)$$\in \Omega$,

$$|A_1| \geq |C_1| \text{ and } |A_2| \geq |C_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; C_1, C_2) \tag{16}$$

Moreover, from Theorem6, since $\succeq$ is extremely nonconsequential from Theorem6, we have the following. For all $(x_1, x_2; C_1, C_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$,

$$|C_1| = |B_1| \text{ and } |C_2| = |B_2| \Rightarrow (x_1, x_2; C_1, C_2) \sim (y_1, y_2; B_1, B_2) \tag{17}$$

From equation(16), equation(17) and the transitivity, for all $(x_1, x_2; A_1, A_1)$, $y_1, y_2; B_1, B_1)$ $\in \Omega$, $|A_1| \geq |B_1|$ and $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$. Q.E.D.

Now, we propose concepts by which we can perfectly judge two choice situations. We introduce four concepts, Part-ranking extreme nonconsequentialism, Sum-ranking extreme nonconsequentialism and Weighted sum-ranking extreme nonconsequentialism, lexicographic extreme nonconsequentialism.

The concept of part-ranking extreme nonconsequentialism requires that the ordering base on one opportunity set and ignore another. There are two type of part-ranking extremely nonconsequentialism, first opportunity set ranking extremely nonconsequentialism and second opportunity set ranking extremely nonconsequentialism.

According to first opportunity set-ranking extreme nonconsequentialism, the extended alternatives ranked according to the cardinality of first opportunity set, and second opportunity set and consequence don't matters. Similarly, second opportunity set-ranking extreme nonconsequentialism requires that the extended alternatives ranked according to the cardinality of second opportunity set, Their definitions are given below.

Definition 7. (First opportunity set ranking extreme nonconsequentialism)
$\succeq$ is said to be first opportunity set ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \geq |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$
.

Definition 8. (Second opportunity set ranking extreme nonconsequentialism)
$\succeq$ is said to be second opportunity set ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(x_1, x_2; B_1, B_1) \in \Omega$, $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Now, we state the full characterization of first opportunity set ranking extreme nonconsequentialism.

Theorem 7. *If $\succeq$ satisfy (IND), (BI), (INS), (LI2) and (LSM1), then, it is first opportunity set ranking extreme nonconsequential, that is, for all* $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $|A_1| \geq |B_1| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Proof. If $\succeq$ is first opportunity set ranking extremely nonconsequential, then, it is satisfy (IND), (BI), (INS), (LI2) and (LSM1). Therefore, we have only to show that, if $\succeq$ satisfy (IND), (BI), (INS), (LI2) and (LSM1), then, it is first opportunity set ranking extremely nonconsequential, that is, for all $(x_1, x_2; A_1, A_2)$,$(y_1, y_2; B_1, B_2) \in \Omega$, $|A_1| \geq |B_1| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Let $\succeq$ satisfy (IND), (BI), (INS), (LI2) and (LSM1). To begin with, by Theorem6, we have the following. for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$,

$$|A_1| = |B_1| \text{ and } |A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{18}$$

From (IND), (BI), (LI2), by the argument similar to Theorem2, we obtain the following.

$$\forall A_2 \in K_2, \forall x_1 \in X_1, (x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \tag{19}$$

By using (IND) and equation(19), we can show the following. for all $x_1 \in X_1$, all $x_2 \in X_2$, all $A_1 \in K_1$ and $A_2 \in K_2$,

$$\forall x_2 \in X_2, \forall A_1 \in K_1, \forall A_2 \in K_2, (x_1, x_2; A_1, A_2) \sim (x_1, x_2; A_1, \{x_2\}) \tag{20}$$

From equation(18) and equation(20), we obtain the following. For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$,

$$|A_1| = |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{21}$$

Moreover, from lemma2, we have following. For all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2)$ $\in \Omega$,

$$|A_1| > |B_1| \text{ and } |A_2| = |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2) \tag{22}$$

By equation(21) and equation(22), for all $(x_1, x_2; A_1, A_2)$,$(y_1, y_2; B_1, B_2) \in \Omega$, $|A_1| \geq |B_1| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$. Q.E.D.

Theorem 8. *$\succeq$ satisfy (IND), (BI), (INS), (LI1) and (LSM2) if and only if it is second opportunity set-ranking extremely nonconsequential, that is, for all* $(x_1, x_2; A_1, A_2)$,$(y_1, y_2; B_1, B_2) \in \Omega$, $|A_2| \geq |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Proof. Exactly analogous to that of Theorem7. Q.E.D.

Next, we define sum-ranking extreme nonconsequentialism.

Definition 9. (Sum-ranking extreme nonconsequentialism)
$\succeq$ is said to be sum-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Sum-ranking extreme nonconsequentialist evaluate all opportunity as same. Sum-ranking extreme nonconsequentialism requires that in two choice situation, the individual ranks extended alternatives according to the summation of each cardinality of opportunity sets.

Theorem 9. *$\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (TI) if and only if it is sum-ranking extremely nonconsequential.*

Proof. If $\succeq$ is sum-ranking extremely nonconsequential, then it satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (TI). Therefore, we have only to show that, if $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (TI), then it is sum-ranking extremely nonconsequential.

Let $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (TI). First, by theorem6, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$,

$$|A_1| = |B_1| \text{ and } |A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{23}$$

Then, this relation implies the following. For all $x_1, y_1, z_1 \in X_1$ and all $x_2, y_2, z_1 \in X_2$,

$$(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1, z_1\}, \{y_2\}) \tag{24}$$

From (TI), for all $y_1, z_1 \in X_1$ and all $y_2, z_2 \in X_2$, $(y_1, y_2; \{y_1, z_1\}, \{y_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2, z_2\})$ By using this and equation(24), for all $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$,

$$(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2, z_2\}) \tag{25}$$

Therefore, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$ where $|A_1| + |A_2| = |B_1| + |B_2| = 3$, alternatives are indifferent.

From simple application of (TI) and (IND), for all distinct $x_1, y_1, z_1 \in X_1$ and all distinct $x_2, y_2, z_2 \in X_2$,

$$(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \sim (x_1, x_2; \{x_1, y_1\}, \{x_2, y_2\}) \sim (x_1, x_2; \{x_1, y_1\}, \{x_2, y_2, z_2\}) \tag{26}$$

By using equation(23) and equation(24), for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$ where $|A_1| + |A_2| = |B_1| + |B_2| = 4$, extended alternatives are indifferent.

Since opportunity sets is finite, by similar argument, we obtain the following. For all $(x_1, x_2; A_1, A_2), (y_{1,2}; B_1, B_2) \in \Omega$,

$$|A_1| + |B_1| = |A_2| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{27}$$

From equation(24) and Lemma2, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \sim (y_1, y_2; \{x_1\}, \{y_1, y_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. Since opportunity sets are finite, using equation(27) and Lemma2 according to similar argument, we obtain that, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| \geq |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$. Q.E.D.

Definition 10. (Weighted sum-ranking extremely nonconsequentialism)
$\succeq$ is said to be weighted sum-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $\alpha|A_1| + \beta|A_2| \geq \alpha|B_1| + \beta|B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; A_1, A_2)$.

Weighted sum-ranking extremely nonconsequentialism is general case of sum-ranking extremely nonconsequentialism. If $\alpha, \beta = 1$, then $\succeq$ is sum-ranking extremely nonconsequential. Since opportunity set is finite, this concept don't represent part-ranking extremely consequentialism.

Axiom 11. Proportional Indifference(PI)
For all $x_1 \in X_1$ and all $x_2 \in X_2$, there exists A_1, A_2, B_1 and B_2 such that $\alpha|A_1| + \beta|A_2| = \alpha|B_1| + \beta|B_2|$, $|A_1| \neq |B_1|$ and $|A_2| \neq |B_2|$, and $(x_1, x_2; A_1, A_1) \sim (x_1, x_2; B_1, B_2)$

This axiom is general case of (TI). If $\alpha, \beta = 1$, then (PI) is correspondent with (TI). Changing (TI) for (PI) in conditions of Theorem9, we can get weighted sum-ranking extremely nonconsequential ordering.

Theorem 10. *$\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (PI) if and only if it is weighted sum-ranking extremely nonconsequential.*

Proof. Exactly analogous to that of Theorem9. Q.E.D.

Definition 11. (Lexicographic extreme nonconsequentialism for first opportunity set)
$\succeq$ is said to be lexicographic extremely nonconsequential for first opportunity set if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| = |B_1| \Rightarrow [\,|A_2| \geq |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Thus, according to lexicographic extreme nonconsequentialism for first opportunity set, consequence (x_1, x_2) do not matter and opportunities matter, and if $|A_1| > |B_2|$, then $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and second opportunity set do not matter. Only when $|A_1| = |B_2|$, the individual compares the cardinalities of second opportunity set. Similarly, we can consider lexicographic extreme nonconsequentialism for second opportunity set.

To characterize lexicographic extreme nonconsequentialism for first opportunity set, we need the next axiom.

Axiom 12. Weakly Robustness for first opportunity set (WROB1)
For all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$ and all $z_2 \in X_2$, $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$, then $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2 \cup \{z_2\})$.

This axiom is similar to (ROB). If individual ranks $(x_1, x_2; A_1, A_2)$ higher than $(x_1, x_2; B_1, B_2)$, then adding $\{z_2\}$ to B_1 while maintaining y_1 from new opportunity set will not affect individual's ranking.

Theorem 11. *$\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (WROB1) if and only if it is lexicographic extremely nonconsequential.*

Proof. If $\succeq$ is lexicographic extremely nonconsequential, then it satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (WROB1). Hence, we have only to show that if $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (WROB1), it is lexicographic extremely nonconsequential.

Let $\succeq$ satisfy (IND), (BI), (LSM1), (LSM2), (INS) and (WROB1). First, we consider the case that $|A_1| > |B_1|$ for $A_1, B_1 \in X_1$. By Lemma2, $(x_1, x_2; A_1, \{x_1\}) \succ (x_1, x_2; B_1, \{x_2\})$ since $|A_1| > |B_1|$ and $|\{x_1\}| = |\{x_2\}| = 1$. By using (WROB1), $(x_1, x_2; A_1, \{x_1\}) \succ (x_1, x_2; B_1, \{x_2, z_1\})$. Repeatedly, we obtain that, for all $B_2 \in X_2$, $(x_1, x_2; A_1, \{x_1\}) \succ (x_1, x_2; B_1, B_2)$. Lemma2 and Theorem1 imply $(x_1, x_2; A_1, A_2) \succeq (x_1, x_2; A_1, \{x_1\})$ for all $A_2 \in X_2$. By the transitivity, $(x_1, x_2; A_1, A_2) \succ (x_1, x_2; B_1, B_2)$. Moreover, Theorem6 implies $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ for all $x_1, y_1 \in X_1$, all $x_2, y_2 \in X_2$ and all $A_2, B_2 \in X_2$.

Next, suppose that $|A_1| = |B_1|$ for $A_1, B_1 \in X_1$. By Lemma3, we have the following.

$$|A_1| = |C_1| \text{ and } |A_2| > |C_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (x_1, x_2; C_1, C_2) \tag{28}$$

Moreover, Theorem6 implies the following.

$$|C_1| = |B_1| \text{ and } |C_2| = |B_2| \Rightarrow (x_1, x_2; C_1, C_2) \sim (y_1, y_2; B_1, B_2) \tag{29}$$

Combining equation(28) and equation(29), if $|A_1| = |B_1|$, then $|A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$. By Theorem6, $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2)$. Therefore, if $|A_1| = |B_1|$, then $|A_2| \geq |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$. Q.E.D.

4.2 Strong nonconsequentialism

In this subsection, we define and characterize the strong nonconsequentialism. First, we propose the following axiom before defining the strong nonconsequentialism.

Axiom 13. Simple Preference for First Opportunities(SPO1)
For all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$

Axiom 14. Simple Preference for Second Opportunities(SPO2)
For all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$

We note that the following lemma that will prove useful to characterize the strong nonconsequentialism.

Lemma 6. *If $\succeq$ satisfies (IND),(SI) and (SPO1), then it also satisfies (LSM1).*

Proof. Let $\succeq$ satisfies (IND),(SI) and (SPO). By (SPO), for all distinct $x_1, y_1 \in X_1$ and all $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. Combining with (IND), for all $z_1 \in X_1/\{x_1, y_1\}$, $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1, z_1\}, \{y_2\})$. (SI) implies $(y_1, y_2; \{y_1, z_1\}, \{y_2\}) \sim (y_1, y_2; \{x_1, y_1\}, \{y_2\})$. Therefore, by the transitivity $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succ y_1, y_2; \{x_1, y_1\}, \{y_2\})$. By (SPO), $(y_1, y_2; \{x_1, y_1\}, \{y_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$. Hence, we obtain that, $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succ (x_1, x_2; \{x_1\}, \{x_2\})$ Q.E.D.

Lemma 7. *If $\succeq$ satisfies (IND),(SI) and (SPO2), then it also satisfies (LSM2).*

Proof. Exactly analogous to that of Lemma6 Q.E.D.

Corollary 2. *If $\succeq$ satisfies (IND), (BI), (SPO1) and (SPO2), then it also satisfies (LSM1) and (LSM2).*

Proof. It is straightforward to show this by lemma1 and lemma6. Q.E.D.

Now, we define four types of the strong nonconsequentialism.

Definition 12. (First opportunity set ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| = |B_1| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 13. (Second opportunity set ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_2| = |B_2| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 14. (Sum-ranking Strong nonconsequentialism)
$\succeq$ is said to be sum-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| + |A_2| > |B_1| + |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ and $|A_1| + |A_2| = |B_1| + |B_2| \Rightarrow [(x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Definition 15. (Lexicographic strong nonconsequentialism for first opportunity set)
$\succeq$ is said to be lexicographic strongly nonconsequential for first opportunity set if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, $|A_1| = |B_1| \Rightarrow [\ |A_2| > |B_2| \Leftrightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)]$, and $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow [\ (x_1, x_2; \{x_1\}, \{x_2\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Leftrightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)]$.

Sum-ranking Strong nonconsequentialism require that consequence do not matter and $(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$ when $|A_1| + |A_2| > |B_1| + |B_2|$. Only when $|A_1| + |A_2| = |B_1| + |B_2|$, consequence matters and preference correspond to one over choice situation from singleton opportunity sets.

We are now ready to put forward the full characterization of the concepts of strong nonconsequentialism as follows.

Theorem 12. *$\succeq$ satisfy (IND), (BI), (SPO1) and (LI2) if and only if it is sum-ranking strongly nonconsequential.*

Proof. If $\succeq$ is sum-ranking strong nonconsequential, then it satisfy (IND), (BI), (SPO1) and (LI2). Hence, we have only to show that if $\succeq$ satisfy (IND), (BI), (SPO1), (LI2), it is sum-ranking strongly nonconsequential.

Let $\succeq$ satisfy (IND), (BI), (SPO1), (LI2). To begin with, since $\succeq$ satisfies (IND), (BI) and (LI2), by a similar argument in Theorem2, for all $A_2 \in K_2$, $(x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_1\})$. By using (IND) repeatedly, for all $A_2 \in K_2$ and $A_1 \in K_1$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; A_1, \{x_1\})$. From Theorem1, we have that,

$$\forall A_2 \in K_2 \text{ and } \forall A_1, B_1 \in K_1, |A_1| = |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, \{x_1\}) \quad (30)$$

By (SPO1), for all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By Theorem1, for all $z_1 \in X_1 \setminus \{x_1\}$, $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. Combining with equation(30), for all $A_2, B_2 \in K_2$ $(x_1, x_2; \{x_1, y_1\}, A_2) \succ (y_1, y_2; \{y_1\}, B_2)$. So, we obtain that

$$|A_1| = 2 \text{ and } |B_2| = 1 \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2) \quad (31)$$

Now, suppose that $|A_1| = k + 1$ and $|B_2| = k \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$. By using (IND) and Theorem1, $|A_1| = k + 2$ and $|B_2| = k + 1 \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$. By mathematical induction, we obtain the following.

$$|A_1| = |B_2| + 1 \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2) \quad (32)$$

From equation(32), by the transitivity of $\succeq$, we have the following.

$$\text{for all } (x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega,\ |A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$$

Now, consider that, for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. Equation(30), (IND) and Theorem1 lead us obtain the following.

$$\text{for all } (x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega,\ |A_1| = |B_1| \text{ and } (x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}), \text{ then}(x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2)$$

Next, consider that, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By an argument similar to the proof of Theorem5, we have the following.

$$\text{for all } (x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega,\ |A_1| = |B_1| \text{ and } (x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}), \text{ then}(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$$

Q.E.D.

Theorem 13. *$\succeq$ satisfy (IND), (BI), (LI1) and (SPO2) if and only if it is sum-ranking strongly nonconsequential.*

Proof. Exactly analogous to that of Theorem12. Q.E.D.

Theorem 14. *$\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (TI) if and only if it is sum-ranking strongly nonconsequential.*

Proof. If $\succeq$ is sum-ranking strong nonconsequential, then it satisfy (IND), (BI), (SPO1), (SPO2) and (TI). Hence, we have only to show that if $\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (TI), it is sum-ranking strongly nonconsequential.

Let $\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (TI). To begin with, by (SPO1) and (SPO2), for all distinct $x_1, y_1 \in X_1$ and all distinct $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1, y_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$ and $(x_1, x_2; \{x_1\}, \{x_2, y_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By Theorem1, for all $z_1 \in X_1 \setminus \{x_1\}$ and all $z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1, z_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$ and $(x_1, x_2; \{x_1\}, \{x_2, z_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. Then, by Lemma4, we obtain that

$$\text{for all } (x_1,x_2;A_1,A_1),(x_1,x_2;B_1,B_1)\in\Omega,\ |A_1|+|A_2|=3 \text{ and } |B_1|+|B_2|=2 \Rightarrow \\ (x_1,x_2;A_1,A_2)\succ(y_1,y_2;B_1,B_2)$$

By using (IND), for all $w_1 \in X_1\backslash\{x_1,z_1\}$, $(x_1,x_2;\{x_1,z_1,w_1\},\{x_2\}) \succ (y_1,y_2;\{y_1,w_1\},\{y_2\})$ and $(x_1,x_2;\{x_1,w_1\},\{x_2,z_2\}) \succ (y_1,y_2;\{y_1,w_1\},\{y_2\})$. Similarly, for all $w_1 \in X_1 \setminus \{x_2,z_2\}$, $(x_1,x_2;\{x_1,z_1\},\{x_2,w_2\}) \succ (y_1,y_2;\{y_1\},\{y_2,w_2\})$ and $(x_1,x_2;\{x_1\},\{x_2,z_2,w_2\}) \succ (y_1,y_2;\{y_1\},\{y_2,w_2\})$.

By the transitivity and Lemma4, we have the following.

$$\text{for all } (x_1,x_2;A_1,A_1),(y_1,y_2;B_1,B_1)\in\Omega,\ |A_1|+|A_2|=4 \text{ and } |B_1|+|B_2|=3 \Rightarrow \\ (x_1,x_2;A_1,A_2)\succ(y_1,y_2;B_1,B_2)$$

Since opportunity sets is finite, by repeated using an above argument, we get the following. By Theorem1 and Lemma4, we obtain that, for all $(x_1,x_2;A_1,A_1)$, $(y_1,y_2;B_1,B_1) \in \Omega$,

$$|A_1|+|A_2|=|B_1|+|B_2|+1 \Rightarrow (x_1,x_2;A_1,A_2)\succ(y_1,y_2;B_1,B_2) \tag{33}$$

From equation(33), by the transitivity of $\succeq$ and Lemma4, we have the following.

$$\text{for all } (x_1,x_2;A_1,A_1),(y_1,y_2;B_1,B_1)\in\Omega,\ |A_1|+|A_2|>|B_1|+|B_2| \Rightarrow \\ (x_1,x_2;A_1,A_2)\succ(y_1,y_2;B_1,B_2)$$

Now, suppose that, for all $x_1,y_1 \in X_1$ and $x_2,y_2 \in X_2$, $(x_1,x_2;\{x_1\},\{x_2\}) \sim (y_1,y_2;\{y_1\},\{y_2\})$. An argument similar to the proof of Theorem5 makes us obtain the following.

$$\text{for all } (x_1,x_2;A_1,A_1),(y_1,y_2;B_1,B_1)\in\Omega,\ |A_1|+|A_2|=|B_1|+|B_2| \text{ and} \\ (x_1,x_2;\{x_1\},\{x_2\})\sim(y_1,y_2;\{y_1\},\{y_2\}), \text{ then}(x_1,x_2;A_1,A_2)\sim(y_1,y_2;B_1,B_2)$$

Next, consider that, for all $(x_1,x_2;A_1,A_2)$, $(y_1,y_2;B_1,B_2)\in\Omega$,$(x_1,x_2;\{x_1\},\{x_2\}) \succ (y_1,y_2;\{y_1\},\{y_2\})$. By an argument similar to the proof of Theorem5, we have the following.

$$\text{for all } (x_1,x_2;A_1,A_1),(y_1,y_2;B_1,B_1)\in\Omega,\ |A_1|+|A_2|=|B_1|+|B_2| \text{ and} \\ (x_1,x_2;\{x_1\},\{x_2\})\succ(y_1,y_2;\{y_1\},\{y_2\}), \text{ then}(x_1,x_2;A_1,A_2)\succ(y_1,y_2;B_1,B_2)$$

Q.E.D.

To characterize lexicographic strong nonconsequentialism for first opportunity set, we present the following axiom. Note that (WROB1) is not sufficient to give complete characterization of lexicographic strong nonconsequentialism for first opportunity set.

Axiom 15. Strongly Robustness for first opportunity set (SROB1)
For all $(x_1,x_2;A_1,A_2)$, $(y_1,y_2;B_1,B_2) \in \Omega$ and all $z_1 \in X_1$,$z_2 \in X_2$, $(x_1,x_2;A_1,A_2) \succ (y_1,y_2;B_1,B_2)$, then $(x_1,x_2;A_1,A_2) \succ (y_1,y_2;B_1,B_2\cup\{z_2\})$.

Note that (SROB1) imply (WROB1).

Theorem 15. *$\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (SROB1) if and only if it is lexicographic strong nonconsequentialism for first opportunity set.*

Proof. If $\succeq$ is lexicographic strong nonconsequentialism for first opportunity set, then it satisfy (IND), (BI), (SPO1), (SPO2) and (SROB1). Hense, we have only to show that if $\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (SROB1), it is lexicographic strong nonconsequentialism for first opportunity set.

Let $\succeq$ satisfy (IND), (BI), (SPO1), (SPO2) and (SROB1). By (SPO1) and Theorem1, for all $z_1 \in X_1 \setminus \{x_1\}$, $(x_1,x_2;\{x_1,z_1\},\{x_2\}) \succ (y_1,y_2;\{y_1\},\{y_2\})$. By using (SROB1) repeatedly, for all $z_1 \in X_1\backslash\{x_1\}$ and all $B_2 \in K_2$, $(x_1,x_2;\{x_1,z_1\},\{x_2\}) \succ (y_1,y_2;\{y_1\},B_2)$. From Lemma2, by the transitivity, we obtain that,

$$\text{for all } z_1\in X_1\setminus\{x_1\} \text{ and all } A_2,B_2\in K_2, (x_1,x_2;\{x_1,z_1\},A_2)\succ(y_1,y_2;\{y_1\},B_2) \tag{34}$$

From (IND) and Theorem1, by similar argument in Theorem12, we obtain the following. For all $(x_1, x_2; \{x_1, z_1\}, A_2), (y_1, y_2; \{y_1\}, B_2) \in \Omega$,

$$|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; A_2, B_2) \tag{35}$$

Next, we consider the case of $|A_1| = |B_1|$. By (SPO2) and Theorem1, for all $z_1 \in X_1 \setminus \{x_1\}$ and all $z_2 \in X_2 \setminus \{x_2\}$, $(x_1, x_2; \{x_1\}, \{x_2, z_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. From (IND) and Theorem1, we obtain the following. For all $(x_1, x_2; A_1, \{x_2, z_2\}), (y_1, y_2; B_1, \{y_2\}) \in \Omega$,

$$|A_1| = |B_1| \Rightarrow (x_1, x_2; A_1, \{x_2, z_2\}) \succ (y_1, y_2; B_2, \{y_2\}) \tag{36}$$

From (IND) and Theorem1, by similar argument in Theorem12, we obtain the following. For all $(x_1, x_2; A_1, A_2), (y_1, y_2; B_1, B_2) \in \Omega$,

$$|A_1| = |B_1| \text{ and } |A_2| > |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; A_2, B_2) \tag{37}$$

Now, suppose that, for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. Clearly, by (IND) and the transitivity, for all $A_1 \in K_1$ and $A_2 \in K_2$, $(x_1, x_2; A_1, A_2) \sim (y_1, y_2; A_1, A_2)$. By Theorem1, $|A_1| = |B_1|$ and $|A_2| = |B_2| \Rightarrow (y_1, y_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2)$ By the transitivity of $\succeq$, we obtain that, for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| = |B_1|$, $|A_2| = |B_2|$ and $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$, then $(x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2)$

Next, consider that, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$,$(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By an argument similar to the proof of Theorem5, we have the following.

for all $(x_1, x_2; A_1, A_1), (y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| = |B_1|$ and $|A_2| = |B_2|$ and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$, then$(x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$

Q.E.D.

5 The Case of Multiplicative-ranking

In Suzumura and Xu(2001), choice from single opportunity set (x, A) is considered. (x, A) means choosing x from opportunity set A. If we set $X = X_1 \times X_2$ and $K = K_1 \times K_2$, our framework become (x, A). For example, extreme nonconsequentialism in Suzumura and Xu(2001) is as follows.

For all (x, A) and (y, B), $|A| \geq |B| \Rightarrow (x, A) \succeq (y, B)$

The following definition of consequentialism and nonconsequentialism corresponds to concepts in Suzumura and Xu(2001).

Definition 16. (Multiplicative-ranking strong consequentialism)
$\succeq$ is said to be sum-ranking strongly consequentialism if, for all $(x_1, x_2; A_1, A_2)$, $(y_1, y_2; B_1, B_2) \in \Omega$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow [\ (x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2) \Leftrightarrow |A_1| \times |A_2| \geq |B_1| \times |B_2|\ ]$, and $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Definition 17. (Multiplicative-ranking extreme nonconsequentialism)
$\succeq$ is said to be multiplicative-ranking extremely nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \times |A_2| \geq |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2)$.

Definition 18. (Multiplicative-ranking strong nonconsequentialism)
$\succeq$ is said to be multiplicative-ranking strongly nonconsequential if, for all $(x_1, x_2; A_1, A_1)$, $(y_1, y_2; B_1, B_1) \in \Omega$, $|A_1| \times |A_2| > |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$, and $|A_1| \times |A_2| = |B_1| \times |B_2|$, then $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$.

Multiplicative-ranking strong consequentialism, multiplicative-ranking extreme nonconsequentialism and multiplicative-ranking strong nonconsequentialism correspond to strong consequentialism, extreme nonconsequentialism and strong nonconsequentialism in Suzumura and Xu(2001) respectively.[10]

These concepts evaluate the freedom of choice by the multiplication of the cardinality of opportunity sets. Note that these multiplicative-ranking case do not satisfy (IND). For example, suppose that $A_1 = \{x_1, y_1, z_1\}$, $A_2 = \{x_2, y_1\}$, $B_1 = \{x_1, y_1\}$, $B_2 = \{x_2, y_2, z_2\}$, and $(x_1, x_2, \{x_1, y_1, z_1\}, \{x_2, y_1\}) \sim (x_1, x_2, \{x_1, y_1\}, \{x_2, y_2, z_2\})$. In this case, $|A_1| \times |A_2| = |B_1| \times |B_2| = 6$, then this example satisfies the requirement of above multiplicative-ranking concepts. Moreover, suppose that $\succeq$ satisfies (IND). If $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$, $(x_1, x_2; A_1 \cup \{w_1\}, A_2) \sim (x_1, x_2; B_1 \cup \{w_1\}, B_2)$. Then, in this case, $(x_1, x_2, \{x_1, y_1, z_1, w_1\}, \{x_2, y_1\}) \sim (x_1, x_2, \{x_1, y_1, w_1\}, \{x_2, y_2, z_2\})$. Since $|\{x_1, y_1, z_1, w_1\}| \times |\{x_2, y_1\}| = 8$ and $|\{x_1, y_1, w_1\}| \times |\{x_2, y_2, z_2\}| = 9$, this relation contradicts the requirement of multiplicative-ranking concepts.

So far, (IND) have important role in our framework. But, to characterize the concepts of multiplicative case, we need other axioms. Note that $(x_2, x_1; A_2, A_1)$ and $(x_1, x_2; A_1, A_2)$ means the same.

Axiom 16. Indifference for Multiplication (INDM)
$n \in \mathbb{N}$ and $i, j \in \{1, 2\}$. For all $(x_1, x_2; A_1, A_2)$,$(x_1, x_2; B_1, B_2) \in \Omega$ and all C_i,D_j such that $A_1 \cap C_1 = \emptyset$, $B_1 \cap D_1 = \emptyset$, $n \times |A_i| = |A_i \cup C_i|$ and $n \times |B_j| = |B_j \cup D_j|$, $(x_1, x_2; A_1, A_2) \succeq (x_1, x_2; B_1, B_2) \Leftrightarrow (x_i, x_{3-i}; A_i \cup C_i, A_{3-i}) \succeq (x_j, x_{3-j}; B_j \cup D_j, B_{3-j})$.

(INDM) have a similar role to (IND). This axiom is very strong, because (INDM) implies (BI) and (SI). For example, suppose that $A_1 = \{x_1, y_1, z_1\}$, $A_2 = \{x_2\}$, $B_1 = \{x_1, y_1\}$, $B_2 = \{x_2\}$, and $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, y_1\}, \{x_2\})$. If $\succeq$ satisfies (INDM), then we have $(x_1, x_2; \{x_1, y_1, z_1, w_1, a_1, b_1\}, \{x_2\}) \succeq (x_1, x_2; \{x_1, y_1, c_1, d_1\}, \{x_2\})$.[11] In this case, $|\{x_1, y_1, z_1, w_1, a_1, b_1\}| = 2|A_1|$ and $|\{x_1, y_1, c_1, d_1\}| = 2|B_1|$. (INDM) implies that $(x_1, x_2, \{x_1, y_1, z_1\}; \{x_2, y_2, z_2\}) \succeq (x_1, x_2; \{x_1, y_1\}, \{x_2, a_2, b_2\})$.[12] As another case, by (INDM), we have $(x_1, x_2; \{x_1, y_1, z_1\}, \{x_2, y_2\}) \succeq (x_1, x_2; \{x_1, y_1, a_1, b_1\}, \{x_2\})$. In this case, $|\{x_2, y_2\}| = 2|A_1|$ and $|\{x_1, y_1, a_1, b_1\}| = 2|B_2|$.

Next, we present three more axioms as follows.

Axiom 17. Semi-Local Indifference(SLI)
For all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1 \in K_1$, $(x_1, x_2; A_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\})$ or, for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_2 \in K_2$, $(x_1, x_2; \{x_1\}, A_2) \sim (x_1, x_2; \{x_1\}, \{x_2\})$

Axiom 18. Semi-Local Strict Monotonicity(SLSM)
For all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\})$ or, for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_2, B_2 \in K_2$, $A_2 \supset B_2 \Rightarrow (x_1, x_2; \{x_1\}, A_2) \succ (x_1, x_2; \{x_1\}, B_2)$

Axiom 19. Semi-Strict Preference for Opportunity(SSPO)
For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ where $x_1 \neq y_1$ and $x_2 \neq y_2$, and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (y_1, y_2; B_1, \{y_2\})$ or, for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ where $x_1 \neq y_1$ and $x_2 \neq y_2$, and all $A_2, B_2 \in K_2$, $A_2 \supset B_2 \Rightarrow (x_1, x_2; \{x_1\}, A_2) \succ (y_1, y_2; \{y_1\}, B_2)$

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[10] Now let $x, y, z \ldots$ and $A, B, C \ldots$ denote a element of $X_1 \times X_2$ and $K_1 \times K_2$. Then, (x, A) represents $(x_1, x_2; A_1, A_2)$. The definition of our concepts rewritten as follows.

Extreme consequentialism; For all $(x, A), (x, B) \in \Omega$, $(x, A) \sim (x, B)$

Multiplicative-ranking strong consequentialism; For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (x, B) \Leftrightarrow [\ (x, \{x\}) \succ (y, \{y\})$ or $(\ (x, \{x\}) \sim (y, \{y\}) \wedge |A| \geq |B|\ )\ ]$

Multiplicative-ranking extreme nonconsequentialism; For all $(x, A), (y, B) \in \Omega$, $|A| \geq |B| \Leftrightarrow (x, A) \succeq (x, B)$

Multiplicative-ranking strong nonconsequentialism; For all $(x, A), (y, B) \in \Omega$, $(x, A) \succeq (x, B) \Leftrightarrow [\ |A| > |B|$ or $(\ |A| = |B| \wedge (x, \{x\}) \succeq (y, \{y\})\ )\ ]$

[11] In this case, $n = 2$ and we adopt (INDM) for first opportunity set.

[12] In this case, $n = 3$ and we adopt (INDM) for second opportunity set.

(SLI), (SLSM) and (SSPO) have a similar role to (LIi), (LSMi) and (SPOi) respectively.[13] (SLI) requires that there exists $i \in \{1,2\}$ such that, for any A_i, choosing x_i from A_i and x_j from $\{x_j\}$ where $i \neq j$ is regarded as being indifferent to choosing x_1 and x_2 from singleton sets. According to (SLMS), there $i \in \{1,2\}$ such that, for any $A_i, B_i \in K_i$ where $A_i \supset B_i$, choosing (x_1, x_2) from A_i and $\{x_j\}$ is regarded as being preferred to choosing (x_1, x_2) from B_i and $\{x_j\}$, where $i \neq j$. According to (SSPO), there $i \in \{1,2\}$ such that, for any $A_i, B_i \in K_i$ where $A_i \supset B_i$ and for all distinct $x_1, y_1 \in X_1$ and distinct $x_2, y_2 \in X_2$, choosing (x_1, x_2) from A_i and $\{x_j\}$ is regarded as being preferred to choosing (y_1, y_2) from B_i and $\{y_j\}$, where $i \neq j$.

Theorem 16. *$\succeq$ satisfy (INDM), (SLI) if and only if it is extremely consequential.*

Proof. If $\succeq$ is extreme consequential, it satisfy (INDM) and (SLI). Therefore, it is only to show that if $\succeq$ satisfy (INDM) and (SLI), it is extreme consequential, that is, for all $(x_1, x_2; A_1, A_2)$,$(x_1, x_2; B_1, B_2) \in \Omega$, $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$.

Let $\succeq$ satisfy (INDM) and (SLI). We have only to show the following relation.

$$\text{For all } x_1 \in X_1 \text{ and all } x_2 \in X_2 \text{ and all } A_1 \in K_1 \text{ and all } A_2 \in K_2,$$
$$(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; A_1, A_2)$$

First, without loss of generality, by (SLI), for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $B_1 \in K_1$, $(x_1, x_2; B_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\})$. By (INDM), we have the following. For all $x_1 \in X_1$ and all $x_2 \in X_2$ and all $A_1, B_1 \in K_1$ and all $A_2 \in K_2$,

$$|A_1| \times |A_2| = n, |B_1| = n, n < \infty \Leftrightarrow (x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, \{x_2\}) \tag{38}$$

Moreover, by (SLI), we have the following. For all $B_1 \in K_1$ and $x_1 \in X_1$ and $x_2 \in X_2$,

$$(x_1, x_2; B_1, \{x_2\}) \sim (x_1, x_2; \{x_1\}, \{x_2\}) \tag{39}$$

Therefore, by equation(38), equation(39) and the transitivity of $\succeq$, for all $x_1 \in X_1$ and all $x_2 \in X_2$ and all $A_1 \in K_1$ and all $A_2 \in K_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (x_1, x_2; A_1, A_2)$. Q.E.D.

Note that, by this theorem and Theorem2, we obtain two characterization of extreme consequentialism.

Theorem 17. *$\succeq$ satisfy (INDM), (SLSM) and (ROB) if and only if it is multiplicative-ranking strongly consequential.*

Proof. If $\succeq$ is multiplicative-ranking strongly consequential, it satisfy (INDM), (SLSM) and (ROB). Therefore, it is only to show that if $\succeq$ satisfy (INDM), (SLSM) and (ROB), it is multiplicative-ranking strongly consequential.

Let $\succeq$ satisfy (INDM), (SLSM) and (ROB). First, we consider the case that $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$. To begin with, by (INDM), we have the following. for all $(x_1, x_2; A_1, A_2)$,$(y_1, y_2; B_1, B_2)$,

$$\text{if } |A_1| = |B_1| and |A_2| = |B_2|, \text{ then } (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{40}$$

In Theorem1, we get same result by assuming (IND) and (BI). Without loss of generality, by (SLSM), for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\})$.

By using (SLMS) and equation(40), we obtain that for all $x_1, y_1 \in X_1$ and $x_2, y_1 \in X_2$ and all $A_1, B_1 \in K_1$,

$$|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (y_1, y_2; B_1, \{x_2\}) \tag{41}$$

[13] Of course, $i \in \{1,2\}$.

By using equation(41) and (INDM), we obtain the following equation. For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ and all $A_1, B_1 \in K_1$ and all $A_2, B_2 \in K_2$, if $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$, then $|A_1|\times|A_2| > |B_1|\times|B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2)$. Combining equation(40) implies that, if $(x_1, x_2; \{x_1\}, \{x_2\}) \sim (y_1, y_2; \{y_1\}, \{y_2\})$,

$$|A_1| \times |A_2| \geq |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_2) \succeq (y_1, y_2; B_1, B_2) \tag{42}$$

Next, we consider the case that $\forall x_1, y_1 \in X_1, \forall x_2, y_2 \in X_2, (x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\})$. By using (ROB), for all $z_1 \in X_1$ and all $z_2 \in X_2$, $(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1, z_1\}, \{y_2, z_2\})$. Moreover, By the repeated use of (ROB), for all $B_1 \in K_1$ and all $B_2 \in K_2$,

$$(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; B_1, B_2) \tag{43}$$

From equation (42) and (44) with the transitivity, we can obtain the following equation. for all $(x_1, x_2; A_1, A_2)$, $(x_1, x_2; B_1, B_2) \in \Omega$,

$$(x_1, x_2; \{x_1\}, \{x_2\}) \succ (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_2) \succ (y_1, y_2; B_1, B_2) \tag{44}$$

Q.E.D.

Theorem 18. *$\succeq$ satisfy (INDM), (SLSM) and (INS) if and only if it is multiplicative-ranking extremely nonconsequential.*

Proof. If $\succeq$ is multiplicative-ranking extremely nonconsequential, it satisfy (INDM), (SLSM) and (INS). Therefore, it is only to show that if $\succeq$ satisfy (INDM), (SLSM) and (INS), it is multiplicative-ranking extremely nonconsequential.

Let $\succeq$ satisfy (INDM), (SLSM) and (INS). To begin with, by using (INDM) and (INS), we have the following. for all $(x_1, x_2; A_1, A_2)$,$(y_1, y_2; B_1, B_2)$,

$$\text{if } |A_1| \times |A_2| = |B_1| \times |B_2|, \text{ then } (x_1, x_2; A_1, A_2) \sim (y_1, y_2; B_1, B_2) \tag{45}$$

Without loss of generality, by (SLSM), for all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\})$. Since (INDM) implies that if $|A_1| = |B_1|$ and $|A_2| = |B_2|$, then $(x_1, x_2; A_1, A_2) \sim (x_1, x_2; B_1, B_2)$, we have the following equation. For all $x_1 \in X_1$ and $x_2 \in X_2$ and all $A_1, B_1 \in K_1$,

$$|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (x_1, x_2; B_1, \{x_2\}) \tag{46}$$

Equation(45) and equation(46) imply that $\succeq$ is multiplicative-ranking extremely nonconsequential. Q.E.D.

Theorem 19. *$\succeq$ satisfy (INDM), (SLSM) and (SSPO) if and only if it is multiplicative-ranking strongly nonconsequential.*

Proof. If $\succeq$ is multiplicative-ranking strongly nonconsequential, it satisfy (INDM), (SLSM) and (SSPO). Therefore, it is only to show that if $\succeq$ satisfy (INDM), (SLSM) and (SSPO), it is multiplicative-ranking strongly nonconsequential.

Let $\succeq$ satisfy (INDM), (SLSM) and (SSPO). Without loss of generality, by (SLSM), for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ and all $A_1, B_1 \in K_1$, $A_1 \supset B_1 \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (y_1, y_2; B_1, \{y_2\})$. Since (SSPO) correspond to (SLMS) when$x_1 = y_1$ and $x_2 = y_2$, then (SSPO) implies (SLMS). By using (SSPO) and (INDM), we obtain the following relation. For all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ and all $A_1, B_1 \in K_1$,

$$|A_1| > |B_1| \Rightarrow (x_1, x_2; A_1, \{x_2\}) \succ (y_1, y_2; B_1, \{y_2\}) \tag{47}$$

By using equation(47) and (INDM), we obtain that for all $x_1, y_1 \in X_1$ and $x_2, y_2 \in X_2$ and all $A_1, B_1 \in K_1$ and all $A_2, B_2 \in K_2$,

$$|A_1| \times |A_2| > |B_1| \times |B_2| \Rightarrow (x_1, x_2; A_1, A_1) \succ (y_1, y_2; B_1, B_2) \tag{48}$$

Next, consider that $|A_1| \times |A_2| = |B_1| \times |B_2|$ for $A_1, B_1 \in X_1$ and $A_2, B_2 \in X_2$. From (INDM), we obtain that for all $A_1, B_1 \in K_1$ and all $A_2, B_2 \in K_2$,

$$\text{If } |A_1| \times |A_2| = |B_1| \times |B_2|,$$
$$\text{then } (x_1, x_2; \{x_1\}, \{x_1\}) \succeq (y_1, y_2; \{y_1\}, \{y_2\}) \Rightarrow (x_1, x_2; A_1, A_1) \succeq (y_1, y_2; B_1, B_2)$$

Q.E.D.

6 Extension to n Opportunity Sets

In this section, we describe the general case that there are n opportunities of choice.

6.1 Notation

Let N denote $\{1, 2, \cdots, n\}$, $n < \infty$. Moreover, for all $i \in N$, let X_i $i \in N$, $|X_i| \geq 3$, be a set of all mutually and jointly exclusive alternatives in category i. The elements of X_i will be denoted by $x_i, y_i, z_i, \cdots$.. K_i denotes a collection of non-empty subset of X_i and the elements in K_i will be denoted by $A_i, B_i, C_i, \cdots$. Extended alternative is $(x_1, \cdots, x_n; A_1, \cdots, A_n), (y_1, \cdots, y_n; B_1, \cdots, B_n), \cdots$. Let x denote $(x_1, x_2, \cdots, x_n)$ and A denote $(A_1, A_2, \cdots, A_n)$. Therefore, $(x; A)$ and $(y; B)$ represent $(x_1, \cdots, x_n; A_1, \cdots, A_n)$ and $(y_1, \cdots, y_n; B_1, \cdots, B_n)$, respectively.

Let $\succeq$ be a reflexive, complete and transitive binary relation over Ω. The asymmetric and symmetric part of $\succeq$ will be denoted by $\succ$ and $\sim$ respectively.

6.2 The Results

In this subsection, we only present the general case of extreme consequentialism and extreme nonconsequentialism. However, by simple application, we can obtain the general case of extreme consequentialism, strong consequentialism and strong nonconsequentialism.

First, we state the general case of basic axioms.

Axiom 20. Independence for Addition'(IND')
We have the following for all $i \in N$. For all $(x_1, \cdots, x_n; A_1, \cdots, A_n)$, $(y_1, \cdots, y_n; B_1, \cdots, B_n)$ $\in \Omega$ and all $z_i \in X_i \backslash \{A_i \cup B_i\}$, $(x_1, x_2, \cdots, x_n; A_1, A_2, \cdots, A_n) \succeq (y_1, y_2, \cdots, y_n; B_1, B_2, \cdots, B_n)$ $\Leftrightarrow (x_1, \cdots, x_n; A_1, \cdots, A_i \cup \{z_i\}, \cdots, A_n) \succeq (y_1, \cdots, y_n; B_1, \cdots, B_i \cup \{z_i\}, \cdots, B_n)$

Axiom 21. Baseline Indifference(BI')
For all $x_i \in X_i$ and $y_i, z_i \in X_i \backslash \{x_i\}$, $\forall i \in N$, $(x_1, x_2, \cdots, x_n; \{x_1, y_1\}, \{x_2, y_2\}, \cdots, \{x_n, y_n\})$ $\sim (x_1, x_2, \cdots, x_n; \{x_1, z_1\}, \{x_2, z_2\}, \cdots, \{x_n, z_n\})$

If $\succeq$ satisfies (BI') and (IND'), then it satisfies (SI'). The following result correspond to Theorem1.

Theorem 20. *Suppose $\succeq$ satisfy (IND') and (BI'). If $|A_i| = |B_i|$ for all $i \in N$, then $(x_1, x_2, \cdots, x_n; A_1, A_2, \cdots, A_n) \sim (x_1, x_2, \cdots, x_n; B_1, B_2, \cdots, B_n)$.*

Proof. Exactly analogous to that of Theorem1 Q.E.D.

Next, we define the general statement of extreme consequentialism, sum-ranking extreme nonconsequentialism, multiplicative-ranking extreme nonconsequentialism.

Definition 19. (Generalized extreme consequentialism)
$\succeq$ is said to be extremely consequential if, for all $(x_1, x_2, \cdots, x_n; A_1, A_2, \cdots, A_n)$, $(x_1, \cdots, x_n; B_1, B_2, \cdots, B_n) \in \Omega$, $(x_1, \cdots, x_n; A_1, A_2, \cdots, A_n) \sim (x_1, \cdots, x_n; B_1, B_2, \cdots, B_n)$.

Definition 20. (Generalized sum-ranking extreme nonconsequentialism)
$\succeq$ is said to be sum-ranking extremely nonconsequential if, for all $(x_1, \cdots, x_n; A_1, \cdots, A_n)$, $(y_1, \cdots, y_n; B_1, \cdots, B_n) \in \Omega$, $\sum_i |A_i| \geq \sum_i |B_i| \Rightarrow (x_1, \cdots, x_n; A_1, A_2, \cdots, A_n) \succeq (y_1, \cdots, y_n; B_1, B_2, \cdots, B_n)$.

Definition 21. (Generalized multiplicative-ranking extreme nonconsequentialism) $\succeq$ is said to be sum-ranking extremely nonconsequential if, for all $(x_1, \cdots, x_n; A_1, \cdots, A_n)$, $(y_1, \cdots, y_n; B_1, \cdots, B_n) \in \Omega$, $\prod_i |A_i| \geq \prod_i |B_1| \Rightarrow (x_1, \cdots, x_n;\ A_1, A_2, \cdots, A_n) \succeq (y_1, \cdots, y_n;\ B_1, B_2, \cdots, B_n)$.

Generalized extreme consequentialism, generalized sum-ranking extreme nonconsequentialism, generalized multiplicative extreme nonconsequentialism is the general case of extreme consequentialism, sum-ranking extreme nonconsequentialism, multiplicative-ranking extreme nonconsequentialism respectively. To characterize these concepts, we need the following axioms.

Axiom 22. Local Indifference' i (LI'i)
For $i \in N$ and all $x_j \in X_j$ where $j = 1, \ldots n$, there exist $A_i \in X_i \setminus \{x_i\}$ such that $(x_1, \cdots, x_i, \cdots, x_n; \{x_1\}, \cdots, \{x_i\}, \cdots, \{x_n\}) \sim (x_1, \cdots, x_i, \cdots, x_n;, \{x_1\}, \cdots, A_i, \cdots, \{x_n\})$.

Axiom 23. Local Strict Monotonicity' i (LSM'i)
For $i \in N$ and all $x_j \in X_j$ where $j = 1, \ldots n$, there exists $A_i \in K_i \setminus \{x_i\}$ such that $(x_1, \cdots, x_n; \{x_1\}, \cdots, A_i, \cdots, \{x_n\}) \succ (x_1, , \cdots, x_n; \{x_1\}, \{x_2\}, \cdots . \{x_n\})$.

Axiom 24. Semi-Local Strict Monotonicity' (SLSM')
There exists $i \in N$, for all $x_j \in X_j$ where $j = \{1, \cdots, n\}$ and all $A_i, B_i \in X_i$, $A_i \supset B_i \Rightarrow (x_1, \cdots, x_n; \{x_1\}, \cdots, A_i, \cdots, \{x_n\}) \succ (x_1, , \cdots, x_n; \{x_1\}, \cdots, B_i, \cdots . \{x_n\})$
.

Axiom 25. Indifference of No-choice Situation'(INS')
For all $x_i, y_i \in X_i$ where $i = 1, \cdots n$, $(x_1, x_2, \cdots, x_n; \{x_1\}, \{x_2\}, \cdots, \{x_n\}) \sim (y_1, y_2, \cdots, y_n; \{y_1\}, \{y_2\}, \cdots, \{y_n\})$.

Axiom 26. Trinary Indifference(TI')
For all $i, j = 1, \cdots, n$ and all $x_1 \in X_1$, $x_2 \in X_2 \cdots$,$x_n \in X_n$, $y_i \in X_i \setminus x_i$ and $y_j \in X_j \setminus x_j$, $(x_1, \cdots, x_i, \cdots, x_n; \{x_1\}, \cdots, \{x_i, y_i\}, \cdots, \{x_j\}, \{x_n\}) \sim (x_1, \cdots, x_i, \cdots, x_n;$ $\{x_1\}, \cdots \{x_i\}, \cdots, \{x_j, y_j\}, \cdots, \{x_n\})$.

Axiom 27. Independence for Multiplication'(INDM')
$m \in \mathbb{N}$. For all $i, j = 1, \cdots, n$ and all $(x_1, x_2, \cdots, x_n; A_1, A_2, \cdots, A_n)$,$(x_1, x_2, \cdots, x_n;$ $B_1, B_2, \cdots, B_n) \in \Omega$ and all $C_i \in X_i$,$D_j \in X_j$ such that $A_i \cap C_i = \emptyset$, $B_i \cap D_j = \emptyset$, $m \times |A_i| = |A_i \cup C_i|$ and $m \times |B_j| = |B_j \cup D_j|$, $(x_1, x_2, \cdots, x_n; A_1, \cdots, A_i, \cdots, A_j, \cdots, A_n)$ $\succeq (x_1, x_2, \cdots, x_n; B_1, \cdots, B_i, \cdots, B_j, \cdots, B_n) \Leftrightarrow (x_1, x_2, \cdots, x_n; A_1, \cdots, A_i \cup C_i, \cdots, A_j, \cdots, A_n)$ $\succeq (x_1, x_2, \cdots, x_n; B_1, \cdots, B_i \cdots, B_j \cup D_j, \cdots, B_n)$.

Main Results is as follows. All results are proved by similar argument in $n = 2$ case.

Theorem 21. *$\succeq$ satisfy (IND'),(BI'), (LI') if and only if it is generalized extreme consequential.*

Proof. Exactly analogous to that of Theorem2 Q.E.D.

Theorem 22. *$\succeq$ satisfy (IND'), (BI'), (LSM'), (INS') and (TI') if and only if it is generalized sum-ranking extreme nonconsequential.*

Proof. Exactly analogous to that of Theorem9 Q.E.D.

Theorem 23. *$\succeq$ satisfy (INDM'), (SLSM') and (INS') if and only if it is generalized multiplicative-ranking extreme nonconsequential.*

Proof. Exactly analogous to that of Theorem18 Q.E.D.

7 Conclusion

In this paper, by extending multiple opportunity sets, we construct the framework that can make us treat the diversity of freedom of choice, and characterize various type of consequentialism and nonconsequentialism. There are two frameworks, additive-ranking case and multiplicative-ranking case. In additive-ranking case, (IND) and (BI) is basic axioms, additive-ranking consequentialism and nonconsequentialism all satisfy (IND) and (BI). In multiplicative-ranking case, (INDM) is basic axiom. In both case, we can characterize the traditional concepts of economics, extreme consequentialism.

First, we argue the additive-ranking case. In addition to (IND) and (BI), each opportunity sets $A_i, i \in \{1, \cdots, n\}$ satisfy (LI i) or (LSM i). If opportunity sets all satisfy Local Indifference axiom, (LI i) for all $i \in \{1, \cdots, n\}$, then all opportunity set don't have the intrinsic value and $\succeq$ is extremely consequential. If opportunity sets all satisfy Local Strictly Monotonicity, (LSM i) for all $\in \{1, \cdots, n\}$, the freedom of choice of all have the intrinsic value. In the intermediate case, some opportunity sets satisfy Local Indifference axiom and others satisfy Local Strictly Monotonicity, opportunity sets that Local Strictly Monotonicity have the intrinsic value.

With Local Strictly Monotonicity, (IND) and (BI), (ROB) and (INS) and (SPO) is crucial for which the intrinsic value of opportunity sets and the value of consequence is important. (ROB) requires that the value of consequence is more important than the intrinsic value of opportunity sets. According to (INS), consequence has no value. With Local Strictly Monotonicity, (IND) and (BI), (SPO) requires that the intrinsic value of opportunity sets is important than the intrinsic value of opportunity sets. Moreover, how evaluate various freedom of choices is depend on (TI), (PI), (WROB1), (SROB1).

Next, we argue the multiplicative-ranking case, which correspond to the framework of Suzumura and Xu(2001). In this case, (SLI) and (SLSM) have to same role as Local Indifferent and Local strict Monotonicity respectively. (SLI) and (SLSM), however, is stronger axioms than Local Indifferent and Local strict Monotonicity. With (SLI), freedom of choice don't matters, and with (SLSM), freedom of choice have the intrinsic value. In this case, (INS) and (ROB) have the critical role to characterize extreme nonconsequentialism and strong consequentialism respectively. To characterize strong nonconsequentialism, however, (SPO) is not sufficient, and we need another stronger axiom, (SSPO).

Finally, we note that our axiomatization of this paper is special case of this framework. Allowing trade-off between the value of consequence and the intrinsic value of opportunities, we can construct more general characterization. These task must be left for our future research.

$$
\begin{cases}
(IND)\oplus(BI)\begin{cases}
\oplus(LIi)\forall i\in N = \text{extreme consequentialism}\\[1em]
\oplus(LIi) \text{ for } i\in S\subset N \text{ and } (LSMi) \text{ for } \in N\setminus S\\
(=\text{partial consequentialism and nonconsequentialism})\\[1em]
\oplus(LSMi)\forall i\in N\\
\begin{cases}
\oplus(ROB)\begin{cases}\oplus(TI) = \text{sum-ranking strong consequentialism}\\ \oplus(PI) = \text{weighted sum-ranking strong consequentialism}\end{cases}\\
\oplus(INS)\begin{cases}\oplus(TI) = \text{sum-ranking extreme nonconsequentialism}\\ \oplus(PI) = \text{weighted sum-ranking extreme nonconsequentialism}\\ \oplus(WROB) = \text{lexicographic extreme nonconsequentialism}\end{cases}\\
\oplus(SPO)\begin{cases}\oplus(TI) = \text{sum-ranking strong nonconsequentialism}\\ \oplus(PI) = \text{weighted sum-ranking strong nonconsequentialism}\oplus(SRO\end{cases}
\end{cases}
\end{cases}\\[2em]
(INDM)\begin{cases}
\oplus(SLI) = \text{extreme consequentialism}\\
\oplus(SLSM)\begin{cases}\oplus(ROB) = \text{multiplicative strong consequentialism}\\ \oplus(INS) = \text{multiplicative extreme nonconsequentialism}\\ \oplus(SSPO) = \text{multiplicative strong nonconsequentialism}\end{cases}
\end{cases}
\end{cases}
$$

IND: Independence for addition
INDM: Indifference for Multiplication
BI: Baseline Indifference
LIi: Local Indifference for ith opportunity set
LSMi: Local Strict Monotonicity for ith opportunity set
SPO: Strong Preference for Opportunities
INS: Indifference of No-choice Situations
ROB: Robustness
TI: Trinary Indifference
PI: Proportional Indifference
WROB: Weakly Robustness
SROB: Strongly Robustness
SLI: Semi-Local Indifference
SLSM: Semi-Local Strict Monotonicity
SSPO: Semi-Strong Preference for Opportunities
*note; $(A)\oplus(B)$ indicates the logical combination of the two axioms A and B.

Appendix: Figure for 2 opportunity sets

$$
\begin{cases}
(IND) \oplus (BI) \begin{cases}
\oplus(LI1) \oplus (LI2) = \text{ extreme consequentialism} \\
\oplus(LI1) \oplus (LSM2) \begin{cases} \oplus(ROB) = \text{ 2nd opp. set ranking strong conseq.} \\ \oplus(INS) = \text{ 2nd opp. set ranking extreme nonconseq.} \\ \oplus(SPO2) = \text{ 2nd opp. set ranking strong nonconseq.} \end{cases} \\
\oplus(LI2) \oplus (LSM1) \begin{cases} \oplus(ROB) = \text{ 1st opp. set ranking strong conseq.} \\ \oplus(INS) = \text{ 1st opp. set ranking extreme nonconseq.} \\ \oplus(SPO1) = \text{ 1st opp. set ranking strong nonconseq.} \end{cases} \\
\oplus(LSM1) \oplus (LSM2) \\
\begin{cases}
\oplus(ROB) \begin{cases} \oplus(TI) = \text{ sum-ranking strong conseq.} \\ \oplus(PI) = \text{ weighted-sum ranking strong conseq.} \end{cases} \\
\oplus(INS) \begin{cases} \oplus(TI) = \text{ sum-ranking extreme nonconseq.} \\ \oplus(PI) = \text{ weighted sum-ranking extreme nonconseq.} \\ \oplus(WROB1) = \text{ lexioco. extreme nonconseq.} \end{cases} \\
\oplus(SPO1) \oplus (SPO2) \\
\begin{cases} \oplus(TI) = \text{ sum-ranking strong nonconseq.} \\ \oplus(PI) = \text{ weighted sum-ranking strong nonconseq.} \\ \oplus(SROB1) = \text{ lexioco. strong nonconseq.} \end{cases}
\end{cases}
\end{cases} \\
(INDM) \begin{cases}
\oplus(SLI) = \text{ extreme consequentialism} \\
\oplus(SLSM) \begin{cases} \oplus(ROB) = \text{ multiplicative-ranking strong consequentialism} \\ \oplus(INS) = \text{ multiplicative-ranking extreme nonconsequentialism} \\ \oplus(SSPO) = \text{ multiplicative-ranking strong nonconsequentialism} \end{cases}
\end{cases}
\end{cases}
$$

IND: Independence for addition
INDM: Indifference for Multiplication
BI: Baseline Indifference
LI1: Local Indifference for first opportunity set
LI2: Local Indifference for second opportunity set
LSM1: Local Strict Monotonicity for first opportunity set
LSM2: Local Strict Monotonicity for second opportunity set
SPO: Strong Preference for Opportunities
INS: Indifference of No-choice Situations
ROB: Robustness
TI: Trinary Indifference
PI: Proportional Indifference
WROB1: Weakly Robustness for first opportunity set
SROB1: Strongly Robustness for first opportunity set
SLI: Semi-Local Indifference
SLSM: Semi-Local Strict Monotonicity
SSPO: Semi-Strong Preference for Opportunities
*note; $(A) \oplus (B)$ indicates the logical combination of the two axioms A and B.

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