Business Cycle Uncertainty and Economic Welfare

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Abstract

We study the welfare implications of uncertainty in business cycle models. In the modern business cycle literature, multiplicative real shocks to production and/or preferences play an important role as the impulses that produce aggregate fluctuations. Introducing shocks in this way has the implication that fluctuating economies may enjoy higher welfare than their steady state counterparts. This occurs because purposeful agents make use of uncertainty in their favor. The result holds for a range of reasonable parameter values and in many models considered in the business cycle literature. One implication is that the welfare cost estimates which have been obtained in the literature may be biased and possibly seriously.

1. Introduction

Robert Lucas(1987) obtained the upper bound estimate of the welfare gain from eliminating consumption risk by replacing postwar U.S. consumption with a consumption series without fluctuations. He assumed a representative agent with a constant relative risk aversion (CRRA) utility function. His estimates of the welfare cost of consumption fluctuations are very small, no more than 0.00008 percent of aggregate consumption assuming logarithmic preferences. The fact that these estimates were so small stimulated interest in the issue of whether other features of the economy would significantly increase the estimated magnitude of the cost of business cycle fluctuations. Imrohoroglu (1989) and Krusell and Smith (1999) introduced incomplete markets and uninsurable individual risk and found higher welfare costs. Cho, Cooley and Phaneuf (1997) calculated the welfare cost of business cycle fluctuations in a model with nominal wage contracts. In their model, the welfare loss derives entirely from labor supply risk and the costs are higher than those found by Lucas. Obstfeld (1994) and Dolmas (1998) introduced non-expected utility type preferences and found much larger welfare costs associated with business cycles.

This paper considers the welfare consequences of the shocks that generate business cycles. We argue that the technology shock in the real business cycle literature is not always detrimental to economic welfare. Since there are no distortions in prototype real business cycle models like Kydland and Prescott (1982), Long and Plosser (1983), and Hansen (1985), aggregate fluctuations in these models still result in Pareto optimal allocations. It may seem natural to think that these fluctuating economies obtain lower welfare than their steady state counterparts, because the latter does not suffer from any uncertainty while the former does. We argue that this is not always correct. That is, economies with business cycle fluctuations may enjoy higher welfare than their steady state counterparts.

We understand that the last statement sounds counterintuitive. But, if we think of the way productivity shocks enter real business cycle models, the result is quite natural. The key to understanding how welfare could increase with uncertainty is to realize that the shocks to production are multiplicative and productive inputs like labor are variable . If there is a favorable productivity shock, output increases one-for-one, given the inputs. In addition, firms may employ more inputs with an increase in productivity so output can increase further. In other words, an increase in productivity will raise

output more than proportionally and thus the reduced form (equilibrium) production function is convex with respect to the shock. Accordingly, introducing uncertainty through multiplicative productivity shocks increases the expected value of production implying that increasing the uncertainty *raises* average output.

The conventional way of thinking about the welfare costs of business cycles is this. Imagine that consumers are risk averse. Offer these consumers two possible consumption streams, one which is constant and the other which has the same mean but fluctuates around that mean with some variance. Risk averse consumers would always prefer the smooth consumption stream and would require some additonal average consumption to be indifferent between the two. This is the logic of the Lucas experiment and it is uncontroversial. This effect is the **fluctuations effect** of the uncertainty and it is always deterimental to welfare. But, suppose that consumers can take advantage of the uncertainty by working harder and investing more when productivity is high. In that case, the mean values of equilibrium ouput and consumption change with the uncertainty because agents try to use the uncertainty in their favor. We call this the **mean effect** of the business cycle uncertainty. If the mean decreases with uncertainty, economic welfare worsens and the uncertainty unequivocally lowers welfare. However, if the mean increases, and if the mean effect dominates the fluctuations effect, welfare increases with uncertainty. To correctly measure the welfare cost of business cycles, we have to know something about the size of the two effects. That, in turn, depends on how risk averse the agents are and how the uncertainty enters the model economy. Note, however, that the conventional approach is to look only at the fluctuations effect and that would always lead one to conclude that business cycle uncertainty reduces economic welfare.

We emphasize that, for uncertainty to increase the economic welfare, it has to be multiplicative to the choices which can be adjusted in response to it. That is, the mean effect is positive in the case of multiplicative shocks so there is a possibility of welfare increasing with the shocks. In the case of additive shocks the mean effect is negative in most of the cases we can think of and so there is no possibility of welfare increasing with them. Shocks which are multiplicative include technology shocks, used extensively in the literature, preference shocks, seasonal shocks, investment specific shocks, shocks to income tax rates etc.. Examples of shocks that are usually additive include monetary shocks, government expenditure shocks etc. In sum, economic wel-

fare may increase with uncertainty because purposeful agents can make use of shocks to the economic environment in their favor, which is possible when the shocks are multiplicative to endogenous choices.

The idea that uncertainty may increase the welfare has been around a while. It was first presented by Frederick V. Waugh (1944, 1945) in the context of a consumer's utility maximization. The following is the result obtained by Waugh.

"··· if a consumer has a given sum of money to spend for all goods and services, and if he can distribute this expenditure as he pleases among n equal periods of time, he will be better off if all prices vary than he would be if all prices were stabilized at their respective arithmetic means." [Waugh (1944, p608)]

However, this is a restatement in the context of the economics of uncertainty of the microeconomic result well known by now that the indirect utility function is nonincreasing and quasiconvex in prices. Walter Y. Oi (1961) showed that a firm's profit is increasing with price uncertainty because of the convexity of the profit function in the context of uncertainty.[1] These results were all shown in partial equilibrium setups, and they may not hold true in general equilibrium. For example, one might be tempted to combine the Waugh and Oi results and argue that since price uncertainty raises the profit of firms and at the same time the consumers' utility, it will always be welfare improving. But, Waugh's result hinges on income being independent of price uncertainty and Oi's result hinges on factor prices being independent of the output price uncertainty.[2] All of this points to the need to consider the effects of uncertainty only in well specified models where the role of shocks and their general equilibrium implications can be captured.

We examine the welfare costs of uncertainty in dynamic general equilibrium models where shocks are a source of fluctuations. In each of the cases considered in the paper we contrast two Pareto optimal economies; one subject to uncertainty and hence fluctuating, and the other one at its steady state. We then see which economy obtains higher utility. We show many

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[1] Oi's proposition is proved without reference to Oi (1961) in an example in the graduate microeconomics textbook by Hal R. Varian [Varian (1992, p43)].

[2] This point was made in a somewhat confused way by Samuelson (1972a) and that led to a further exchange between Samuelson (1972b) and Oi(1972).

cases where the economy with uncertainty has higher utility than the counterpart steady state economy. This result is robust over a range of reasonable parameter values that are often considered in the literature.

The next section presents two examples, which will clarify the issue. Section 3 studies the welfare issue in a prototype real business cycle model and looks for the range of parameter values that yield a welfare gain with business cycle uncertainty. Section 4 examines the welfare consequences of preference shocks and Section 5 discusses several issues related to the computation of the welfare cost of the business cycles in the literature. Section 6 concludes.

2. Examples

The welfare cost of uncertainty depends on whether economic agents have some means to make use of the uncertainty in their favor. The first example shows that if there are no such means, uncertainty is certainly detrimental to economic welfare as in Lucas (1987). But the second example shows that if the agents have some endogenous choices that allow them to use the uncertainty in their favor, an increase of economic welfare with uncertainty is a possibility.

2.1 An Endowment Economy

Conside the following endowment economy, which is basically identical to the one considered by Lucas (1987). The representative agent maximizes the following lifetime utility.

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \cdot \frac{1}{1-\sigma} \cdot c_t^{1-\sigma} \right\}, \tag{1}$$

where E_0 is the expectations operator conditional on the initial period information Ω_0, c_t is the period t consumption, $0 < \beta < 1$ is the utility discounting factor, and $\sigma > 0$ is the relative risk aversion parameter. The agent faces the following constraint for consumption in each period:

$$c_t \leq e_t, \tag{2}$$

where e_t is the endowment in period t. Assume that e_t follows an i.i.d process.

$$\ln(e_t) = \varepsilon_t, \tag{3}$$

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where $\varepsilon_t \sim i.i.d.$ $N(-\gamma_\varepsilon^2/2, \gamma_\varepsilon^2)$. That is, e_t follows an i.i.d. log-normal process.[3] If we assume (3)[4], we have $E(e_t) = 1$ and $Var(e_t) = \exp(\gamma_\varepsilon^2) - 1$. Hence a change in the variance of ε_t is a mean-preserving spread of the endowment shocks.

If we use the endowment process (3) in the lifetime utility (1), we have the lifetime utility:

$$U_0 = \frac{1}{(1-\sigma)(1-\beta)} \cdot \exp\left(-\frac{\sigma(1-\sigma)\gamma_\varepsilon^2}{2}\right). \tag{4}$$

Now it is straightforward to take the derivative:

$$\frac{\partial U_0}{\partial \gamma_\varepsilon^2} = -\frac{\sigma}{2(1-\beta)} \cdot \exp\left(-\frac{\sigma(1-\sigma)\gamma_\varepsilon^2}{2}\right) < 0. \tag{5}$$

Economic welfare decreases unequivocally with the uncertainty.[5] Note that the agent cannot alter anything in this setup in response to uncertainty and

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[3]Assuming more realistic process for e_t does not change the result qualitatively. The key is that the agent does not have any means to make use of the uncertainty.

[4]In the distribution, we assume the mean to represent the mean preserving spread of e_t as changes in the variance of $\ln(e_t)$, γ_ε^2. To see this, suppose a random variable X has a log-normal distribution as:
$$\ln(X) \sim N(\mu, \gamma^2).$$
Then the mean and variance of X can be obtained as:

$$E(X) = \exp(\mu + \frac{\gamma^2}{2}), \ Var(X) = \exp(2\mu + \gamma^2)[\exp(\gamma^2) - 1]$$

Hence the mean of X changes whenever the value of γ^2 changes. To have a distribution of X whose mean does not depend on γ^2, we have to change the distribution of $\ln(X)$ as:

$$\ln(X) \sim N(\Gamma - \frac{\gamma^2}{2}, \gamma^2),$$

where Γ is the target mean of X. Then we can have the mean and variance of X as folows:

$$E(X) = \exp(\Gamma), \ Var(X) = \exp(\Gamma)[\exp(\gamma^2) - 1],$$

and hence the mean of X does not depend on γ^2. Now a change in γ^2 means a mean preserving spread of the random variable X.

[5]As Lucas (1987) shows, a deterministic trend in endowment (and hence in consumption) does not change the result.

so is not able to make use of it. The next example is one where an agent can alter the labor input.

2.2 An Economy with Endogenous Labor

This example has endogenous labor so the agent can choose to supply labor in the way that makes use of multiplicative productivity shocks. Consider the following real business cycle model. The representative agent is assumed to have the preferences:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \cdot \frac{1}{1-\sigma} \cdot (c_t - \alpha \cdot n_t)^{1-\sigma} \right\}, \tag{6}$$

where E_0 is the expectations operator conditional on the initial period information Ω_0, c_t is the period t consumption and n_t represents hours of work in the period. β is the utility discounting factor, and σ and α are preference parameters. We asume that $0 < \beta < 1$, $\sigma > 0$ and $\alpha > 0$. Because this preference specification abstracts from wealth effects it keeps the problem simple. It has been used by many authors including Greenwood, Hercowitz and Huffman (1988) and Hercowitz and Samson (1992). We assume that output is produced according to the production function:

$$y_t = A_t k_t^{\theta} n_t^{1-\theta}, \tag{7}$$

where y_t denotes output, A_t is the productivity shock, k_t is the capital stock in period t and $0 < \theta < 1$. We assume that the productivity shock follows an *i.i.d.* log-normal process. Specifically, assume:[6]

$$\ln(A_t) \sim N(-\frac{\tau^2}{2}, \tau^2). \tag{8}$$

That is, as in the previous example, A_t follows an *i.i.d.* log-normal process. Now the mean and variance of A_t can be obtained as in the previous example and we write them here for later reference.

$$E(A_t) = 1, \ Var(A_t) = \exp(\tau^2) - 1. \tag{9}$$

Hence a change in τ^2 implies a mean preserving spread of A_t. In addition, to make the example as simple as possible and to have an analytical solution, we

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[6]The result in this example does not depend on the log-normality of the shock.

assume that capital depreciates completely each period. The firm is owned by the representative agent.

The resource constraint for the economy is:

$$c_t + k_{t+1} = A_t k_t^\theta n_t^{1-\theta}. \tag{10}$$

Now the problem facing the representative agent is to maximize (6) subject to the resource constraint (10).

The first order conditions for the utility maximization are:

$$\alpha \cdot (c_t - \alpha \cdot n_t)^{-\sigma} = (1 - \theta) A_t k_t^\theta n_t^{-\theta} \cdot (c_t - \alpha \cdot n_t)^{-\sigma}, \tag{11}$$

$$(c_t - \alpha \cdot n_t)^{-\sigma} = \beta \theta E_t \left\{ A_{t+1} k_{t+1}^{\theta-1} n_{t+1}^{1-\theta} \cdot (c_{t+1} - \alpha \cdot n_{t+1})^{-\sigma} \right\}. \tag{12}$$

Solving (11) in terms of the working hours, we have,

$$n_t = b A_t^{1/\theta} k_t, \tag{13}$$

where $b = [(1-\theta)/\alpha]^{1/\theta}$. Using (13) in (7), we get the reduced form production function in terms of the real shock and the capital stock.

$$y_t = d A_t^{1/\theta} k_t, \tag{14}$$

where $d = b^{1-\theta}$. The reduced form (equilibrium) solution for output is convex in the shock since $0 < \theta < 1$, and production smoothing is not optimal when there are shocks to the technology. In addition, increasing uncertainty raises the expected output. This is what we refer to as the mean effect of the uncertainty.

On the other hand, solving (12) is a little messy. Using the guess for the solution:

$$c_t = \lambda y_t, \quad k_{t+1} = (1 - \lambda) y_t, \tag{15}$$

we get the following solution for λ:

$$\lambda = 1 - \left\{ \beta \theta d^{1-\sigma} E \left[A_t^{(1-\sigma)/\theta} \right] \right\}^{1/\sigma}, \tag{16}$$

where $d = b^{1-\theta}$ and the definiton is used to obtain (16). The coefficient λ is a function of the size of the uncertainty in the economy.[7] That is, precautionary

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[7]If we use the fact that A_t follows an *i.i.d.* log-normal process, we can solve for λ as follows.

$$\lambda = 1 - \left\{ \beta \theta d^{1-\sigma} \exp \left\{ \left(\frac{1-\sigma}{\theta} - 1 \right) \left(\frac{1-\sigma}{\theta} \right) \frac{\tau^2}{2} \right\} \right\}^{1/\sigma}.$$

Hence the fraction of consumption out of output decreases with τ^2 and that of investment increases with it if $\sigma > 1$ or $\sigma + \theta < 1$.

savings depends on the value of the preference and production parameters. If $\theta + \sigma = 1$, the size of the uncertainty does not affect the savings rate $1 - \lambda$, i.e. capital accumulation.[8] However, if $\theta + \sigma < 1$, increasing uncertainty implies more precautionary savings and vice versa.[9]

Although the multiplicative productivity shock increases the expected value of equilibrium output, it is not clear whether the uncertainty raises economic welfare since it causes the economy to fluctuate. To check if the utility increases with the uncertainty, define the period utility function as:

$$u_t = \frac{1}{1 - \sigma} \cdot (c_t - \alpha \cdot n_t)^{1 - \sigma}. \tag{17}$$

Then using the analytical solution, (15), we have:

$$u_t = \frac{(\lambda d - \alpha b)^{1 - \sigma}}{1 - \sigma} \cdot [(1 - \lambda)d]^{(1 - \sigma)t} A_t^{(1 - \sigma)/\theta} A_{t-1}^{(1 - \sigma)/\theta} \cdots A_0^{(1 - \sigma)/\theta} k_0^{1 - \sigma} \tag{18}$$

If we assume the expected lifetime utility is finite[10], given the initial capital

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[8]If capital stock grows in the economy, working hours also grow and hence the constraint on the total available hours will be violated. Using (14) in (15), we have the following.

$$\frac{k_{t+1}}{k_t} = (1 - \lambda)dA_t^{1/\theta}$$

To guarantee that the economy is fluctuating around the steady state, we need to assume that:

$$(1 - \lambda)dE\left[A_t^{1/\theta}\right] = d\left\{\beta\theta d^{1 - \sigma}E\left[A_t^{(1 - \sigma)/\theta}\right]\right\}^{1/\sigma} \cdot E\left[A_t^{1/\theta}\right] = 1$$

This condition keeps capital stock from growing or from shrinking in the long run. See King, Plosser and Rebelo (1988) for more detailed discussion. We do not have growth in the example so explosive growth is not an issue.

[9]Sandmo(1970) showed that when there are changes in the degree of income uncertainty, the response of precautionary savings depends on the sign of the third derivative of the utility function. In our case, the type of uncertainty is different from that studied by Sandmo. In our case production parameters matters together with preference parameters.

[10]The condition is the following:

$$\beta[(1 - \lambda)d]^{1 - \sigma}E\left(A_t^{(1 - \sigma)/\theta}\right) < 1.$$

stock, it can be obtained as:

$$
\begin{aligned}
EU &= \frac{(\lambda d - \alpha b)^{1-\sigma} k_0^{1-\sigma}}{1-\sigma} \cdot \sum_{t=0}^{\infty} \left\{ \left[\beta[(1-\lambda)d]^{(1-\sigma)} \right]^t \left[E\left(A_t^{(1-\sigma)/\theta} \right) \right]^{t+1} \right\} \\
&= \frac{(\lambda d - \alpha b)^{1-\sigma} k_0^{1-\sigma} E(A_t^{(1-\sigma)/\theta})}{(1-\sigma) \left\{ 1 - \beta[(1-\lambda)d]^{(1-\sigma)} \left[E\left(A_t^{(1-\sigma)/\theta} \right) \right] \right\}},
\end{aligned}
$$

(19)

where we used the assumption that the shock follows an *i.i.d.* process. Now the effect of an increase in the uncertainty is not so straightforward but the critical value for the parameters to imply increasing utility with uncertainty can be obtained in the following way. First, we express (19) in terms of λ using the definition (16). We can rewrite (16) as follows.

$$
E\left(A_t^{(1-\sigma)/\theta} \right) = \frac{(1-\lambda)^{\sigma}}{\beta \theta d^{1-\sigma}}
$$

(20)

If we substitute this expression in (19), we obtain the following expression for expected utility[11].

$$
EU = \frac{k_0^{1-\sigma}(\lambda d - \alpha b)^{1-\sigma}(1-\lambda)^{\sigma}}{\beta(1-\sigma)d^{1-\sigma}[\lambda - (1-\theta)]} = \frac{k_0^{1-\sigma}[\lambda - (1-\theta)]^{-\sigma}(1-\lambda)^{\sigma}}{\beta(1-\sigma)}
$$

(21)

The expected utility depends on λ and σ. First, consider the case that $0 < \sigma < 1$. If λ goes up in this case, utility in (21) will decrease and vice versa. However, as was mentioned previously in a footnote, λ is related to the variance of the shock τ^2 inversely when $\theta + \sigma < 1$, which means that utility is increasing with τ^2. If $\theta + \sigma = 1$, λ does not respond to the changes in the variance, τ^2, and thus the fluctuations and the mean effect are balanced, which means that business cycle uncertainty does not affect expected lifetime utility. But if $\theta + \sigma > 1$, λ is increasing with τ^2, which means that utility is decreasing with τ^2. Now consider the case that $\sigma > 1$. If λ goes up in this case, utility in (21) will increase and vice versa. In addition, we can show

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[11]Note the following in the derivation.

$$
\frac{\alpha b}{d} = \frac{\alpha b}{b^{1-\theta}} = \alpha b^{\theta} = \left\{ \left(\frac{1-\theta}{\alpha} \right)^{\theta} \right\}^{1/\theta} \quad \alpha = 1 - \theta
$$

from (16) that λ goes down as the variance of the shock τ^2 increases[12] and thus utility will always decrease with the uncertainty. The same is true in the case that $\sigma = 1$.

Hence if we define an increase in uncertainty by mean preserving spread of the distribution following Rothchild and Stiglitz (1970, 1971)[13], the critical value for the uncertainty to increase the utility can be obtained as:

$$\frac{1-\sigma}{\theta} = 1 \quad \Longleftrightarrow \quad \theta + \sigma = 1. \tag{22}$$

If (22) holds, the lifetime utility function does not depends on τ^2 and hence the mean preserving spread of the distribution of the shock does not affect the expected lifetime utility. In other words, the fluctuations and the mean effect are balanced. However, if $\theta + \sigma > 1$, the lifetime utility function is concave in the shock and hence the fluctuations effect dominates the mean effect. In this case, the conventional wisdom holds, i.e. the uncertainty reduces welfare. On the other hand, if $\theta + \sigma < 1$, the lifetime utility function is convex in the shock and the mean effect dominates the fluctuations effect. That is, welfare increses with uncertainty. This result confirms that the effect of uncertainty on the economy depends critically on the parameters

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[12]Note the following in (16). If we let

$$Z_t = A_t^{(1-\sigma)/\theta},$$

we have

$$\frac{\partial Z_t}{\partial A_t} = \left(\frac{1-\sigma}{\theta}\right) A_t^{[\{(1-\sigma)/\theta\}-1]}$$

$$\frac{\partial^2 Z_t}{\partial A_t^2} = \left(\frac{1-\sigma}{\theta}\right)\left(\frac{1-\sigma}{\theta} - 1\right) A_t^{[\{(1-\sigma)/\theta\}-2]}.$$

Hence if $\sigma > 1$, we can conclude that Z_t is convex with respect to A_t. This means that the expected value of Z_t is increasing with τ^2.

[13]Using the assumption that A_t follows an *i.i.d.* log-normal distribution as (8), the unconditional mean of the lifetime utility can be obtained as:

$$EU = \frac{(d - \alpha b)^{1-\sigma}}{(1-\sigma)(1-\beta)} \cdot \exp\left\{\left(\frac{1-\sigma}{\theta} - 1\right)\left(\frac{1-\sigma}{\theta}\right)\frac{\tau^2}{2}\right\}.$$

Hence if we define the degree of uncertainty with the variance of normal distribution, τ^2, we have the same condition for welfare increase with the uncertainty.

determining the elasticity of labor demand, i.e. θ, and risk aversion, i.e. σ.[14] In other words, if the elasticity of labor demand is large and/or if the degree of risk aversion is small, the possibility that uncertainty increases economic welfare is higher.

3. Productivity Shocks and Welfare

The examples just presented make it clear that the welfare of an economy can , in principal, increase with the introduction of uncertainty, even if the parameter values required for this to happen make it seem unlikely. In addition, welfare can be increasing as the shock becomes more uncertain. The key to the result is the endogenous choice, i.e. labor choice in the previous examples, which can be made by the agents to make use of the uncertainty in their favor. We now consider an economy with a more general preference specification and with more realistic capital accumulation. Now, unfortunately, we cannot solve the model analytically so we will solve it numerically as is common in the real business cycle literature.

3.1 The Economy

The economy is a very standard real business cycle model. The representative agent is endowed with initial capital stock k_0 and with one unit of time each period. The preferences of the agent are:

$$U_0 = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \frac{[c_t^\alpha (1 - n_t)^{1-\alpha}]^{1-\sigma}}{1 - \sigma} \right\}, \quad 0 < \alpha < 1, \ \sigma > 0, \qquad (23)$$

and production takes place with the Cobb-Douglas technology in (**??**).

We assume that the productivity shock follows an AR(1) process:

$$\ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t, \qquad (24)$$

where ε_t has an *i.i.d.* normal distribution $N(-\frac{\tau_\varepsilon^2}{2(1+\rho)}, \tau_\varepsilon^2)$. If we characterize the mean and variance of the shock innovation ε_t in this way, the unconditional mean and variance of A_t is as follows. First, if we represent (24) with an MA process, we have:

$$A_t = \exp \left\{ \sum_{j=0}^{\infty} \rho^j \varepsilon_{t-j} \right\}. \qquad (25)$$

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[14]The elasticity of labor demand is $1/\theta$. Note also that the elasticity of labor supply matters in more general cases which will be studied later.

For a normal random variable X, we can have the following.

$$E[\exp(aX)] = \exp\left\{ aE(X) + \frac{a^2 Var(X)}{2} \right\}, \tag{26}$$

where a is an arbitrary parameter. Hence the first two moments of A_t can be obtained as:

$$
\begin{aligned}
E(A_t) &= \Pi_{j=0}^{\infty} \exp\left[\rho^j E(\varepsilon_t) + \frac{\rho^{2j} Var(\varepsilon_t)}{2} \right] \\
&= \exp\left\{ \sum_{j=0}^{\infty} \left[\rho^j E(\varepsilon_t) + \frac{\rho^{2j} Var(\varepsilon_t)}{2} \right] \right\},
\end{aligned} \tag{27}
$$

$$
\begin{aligned}
E(A_t^2) &= \Pi_{j=0}^{\infty} \exp\left[2\rho^j E(\varepsilon_t) + 2\rho^{2j} Var(\varepsilon_t) \right] \\
&= \exp\left\{ \sum_{j=0}^{\infty} \left[2\rho^j E(\varepsilon_t) + 2\rho^{2j} Var(\varepsilon_t) \right] \right\}.
\end{aligned} \tag{28}
$$

Now if we substitute the mean and the variance of ε_t in the above expression, we can verify that the mean of A_t is 1. In addition, since,

$$Var(A_t) = E(A_t^2) - [E(A_t)]^2, \tag{29}$$

we can write the variance of A_t as:

$$Var(A_t) = \sum_{j=0}^{\infty} \left[\exp(\rho^{2j} \tau_\varepsilon^2) - 1 \right]. \tag{30}$$

A change in the variance of ε_t, τ_ε^2, is a mean preserving spread of the random variable A_t in the sense of Rothchild and Stiglitz (1970, 1971)[15].

Output is consumed or saved as capital and capital follows the law of motion:

$$k_{t+1} = (1 - \delta)k_t + i_t, \tag{31}$$

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[15]In the real business cycle literature, it is usually assumed that:

$$\varepsilon_t \ \tilde{} \ i.i.d. \ N(0, \tau_\varepsilon^2),$$

and hence the mean of A_t increases with an increase in τ_ε^2, which means that the change does not mean a mean preserving spread of the shock A_t. In this case, the shock in the economy without uncertainty and that in the counterpart steady state economy do not share the same mean. However, there is a good reason that they assume this, which will be made clear later.

where i_t is the investment in period t and δ is the rate of capital depreciation. For later reference, we write the problem facing the representative agent using the Bellman equation as:

$$V(A_t, k_t) = \max \left\{ \left(\tfrac{1}{1-\sigma}\right) \left[c_t^\alpha (1-n_t)^{1-\alpha}\right]^{1-\sigma} + \beta E_t \left[V(A_{t+1}, k_{t+1})\right]\right\}$$

$$\text{s.t.} \quad (1)\ c_t + i_t = A_t k_t^\theta n_t^{1-\theta}$$
$$(2)\ k_{t+1} = (1-\delta)k_t + i_t$$
$$(3)\ \ln(A_t) = \rho \ln(A_{t-1}) + \varepsilon_t$$
$$(4)\ c_t,\ i_t \geq 0,\ \ 0 \leq n_t \leq 1,\ \ k_0 \text{ is given.}$$

$$(32)$$

Note here that we are invoking the second welfare theorem to get the equilibrium allocation by solving a programming problem.

3.2 Calibration

We set the parameter values as in the real business cycle literature. Assume a period is a quarter. The utility discounting factor is assumed to be $\beta = 0.99$, which implies 4% real interest rate per annum, and the preference parameter determining the substitution between consumption and leisure is set to be $\alpha = 0.35$, which implies the hours of work to be about one third of the total endowment of time. We vary σ to see how it affects the relationship between uncertainty and welfare. The value of the capital share parameter in production is assumed to be $\theta = 0.36$, which is roughly the share in U.S. economy. Capital depreciates at the rate that $\delta = 0.025$, which means 10% capital depreciation per annum. The remaining parameters are related to t'he technology shock. We assume that $\rho = 0.95$ when we vary τ_ε^2 to see its effect on the welfare cost. These numbers, or ones close to them, are used by many authors in the real business cycle literature including Kydland and Prescott (1982), Prescott (1986), Hansen (1985), and Cooley and Prescott (1995).

3.3 Steady State

The steady state of the economy can be obtained from the first order conditions.

$$\frac{1-\alpha}{1-n} = \alpha(1-\theta)Ak^\theta n^{-\theta} \cdot \frac{1}{c} \tag{33}$$

$$\theta Ak^{\theta-1}n^{\theta-1} = \frac{1}{\beta} - 1 + \delta \tag{34}$$

$$c + \delta k = A k^\theta n^{1-\theta}, \tag{35}$$

where we used the fact that $i = \delta k$ in a steady state and the variables without time subscript are the steady state values[16].

3.4 Numerical Simulation

The simulation method used for the calculation is the one developed by Kydland and Prescott (1982), which substitutes the non-linear constraints into the preferences, approximates the temporal utility around the steady state with a quadratic function and then invokes certainty equivalence[17].

Note that once the approximation is made, the distribution itself is not used in the decision making at all but the mean is the only moment used in the optimization. However, this implies that keeping the first moment of the approximated shock in the approximated economy the same as that of the shock in the original economy is crucial to get the correct welfare evaluation. Since the approximation involves a subtle random variable transformation, the mean is not preserved until we modify the mean of the shock in the approximation and in the approximated economy. Specifically, the key step in the approximation is the following. To achieve the linear-quadratic structure by approximation and to use the linearity of the law of motion for the technology shock, (24), we transform the technology shock as follows:

$$y_t = A_t k_t^\theta n_t^{1-\theta} = \exp(x_t) k_t^\theta n_t^{1-\theta}, \text{ where } x_t = \ln(A_t), \tag{36}$$

and we approximate the shock around the mean[18] of x_t. However, note that

$$E(A_t) = E[\exp(x_t)] \neq \exp[E(x_t)], \tag{37}$$

so approximating the technology shock around the mean of x_t does not preserve the mean of the original shock A_t. The first order approximation around

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[16]The steady state does not depend on the parameter σ, since intertemporal substitution does not takes place in a steady state.

[17]For an excellent survey of the method, see Hansen and Prescott (1995).

[18]Under the assumption (24), we have:

$$E(x_t) = -\frac{\tau_\varepsilon^2}{2(1 - \rho^2)}.$$

the mean produces:

$$A_t = \exp(x_t) \doteq \exp[E(x_t)] + \exp[E(x_t)](x_t - E(x_t)) = G(x_t), \qquad (38)$$

and the righthand side of (38), $G(x_t)$, is the shock used in the approximated economy. Hence keeping the mean of this quantity the same as the mean of the technology shock in the original model is important. However, the mean of this approximated quantity, $G(x_t)$, is not the same as the mean of the original technology shock.

$$E\left[G(x_t)\right] = \exp[E(x_t)] = \exp\left\{-\frac{\tau_\varepsilon^2}{2(1-\rho^2)}\right\} < 1 = E(A_t). \qquad (39)$$

To have the mean preserved in the approximated economy, we have to adjust the mean of the technology shock process in the approximation and in the approximated economy as follows.

$$x_{t+1} = \rho x_t + \varepsilon_{t+1}, \ \varepsilon_{t+1} \sim i.i.d. \ N(0, \nu_\varepsilon^2) \qquad (40)$$

Then $E(x_t) = 0$ and hence we can have the following[19].

$$E\left[G(x_t)\right] = \exp\left[E(x_t)\right] = 1 = E(A_t) \qquad (41)$$

Now the approximated technology shock in the approximated economy, $G(x_t)$, shares the same mean as the original technology shock, A_t.

Although the variance is not used in the optimization in the approximated economy, it has to be used in the welfare evaluation so we have to check if is preserved through the approximation. The variance of the approximated technology shock is:

$$Var[G(x_t)] = [\exp[E(x_t)]]^2 \cdot Var(x_t) = \frac{\nu_\varepsilon^2}{1-\rho^2}, \qquad (42)$$

where we used (40). Note here that (42) is the approximation to the variance of A_t and hence if we let

$$\nu_\varepsilon^2 = (1-\rho^2)\left[\sum_{j=0}^{\infty}\left(\exp(\rho^{2j}\tau_\varepsilon^2) - 1\right)\right], \qquad (43)$$

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[19]If we use (24) rather than (40) in the approximated economy, then we always have a lower mean of the shock in the approximated economy. See Kim and Kim (1998) for the details and for related issues.

then the variance is preserved through the approximation[20]. However, note also that there exists a one-to-one correspondence between τ_ε^2 and ν_ε^2 and thus the mean preserving spread can be represented by the changes in ν_ε^2 rather than in τ_ε^2.

Once the quadratic value function is obtained, we use the mean state as the initial value to obtain lifetime utility. That is, if we let U denote this lifetime utility under uncertainty, we have $U = V(x, k)$, where the variables without time subscript denote the steady state values of the counterpart varibles with time subscript as in (33)-(35). We compare this lifetime utility to that in the steady state economy in the following way. Suppose U^s denotes the lifetime utility in the steady state economy. Then U^s can be written as follows.

$$U^s = \frac{1}{(1-\beta)(1-\sigma)} \cdot \left[c^\alpha (1-n)^{1-\alpha}\right]^{1-\sigma}. \tag{44}$$

If we let Δc denote the utility gain or loss due to the uncertainty in terms of consumption, we have that,

$$\frac{1}{(1-\beta)(1-\sigma)} \cdot \left[(c+\Delta c)^\alpha (1-n)^{1-\alpha}\right]^{1-\sigma} = U \tag{45}$$

Hence the utility gain (or loss) due to uncertainty can be obtained as:

$$\Delta c = \left\{[(1-\beta)(1-\sigma)U]^{1/(1-\sigma)}/(1-n)^{1-\alpha}\right\}^{1/\alpha} - c. \tag{46}$$

If Δc is positive, it is a utility gain from uncertainty and vice versa. We present the ratio of Δc relative to the steady state output y.

3.5 Results

The welfare gain or loss depends critically on the factor share parameter in production, θ, and the risk aversion parameter, σ. The the factor share parameter in the Cobb-Douglas production function can be identified without difficulty and it is known in the U.S. economy not to have moved very much over a long period of time.[21] It is not as easy to identify the value of the risk aversion (and at the same time intertemporal substitution) parameter.

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[20]Note that the mean of the approximated technology shock $G(x_t)$ obtained in (39) does not depend on the variance ν_ε^2.

[21]See Solow (1970) and Cooley and Prescott (1995).

Hence we vary the value of the parameter and measure the welfare gain or loss due to uncertainty.

Figure 1 shows the welfare gain which depends on the risk aversion and the size of the uncertainty, namely ν_ε. Given the size of the uncertainty, ν_ε, the welfare gain decreases with the risk aversion parameter σ. The critical value of σ for which the effect of the uncertainty turns from beneficial to detrimental is about 3.9. At this value of σ, uncertainty is neither beneficial nor detrimental and hence an increase in the uncertainty does not affect the welfare of the economy. Given a value of σ which is smaller than 3.9, an increase in the size of uncertainty, ν_ε, raises welfare.

Note that the critical value for σ to imply a welfare gain with uncertainty is much higher than in the simple examples.[22] This model economy differs in two important ways from the simple examples. First, the shock is now persistent. Second, there is realistic capital accumulation so intertemporal substitution is more meaningful. If there is a favorable realization of the shock, they will work harder, produce more and save more. This means higher production efficiency because of the convexity of the reduced form production function with respect to the shock. Since most of the models in the real business cycle literature have assumed a value of σ around 2, uncertainty is beneficial in those models.[23] Given any value of σ which is less than 3.9, increasing uncertainty raises the welfare of the economy. On the other hand, if σ is greater than 3.9, increasing uncertainty means lower utility.

3.6 A Few More Results

Many authors in the quantitative literature have assumed preferences which are separable between consumption and leisure. To understand the welfare effect of uncertainty in those cases, we consider the following separable preferences.

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \cdot c_t^{1-\sigma} - \frac{B}{1+\gamma} \cdot n_t^{1+\gamma} \right) \right\}, \quad \sigma, \, \gamma > 0 \qquad (47)$$

Using these preferences, we obtain the results shown in Figure 2.[24] One

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[22]Since $\theta = 0.36$, the critical value in the examples is $\sigma = 0.64$.

[23]See, for example, Stockman and Tezar (1995), Backus, Kehoe and Kydland (1992).

[24]We set the parameter values except those of the preference parameters as in the

benefit of separable preferences is that we can disentangle the welfare effects of uncertainty associated with consumption and labor supply.

Figure 2a shows the welfare effect of changes in the risk aversion parameter σ. Given the size of the uncertainty, ν_ε, the welfare gain decreases with the value of σ. The critical value for σ at which the effect of uncertainty turns from beneficial to detrimental is about 2. It takes place at the value where the curves intersect in figure 2a. Models with preferences that are separable between consumption and leisure usually assume (at least in the real business cycle literature) that prefernces are logarithmic in consumption ($\sigma = 1$) implying that uncertainty is beneficial.

Figure 2b shows the welfare effect of the intertemporal substitution elasticity of labor supply. Since the elasticity is $1/\gamma$, it decreases with the value of γ. The parameter γ also represents the attitude toward labor supply risk. That is, as the value of γ increases, the degree of risk aversion associated with labor supply uncertainty increases. Figure 2b has two features that are worth noting. First, the change in welfare is very slow after $\gamma = 1$. Second, if the utility of consumption is logarithmic, the welfare gain from labor supply uncertainty is always positive over a plausible range of the value of γ. This means that the curvature of the reduced form (equilibrium) output as a convex function of the productivity shock is not very sensitive to the intertemporal elasticity of labor supply. However, we saw in Figure 2a that the curvature is reasonably sensitive to the risk aversion toward consumption risk. This can be explained as follows. First of all, changes in γ, given the

previous economy. The values of the preference parameters are set in the following way. First of all, we assume that $B = 3$, which implies the hours of work are about one third of endowment of time when $\sigma = 1$ and $\gamma = 1$. The values of the other two preference parameters are allowed to vary to see the impact of the changes on the welfare loss of the business cycle. When the value of the risk aversion parameter σ varies, that of γ is assumed to be 1. This value implies the elasticity of intertemporal substitution to be 1. Although the estimates of the elasticity for women and youth are much higher than 1 according to the micro studies of labor supply, this value is a bit higher than the estimates for men obtained in the literature (see Pencaval (1986), Killingworth and Heckman (1986)). However, recent aggregate labor studies like Alogoskoufis (1987) and Cho, Merrigan and Phaneuf (1998) obtained the estimate of the elasticity higher than 1. In fact, the intertemporal elasticity of labor supply is assumed to be much higher than 1 in most real business cycle models. On the other hand, when the value of the elascicity of labor supply parameter is allowed to vary, we assume the value of σ to be 1. This value implies the logarithmic preferences and it has been used in the literature munerous times (for example, see Burnside and Eichenbaum (1996)).

value of σ, affect the aggregate fluctuations more than changes in σ, given the value of γ. Hence aggregate flucuations are dampened a lot faster with the value of γ, given the value of σ, than with the value of σ, given the value of γ. This implies that the effect of changes in σ on welfare are larger than changes in γ.

A standard specification that has been used widely is one where labor supply is indivisible because, for example, of fixed costs associated with labor supply.[25] The aggregate implications of indivisible labor supply were studied by Rogerson (1988). He showed that indivisible labor combined with employment lotteries implies preferences linear in labor supply,

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(\frac{1}{1-\sigma} \cdot c_t^{1-\sigma} - D \cdot n_t \right) \right\}, \quad \sigma > 0, \tag{48}$$

where D is a preference parameter. Hansen (1985) used these preferences to sstudy the effect of the indivisibility on the business cycle. Now we use (48) to see the effect of indivisibility on the welfare cost of uncertainty. We follow the same procedure as in the previous economies to solve the model and obtained the lifetime utilities with and without uncertainty for comparison.[26]

Figure 3 depicts the welfare gain in the economy with the labor supply indivisibility. Compared to Figure 2a, there are some notable differences.[27] First, the size of the welfare gain increases considerably. This result is not difficult to understand. Since the disutility of labor is linearized because of the indivisibility and employment lotteries, the expected disutility of work is not affected by the fluctuations in labor supply. Second, the critical value for σ at which the welfare gain turns to being negative with uncertainty has increased from about 2 to about 2.3. Third, even when the business cycle is detrimental to welfare, the magnitudes are smaller with indivisible labor than with divisible labor.

4. Preference Shocks and Welfare

[25]The economy with fixed costs of labor supply is observationally equivalent to the indivisible labor economy. See Rogerson and Wright (1988).

[26]We assumed that $D = 2.6$, which implies that the hours of work in the steady state are about one third of the endowment of time, and that the values of the other parameters are the same in the previous economies.

[27]Since the economy blows up when $\sigma = 0$, the simulation starts with $\sigma = 0.1$.

The key to the results in the previous sections is that the productivity shock is multiplicative and the variable factor(s) of production (labor in the previous cases) respond positively to the shock. With this construct, we can show that purposeful agents in the economy make use of the uncertainty in their favor. That is, the agents work harder when the realized shock is favorable and insure by saving more against the unfavorable realization of the shock in the future. This kind of response of the economy to uncertainty is not restricted to technology shocks. In the case of preference shocks, we can also that welfare increases with uncertainty.

Bencivenga (1991) introduced preference shocks in a real business cycle model and studied their implications. We will show that welfare may increase with the uncertainty associated with preference shocks through an example. Following Long and Plosser (1983), consider the simplest real business cycle model with log linear preferences, Cobb-Douglas production function, and 100 percent capital depreciation.

$$
\begin{aligned}
V(B_t, k_t) = \quad & \max \quad \{\ln(c_t) + B_t \ln(1 - n_t) + \beta E_t \left[V(B_{t+1}, k_{t+1})\right]\} \\
& \text{s.t.} \quad (1) \; c_t + k_{t+1} = k_t^\theta n_t^{1-\theta} \\
& \qquad\quad (2) \; k_{t+1} = (1 - \delta)k_t + i_t \\
& \qquad\quad (3) \; \ln(B_t) \; \tilde{} \; i.i.d. \; N \left[\ln(B) - \frac{\tau_B^2}{2}, \tau_B^2\right] \\
& \qquad\quad (4) \; c_t, \; k_{t+1} \geq 0, \; 0 \leq n_t \leq 1, \; k_0 \text{ is given,}
\end{aligned}
\tag{49}
$$

where B is a constant. Here, B_t is the preference shock with constant mean B[28] and it is the only source of the aggregate fluctuations. To make the example as simple as possible, we assume that there is no uncertainty associated with the technology. The equilibrium solution for the problem can be obtained analytically as follows.

$$
c_t = (1 - \beta\theta)k_t^\theta n_t^{1-\theta}
\tag{50}
$$

$$
k_{t+1} = \beta\theta k_t^\theta n_t^{1-\theta}
\tag{51}
$$

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[28]The mean and variance of the shock can be obtained as follows.

$$
E(B_t) = B, \; Var(B_t) = \exp[\ln(B) + \tau_B^2] - B^2
$$

Hence the mean of the random variable B_t is constant and a change in τ_B^2 is equivalent to its mean preserving spread.

$$n_t = \frac{1 - \theta}{(1 - \theta) + (1 - \beta\theta)B_t} \tag{52}$$

Note two features of this solution. First, the hours of work respond negatively to the preference shock. Second, changes in the preference shock in period t affect the consumption in period t through n_t and the future consumption, say c_{t+j}, $j = 1, 2, \cdots$, through capital accumulation.

To see the welfare consequences of introducing the preference shock, define the following.

$$U_t = \sum_{j=0}^{\infty} \beta^j [\ln(c_{t+j}) + B_t \ln(1 - n_{t+j})] \tag{53}$$

Using the equilibrium consumption (50) and capital investment (51), (53) can be rewritten as follows.

$$
\begin{aligned}
U_t &= \frac{\ln(1-\beta\theta)}{1-\beta} + \sum_{j=0}^{\infty}\left[(\beta\theta)^{j+1}\left(\frac{1-\theta^{j+1}}{1-\theta}\right)\cdot\ln(\beta\theta)\right] \\
&\quad + \left(\frac{\theta}{1-\beta\theta}\right)\cdot\ln(k_t) + \left(\frac{1-\theta}{1-\beta\theta}\right)\cdot\ln(n_t) + B_t\cdot\ln(1-n_t) \\
&\quad + \sum_{j=1}^{\infty}\beta^j\left\{\left(\frac{1-\theta}{1-\beta\theta}\right)\cdot\ln(n_{t+j}) + B_{t+j}\cdot\ln(1-n_{t+j})\right\}
\end{aligned}
\tag{54}
$$

Note that because the preference shock is *i.i.d.* the shock in period t affects only n_t in (54) and B_t and n_t are related as in (52). Hence we can take the first and the second derivative of U_t with respect to the preference shock to get:

$$\frac{\partial U_t}{\partial B_t} = \ln\left\{\frac{(1-\beta\theta)B_t}{(1-\theta)+(1-\beta\theta)B_t}\right\} < 0 \tag{55}$$

$$\frac{\partial^2 U_t}{\partial B_t^2} = \frac{1-\theta}{[(1-\theta)+(1-\beta\theta)B_t]B_t} > 0 \tag{56}$$

That is, U_t is decreasing in the shock B_t but at a decreasing rate. In other words, U_t is convex in B_t so introducing uncertainty increases welfare. Further, the welfare is increasing with uncertainty. In sum, multiplicative preference shocks may not be detrimental to welfare just as with technology shocks.

5. The Welfare Cost of Business Cycles

In a celebrated paper, Lucas (1976) criticized the use of reduced form models rather than structural ones to obtain policy implications. His argument was so convinicng that it is hard nowadays to find academic papers

using reduced form models. Oddly though, many have used reduced form models to study the welfare cost of business cycles. Examples are the partial equilibrium model of Lucas (1987), Obstfeld (1994), and Dolmas (1998).[29] The conventional wisdom is that uncertainty is welfare reducing in a concave economic environment. This common-sense intuition exists partly because of these partial equilibrium analyses. For example, Lucas (1987) looked at the time series of consumption and inferred the welfare costs of the business cycle, which is the traditional way of analyzing the welfare loss associated with uncertainty.

But, analyzing the welfare cost of business cycles by looking at the time series data of a few specific macroeconomic variables may lead to the wrong conclusion. As we saw in the previous sections, any business cycle uncertainty has two effects, the fluctuations and the mean effect. The method of obtaining the cost of the business cycle by comparing the utility of the actual consumption series to the utility of the mean of the actual series ignores the mean effect of business cycle uncertainty so the estimate of the welfare cost can be correct only when the mean effect happens to be zero.

The notion of "making hay while the sun shines" is well enshrined as a principle of the business cycle. Rational economic agents would respond to favorable shocks and it is this that produces what we have called the "mean effect". If the mean effect is positive enough to dominate the fluctuations effect as in many real business cycle models, the business cycle is welfare improving.

This means that the estimates of the welfare cost of the business cycles obtained using the consumption series are often biased. Fluctuating economies may sometimes enjoy higher economic welfare than non-fluctuating economies. The direction and the size of the bias depend on the nature of the shocks and the means the agents use to take advantage of the shocks. If the shocks are multiplicative like productivity and taste shocks, the possibility of over-estimating the cost of business cycles is pretty high. If shocks are additive like monetary and government expenditure shocks, then it is not likely to be an issue.

There is of course a big differences between the aggregate cost of business cycles and the cost to individuals. Heterogeneity, especially employment

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[29]The meaning of 'partial equilibrium' here is that one obtains the welfare cost by looking at a few time series like consumption, leisure etc.

status, matters in terms of individual specific costs of the business cycle. This point was well established by Imrohoroglu (1989). Of course, the issue of what factors determine the employment status is important, but we beg the question. But, even in a world with heterogeneity the issue raised in this paper is important. Suppose an economy consists of two groups of agents, employed and unemployed, and that productivity shocks drive the economy. The agents who are employed in the market can effectively make use of the shocks in their favor, but the unemployed agents cannot. That is, the employment status of an agent decides whether he/she has a means of making use of the business cycle uncertainty in their favor or not. Unemployed agents experience only the fluctuations effect so they bear the brunt of the business cycle. Krusell and Smith (1999) document this result in an economy where agents face idiosyncratic risk. This line of research is clearly important and the distribution of the cost of business cycles across the agents may be more important than the average cost of the business cycle fluctuations.

Gomes, Greenwood and Rebelo (1998) introduced job search in a real business cycle model. They also found a welfare gain from business cycle fluctuations. In their model, the welfare gain results from two factors. Although they do not mention it in the paper, the first factor is the mean effect due to the multiplicative productivity shock. The second factor is the option value of job search which depends positively on the size of the uncertainty. Since their main concern is with the welfare gain due to job search, it seems desirable to disentangle the welfare effect of the two factors.

6. Conclusion

This paper considers the welfare effect of uncertainty in business cycle models. We showed that when the uncertatinties are multiplicative, as they always are in the real business cycle literature, welfare may be higher in an economy with aggregate fluctuations than in the counterpart economy without uncertainty. This finding holds true over the range of parameter values that have been used many times in the literature.

The findings in this paper may have some implications for stabilization policies. If the shocks initiating the business cycles are additive, then there is no possibility of fluctuations being beneficial and hence stabilizing the fluctuations is welfare improving.[30] However, if the shocks are multiplicative

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[30]Of course, the policy should not involve any distortions. If the policy involves welfare

as in the real business cycle literature and as in the case of some seasonal fluctuations or some preference shocks, stabilizing the fluctuations may not be welfare improving. Policies that respond to shocks have to take account of the source of shocks and often will have the implication that the optimal policy will cause the economy to fluctuate more. This, for example, is the nature of the optimal and time-consistent monetary policy in Cooley and Quadrini(2000).

The magnitude of welfare changes with uncertainty depends on the means which the agents can use to make use of the uncertainty in their favor. The only means in the previous sections is labor input. There can be many other means which can be used by the agent in an uncertain economic environment. The first such means that we can think of is cyclical factor utilization (for example see Bils and Cho (1995), Greenwood, Hercowitz and Huffman (1988)). If the intensity of the use of a production input can be adjusted procyclically, the welfare gain can be larger. For example, consider an agent with the same preference as in the second example in section 2, (6), but with the following production function.

$$y_t = A_t(n_t^\alpha k_t)^\theta n_t^{1-\theta}, \ 0 < \alpha < 1, \tag{57}$$

where n_t^α is a rate of capital utilization as a function of labor input. If the agent has this additional margin of adjustment, the equilibrium labor input and production can be obtained as:

$$n_t = \left(\frac{1-\theta(1-\alpha)}{\alpha}\right)^{1/[\theta(1-\alpha)]} A_t^{1/[\theta(1-\alpha)]} k_t^{1/(1-\alpha)}, \tag{58}$$

$$y_t = \left(\frac{1-\theta(1-\alpha)}{\alpha}\right)^{[1-\theta(1-\alpha)]/[\theta(1-\alpha)]} A_t^{1/[\theta(1-\alpha)]} k_t^{1/(1-\alpha)}. \tag{59}$$

This equilibrium production function exhibits more curvature in the shock A_t, which means that the mean effect can be much larger with the factor utilization. Procyclical factor utilization can also take place along labor effort margin, which has a similar effect.

Another margin that can be used by the agents is home production (for example, see Benhabib, Rogerson and Wright (1991), Greenwood and Hercowitz (1991), Greenwood, Rogerson and Wright (1995)). First of all, if the

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costs, it is not clear whether the stabilization is welfare improving.

total working hours at home and in the marketplace stay stable and only the composition changes in response to shocks to market production, the welfare loss due to hours and consumption fluctuations will be minimized and hence the possibility of welfare increasing with business fluctuations will rise. Further, shocks to home production can be interpreted as a multiplicative shocks to preferences. However, as we saw in the previous section, multiplicative preference shocks can increase welfare. In sum, home production can be a means of buffering the fluctuations effect of shocks to market production and ,at the same time, shocks to home production themselves may increase the welfare.

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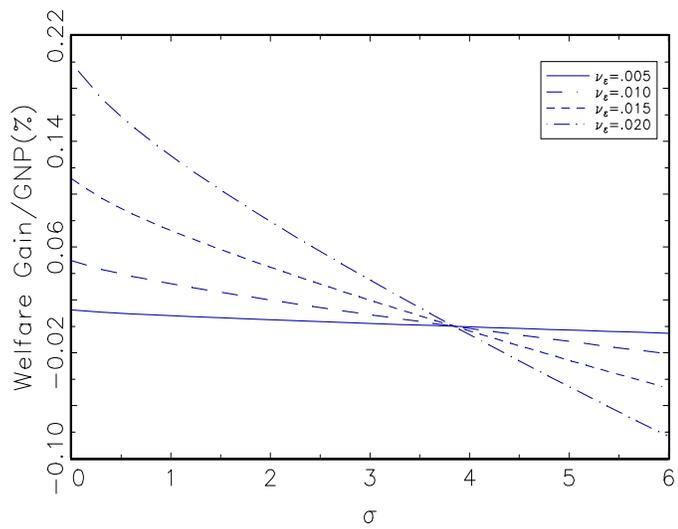
Figure 1: Risk Aversion and Welfare Cost of Uncertainty

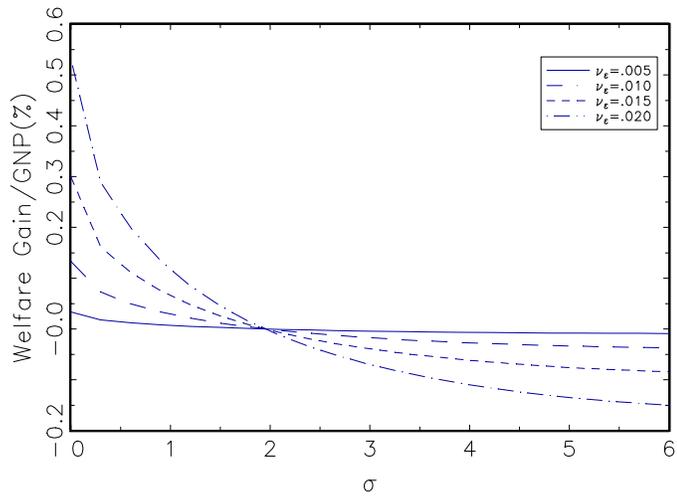Figure 2(a): Separable Preference (Risk Aversion)
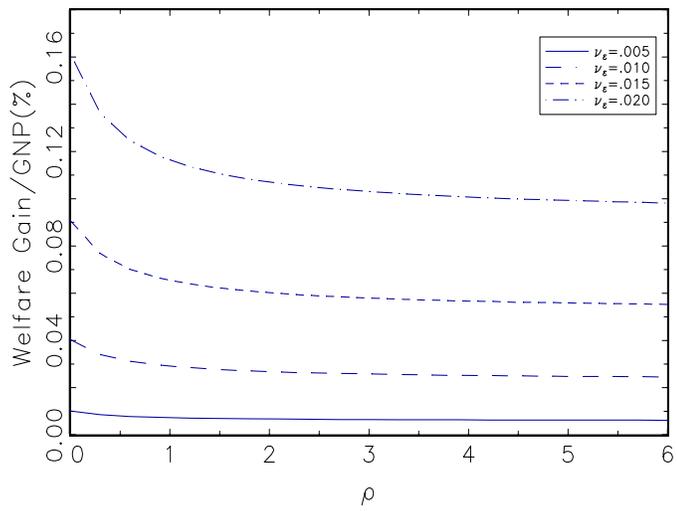

Figure 2(b): Separable Preference (Labor Supply Elasticity)

Figure 3: Indivisible Labor (Risk Aversion)