Vintage Capital in Frictional Labor Markets*

Andreas Hornstein† Per Krusell‡ Giovanni L. Violante§

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Abstract

Does capital-embodied technological change play an important role in shaping labor market outcomes? This paper addresses the question in a model with vintage capital and search-matching frictions where costly capital investment leads to heterogeneity in productivity among vacancies in equilibrium. The paper first demonstrates analytically 1) how technology growth affects equilibrium unemployment, and 2) how its impact depends on the presence of labor market institutions (welfare benefits, firing taxes). Next, it applies the model to a quantitative evaluation the U.S.-Europe unemployment comparison: a common embodied technological acceleration interacted with different labor market institutions can explain over half of the differential rise in the unemployment rate between the two regions. In the model, shocks and policies interact through a reduction of firms' demand for labor. Hence, our model represents an alternative explanation with respect to a class of "labor supply" models where the rise in European unemployment is associated to the unwillingness of low-skilled workers to accept poor job offers, in presence of a generous welfare state. Our model has implications for the evolution of the labor share, the age of capital and the vacancy-employment ratio that are broadly supported empirically.

*Krusell thanks the NSF for research support. Any opinions expressed are those of the authors and do not necessarily reflect those of the Federal Reserve Bank of Richmond or the Federal Reserve System. For correspondence, our e-mail addresses are: andreas.hornstein@rich.frb.org, pekr@troi.cc.rochester.edu, and glv2@nyu.edu.

†Federal Reserve Bank of Richmond.

‡University of Rochester, Institute for International Economic Studies, CAERP, CEPR, and NBER.

§New York University, and CEPR.

1 Introduction

Whether or not one believes that variations in technology-based productivity are an important driving force for business cycles, there is no doubt that medium- and long-run changes in technology are quantitatively significant and affect deeply the economic conditions, and the organization of work. The questions of how technological change influences labor market outcomes, and how different institutions may produce different responses to technology, are central ones in macroeconomics. Specifically, the evolution over time and across countries of unemployment rates is frequently, at least on an informal level, interpreted with reference to the interaction between technological change and labor market policies.

Krugman (1994) was the first to propose the technology-institutions perspective for studying the differential unemployment dynamics of the U.S. and Europe over the past 30 years.[1] Since then, this perspective has been dominant in the literature. Several cross-country regression studies concluded that "shock-policy interactions" help considerably in explaining the observed variation in the unemployment rate over time and across countries (Blanchard and Wolfers 2000, Bertola, Blau, and Kahn 2001). A major shortcoming of these empirical implementations of the Krugman hypothesis, however, is that they do not allow an explicit identification of the structural shocks; relatedly, they are agnostic about the economic mechanism behind the interaction.

Ljungqvist and Sargent (1998) offer the first rigorous formalization of this view.[2] Their explanation is based on the interaction between a shock, common to the two continents, and a difference in the degree of generosity of the respective welfare states. The primitive shock is a rise in "economic turbulence" which is modelled as the degree of skill depreciation that takes place for unemployed workers. Thus, the idea is that technological change led to changes in the nature of skill depreciation. The welfare state is modelled as a system of unemployment benefits linked to the workers' past earnings (and their past skills). Within a standard McCall (1970) search model where unemployed workers sample wage offers proportional to their current skills, Ljungqvist and Sargent then show that faster skill depreciation worsens the

[1] The first panel of Figure 1 reminds the reader about the size of the unemployment differential. As documented extensively (e.g., Machin and Manning, 1999) longer unemployment durations account for the entire rise in European unemployment, whereas separation rates have barely changed over the period.

[2] Bertola and Ichino (1995), Caballero and Hammour (1998), Mortensen and Pissarides (1999), den Haan, Haefke, and Ramey (2001), among others, also study the technology-policy interaction in the context of European unemployment. In section 5.3 we review the literature in detail and contrast it with our approach.

value of the average wage offer compared to the value of unemployment, thereby increasing the reservation wage and, in turn, unemployment duration. Quantitatively, they show that the complementarity between turbulence shocks and the welfare state can be massive.

In the Ljungqvist-Sargent framework of analysis, the shock-policy interaction operates entirely through vintage human capital, and therefore through the *labor supply* side. In other words, the argument is that unemployment in Europe went up because, for the jobless workers it was more beneficial to collect unemployment insurance than to work at a low wage, given their new, low productivity induced by technological change. Though this is an hypothesis that should be considered very seriously, it is not the only possible interpretation of events.

Technological change, especially if embodied in new vintages of capital, may well have important effects that operate more directly through *labor demand*. The purpose of the present paper, thus, is to explore this alternative mechanism in some detail: different labor-market institutions in Europe, under a burst of capital-embodied technological change, led to a decrease in firms' willingness to employ workers. With frictional labor markets, the fall in job creation induces a rise in unemployment. Such an hypothesis would complement, and possibly challenge, the view that European workers chose unemployment over employment due to the generous welfare state. Specifically, the case for developing a model of vintage physical capital in frictional labor markets applicable to a quantitative analysis of the U.S-Europe unemployment experience can be built on three distinct arguments.

First, over the past three decades the technological innovations embodied in new and more productive capital goods—especially in information and communication equipment and software—have represented the major source of productivity growth in U.S. output per capita (Jorgenson, 2001) and their contribution to growth is also major in several European countries (Colecchia and Schreyer, 2002). Productivity growth embodied in new vintages of capital has accelerated significantly over the past 30 years, from 2% per year in the 1960s to 4.5% in the 1990s (Cummins and Violante, 2002). The rapid investment in the new capital goods triggered by this faster obsolescence process is witnessed by the significant decline in the average age of capital: Figure 1 (fourth panel) documents a fall of 2 years (over 17%) in the period 1960-1995. Overall, these facts suggest that the magnitude of this productivity shock is large, so potentially its interaction with the rigid European-type labor market institution can be substantial.

Second, capital-embodied technical change together with that the observation that the new technologies are skill-biased has proved to be useful to account for the major transformation that occurred in the U.S. labor market over the past 30 years– the rise of wage inequality (Krusell, Ohanian, Rios-Rull, and Violante, 2000; Acemoglu 2002)– thus it is also a natural candidate driving force behind the rise in European unemployment, possibly with the help of institutional rigidities.

Third, the Ljungqvist and Sargent approach is based on a search model and, as such, is silent on two important dimensions of the labor market: job creation rates and labor share of output. Figure 1 (second and third panels) shows that the worse unemployment performance of Europe has a counterpart in the relative decline of both the labor share and the vacancy creation rate. In the U.S., the labor share shows only a modest decline since 1960 (around 3 percentage points), while in Europe the decline is twofold, and more than threefold if one measures it from its peak in 1975.[3] The vacancy-employment ratio was roughly constant in the U.S., while it declined by 40% in Europe.[4] The latter fact, in particular, supports the existence of economic forces at work through labor demand: the higher European unemployment is due not just to the unwillingness of workers to accept low-paid jobs, but also to the lack of job creation by firms.

There is a long tradition in modelling the relation between embodied productivity growth and unemployment in equilibrium models of the labor market. Aghion and Howitt (1994) pioneered the research in this area by incorporating aspects of vintage capital into the traditional Diamond-Mortensen-Pissarides matching model. The model we develop builds on this approach; in common with these studies, we share the matching-function approach to labor market frictions and the Nash bargaining solution to the bilateral monopoly problem between firms and workers. However, we model vintage capital differently from the tradition, and this difference is crucial.

In the standard matching model new capital is costless to buy and, as a result, vacancies all consist of the newest capital and have always zero value under free entry. In the spirit of the traditional vintage capital growth model, instead, we view capital investment as the

[3]This graph uses the same accounting procedures to compute the labor share across several countries (see the Appendix for details). Krueger (1999) estimates the labor share of income in the U.S. in many alternative ways. All the different estimates (Krueger, 1999, Figure 1, page 46) show a decline from 1970-1995, between 2% and 6%.

[4]Appendix A.1 provides a detailed explanation of the data sources used to construct the time-series in Figure 1.

costly activity: when creating a job requires purchasing capital, vacant jobs with installed capital can have positive value in equilibrium since the cost of the machine is sunk, and vacant machines of different vintages can coexist in the labor market, even with free entry. The result is a natural framework for studying how the interaction between capital-embodied technical change and policies affects the unemployment rate, job creation, and the determination of the labor share in equilibrium.

Notwithstanding the additional layer of complexity due to the non-degenerate equilibrium distribution of vacancies and the non-zero value of vacant firms, our qualitative analysis shows that it is possible to maintain analytical tractability in characterizing the chief features of a labor market equilibrium. We identify conditions for the existence and uniqueness of a stationary equilibrium, and we show that the equilibrium of the economy can still be represented in a two-dimensional graph—we identify a job creation curve and a job destruction curve—in the space defined by the age of capital at destruction and the labor market tightness, similarly to how the standard model is analyzed. In all the interesting comparative statics with respect to the "deep" technological and policy parameters of the environment, the shifts of the two curves are unambiguous.

This analysis allows us to build the intuition for the second part of the paper, where we study whether an acceleration of capital-embodied growth interacted with certain labor market institutions (i.e., welfare benefits and firing taxes), whose strength differs between U.S. and Europe, can account for the different evolution of U.S. and European labor markets. The main result of our quantitative exercise is that the calibrated model generates over half of the observed differential rise in unemployment: a permanent rise in the rate of capital-embodied productivity growth of the magnitude observed in the data increases unemployment rate by 0.6% in the U.S.-type economy and by 4.5% in the rigid European-type economy, with all the increase taking place along the unemployment duration margin (from 8.5 to 18 weeks in the rigid economy), as in the data.

Moreover, the model is quantitatively consistent with the other facts reported in Figure 1: the labor share falls by 7% in the European-type economy and only by half in the flexible U.S.-like economy, and the vacancy rate remains fairly stable in the flexible economy while it plummets in the rigid economy. Finally, the decline in the age of capital implied by the model is perfectly in line with the U.S. data.

Several forces play a role in our model to shape how the U.S. and Europe respond to a faster rate of capital-embodied technology growth. As the rate of productivity growth of new capital accelerates, existing capital-worker matches –which have old vintages of capital– become obsolete faster. In the U.S., this loss of economic value is to partly borne by workers, whose wages fall in order to keep firms from scrapping capital and breaking up matches earlier. In Europe, in contrast, because firing regulations and high unemployment benefits keep labor costs "artificially" isolated from productivity, this kind of wage adjustment is much less effective; instead, firms bear the adjustment by destroying matches earlier and creating fewer jobs. The corresponding sharp increase in unemployment greatly improves the bargaining position of firms, which can now force workers closer to their outside option, thus reducing the wage, and the labor share of output. Thus, in response to a technological acceleration, an economy with rigid, European-like institutions would experience a higher unemployment rate, a more pronounced decline in the labor share, and a lower job creation rate than a flexible economy, like the U.S., would.

The remainder of the paper is organized as follows. In Section 2, we start our analysis with the frictionless economy that represents a useful benchmark for the general model. In Section 3 we move to the frictional environment with costly capital investment, solve the model, and prove the existence and uniqueness of equilibrium. In Section 4 we characterize the comparative statics that build the intuition for the quantitative part. Section 5 presents the calibration of the model and the results of our quantitative exercises, and discusses the related literature in detail. Section 6 concludes the paper.

2 The frictionless economy

We present here a version of the Solow (1960) frictionless vintage capital model where production is decentralized into worker-machine pairs operating Leontief technologies: this decentralized production structure is typical in frictional economies. The competitive economy displays no unemployment, so it cannot serve as a tool to analyze the facts we described. However, it is a useful starting point for our analysis since it embeds many of the economic forces present in the richer (and more complex) frictional model.

Environment– Time is continuous. The economy is populated by a stationary measure 1 of ex-ante equal, infinitely lived workers who supply one unit of labor inelastically. The workers are risk-neutral and discount the future at rate r. Production requires pairing one machine and one worker. Machines (or jobs, or firms, or production units) are characterized by the amount of efficiency units of capital k they embody. A matched worker-machine pair produces a homogeneous output good.

There is embodied and disembodied technical change. The economy-wide disembodied productivity level $z(t)$ grows at a constant rate $\psi > 0$. Technological progress is also embodied in capital and the amount of efficiency units embodied in new machines grows at the rate $\gamma > 0$. Once capital is installed in a machine it is subject to physical depreciation at the rate $\delta > 0$. A production unit that at time t has age a and is paired with a worker has output

$$y(t,a) = z(t)\, k(t,a)^{\omega} = z_0 e^{\psi t} \left[k_0 e^{\gamma(t-a)} e^{-\delta a}\right]^{\omega}, \tag{1}$$

where $\omega > 0$. In what follows we set, without loss of generality, $z(0) = k(0) = 1$.

At any time t firms can freely enter the market upon payment of the initial installation cost $I(t-a)$ for a machine of vintage $t-a$, i.e. firms can choose whether to purchase the newest vintage machine or a machine of any older existing vintage: newer vintages are relatively more expensive to set up, but they are also relatively more productive. The cost of new vintages grows at the rate g.

Rendering the growth model stationary– We will focus on the steady state of the normalized economy; this corresponds to a balanced growth path of the actual economy. It is immediate that for a balanced growth path to exist, we need $g = \psi + \omega\gamma$. In order to make the model stationary we normalize all variables dividing by the growth factor e^{gt}. The normalized cost of a new production unit is then constant at I, and the normalized output of a production unit of age a which is paired with a worker is $e^{-\phi a}$, where $\phi \equiv \omega(\gamma + \delta)$, thus output is defined relative to the newest production unit. Note that the parameter ϕ represents the effective depreciation rate of capital obtained as the sum of physical depreciation δ and technological obsolescence γ. In Appendix A.1, we describe the normalization procedure in detail.

Competitive equilibrium– Assume that the labor market is frictionless and competitive, so that there is a unique market-clearing wage. We start by arguing that profit-maximizing firms always choose the newest capital vintage. The key behind this argument is that the labor required to operate new machines is constant over time, which is why new technologies are better: in fact, technological change allows firms to pair their worker with more and more efficiency units of capital over time by using newer and newer equipment. A firm choosing to invest in old capital would, once in operation, generate lower output at the same wage cost. The lower initial installation cost of the old machine would compensate these losses only partially.[5]

In the steady state the wage rate w, now measured relative to the output of the newest vintage, is constant. Consider a price-taker firm that plans to set up a new vintage machine. The firm optimally chooses the exit age $\bar{a}$ that maximizes the present value of machine lifetime profits

$$\Pi(w) = \max_{\bar{a}} \int_0^{\bar{a}} e^{-(r-g)a} \left[e^{-\phi a} - w\right] da,$$

where $\Pi(w)$ is the profit function. Since flow profits are monotonically declining in age and eventually become negative, there is a unique exit age for new vintages. The intuition is that while the wage w is the same for all firms, output falls compared to the new production units, because of depreciation and obsolescence. Profit maximization leads to the condition

$$w = e^{-\phi\bar{a}}, \tag{2}$$

stating that the price of labor has to equal the relative productivity of the oldest machine, which is also the marginal productivity of labor. The higher the wage, the shorter the life-length of capital since (normalized) profits per period fall and thus reach zero sooner.

Free entry of firms requires that in equilibrium $I = \Pi(w)$. This is the key condition that determines exit age $\bar{a}$, and hence wages. Using the profit-maximization condition (2), the free entry condition can be written as

$$I = \int_0^{\bar{a}} e^{-(r-g+\phi)a} \left[1 - e^{-\phi(\bar{a}-a)}\right] da. \tag{3}$$

Equation (3) allows us to discuss existence and uniqueness of the equilibrium as well as comparative statics. It is straightforward to solve for efficient allocations and show that a stationary solution to the planner's problem reproduces the competitive allocations (see Appendix A.2).

Existence and uniqueness– The right-hand side of the equilibrium condition (3) is strictly increasing in the exit age $\bar{a}$ for two reasons. First, in an equilibrium with older firms, the relative productivity of the marginal operating firm is lower and therefore wages have to

[5] This argument is easy to verify mathematically, so we omit its proof in the text.

be lower and profits higher. Second, a longer machine life increases the time-span for which profits are accumulated. Define $\bar{r} \equiv r - g + \phi = r - \psi + \omega\delta$. The right-hand side of (3) increases from 0 to $1/\bar{r}$ as $\bar{a}$ goes from 0 to infinity. Taken together, these facts mean that there exists a unique steady state exit age $\bar{a}^{CE}$ whenever $I < 1/\bar{r}$. This condition is natural: unless you can recover the initial capital investment at zero wages using an infinite lifetime ($\int_0^\infty e^{-\bar{r}a} da = 1/\bar{r}$ being the net profit from such an operation), it is not profitable to start *any* firm. It is useful, for future reference, to state explicitly

Assumption A1 (existence of the frictionless equilibrium): $\bar{r}I < 1$.

Finally, with a unit mass of workers, all employed, the firm distribution is uniform with density $1/\bar{a}^{CE}$, which is also the measure of entrant firms e_f.

Comparative statics– A larger interest rate r decreases present-value profits, thus lowering entry and increasing the life span of the machine $\bar{a}$. A higher rate of disembodied technical change ψ acts just like a reduced interest rate. An increase in the cost of purchasing a new machine I will raise the life span: fewer machines enter and they stay in operation longer to recover the fixed cost. Conversely, a higher rate of embodied technical change γ lowers the cost of hiring labor because it reduces the relative productivity of the least productive firm. Higher profits imply an increase in entry at the expense of older machines that are forced to exit earlier.[6] It is straightforward to see that the labor share in the competitive economy equals

$$\frac{\phi\bar{a}^{CE}}{e^{\phi\bar{a}^{CE}} - 1}, \tag{4}$$

and thus an increase in the rate of embodied technical change ϕ, a rise in the interest rate r, a fall in disembodied productivity ψ, or a fall in the entry cost I all lead to a reduction in the labor share, one of the key facts of Figure 1.

3 The economy with matching frictions

Frictions and vacancy heterogeneity– Consider an economy with same preferences, demographics and technology, but where the labor market is frictional, in the sense of Pissarides (1990): an aggregate matching function governs job creation. The nature of the firm's decision process remains the same as in the frictionless economy: there is free entry

[6] More formally, the right-hand side of equation (3) is increasing in the growth rate γ since $\bar{r}$ is independent of γ and ϕ is increasing in γ.

of firms which buy a new piece of capital, participate to the search process, start producing upon matching with a worker, and finally exit when the capital is so old that it no longer generates positive profit flows. Searching is costless: it only takes time. Existing matches dissolve exogenously at the rate σ: upon dissolution, the worker and the firm are thrown into the pool of searchers.[7]

In this environment there is a distribution of vacant firms heterogeneous in the vintage of their capital, for two reasons: first, newly created firms do not match instantaneously, but stay idle until they find a worker; second, firms hit by exogenous separation will also become idle.

Note here a key difference with the classical search-matching framework (Mortensen and Pissarides, 1998): traditional search-matching models assume that a new machine can be purchased at no cost and that only posting a vacancy entails a flow cost. This assumption implies that the pool of vacancies consists of the newest machines only, and that only matched machines age over time.[8] Our setup is built on the opposite assumption: purchasing and installing capital is costly –an expense which is sunk when the vacant firm starts searching– whereas posting a vacancy is costless.[9] As a result, it can be optimal even for firms with old capital to remain idle.

This class of economic environments is a hybrid between vintage models and matching models. The traditional assumption emphasizes the matching features of the environment, while the explicit distinction between a "large" purchase/setup cost for the machine and a "smaller" search/recruiting flow cost (zero in our model) fits more naturally with its vintage capital aspects, whose emphasis is on capital investment expenditures as a way of improving productivity. In actual economies, new and old vacancies coexist, as in our set-up. Moreover, there are additional reasons that make our assumption more natural. We return on this point later in this section.

Random matching– The meeting process between workers and production units is

[7]We omitted separations from the description of the competitive equilibrium because, without frictions, it is immaterial whether the match dissolves exogenously or not as the worker can be replaced instantaneously by the firm at no cost.

[8]Aghion and Howitt (1994) also describe a vintage capital model with initial setup costs for capital, but they assume that matching is "deterministic": at the time a new machine is set up, a worker queues up for the machine, and after a fixed amount of time the worker and firm start operations. Hence, in the matching process, all vacant firms are equal (although they do not embody the leading-edge technology).

[9]Vacancy heterogeneity will survive the addition of a flow search cost, as long as this cost is strictly less than the initial set-up cost I.

random and takes place in one pool comprising all unemployed workers (i.e., there is no on-the-job search) and all vacant firms: vacant firms are distinguished by the age of their capital, but this characteristic is not observable to workers who therefore cannot direct their search. We assume that the number of matches in any moment is determined by a matching function $m(v,u)$ with constant returns to scale, where $v \equiv \int_0^{\bar{a}} \nu(a)da$ is the total number of vacancies, $\nu(a)$ denotes the measure of vacant firms of age a, and u is the total number of unemployed workers. We also assume that $m(v,u)$ is strictly increasing in both arguments and satisfies some standard regularity conditions.[10]

A firm meets a worker at the rate λ_f. The rate at which a worker meets a firm with capital of age a is $\lambda_w(a)$ and the unconditional rate at which she meets any firm is $\lambda_w \equiv \int_0^{\bar{a}} \lambda_w(a)da$, where $\bar{a}$ is the job-destruction age. Using the notation $\theta = v/u$ to denote labor market tightness, we then have that

$$\lambda_f = \frac{m(\theta,1)}{\theta}, \qquad (5)$$

$$\lambda_w(a) = m(\theta,1)\frac{\nu(a)}{v}. \qquad (6)$$

The expression for the meeting probability in (5) provides a one-to-one (strictly decreasing) mapping between λ_f and θ. Thereafter, when we discuss changes in λ_f, we imagine changes in θ. Throughout, and for tractability, we will focus on steady-state analysis; thus, the notation presumes no time-dependence. In particular, all distributions are stationary over time.

Institutions– We introduce two labor market institutions that will play a crucial role in the interpretation of the divergence of outcomes between U.S. and Europe. First, we assume that unemployed workers receive a welfare benefit b every period. Second, we model an employment protection policy that combines a firing tax with a hiring subsidy. In particular, we assume that a vacant firm that hires a worker receives a hiring subsidy T, but upon separation the same amount T has to be paid by the firm as a firing tax. It is easily understood that the policy T is equivalent to a gift of a coupon bond from the government

[10]In particular:

$$\begin{aligned} m(0,u) &= m(v,0) = 0, \\ \lim_{u\to\infty} m_u(v,u) &= \lim_{v\to\infty} m_v(v,u) = 0, \\ \lim_{u\to 0} m_u(v,u) &= \lim_{v\to 0} m_v(v,u) = +\infty. \end{aligned}$$

to the firm upon hiring. The firm is therefore entitled to receive the associated growth-adjusted return, $r-g$, for the duration of the match. At the time of the job separation, the bond must be returned to the government.[11]

Values– Values for the market participants are $J(a)$ and $W(a)$ for matched firms and workers, respectively, $V(a)$ for vacant firms, and U for unemployed workers. Let $w(a)$ denote the wage paid to a worker by an age a firm. The values solve the following differential equation system, which summarizes the flow payoffs of workers and firms:

$$\begin{aligned}
(r-g)V(a) &= \max\{\lambda_f\left[J(a)-V(a)+T\right]+V'(a),0\} && (7)\\
(r-g)J(a) &= \max\{e^{-\phi a}-w(a)-\sigma\left[J(a)-V(a)+T\right]+J'(a),(r-g)\left[V(a)-T\right]\} && (8)\\
(r-g)U &= b+\int_0^{\bar{a}}\lambda_w(a)\left[W(a)-U\right]da && (9)\\
(r-g)W(a) &= \max\{w(a)-\sigma\left[W(a)-U\right]+W'(a),(r-g)U\}. && (10)
\end{aligned}$$

The derivatives of the value functions with respect to a will be negative and are flow losses due to the aging of capital.[12]

Wage determination– In the presence of frictions, a bilateral monopoly problem between the firm and the worker arises, and thus wages are not competitive. As is standard in the literature, we choose a Nash bargaining solution for wages whereby the wage at every instant maximizes the total joint surplus from the match

$$S\left(a\right)\equiv J\left(a\right)+W\left(a\right)-V\left(a\right)-U+T. \tag{11}$$

Note that the surplus function is the same both for a new meeting and for an ongoing relationship. The term T adds to the value of a job $J\left(a\right)$ and plays the role of the subsidy in the first case, whereas subtracts to the outside option $V\left(a\right)$ and plays the role of the tax in the second case: we can therefore avoid a two-tier structure. With outside options as in the above equations, the wage is such that at every instant a fraction β of the total surplus $S\left(a\right)$ of a type a match goes to the worker and a fraction $(1-\beta)$ goes to the firm, implying

$$W(a)=U+\beta S\left(a\right) \text{ and } J(a)=V(a)-T+(1-\beta)S\left(a\right). \tag{12}$$

[11]In general a pure firing tax complicates the analysis significantly. This is a simple way to introduce an employment protection policy without affecting the structure of our equilibrium. We explain this point in detail below.

[12]In Appendix A.3 we describe a typical derivation of the differential equations above.

Using the surplus-based definition (12) of the value of an employed worker $W(a)$ in equation (10) and rearranging terms, we obtain the wage rate as

$$w(a) = (r-g)U + \beta \left[(r-g+\sigma) S(a) - S'(a) \right].$$

Since the wage is not needed in solving the matching model, we postpone the discussion of its determinants to Section 3.2.4.

3.1 Solving the matching model

We characterize the equilibrium of the matching model in terms of two variables: the exit age and the rate at which vacant firms meet workers: $(\bar{a}, \lambda_f)$. The two variables are jointly determined by two key conditions. The first condition, labelled the job destruction condition, expresses the indifference between carrying on and separating for a match with capital of age $\bar{a}$. The second condition, labelled the job creation condition, expresses the indifference for outside firms between creating a vacancy with the newest vintage and not entering. We then demonstrate that a solution to these two equations exists and is unique.

3.1.1 The surplus function

In this class of models all decisions are surplus-maximizing. Thus, it is useful to start by stating the (flow version of the) surplus equation. Using (11) this equation can be described by

$$(r-g)S(a) = \max\{e^{-\phi a} - \sigma S(a) - \lambda_f(1-\beta)S(a) - (r-g)(U-T) + S'(a), 0\}. \tag{13}$$

This asset-pricing-like equation is obtained by combining equations (7)-(12): the growth-adjusted return on surplus on the left-hand side equals the flow gain on the right-hand side, where the flow gain is the maximum of zero and the difference between total inside minus total outside flow values. The inside value includes: a production flow $e^{-\phi a}$, a flow loss due to the probability of a separation of the match $\sigma S(a)$, the opportunity cost of separation $(r-g)T$, and changes in the value for the matched parties, $J'(a) + W'(a)$. The outside option flows are: the flow gain from the chance that a vacant firm matches $\lambda_f(1-\beta)S(a)$, the change in the value for the vacant firm $V'(a)$, and the flow value of unemployment $(r-g)U$. Note a key difference with the traditional model: the value of a vacancy is positive, and it contributes towards a reduction of the rents created by the match.

The solution of the first-order linear differential equation (13) is the function

$$S(a) = \int_{a}^{\overline{a}} e^{-(\bar{r}+\sigma+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - (r-g)(U-T)\right] d\tilde{a}, \tag{14}$$

where we have used the boundary condition associated with the fact that the surplus-maximizing decision is to keep the match alive until an age $\bar{a}$ such that $S'(\bar{a}) = 0$. For lower a's the match will have strictly positive surplus, and for values of a above $\bar{a}$ the surplus will be equal to zero.[13] Intuitively, the surplus is decreasing in age a for two reasons: first, the time-horizon over which the flow surplus accrues to the pair shortens with a; second, the value of a job's output declines with age relative to that of the new vacant jobs.

Equation (14) contains a non-standard term associated to the non-degenerate distribution of vacancies: the non-zero firm's outside option of remaining vacant with its machine reduces the surplus by increasing the "effective" discount rater through the term $(1-\beta)\lambda_f$. Everything else being equal, the quasi-rents in the match are decreasing as the bargaining power of the idle firm or its meeting rate is increasing.

3.1.2 The separation decision

The optimal separation rule $S'(\bar{a}) = 0$ together with equation (14) implies that the exit age $\bar{a}$ satisfies

$$e^{-\phi\bar{a}} = (r-g)(U-T), \tag{15}$$

for a given value of unemployment U. The left hand side of (15) is the output of the oldest match in operation, whereas the right hand side is the flow-value of an idle worker, net of the separation tax. The idea is simple: firms with old enough capital shut down because workers have become too expensive, since the average productivity of vacancies and, therefore, the workers' outside option of searching, is growing at a constant rate. The firing tax, as expected, delays separation. Note that this equation (with $T = 0$) resembles the profit-maximization condition in the frictionless economy, with the worker's flow outside option, $(r-g)U$, playing the role of the competitive wage rate.[14]

We can now rewrite the surplus function (14) in terms of the two endogenous variables

[13] Straightforward integration of the right-hand side in (14) and further differentiation shows that, over the range $[0, \bar{a})$, the function $S(a)$ is strictly decreasing and convex; moreover, $S(a)$ will approach 0 for $a = \bar{a}$.

[14] In fact, later we show that the lowest wage paid in the economy (on machines of age $\overline{a}$) exactly equals the flow value of unemployment.

$(\bar{a}, \lambda_f)$ only, by substituting for $(r-g)(U-T)$ from (15):

$$S(a;\bar{a},\lambda_f) = \int_a^{\bar{a}} e^{-(\bar{r}+\sigma+(1-\beta)\lambda_f)(\tilde{a}-a)} \left[e^{-\phi\tilde{a}} - e^{-\phi\bar{a}}\right] d\tilde{a}. \tag{16}$$

In this equation, and occasionally below, we use a notation of values (the surplus in this case) that shows an explicit dependence of $\bar{a}$ and λ_f. From (16) it is immediately clear that $S(a;\bar{a},\lambda_f)$ is strictly increasing in $\bar{a}$ and decreasing in λ_f. A longer life-span of capital $\bar{a}$ increases the surplus at each age because it lowers the flow value of the worker's outside option, as evident from (15). A higher rate at which firms, when idle, meet workers reduces the surplus because it increases the outside option for a firm and shrinks the rents accruing to the matched pair.

Using (9) and (12) we obtain the optimal separation (or *job destruction*) condition

$$e^{-\phi\bar{a}} + (r-g)T = b + \beta \int_0^{\bar{a}} \lambda_w(a;\bar{a},\lambda_f) S(a;\bar{a},\lambda_f) da, \tag{JD}$$

which is an equation in the two unknowns $(\bar{a}, \lambda_f)$. The rates $\lambda_w(a)$ at which unemployed workers are matched with firms also depend on the two endogenous variables.

3.1.3 The free-entry condition

We define the value of a vacancy of age a using the new expression (16) for the surplus of a match $S(a;\bar{a},\lambda_f)$ together with (12). The differential equation for a vacant firm (7) then implies that the net-present-value of a vacant firm equals

$$V(a;\bar{a},\lambda_f) = \lambda_f(1-\beta) \int_a^{\hat{a}} e^{-(r-g)(\tilde{a}-a)} S(\tilde{a};\bar{a},\lambda_f) d\tilde{a}, \tag{17}$$

where $\hat{a}$ equals the age at which the vacant firm exits. Since vacant firms do not incur in any direct search cost, they will exit the market at an age such that this expression equals 0, from which it follows immediately that $\hat{a} = \bar{a}$. Since in equilibrium there are no profits from entry, we must have that $V(0;\bar{a},\lambda_f) = I$, and we thus have the free-entry (or *job creation*) condition

$$I = \lambda_f(1-\beta) \int_0^{\bar{a}} e^{-(r-g)a} S(a;\bar{a},\lambda_f) da. \tag{JC}$$

This condition requires that the cost of creating a new job I equals the value of a vacant firm at age zero, which is the expected present value of the profits it will generate: a share $(1-\beta)$ of the discounted future surpluses produced by a match occurring at the instantaneous rate

λ_f. The job creation condition is the second equation in the two unknowns $(\bar{a}, \lambda_f)$. It is interesting to point out that the two policies—welfare benefits b and employment protection T—do not directly affect the job creation condition (JC), whereas they enter the job destruction equation (JD) with opposite signs.[15]

3.1.4 The stationary distributions

We now complete the characterization of the equilibrium and derive explicit expressions for the matching probabilities in terms of the endogenous variables $(\bar{a}, \lambda_f)$. Denote with $\mu(a)$ the measure of matches between an a firm and a worker, and denote total employment with μ. The probabilities $\lambda_w(a)$ depend on the steady-state distributions of vacant firms. The inflow of new firms is $\nu(0)$: new firms acquire the new capital and proceed to the vacancy pool. Thereafter, these firms transit stochastically back and forth between vacancy and match (after matched, a firm can become vacant at age $a < \bar{a}$ at rate σ), and they exit at $a = \bar{a}$, whether vacant or matched. This means that $\nu(a) + \mu(a) = \nu(0)$ for all $a \in [0, \bar{a})$. The functions $\nu(a)$ and $\mu(a)$ jump down to 0 discontinuously at $\bar{a}$. For $a \in [0, \bar{a})$, the evolution of $\mu(a)$ therefore follows

$$\dot{\mu}(a) = -\sigma\mu(a) + \lambda_f \nu(a) = \lambda_f \nu(0) - (\sigma + \lambda_f)\mu(a). \tag{18}$$

Exogenous separations $\sigma\mu(a)$ reduce employment, and filled vacancies $\lambda_f \nu(a)$ increase employment.[16] It is easy to demonstrate that

$$\frac{\mu(a)}{\mu} = \frac{1 - e^{-(\sigma+\lambda_f)a}}{\bar{a} - \frac{1}{\sigma+\lambda_f}\left(1 - e^{-(\sigma+\lambda_f)\bar{a}}\right)}, \text{ and} \tag{19}$$

$$\frac{\nu(a)}{v} = \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}\left(1 - e^{-(\sigma+\lambda_f)\bar{a}}\right)}. \tag{20}$$

[15] At this point we can explain why we chose to model employment protection as a combination of firing tax and hiring subsidy. With a pure firing tax T without a hiring subsidy, the destruction age for a vacancy, $\hat{a}$, is no longer the same as the destruction age for an existing match, $\bar{a}$. This is because prior to matching, the cost of the dissolution of the match in the future—the firing tax—is not a liability, and it will not become one until the match is formed. An existing match dissolves at an age such that the total surplus of the match equals $-T$: at this point the marginal profit flow from production is equal to zero. A vacancy would not be posted at such an age because the total surplus is negative, so vacant capital withdraws from the market at an earlier age than matched capital: $\hat{a} < \bar{a}$. Formally, the presence of a firing tax leads to two different notions of surplus: surplus upon hiring, where the disagreement in the bargaining does not imply the payment of the tax, and surplus during the match, where it does. As a result, we have a two-tier labor market (with two wage functions) and two destruction thresholds. The structure of the equilibrium in the model with the firing tax/hiring subsidy is therefore much simpler to study and the employment protection policy retains the standard effects (see below for the comparative statics).

[16] In Appendix A.4 we describe in detail how to derive (18), and the stationary measures (19), and (20).

The employment (vacancy) density is therefore increasing and concave (decreasing and convex) in age a. The reason for this is that for every age $a \in [0, \bar{a})$ there is a constant number of machines, and older machines have a larger cumulative probability of having been matched in the past. This feature distinguishes our model from standard-search vintage models where the distribution of vacant jobs is degenerate at zero and the employment density is decreasing in age a at a rate equal to the exogenous destruction rate σ.

With the vacancy distribution in hand, we now have the explicit expression for the value of $\lambda_w(a)$,

$$\lambda_w(a; \bar{a}, \lambda_f) = \lambda_w \frac{\nu(a)}{v} = m(\theta, 1) \frac{\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}}{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}\left(1 - e^{-(\sigma+\lambda_f)\bar{a}}\right)}, \tag{21}$$

which depends only on the pair of endogenous variables $(\bar{a}, \lambda_f)$, given the relation between θ and λ_f.

3.2 Analysis of the equilibrium

We now proceed to show that there exists a unique steady state for the economy with frictions. We characterize the equilibrium in terms of the rate at which firms find workers, λ_f, and the exit age, $\bar{a}$. This pair of variables are jointly determined by the two key firm decisions, entry and exit, through the job creation condition (JC) and the job destruction condition (JD).[17] We begin by studying each of the two steady-state equations in turn. The formal proofs of our arguments are contained in the Appendix.

3.2.1 The job creation condition (JC)

The job creation condition states that a potential entrant makes zero profits from setting up a new machine. We have

Lemma 1. *The job creation condition (JC) describes a curve that is negatively sloped in $(\bar{a}, \lambda_f)$ space.*

Lemma 1 follows from the fact that the vacancy value of new firms is increasing in the exit age $\bar{a}$ and in the rate at which firms find workers λ_f. A longer life-span of capital $\bar{a}$ increases the vacancy value of a new machine by raising the surplus of every potential match. More

[17] Clearly, since the surplus of creating a match between any unemployed worker (they are all equal) and any idle firm is positive, the decision whether to create or not a match upon meeting is trivial. Put differently, vacant firms exit at age $\bar{a}$ because older capital would be turned down by workers who will prefer searching for a better vintage in the pool.

firms need to enter to restore the zero-profit condition.[18] The job creation condition thus defines a curve in $(\bar{a}, \lambda_f)$ space that has a *negative* slope: if the life-length of a machine $\bar{a}$ goes up, the probability of meeting a worker λ_f has to go down so that the value of entry remains at I. This condition is plotted in Figure 2.

Lemma 2. *As $\bar{a} \to \infty$, the (JC) curve asymptotes to a strictly positive value*

$$\lambda_f^{\min} \equiv \frac{\bar{r}I}{1-\bar{r}I} \frac{\bar{r}+\sigma}{1-\beta}. \tag{22}$$

Suppose that λ_f is very close to zero. Even if the life-length of capital is infinite, vacant firms meet workers with a probability that is too low for the initial investment to pay off in expected terms. The asymptote value $\lambda_f^{\min}$ is increasing in I and in the effective discount rate $\bar{r}+\sigma$, as they both make it more difficult to recover the initial investment, and it is decreasing in $(1-\beta)$, the surplus share accruing to the firm. Note that if $\bar{r}I > 1$ (recall that A1 is the condition for existence of the frictionless equilibrium), this asymptote would be negative.

Lemma 3. *As $\lambda_f \to \infty$, the (JC) curve asymptotes to the exit age of the frictionless economy, $\bar{a}^{CE}$.*

Suppose firms live for a very short period: $\bar{a}$ is very close to zero. Even if vacant firms meet workers for sure (with an arbitrarily high rate λ_f), the life-length of capital is too short for the initial investment I to pay off. That is, a minimum life-length is necessary to ensure that the free-entry condition can be satisfied with equality. The asymptote can be worked out to lie exactly at the destruction age for the competitive solution $\bar{a}^{CE}$. Intuitively, as $\lambda_f \to \infty$, the matching frictions disappear for vacancies and the firms' entry problem becomes the competitive problem (3) with solution $\bar{a}^{CE}$.

This result provides a key motivation for our assumption on the set-up cost I. In this type of environments, when the capital is matched with a worker, it ages until a break-up results from the capital becoming too obsolete relative to the cost of keeping the worker. Frictions reduce this cost and extend the life of capital, i.e., in general $\bar{a} > \bar{a}^{CE}$. In the traditional model, as the matching friction is made weaker, the separation occurs earlier

[18] Even though the surplus of a match declines in λ_f as discussed above, it is straightforward to prove that this second-order effect is always dominated.

and earlier and in the limit, with no matching friction, all capital is new, so vintage effects are absent. In other words, in that model frictions are fully at the origin of vintage capital. Lemma 3 shows that in our model instead as labor market frictions vanish, the maximum life of capital converges to the oldest vintage in activity in the competitive model discussed in Section 2. In this sense, our assumption is a more natural way to introduce frictions into the standard Solow (1960) growth model where investment decisions are at the origin of vintage effects, even in the absence of frictions.

3.2.2 The job destruction condition (JD)

The job destruction condition states that the productivity of the marginal match at the cutoff age $\bar{a}$ equals the flow value of the outside option for the worker. The characterization of the job destruction condition (JD) turns out to be slightly more involved, so to make the exposition simpler it is convenient to proceed under two additional assumptions:

Assumption A2 (relative size of policies): a) $b - (r - g)\,T < 1$ *and b)* $b - (r - g)\,T > 0$.

Assumption A3 (shape of the matching function): $m(v, u) \equiv Av^{\alpha}u^{1-\alpha}$, *with* $\alpha > 1/2$.

For the labor market to be viable, we need to impose the restriction $b < 1 + (r - g)\,T$, where the right hand side of the inequality represents the normalized output on the best firm; otherwise no worker would accept any job. The other inequality $b > (r - g)\,T$ is useful for characterizing the slope of the (JD) condition. The Cobb-Douglas specification is chosen in the literature virtually every time a functional form has to be selected, including in our quantitative section.[19] The restriction $\alpha > 1/2$ is again helpful for describing the shape of the (JD) condition.

Lemma 4. *If A1-A2 hold, then the job destruction condition (JD) describes a curve that is positively sloped in* $(\bar{a}, \lambda_f)$ *space.*

Note first that the capital value of being unemployed depends on the expected surplus from a match, and we know that the surplus function decreases in λ_f, as explained earlier. Also, a higher λ_f decreases the unconditional meeting probability for workers λ_w by definition. But there is also a counteracting effect that is unique to our model with an endogenous vacancy distribution: a faster meeting rate for vacant firms shifts the vacancy density towards younger

[19] In their survey of the matching function, Petrongolo and Pissarides (2001, Table I) list 27 recent empirical studies, 24 of which use the log-linear specification.

vintages with larger potential surplus. We show that when the matching technology is Cobb-Douglas with vacancy share α, the value of search is decreasing in λ_f because the decline of the unconditional probability overcomes the counteracting shift in the vacancy distribution. Intuitively, one can write $\lambda_w \simeq A^{\frac{1}{1-\alpha}} (1/\lambda_f)^{\frac{\alpha}{1-\alpha}}$ so the larger is α, the sharper is the decline in λ_w for a given rise in λ_f. Since the value of search—the RHS of the (JD) condition—falls with λ_f, to reestablish equality in that equilibrium condition, $\bar{a}$ has to rise to decrease the value of output on the marginal job.[20] We conclude that the (JD) curve has a positive slope in $(\bar{a}, \lambda_f)$ space (see Figure 2).[21]

Lemma 5. *As $\lambda_f \to \infty$, the (JD) curve asymptotes to $\bar{a}^{\max} = -\ln[b - (r-g)T]/\phi > 0$.*

This result tells us that when the meeting frictions disappear, the surplus goes to zero and output on the marginal job equals the wage, which, in turn, would equal the marginal value of leisure, given by the welfare benefit b. Recall that under A1, the argument of the logarithm is positive.

3.2.3 Existence and uniqueness

Based on our characterization of the (JC) and (JD) curves we can now state a set of conditions that imply the existence and uniqueness of the steady-state equilibrium, depicted in Figure 2.

Proposition 1. If and only if A1 and A2a hold and $\bar{a}^{\max} > \bar{a}^{CE}$, an equilibrium with finite values of the pair $(\bar{a}, \lambda_f)$ exists. If, in addition, A2b and A3 hold, then the equilibrium is unique.

Proof. We first prove the necessity of each condition. If A1 is violated and $\bar{r}I > 1$, then $\bar{a}^{CE} = 0$ and $\lambda_f^{\min} < 0$, thus the (JC) curve lies outside the positive orthant. As $\bar{r}I \to 1$, $\lambda_f^{\min} \to \infty$ and the (JC) and (JD) curves do not intersect for a finite value of λ_f. If A2a is violated, the labor market is not viable and if $\bar{a}^{\max} \leq \bar{a}^{CE}$, then the (JC) curve lies strictly above the (JD) curve, and there is no intersection. Hence, if any of the conditions of the

[20] Note the effect of $\bar{a}$ on the value of search. First, the surplus is increasing with $\bar{a}$. However, the probability of meeting any given vintage—which the surplus function is weighted by—decreases as $\bar{a}$ goes up; in particular, it becomes relatively more likely to meet older vintages, and older vintages have lower surplus than younger ones. The latter effect is unambiguously dominated by the former effect with the assumed aggregate matching function.

[21] If one allows for a general CRS matching function, the proof still goes through if a the elasticity of matching with respect to vacancies is large enough.

Proposition is violated, no equilibrium will exist. To prove sufficiency, it is enough to consider that if $\bar{r}I < 1$, then $\lambda_f^{\min}$ and $\bar{a}^{CE}$ are positive and finite, and if $\bar{a}^{\max} > \bar{a}^{CE}$, then the two curves intersect at least once in the positive orthant and an equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$ exists. Furthermore, under A2b and A3 by Lemma 4 the (JD) curve is monotonically increasing, and since the (JC) curve is monotonically decreasing, the intersection of the two curves (and the equilibrium) is unique. ■

3.2.4 Wage determination

Using the differential equation for the surplus (13), we obtain the wage equation

$$w\left(a\right) = (r-g)U + \beta\left[e^{-\phi a} - \lambda_f\left(1-\beta\right)S\left(a\right) - \left(r-g\right)\left(U-T\right)\right]. \tag{23}$$

The Nash wage rate exceeds the flow value of unemployment by a fraction β of the quasi-rents. This latter term is composed by the production flow $e^{-\phi a}$ plus the capital income $\left(r-g\right)T$, net of the worker's flow outside option $(r-g)U$ and net of the firm's expected surplus share of being in an alternative match $\lambda_f\left(1-\beta\right)S\left(a\right)$. The last term is age-specific and it is intrinsically related to the value to older firms of becoming vacant. In standard models, this value is zero for every firm.

The wage equation also confirms that at the separation age $\bar{a}$, both the firm and the worker are indifferent between continuing the match and separating, i.e., there is no transfer between the parties that can extend the duration of the match. Evaluating (23) at $\bar{a}$ together with the destruction condition demonstrates that

$$e^{-\phi\bar{a}} + (r-g)T = w\left(\bar{a}\right) = (r-g)U,$$

and thus the flow profits are zero and the firm is indifferent between continuing to operate and shutting down (the first equality) and the worker is indifferent between working and entering unemployment (the second equality).

4 Comparative statics

We now study how different parameters of the model $(b, T, \gamma, r, \psi, A)$ shift the job creation and the job destruction curves and affect the equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$. We then study the implications of changes in the two endogenous variables for equilibrium unemployment. Finally, we use these results to suggest a qualitative interpretation of the different evolution of labor markets between U.S. and continental Europe through the interaction between the rate of embodied technological change γ and the institutional pair (b, T).[22]

Lemma 6. *A rise in b does not shift the (JC) curve but shifts the (JD) curve downward, inducing a fall in $\bar{a}^*$ and a rise in λ_f^*. A rise in T does not shift the (JC) curve, but shifts the (JD) curve upward, inducing a rise in a^* and a fall in λ_f^*.*

A simple inspection of the (JC) curve reveals that it is unaffected by either policy. A higher benefit will increase workers' outside options, so in order to restore the (JD) condition, output on the marginal job has to increase. Hence, for a given value of λ_f, the exit age $\bar{a}$ must fall, which induces a downward shift of the job-destruction curve. Workers become more expensive for firms without becoming more productive, and therefore machines are scrapped earlier. The equilibrium moves along the (JC) curve and both the life length of firms and labor market tightness fall unambiguously. In particular, the general equilibrium feedback weakens the fall in the life-length of capital, but transfers part of the impact of the shock on a reduction in firms' entry.[23] A larger firing tax T augments the value of maintaining the job; thus, for a given value of unemployment and fixing λ_f, to restore the (JD) curve, $\bar{a}$ must rise inducing an upward shift of the (JD) curve. As a result, matches last longer and firms increase profits, so more firms enter to restore the ex-ante zero-profits condition and their equilibrium contact rate λ_f^* is reduced.

Lemma 7. *A rise in γ shifts both the (JC) curve and the (JD) curve downward, inducing a fall in $\bar{a}^*$. The change in λ_f^* is ambiguous.*

The comparative statics for γ are somewhat more complicated because an increase in the rate of technological obsolescence γ has two counteracting effects on the surplus function (16). First, a higher γ means that a vintage's output relative to the frontier falls at a faster rate with age, which decreases the surplus of a match. On the other hand, a higher γ reduces the relative output of the marginal technology of age $\bar{a}$ compared to the frontier, thereby shrinks the outside option value of a worker, and increases the surplus of a match. The older

[22]Note that the parameters b and T have also implications for the degree of downward wage rigidity, given that with Nash bargaining the lower bound for wages is the workers' outside option, which is strictly increasing in b and T.

[23]Upon impact, the higher b leads to a higher wage, lower profits, and shorter job duration; the reduction in firms' profits, in turn, decreases their incentive to enter the labor market with new machines (λ_f increases). The implied fall in the meeting rate for workers tends to reduce their outside option and hence their hiring costs and increase profits, therefore making a smaller fall in $\bar{a}$ necessary for the adjustment to the new equilibrium.

a match is, the stronger will be the first effect and the shorter the time period for which it will benefit from the second effect. We show that there is a critical age such that for vintages younger (older) than this threshold the surplus rises (falls) with γ.[24]

Notwithstanding this non-monotonicity of the surplus function with respect to γ, we can prove that the shifts of the (JC) and (JD) curves are unambiguous. The intuition for this result is clear if one thinks of a rise in γ as a "capital obsolescence" shock. For a given λ_f, faster obsolescence leads to a shorter optimal exit age for entrant firms, and thus the (JC) curve shifts down. Turning to the (JD) curve, a rise in γ reduces output on the marginal job relative to the frontier technology but also increases the value of search for an unemployed worker, as waiting is compensated by the expectation of being matched to a more productive firm. Both effects lead to the conclusion that for the (JD) condition to hold for a given market tightness, the marginal machine has to be scrapped earlier so the curve will shift downward.

Taking these two shifts together, we see that the life length of firms declines unambiguously with a higher rate of embodied technological change. Whether labor market tightness goes up or down depends on the location of the initial equilibrium, as explained below.

Lemma 8. *A rise in r (fall in ψ) shifts the (JC) and (JD) curves upward, inducing an increase in $\bar{a}^*$. The effect on λ_f^* is ambiguous.*

An increase in the interest rate (or a decline in disembodied growth) lowers the weight on future profits and thus lowers surplus. This surplus reduction makes the job destruction curve shift up: for a given λ_f, the worker who is indifferent between staying on the job and leaving now needs a longer life on the job to counteract the fall in the surplus of the job. Similarly, the job creation curve shifts up: a lower surplus must be counteracted by a longer life in order for the firm to remain indifferent between entering and not entering. As a result, higher interest rates induce a rise in $\bar{a}^*$, whereas the impact on λ_f^* cannot be signed and once again, depends on the initial location of the equilibrium.

The severity of the matching friction can be regulated with the level of the shift parameter of the matching function (A in the Cobb-Douglas formulation).

Lemma 9. *In the limit, as $A \to \infty$, the frictions disappear and the equilibrium with frictions converges to the competitive equilibrium.*

[24] In particular, at age zero there is no obsolescence effect, so the surplus of a new machine grows unambiguously with γ in the standard Aghion-Howitt/Mortensen-Pissarides framework where all vacancies are of age zero.

As $A \to \infty$, the matching friction vanishes and the equilibrium of the economy entails $\overline{a}^* \to \overline{a}^{CE}$ and $\lambda_f^* \to \infty$. Therefore, frictions extend the life of capital, but are not necessary for old machines to be operated by workers in equilibrium.

Having characterized the changes in the equilibrium pair $\left(\bar{a}^*, \lambda_f^*\right)$ for a given parameter change, we can study how they determine equilibrium unemployment.

Unemployment– In steady state, the flow into unemployment equals the flow out of unemployment. That is,

$$\sigma\mu + \mu(\bar{a}) = \lambda_w u = m(\theta, 1)u. \tag{24}$$

To understand how unemployment responds to changes in the pair $(\bar{a}, \lambda_f)$, it is convenient to use the fact that $\mu = 1 - u$ and restate (24) as

$$\frac{u}{1-u} = \frac{\sigma + \mu(\bar{a})/\mu}{m(\theta, 1)}, \tag{25}$$

which is simply the product of unemployment incidence and duration. The degree of endogenous job destruction $\mu(\overline{a})/\mu$, that is, the fraction of matched jobs destroyed at $\bar{a}$, can be read from (19). A rise in $\bar{a}$ reduces the unemployment rate, since endogenous job destruction is reduced. A rise in λ_f has two counteracting effects. First, a higher λ_f reduces the meeting probability for unemployed workers, which in turn increases unemployment duration. Second, a higher λ_f reduces endogenous job destruction, which in turn reduces unemployment incidence. From what we said, it is clear that one can read changes in the two dimensions of unemployment, incidence and duration, directly from how the curves shift in the $(\bar{a}, \lambda_f)$ space.

Finally, it is useful to discuss the comparative statics of the labor share. A shorter life-length of capital $\bar{a}$ unambiguously increases the labor share through a rise in the equilibrium outside option of the worker $e^{-\phi\bar{a}}$. Instead, a larger firm's meeting rate λ_f reduces the worker's value of search and improves the firm's threat point in the bargaining, thus the wage share of output in each match deteriorates.[25] It is worth remarking that this mechanism is reinforced by the fact that firms' outside options respond to shocks in equilibrium, a unique feature of our environment that is absent in the standard matching model where the value of vacancies is fixed at zero.

[25]The closed-form expression for the labor share in our model (complex, so we omit it) shows a second-order effect following an increase in λ_f or a reduction in $\bar{a}$ that tends to weaken the direct effects: the shift in the employment distribution towards younger vintages which have smaller labor share of output (recall that the labor share is 1 for the oldest firms).

4.1 The U.S.–Europe comparison: a qualitative analysis

Figure 1 shows that until the first half of the 1970s Europe and the U.S. had both low unemployment rates. Our thought experiment is as follows: think of Europe and the U.S. as being different only via the levels of welfare benefits b (more generous in Europe) and employment protection T (stricter in Europe). Can our model generate, at least qualitatively at this stage, a situation where the two economies look indistinguishable for low values of embodied technical change γ, but they dramatically diverge in responding to an obsolescence shock, i.e., an acceleration in the rate of embodied technological change γ?

Institutional differences– A more generous welfare system, higher b, decreases $\bar{a}^*$ and increases λ_f^*, which leads to a higher unemployment rate: both incidence and duration are higher. Interestingly, as they decrease $\bar{a}^*$ and increase λ_f^*, welfare benefits have conflicting effects on the labor share, so one should not expect large differences in the labor share across economies with different b.

A stricter employment protection system, higher T, increases $\bar{a}^*$ and decreases λ_f^*, inducing exactly the opposite effects of those of welfare benefits: an unambiguously lower unemployment rate and higher wage inequality. The larger firing tax will tend to increase the labor share as workers, through the threat of separation in bargaining, can appropriate an additional fraction of the surplus equal to $\beta(r-g)T$, as evident from equation (23). Thus, an economy with high values of *both* welfare benefits and the firing tax can display a higher labor share than an economy with both low benefits and low firing taxes while the unemployment rate is similar: technically, the (JC) curve for the two economies will be the same and the (JD) curve is shifted in offsetting directions by the two policies. This was the state of affairs of Europe and the U.S. in the early 1970s.

Embodied technical change: the acceleration– Irrespectively of the level of policies, a rise in γ shifts the job creation curve by the same amount. The obsolescence shock and the different institutions interact entirely through the shift of the job destruction curve. Intuitively, an obsolescence shock reduces the value of production (compared to the frontier) of every job, including the marginal job. Since output on the marginal job has to equal the flow value of unemployment, either the latter declines through a fall in λ_w or firms scrap their machine earlier ($\bar{a}$ falls) and fewer vintages of capital remain in operation, increasing output on the marginal job mechanically. Simple inspection of equation (JD) shows that as

γ increases, the size of the fall in the LHS of the equation rises with T: in the Europe-like economy the size of the deviation is larger due to higher T. Moreover, in this economy most of the value of unemployment (the RHS of the job destruction condition) is made up by benefits b rather than by the value of search; thus, the decline in either $\bar{a}$ or λ_w, and the consequent shift of the curve, has to be significantly larger to reestablish equality in the optimal destruction condition, with more dramatic implications for unemployment.

Figure 3 portrays the two extreme cases that can arise: in the upper panel, the rigid Europe-like economy adjust through a sharp decline in $\bar{a}$ and ends up with a larger unemployment rate associated to higher job destruction and bigger unemployment incidence. In addition, the fall of $\bar{a}$ will increase the value of unemployment in equilibrium, and the labor share. In the lower panel of Figure 3, the rigid economy instead displays a sharp rise in λ_f, so the higher unemployment rate arises as a consequence of lower job creation and longer unemployment duration. In these economies, where large adjustments take place through the unemployment duration margin, the strong decrease in the worker's meeting rate improves the firm's outside option with the result that the worker's wage is squeezed towards its own outside option. Thus, we should expect a fall in the labor share in such economies.

As argued in the introduction, the differential rise in unemployment across the two regions is attributable to the duration margin; moreover, in Europe the labor share and the vacancy rate declined more dramatically (Figure 1). Overall, the lower panel comparative statics provide an accurate qualitative description of the differential dynamics between U.S. and European labor markets.

5 The quantitative analysis

We now go beyond a study of qualitative comparative statics and calibrate the model economy to answer the following questions: can our model account quantitatively for the differential behavior of unemployment in the United States and Europe over the past thirty years? Can the model generate the rise in unemployment through a surge in unemployment duration and a stable separation rate? Is the model also quantitatively consistent with the additional facts we documented concerning the relative evolution of the labor share and the vacancy rate?

In our experiment we calculate the steady-state responses to the observed increase in the rate of embodied technological change γ for two model economies that only differ with respect to the institutional pair (b, T). This is the same thought experiment as in the qualitative analysis. There, the success of the model depends entirely on the location of the pre-shock equilibrium and only a quantitative analysis can shed light on this point.

5.1 Calibration

Methodology– In the calibration procedure, we aim to match the key labor market variables of Figure 1 in the U.S. and Europe for the 1960s. The objective of the exercise is to account for the change in all these variables until 1995.[26] Overall, we have 12 parameters to calibrate, $(r, A, \alpha, \sigma, \beta, \delta, I, \psi, \omega, \gamma, b, T)$, whose values are summarized in Table 1.

Parameters calibrated "externally"– We set r to match an annual interest rate of 4% (Cooley, 1995). We normalize the scale parameter A of the matching function to 1 and we set the matching elasticity with respect to vacancies $\alpha = 0.5$, an average of the values reported in the comprehensive survey of empirical estimates of matching functions by Petrongolo and Pissarides (2001, Table 3).

Table 1: Calibration of the Model Economy

Parameter	Value	Moment to match
r	0.040	interest rate (Cooley, 1995)
A	1.000	normalization
α	0.500	micro-estimates (Petrongolo-Pissarides, 2001)
σ	0.206	separation rate (Shimer, 2004)
β	0.748	labor share (OECD database)
δ	0.064	average life of capital (BEA, 2004)
I	4.126	unemployment duration (Abrahams and Shimer, 2002)
ψ	0.050	disembodied technical change (Hornstein-Krusell, 1996)
ω	0.312	growth rate of output per capita (OECD database)
γ	0.040-0.077	relative price of new investments (Cummins-Violante, 2002)
b	0.050-0.400	welfare benefits (OECD, 1996 and Hansen, 1998)
T	0.000-7.000	firing tax (OECD, 1999)

Note: A unit time period represents one year.

Parameters calibrated "internally" – We simultaneously calibrate $(\sigma, \beta, \delta, I)$ so that the steady-state of the model in the low technical change regime generates: (1) an annual

[26] The technological shock we model is likely to have hit the economy around the mid 1970s. See Hornstein and Krusell (1998) and Acemoglu (2002) for discussions of the timing of the technological acceleration.

worker's separation rate from employment to unemployment equal to 25%, as reported in Shimer (2004, page 10). This value is also consistent with estimates of the employment-unemployment flow in several European countries (see the report published by the CEPR, 1995).[27] (2) A labor income share of 0.70 (Figure 1). (3) An average age of capital of 11.5 years, as reported by the Bureau of Economic Analysis (2004) for the mid 1960s (Figure 1). And, (4) an average unemployment duration of approximately 8-9 weeks, as reported by Abrahams and Shimer (2002). The separation rate together with the average unemployment duration imply an unemployment rate of 4%, roughly in between the two values of U.S. and Europe in the 1960s (Figure 1).

Embodied technical change– We have two sources of growth in the model: disembodied, at rate ψ and capital-embodied. We set $\psi = 0.8\%$, the estimate of annual disembodied growth in the U.S. for 1954-1993 computed by Hornstein and Krusell (1996). At least since Greenwood et al. (1997), a number of authors have suggested to measure the speed of embodied technical change through the (inverse of the) rate of decline of the quality-adjusted relative price of capital. As documented in Gordon's (1990) influential work on quality-adjusted prices for durable goods, and more recently by Cummins and Violante (2002), the rate of change in the relative price of new capital investments in the U.S. has decreased substantially from -2% before the mid-seventies up to -4.5% in the 1990s, suggesting an acceleration in capital-embodied growth.[28] In our environment embodied technical change is also directly reflected in the relative price of new capital, as demonstrated in the Appendix. Since the cost I of new vintage machines in terms of the output good is growing at the rate g, but the number of capital efficiency units embodied in new vintages is growing at the rate γ, the price of quality-adjusted capital (efficiency units) in terms of output is changing at the rate $g - \gamma$. Given the observed average output growth rate $g = 2\%$, a 2% rate of price decline for capital before 1970 implies $\gamma = 4\%$. From the relation $g - \gamma = \psi - (1 - \omega)\gamma$, we obtain a capital share parameter $\omega = 0.312$. This estimate, together with the assumption that ψ remains constant, means that $\gamma = 7.7\%$ to generate a decline in the relative price of

[27] In the model, the separation rate, i.e., the unconditional probability that a worker separates from a job within the period, is defined as $[\sigma + \mu(\bar{a})]/\mu$. Note that it would be incorrect to match this variable to job destruction rates (i.e., job flows rather than worker flows, as we do) since the event occurring at rate σ involves only a separation of workers and machines, but not the destruction of the job.

[28] This estimate is obtained averaging the quality-adjusted prices for equipment and software with the unadjusted prices for structures (see Cummins and Violante, 2002 for details). Other authors, using measurement techniques different from quality-adjusted relative prices, arrived at very similar conclusions on the pace of embodied technical change in the postwar era (see for example Hobijn (2000)) for the United States.

capital of 4.5% per year.

Institutions– The parameter b is meant to summarize a wide range of benefit policies that vary with unemployment duration and family status (none of which we model). The OECD Employment Outlook (1996) computes replacement rates for unemployment benefits in OECD countries from 1961 to 1995 for two earnings levels, three family types, and three durations of unemployment. The reported average replacement rate for the United States was 10%, whereas for many European countries the replacement rates were 40% or higher (Chart 2.2, page 29). Hansen (1998) argues convincingly that the OECD replacement rates for Europe understate the generosity of benefits, as many European governments offer long-term social assistance schemes in addition to pure unemployment benefits. Our parameter should reflect these policies, since most of them are not earnings-related and have indefinite duration. The corrected replacement ratios to account for social assistance are much larger, around 75% of average wages (Hansen 1998, Graph 3, page 29). [29]

The OECD Employment Outlook (1999, Table 2.A.3) calculates the size of firing costs for a large set of countries: in the U.S. the firing tax T is estimated to be roughly zero, whereas in Europe it varies substantially by tenure length: for tenures around 4 years, the value implied by our separation rate, the firing cost is close to 4 months of salary.

The experiment– We choose to represent Europe as an economy that differs from the U.S. only in terms of institutions. This choice simplifies the interpretation of the results since the diverging outcomes are entirely attributable to differential policies. In the baseline economy, which we interpret as the U.S. economy before the technological acceleration, we set $b = 0.05$, which implies a ratio of welfare benefits to average wage of roughly 10% and $T = 0$, in line with the OECD calculations. To model European-type economies, we gradually increase b to 0.4, which implies a ratio of welfare benefits to average wage of 75% and, at the same time, we raise the firing tax T in order to generate the same initial unemployment rate as the U.S., 4% in the initial steady-state, in line with Table 1. This strategy requires setting $T = 7$ which implies a value for the firing tax around a third of the average yearly wage in the model, well in line with the OECD data. In our experiment, we gradually increase γ to match the observed embodied acceleration and study the response of some key labor market variables in the two economies.

[29]These replacement rates are calculated for a 40-year old single male production worker. Ljungqvist and Sargent (1998, Table 3, page 523) report similar evidence from a different source).

This experiment relies on three maintained hypotheses. First, we treat the data for the late 1960s and mid-1990s as both representing steady states. The dynamics of the standard vintage-capital/matching model have already proven to be very complex (see Postel-Vinay, 2002) and in addition, the dynamics of our generalized environment would have to keep track of the evolution of the entire distribution of machines across vintages. Thus, for simplicity we presume that the dynamics associated with our experiment are roughly consistent with treating these two time periods as steady states. Second, we maintain that the degree of rigidity of institutions in Europe has not changed substantially since the early 1970s. Bertola, Blau, and Kahn (2001) and Blanchard and Wolfers (2000) find that time-variation in labor market institutions is small compared to cross-country differences and to the size of macroeconomic shocks and thus empirically accounts for a minor fraction of unemployment rate differentials. Third, we assume that the acceleration in the rate of capital-embodied technical change has the same magnitude in the U.S. and Europe. A recent OECD study (Colecchia and Schreyer, 2002) measures the decline in relative price for several high-tech equipment items across various countries in Europe from 1980 to 2000. Table 2 shows that in the last decade large European countries experienced an acceleration quantitatively comparable to the United States. Since high-tech goods drove the technological acceleration in the aggregate price index, we can be confident that the aggregate indexes should display similar patterns.[30]

Table 2: Acceleration of capital-embodied technical change

	U.S.	U.K.	France	Germany	Italy	Europe Average
Computers	7.6	7.3	7.4	4.8	8.7	7.1
Communications	4.4	5.1	5.5	2.6	7.4	5.1
Software	1.9	2.8	3.0	2.2	5.1	3.3

Note: Difference between the rate of decline of quality-adjusted relative price of equipment-capital type in the periods 1990-2000 and 1980-1990. Source: Table 4, Colecchia and Schreyer, 2002)

[30]Ideally, one would like to compare growth rates in the 1970s as well, but these are not available for European countries. Table 2 shows that in some continental European countries the measured acceleration is even larger than in the United States, although one should keep in mind that the high-tech goods' share of aggregate equipment in these same countries is likely to be smaller than in the United States.

5.2 Results

The main quantitative results of our experiment are reported in Figure 4 where several equilibrium outcomes of the model (unemployment rate, unemployment duration, separation rate, average age of capital, vacancy rate, and labor share) are plotted for a range of embodied productivity growth rates (from 4% to 8%) and for two different combinations of institutions, that we have labelled "flexible" $(b = 0.05, T = 0)$, and "rigid" $(b = 0.4, T = 7)$. The flexible economy is calibrated to the U.S., the rigid economy corresponds to Europe.

Unemployment– A faster rate of technological change raises unemployment for both values of the policy pair, but the increase is much more pronounced for the rigid economies. If we take the view that the new steady-state level of capital-embodied productivity growth is 4.5%, (corresponding to $\gamma = 7.7\%$) then the model predicts that in the baseline flexible economy unemployment barely changes, rising by 0.6 of a percentage point, whereas in the rigid economy it jumps by 4.4 percentage points. It is clear from Figure 4 that the bulk of the differential increase in unemployment is explained by duration: the separation rate changes only very marginally, by less than 1% in all the economies considered, while the increase in unemployment duration is small in the U.S.-type economy (from 8.5 to 10 weeks) but is substantial in the rigid economies, from 8.5 to over 18 weeks. This result is consistent with the recent experience of European labor markets, where labor turnover did *not* increase significantly, and where most of the increase in the unemployment rate is associated to longer durations (see Machin and Manning, 1999, for a survey).

Labor shares– The labor share of the flexible economy is calibrated to the average U.S. value in 1960s, while the labor share of the rigid economy is implied by the combination of policies that yields the same initial steady-state unemployment as for the U.S. Its value is slightly larger than the actual labor share in Europe in the 1960s, but consistent with the European labor shares of the mid 1970s. The labor share declines as γ increases for every value of the policy parameters: a rise in γ from 4% to 7.7% reduces the labor share in the U.S.-like economy by 4% and in the Europe-like economy by 7%. Although the model slightly overpredicts the reduction of the U.S. labor income share, the magnitude of the decline of the labor share for Europe is well in line with the data in Figure 1.

Vacancy rates– The vacancy-employment ratio in the flexible economy has a modest decline, by less than 10%, whereas the fall in the rigid economy is substantial, close to 50%.

The data presented in Figure 1 showed similar magnitudes. This aspect of our experiment demonstrates clearly that in the model the adjustment following the technology shock occurs through a reduction in firms' job creation, and demand for labor.

Average age of capital– In the qualitative analysis of Section 4.1 we argued that the faster obsolescence due to the rise in the productivity growth embodied in new capital leads, unambiguously, to a fall in the average age of capital. Figure 4 shows that the model generates a change in the age of capital in the flexible economy that is remarkably close in magnitude to the one observed in the U.S. data (see Figure 1): from 11.5 to 9.7 years.

Interpretation– The interpretation of the U.S.–Europe quantitative comparison is straightforward in light of our analysis of Section 4.1. Clearly, the results are in line with the comparative statics in the lower panel of Figure 3: since the initial location of the equilibrium of the parameterized model is on the flat region of the job creation curve, our economy responds to the technological shock through a rise in unemployment duration and only a small decline in the age of capital at destruction.

A faster rate of embodied technical change induces a substitution of labor with better and more productive capital. In a flexible economy, this adjustment is partially absorbed by the price, partially by the quantity of labor. In rigid economies, the value of unemployment (hence the wage) is kept artificially high by the generous benefits and the rents on high firing taxes, so it is sheltered from changes in firms' productivity and in the market value of search. As a result, the adjustment in the aftermath of the shock requires massive changes in the workers' meeting rate and in unemployment duration, though a reduction of firms' job creation rates. The sharp increase in unemployment duration improves the bargaining position of the firms so as to squeeze workers against their outside option. This force induces a fall in the labor share.

5.2.1 Digression: vintage capital as an origin of wage dispersion

Unlike in the frictionless economy, in the economy with frictions capital-embodied technical change is a direct source of wage dispersion: the Nash wage payments supporting the surplus-sharing allocation are strictly increasing in firm's productivity, and embodied technical change generates productivity differences across vintages of capital.

One contribution of our quantitative work is that we can use the calibrated model as a measurement tool to estimate how much wage inequality is generated by vintage capital: we find that a vintage differential of 10 years translates into a wage differential around 3.5% for an intermediate value of embodied productivity growth of 5% per year. This is a fairly small number, but the finding should not be surprising since, as explained, the model is designed to generate inequality among ex-ante (upon labor market entry) equal workers originating from a combination of labor market frictions and technological heterogeneity in capital goods.[31]

The direct empirical estimates measuring the extent of wage dispersion that can be attributed to this source is scant: data on the age of capital are difficult to link to wage data at a very disaggregated level, and firm or plant age are only very crude and indirect measures of the vintage of technology in use. Moreover, any systematic relation between wages and capital quality in cross-sectional data would also be hard to interpret, since workers' unobservable characteristics are likely to be correlated with the capital they are matched with (e.g., one might expect some degree of positive sorting). The key limit is the lack of detailed employer-employee matched panel data where one could sharply distinguish the role of workers' individual characteristics from the role of firms' characteristics in wage determination.

Doms, Dunne, and Troske (1998) provide an interesting benchmark of comparison to gauge the plausibility of our findings. They examine a panel of U.S. plants in 1988 for which they can observe both the degree of technological advancement of the plant (i.e., the technologies recently adopted) and the characteristics of the workers (like education). Their estimate of the wage differential induced exclusively by the technological gap between a plant in the first quintile and a plant in the top quintile of a technological scale based on age of adoption is between 2.5% and 4% (Tables V, page 276), a number close to ours, but also statistically insignificant in their regressions.[32]

[31] To put this number in perspective, consider that Katz-Autor (1999) report that the 90-10 log-wage differential that is left as a residual in a typical wage regression, after controlling for observable characteristics of the workers, including fixed effects to capture "unobserved ability", is about 30%. Our model generates a 90-10 log wage differential just below 6%. We conclude that vintage capital, combined with labor market frictions, can explain up to 1/5 of wage dispersion among ex-ante equal workers.

[32] Doms, Dunne and Troske show that cross-sectional estimates of these effects can be severely upward-biased (up to factors of 2 or 3) because of the assortative assignment of high-ability workers to new technologies. This explains the discrepancy with another well-known finding by Bartel and Lichtenberg (1990) –based on cross-industry evidence– that a vintage differential of 10 years translates into a 16% wage differential. Interestingly, when Bartel and Lichtenberg focus on the young, low-education workers their estimate shrinks to 2% (Table 4).

5.3 Discussion of related results in the literature

As emphasized in the Introduction, Ljungqvist and Sargent (1998) study quantitatively the impact of a rise in "economic turbulence" for the equilibrium unemployment rate in an economy with generous welfare state. Their model has a fixed wage distribution and fixed number of jobs, thus the mechanism operates entirely on the labor supply side. In our model, in contrast, workers accept every job offer, but both wages and labor demand (i.e., the number of jobs created) are endogenous. In this sense, the two papers highlight the complementarity between technological shocks and unemployment benefits along two parallel margins: labor supply and labor demand.

Our paper is not the only one to emphasize the labor demand side of the shock-policy interaction. Bertola and Ichino (1995) argued that a rise in economic uncertainty –or turbulence, in the language of Ljungqvist and Sargent– coupled with severe firing costs, makes firms cautious in hiring and deteriorates labor demand. In their model wages are fixed, hence lower labor demand translates into higher unemployment. The authors sketch a simple framework to illustrate their point and do not attempt a quantitative evaluation of their mechanism. Our model is deterministic, hence there is no interaction between uncertainty and firing restrictions. The existence of firing costs allows workers to extract quasi-rents and represents an endogenous source of wage rigidity through the Nash bargaining.

A different argument was advanced in Caballero and Hammour (1998): the uprise of the labor movement in the 1970s triggered an "appropriation" shock that changed the division of quasi-rents away from capital towards labor. The surge in the cost of labor services induced firms to adopt ever more labor-saving technologies. Once again, in presence of wage rigidities this reduction in labor demand translates into high unemployment. Albeit our model does not allow for any direct substitutability between capital and labor at the level of the individual production unit, substitution takes place at the aggregate level: as capital becomes cheaper in efficiency units, the European economy as a whole produces with more productive capital per worker and fewer workers. In terms of the plausibility of their argument, the job destruction equation (JD) in our model shows clearly that an institutional shift in favor of labor that simultaneously increases both unemployment benefits and firing costs needs not necessarily increase the equilibrium unemployment rate.

Mortensen and Pissarides (1999) build a multi-skill version of their classical matching model (Mortensen and Pissarides, 1994) where skills are observable (e.g., education), thus labor markets for workers with different skill levels are naturally segmented in equilibrium. Their shock is a mean-preserving spread of the productivity distribution across skills, to capture skill-biased technical change. The impact of the shock is analyzed in economies with different levels of unemployment benefits and firing taxes. Their main result is that differential institutions can explain a sizeable fraction of the differential rise in unemployment. However, the experiment has two shortcomings: first, the shock implies a rise in inequality also in the rigid economy, a result not supported by the available evidence on changes in the wage distribution in Europe; second, the rise in unemployment is concentrated among the least skilled, a result at odds with the data which show higher unemployment rates among all the education segments of the population in Europe (see Nickell and Bell, 1996). Our model has homogeneous workers, thus admittedly, is silent on the evolution of unemployment rates across skill levels.[33]

Finally, in a recent paper, den Haan, Haefke, and Ramey (2001) study the quantitative implications of interest-rate and TFP shocks within a calibrated version of the traditional Mortensen and Pissarides (1994) framework. Like us, den Haan et al. also ask if polices can account for the differential response of labor markets to these shocks. They find that labor market institutions are important only if the United States and Europe differ substantially in their cross-sectional distributions of match-specific productivities, a dimension of the data that the authors do not attempt to measure. In our model initially the equilibrium productivity distribution across machines are the same in the two economies and, not surprisingly, we found that a reduction in TFP growth ψ by 1% (or, equivalently, a rise of 1% in the interest rate r) has negligible effects on the equilibrium allocations.

6 Concluding remarks

The past twenty years have been marked by very rapid capital-embodied growth. In this paper, we have made an attempt to understand how this type of technological change affects the equilibrium allocations in the labor market, when the labor market is non-competitive.

Notwithstanding the increased complexity introduced in the model by costly capital investment, our qualitative analysis in the first part of the paper demonstrates that it is

[33] Jovanovic (1998) analyzes a frictionless assignment model of vintage capital with heterogeneous workers. Extending our frictional model to allow for heterogeneous also on the worker side is not a simple task: with two-sided heterogeneity, maintaining the assumption of random matching would induce multiplicity of steady-states. Directed search would be the natural alternative in this case.

possible to maintain analytical tractability in characterizing the chief features of a labor market equilibrium.

Obviously, keeping the analysis tractable has some costs. For example, our theory of capital-embodied technical change relies on the assumption that new capital goods enters the market through new firms only. That is, existing production sites are not given the opportunity to buy new capital or to upgrade the existing capital. The focus of our quantitative analysis is on a period where the introduction of new technologies coincided with substantial firms and industry reorganizations which often implied plants' deaths and vast labor shedding for surviving production units (e.g., Breshnahan, Brynjolfsson and Hitt, 2002), so to some extent the assumption is not too stringent. But this does not mean that upgrading is unimportant, we have maintained this assumption in the present paper for simplicity only.

In ongoing work (Hornstein, Krusell, and Violante, 2003) we have explored the effects of allowing firms to retain workers when upgrading their machines. Theoretical derivations and existence proofs are significantly harder with upgrading, and the analysis needs to rely on numerical solution to a larger extent than in the present paper. Our conclusion is stark: a properly parameterized model has the same quantitative implications for the link between capital-embodied growth and labor market outcomes, independently of the seemingly important details regarding technological advancement (i.e., upgrading vs. replacement). The intuition for this "equivalence result" is that upgrading can be a lot better than creative destruction only if it is very costly for firms to meet workers, but our calibration shows that this meeting friction is minor from the point of view of the firm, due to the fact that the average duration of a vacancy is quite short in the data, and in the calibrated model.

In the second part of the paper, we have applied our framework to a quantitative investigation of the European unemployment experience. We argued that the shock-policy interaction in our model acts through a reduction in labor demand, in contrast with the "mainstream" explanation that focuses on the labor supply side (Ljungqvist and Sargent, 1998). We built the case for a labor-demand model on three arguments: 1) the technological acceleration had a large impact on the macroeconomy; 2) it is successful in explaining the rise in wage inequality in the U.S. labor market; 3) there is independent evidence –through the dynamics of the vacancy rate and the labor share– that labor demand dynamics are important.

Our approach has the advantage, over virtually all of the existing quantitative literature on the topic, of being able to measure the source of the shock independently of labor market variables, through the quality-adjusted relative price of new capital goods. We believe that this parametrization of the shock puts lots of discipline on the quantitative exercise.

Quantitatively, the model can account for roughly half of the rise in European unemployment, and the surge in unemployment is entirely associated to longer durations, like in the data. In addition, the model generates a decline in the labor share and in the vacancy rates of a magnitude similar to the data.

The "labor demand" view of European unemployment represents a challenge to the currently prevailing explanations based on "labor supply". However, the two hypotheses are not mutually exclusive. In a theoretical framework with elements of vintage human capital and vintage physical capital, a technological acceleration will also worsen the rate skill obsolescence –exactly like in Ljungqvist and Sargent. The next generation of investigations of the European unemployment puzzle should bring together supply and demand forces and allow a joint evaluation of their respective strength.

Appendix

A.1 Data Appendix.

The first panel of Figure 1 (standardized unemployment rate) is constructed using the data collected by Blanchard and Wolfers (2000) and available as download from the website http://econ-www.mit.edu/faculty/blanchar/harry_data/. The dataset is constructed from raw OECD data. The countries included under the "Europe" label are: Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, and Switzerland. The U.K. is excluded since its unemployment experience and its labor market institutions are more similar to the U.S. economy. The data are only available at 5-year intervals: the 1960 data-point is an average of 1960-1964, the 1965 is an average of 1965-1969, and so on, until 1995 which is reported by the authors to be an average of 1995 and 1996 only. The Europe aggregate is an unweighted mean of the 15 countries.

The second panel of Figure 1 (labor share) is constructed from Blanchard and Wolfers (2000), except the 1995 entry for Austria, Denmark, Ireland and Portugal (not available in the original Blanchard-Wolfers dataset) which was computed directly from the raw OECD data, averaging out 1995 and 1996. The Europe aggregate is an unweighted mean.

The third panel of Figure 1 (vacancy-employment ratio) is the ratio of job vacancies to total civilian employment. The numerator is computed from the "number of registered vacancies" by country, a set of series available in the OECD Main Economic Indicators. In the United States vacancies come in the form of a "help-wanted" advertising index. The denominator is computed from the OECD Labor Force Statistics. Due to data limitations, only Austria, Belgium, Finland, France, Germany, Netherlands, Norway, Sweden, and Switzerland have complete time series spanning the period 1960-1996. The series for Spain starts in 1977, the series for Portugal in 1974, and the series for Denmark in 1970. Data on vacancies are unavailable for Greece, Ireland, and Italy. The data-points in the panel are constructed using the same time aggregation rule as in the other panels. The value of the ratio is meaningless per se, hence the normalization in 1960. The Europe aggregate is an unweighted mean.

The fourth panel of Figure 1 (average age of capital) is constructed with the data in Table 2.10 labelled "Historical-Cost Average Age at Yearend of Private Fixed Assets; Equipment, Software, and Structures; by Type" available in the Fixed Assets Tables from the Bureau of

Economic Analysis official website. The data-points in the panel are constructed using the same time aggregation rule as in the other panels.

A.2 Rendering the equilibrium stationary.

Aggregate output at time t can be written as

$$Y(t) = \int_0^{\bar{a}} z(t)\, k(t,a)^{\omega}\, \mu(t,da),$$

where $\mu(t,da)$ is the equilibrium distribution of employment across vintages of machines at time t. Along a balanced growth path $\mu(t,da)$ does not depend on time t and the disembodied technology factor $z(t)$ can be modelled as $z(0)\, e^{\psi t}$ and efficiency units of capital $k(t,a)$ as $k(0)\, e^{\gamma(t-a)-\delta a}$. With the innocuous normalization $z(0) = k(0) = 1$, aggregate output becomes

$$Y(t) = e^{(\psi+\omega\gamma)t} \int_0^{\bar{a}} e^{-\omega(\gamma+\delta)a} \mu(da),$$

which implies that aggregate output has to grow at the constant rate $g \equiv (\psi + \omega\gamma)$. Output can be used as consumption or, alternatively, the economy can forego current consumption to increase the stock of productive capital, i.e.

$$Y(t) = C(t) + I(t)\, e_f(t),$$

where $C(t)$ is aggregate consumption and $I(t)\, e_f(t)$ are the total resources devoted to the purchase of new machines at time t. The cost of a new machine at time t is $I(t)$ and $e_f(t)$ is the measure of new entrant firms at time t. Along the balanced growth path, $C(t)$ grows at rate g, $e_f(t) = e_f$ and $I(t) = Ie^{gt}$. The amount $I(t)\, e_f(t)$ of foregone consumption translates into $X(t) \equiv k(t,0)\, e_f$ efficiency units of new investments.

The price of an efficiency unit of capital at time t (relative to the price of the final good, normalized to 1) in this economy is

$$p(t) \equiv \frac{I(t)}{k(t,0)} = Ie^{(g-\gamma)t},$$

implying that along the balanced growth path, the relative price of an efficiency unit capital grows at rate $g - \gamma = \psi - (1-\omega)\gamma$, a number that can be positive or negative.

Finally, notice that the free-entry equilibrium condition at time t

$$I(t) = e^{gt} \int_t^{t+\bar{a}} e^{-(r-g+\phi)a} \left[1 - e^{-\phi(\bar{a}-a)}\right] da,$$

can be made stationary by using the fact that $I\left(t\right)=Ie^{gt}$.

A.3 In the frictionless economy the competitive equilibrium allocation and the Pareto-optimal allocations are the same.

The planner maximizes the discounted value of future consumption, i.e., output net of investment, subject to the constraint that the total number of machines in operation in each period cannot exceed the aggregate labor force:

$$\max_{\left\{e_f(t),\bar{a}(t)\right\}}\int_0^\infty e^{-rt}\left\{e^{gt}\int_0^{\bar{a}(t)}e_f\left(t-a\right)e^{-\phi a}da-e_f\left(t\right)Ie^{gt}\right\}dt$$
$$\text{s.t.}\quad\int_0^{\bar{a}(t)}e_f(t-a)da\leq 1,\ \text{for all } t.$$

The planner chooses the measure of entrants $e_f(t)$ of vintage t and the maximal age $\bar{a}\left(t\right)$ of vintages which are operating at t. Since there are no frictions, the planner can use arbitrary firm measures of any vintage, and since more recent vintages are more productive, the planner uses all firms of the most recent vintages first. At time t the planner operates vintages in $\left[t-\bar{a}\left(t\right),t\right]$. We can write the Lagrangian for the constrained optimization problem in terms of the contributions of the different vintages as

$$\begin{aligned}\mathcal{L} &= \int_0^\infty e^{-(r-g)t}e_f\left(t\right)\left\{\int_0^{\hat{a}(t)}e^{-(r-g+\phi)a}da-I\right\}dt\\ &\quad-\int_{-\bar{a}(0)}^\infty e_f\left(t\right)\left\{\int_t^{t+\hat{a}(t)}\varphi\left(\tau\right)e^{-(r-g)\tau}d\tau\right\}dt+\int_0^\infty e^{-(r-g)t}\varphi\left(t\right)dt\ ,\end{aligned}$$

where $\varphi\left(t\right)$ is the Lagrange multiplier on the labor endowment constraint and $\hat{a}\left(t\right)\equiv\bar{a}\left[t-\hat{a}\left(t\right)\right]$ denotes the age at which a vintage t machine is retired. The first order conditions with respect to $e_f(t)$ and $\hat{a}(t)$ read, respectively,

$$\begin{aligned}\int_0^{\hat{a}(t)}e^{-(r-g+\phi)a}da-I-\int_0^{\hat{a}(t)}e^{-(r-g)a}\varphi(t+a)da &= 0,\\ 1-e^{\phi\hat{a}(t)}\varphi\left[t+\hat{a}\left(t\right)\right] &= 0.\end{aligned}$$

The first condition states that the planner will add new firms until the benefits (present value of additional output) equal the direct installation costs and the indirect costs. This could follow from the fact that the creation of new firms requires the destruction of others, given the fixed amount of labor available. The second condition states that a marginal increase in the destruction age raises the expected output of existing firms but once again requires a reduction in the total number of operating firms.

In steady state, the time subscripts can be omitted and $\hat{a} = \bar{a}$. Moreover, we can impose the condition $e_f = 1/\bar{a}$ which guarantees that the distribution is stationary and all the labor is employed. From the second condition, we obtain that $\varphi = e^{-\phi\bar{a}}$. This expression is easily interpretable: φ is the multiplier on the total labor force constraint, and $e^{-\phi\bar{a}}$ is exactly the value of slackening this constraint, i.e., the marginal contribution of an extra unit of labor (recall that this is also the equilibrium wage rate). Using this result in the first condition, we arrive at

$$I = \int_0^{\bar{a}} e^{-(r-g+\phi)a} \left[1 - e^{-\phi(\bar{a}-a)}\right] da$$

which is the key equilibrium condition (3) in the decentralized economy.

A.4 Derivations of value functions and employment distributions.

The value functions and distributions of our continuous-time model can be derived as limits of a discrete time formulation. A typical derivation of the differential equations for value functions (7)-(10) goes as follows. Consider the value of a vacant firm with capital of age a at time t, $\tilde{V}(t,a)$. For a Poisson matching process, the probability that the vacant firm meets a worker over a small finite time interval $[t, t+\Delta]$ is $\Delta\lambda_f$. We can define the vacancy value recursively as

$$\tilde{V}(t,a) = \Delta\lambda_f \left[\tilde{J}(t+\Delta, a+\Delta) - \tilde{V}(t+\Delta, a+\Delta) + \tilde{T}(t+\Delta)\right] + e^{-r\Delta}\tilde{V}(t+\Delta, a+\Delta),$$

where the first term is the expected capital gain from becoming a matched firm with value $\tilde{J}$ and the second term is the present value of remaining vacant at the end of the time interval. On a balanced growth path all value functions increase at the rate g over time, i.e., $\tilde{V}(t,a) = e^{gt}V(a)$, $\tilde{J}(t,a) = e^{gt}J(a)$ and $\tilde{T}(t) = e^{gt}T$. Subtracting $\tilde{V}(t+\Delta, a)$ from both sides, substituting the balanced growth path expressions for $\tilde{V}$, $\tilde{J}$ and $\tilde{T}$, and dividing by $\Delta e^{g(t+\Delta)}$, we can rearrange the value equation into

$$\begin{aligned} -e^{-g\Delta}V(a)\frac{e^{g\Delta}-1}{\Delta} &= \lambda_f\left[J(a+\Delta) - V(a+\Delta) + T\right] + \frac{e^{-r\Delta}-1}{\Delta}V(a+\Delta) \\ &\quad + \frac{V(a+\Delta) - V(a)}{\Delta}. \end{aligned}$$

As we shorten the length of the time interval and take the limit for $\Delta \to 0$, we obtain the differential equation (7):

$$-gV(a) = \lambda_f\left[J(a) - V(a) + T\right] - rV(a) + V'(a).$$

The equations describing employment dynamics are derived as follows. Consider the measure of matched vintage a firms at time t. Over a short time interval of length Δ, the approximate change in the measure is

$$\mu(t+\Delta, a) = \mu(t, a-\Delta)(1-\Delta\sigma) + \Delta\lambda_f \nu(t, a-\Delta).$$

Subtracting $\mu(t,a)$ from both sides and dividing by Δ we obtain

$$\frac{\mu(t+\Delta, a) - \mu(t,a)}{\Delta} = -\frac{\mu(t,a) - \mu(t, a-\Delta)}{\Delta} - \sigma\mu(t, a-\Delta) + \lambda_f \nu(t, a-\Delta).$$

Taking the limit for $\Delta \to 0$ one obtains

$$\mu_t(t,a) = -\mu_a(t,a) - \sigma\mu(t,a) + \lambda_f \nu(t,a).$$

At steady state, these measures do not change with t, and we obtain the result stated in (18).

Similarly, the differential equation for unemployment can be derived as follows. Over a short time period of length Δ the change in unemployment is

$$u(t+\Delta) = u(t)\left[1 - \int_0^{\bar{a}} \Delta\lambda_w(a)da\right] + \Delta\sigma \int_0^{\bar{a}} \mu(t,a)da + \int_0^{\Delta} \mu(t, \bar{a}-x)dx.$$

The first two terms on the right-hand side are standard: they are flows assuming a Poisson process and these flows are approximately linear in the length of the interval, since the interval is small. The third term sums all those matches that will reach $\bar{a}$ by the end of the period and therefore separate. Subtracting $u(t)$ on both sides, dividing by Δ, taking limits as Δ approaches 0, and assuming steady state yields the result (24). To find $\lim_{\Delta\to 0}\left[\int_0^{\Delta} \mu(\bar{a}-x)dx\right]/\Delta$, use l'Hôpital's rule.

Given the differential equation for employment (18), we can easily determine that

$$\mu(a) = \frac{\lambda_f \nu(0)}{\sigma+\lambda_f}\left[1 - e^{-(\sigma+\lambda_f)a}\right], \quad \text{and} \tag{26}$$

$$\nu(a) = \frac{\nu(0)}{\sigma+\lambda_f}\left[\sigma + \lambda_f e^{-(\sigma+\lambda_f)a}\right]. \tag{27}$$

Thus, the total number of vacancies, v, satisfies

$$v = \int_0^{\bar{a}} \nu(a)da = \frac{\nu(0)}{\sigma+\lambda_f}\left\{\bar{a}\sigma + \frac{\lambda_f}{\sigma+\lambda_f}\left[1 - e^{-(\sigma+\lambda_f)\bar{a}}\right]\right\}. \tag{28}$$

Integrating both sides of the equation $\nu(a)+\mu(a) = \nu(0)$ over the support $[0, \overline{a})$, we conclude that the total number of matched pairs (employment), μ, satisfies $\mu = \nu(0)\bar{a} - v$, or

$$\mu = \frac{\nu(0)\lambda_f}{\sigma + \lambda_f} \left\{ \bar{a} - \frac{1}{\sigma + \lambda_f} \left[1 - e^{-(\sigma+\lambda_f)\bar{a}} \right] \right\}. \tag{29}$$

Solving (24) for u, and substituting in for $\mu(\bar{a})/\mu$, we arrive at

$$u = \frac{1 + \sigma \left(\frac{\bar{a}}{1-e^{-(\sigma+\lambda_f)\bar{a}}} - \frac{1}{\sigma+\lambda_f} \right)}{1 + [\sigma + m(\theta, 1)] \left(\frac{\bar{a}}{1-e^{-(\sigma+\lambda_f)\bar{a}}} - \frac{1}{\sigma+\lambda_f} \right)}. \tag{30}$$

Having found the unemployment rate u, the entry of firms $\nu(0)$ is simply found from equation (29), using the fact that $\mu = 1 - u$. Equations (19) and (20) in the main text can be derived simply using (26) together with (29) and (27) together with (28), respectively.

A.5 Proofs of Lemmas 1, 2, 3 (the job creation curve).

Lemma 1 (the downward sloping (JC) curve): The (JC) curve is implicitly defined by the equation

$$I = (1 - \beta) \lambda_f \int_0^{\bar{a}} e^{-(r-g)a} S(a; \bar{a}, \lambda_f) \, da. \tag{JC}$$

We show that the RHS of this expression is increasing in $\bar{a}$ and λ_f, which implies that the (JC) curve is downward sloping in the $(\lambda_f, \bar{a})$ space.

Straightforward integration of the equation (16) defining the surplus equation yields

$$S(a; \bar{a}, \lambda_f) = e^{-\phi a} \left(1 - e^{-\rho_2(\bar{a}-a)} \right) / \rho_2 - e^{-\phi \bar{a}} \left(1 - e^{-\rho_1(\bar{a}-a)} \right) / \rho_1 \tag{31}$$

with $\rho_0 = r - g + \sigma$, $\rho_1 = \rho_0 + (1 - \beta) \lambda_f$, and $\rho_2 = \rho_1 + \phi$. It is immediate that the surplus function is decreasing in a and increasing in $\bar{a}$. Since the surplus function is increasing in the exit age $\bar{a}$, it is immediate that the RHS of (JC) is increasing in the exit age $\bar{a}$.

To show that the RHS of (JC) is increasing in λ_f, rewrite the integral as

$$I = e^{-\phi \bar{a}} \int_0^{\bar{a}} e^{-(r-g)a} \left\{ \int_0^{\bar{a}-a} \hat{\lambda}_f e^{-\left(\rho_0 + \hat{\lambda}_f\right)\tilde{a}} \left[e^{\phi(\bar{a}-a-\tilde{a})} - 1 \right] d\tilde{a} \right\} da \tag{32}$$

with $\hat{\lambda}_f = (1 - \beta) \lambda_f$. We now show that the integral of the function $f(\tilde{a}; \lambda_f) = \lambda_f e^{-\left(\rho_0 + \lambda_f\right)\tilde{a}}$ with respect to the weighting function $h(\tilde{a}) = e^{\phi(\bar{a}-a-\tilde{a})} - 1$ is increasing in λ_f. The function f is increasing (decreasing) with respect to λ_f for $\tilde{a} < (>) \, \hat{a} = 1/\lambda_f$. The integral of the function f, however, is increasing with λ_f, as

$$\begin{aligned} \int_0^{\bar{a}} f\left(\tilde{a};\lambda_f\right) d\tilde{a} &= \left[1-e^{-\left(\rho_0+\lambda_f\right)\bar{a}}\right]\frac{\lambda_f}{\rho_0+\lambda_f} \\ \frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}} f\left(\tilde{a};\lambda_f\right) d\tilde{a} &= \frac{\rho_0}{\rho_0+\lambda_f}\int_0^{\bar{a}} f\left(\tilde{a};\lambda_f\right) d\tilde{a}+\frac{\lambda_f}{\rho_0+\lambda_f}e^{-\left(\rho_0+\lambda_f\right)\bar{a}}>0. \end{aligned}$$

The integral of f with respect to h is also increasing with λ_f, since the weighting function h is monotonically decreasing in $\tilde{a}$,

$$\begin{aligned} & \frac{\partial}{\partial\lambda_f}\int_0^{\bar{a}-a} f\left(\tilde{a};\lambda_f\right) h\left(\tilde{a}\right) d\tilde{a} \\ = & \int_0^{a_+} f_{\lambda_f}\left(\tilde{a};\lambda_f\right) h\left(\tilde{a}\right) d\tilde{a}+\int_{a_+}^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right) h\left(\tilde{a}\right) d\tilde{a} \\ > & \int_0^{a_+} f_{\lambda_f}\left(\tilde{a};\lambda_f\right) h\left(a_+\right) d\tilde{a}+\int_{a_+}^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right) h\left(a_+\right) d\tilde{a} \\ = & h\left(a_+\right)\int_0^{\bar{a}-a} f_{\lambda_f}\left(\tilde{a};\lambda_f\right) d\tilde{a}>0, \end{aligned}$$

with $a_+ = \min\{\hat{a}, \bar{a}-a\}$.

Lemmas 2 and 3 (the asymptotes of the (JC) curve): Integrating equation (32) yields

$$\begin{aligned} I = & \frac{(1-\beta)\lambda_f}{\rho_2} \\ & \left\{\frac{1-e^{-(r-g+\phi)\bar{a}}}{r-g+\phi}-\frac{\rho_2}{\rho_1}e^{-\phi\bar{a}}\frac{1-e^{-(r-\phi)\bar{a}}}{r-\phi}+e^{-\rho_2\bar{a}}\frac{1-e^{g\bar{a}}}{\sigma+(1-\beta)\lambda_f}\frac{\phi}{\rho_1}\right\}. \end{aligned} \tag{33}$$

Taking the limit of expression (33) as $\lambda_f \to \infty$, we get

$$\begin{aligned} I &= \frac{1-e^{-(r-g+\phi)\bar{a}^{\min}}}{r-g+\phi}-e^{-\phi\bar{a}^{\min}}\frac{1-e^{-(r-g)\bar{a}^{\min}}}{r-g} \\ &= \int_0^{\bar{a}^{\min}} e^{-(r-g+\phi)a}\left[1-e^{-\phi\left(\bar{a}^{\min}-a\right)}\right] da \Rightarrow \bar{a}^{\min}=\bar{a}^{CE}, \end{aligned}$$

where $\bar{a}^{CE}$ is the age cut-off of the frictionless economy, implicitly defined by (3). Alternatively taking the limit of expression (33) as $\bar{a} \to \infty$, we get

$$I=\frac{(1-\beta)\lambda_f^{\min}}{r-g+\sigma+(1-\beta)\lambda_f+\phi}\frac{1}{r-g+\phi}\Rightarrow\lambda_f^{\min}=\frac{(r-g+\phi)I}{1-(r-g+\phi)I}\frac{r-g+\phi+\sigma}{1-\beta}$$

which is expression (22) given the definition $\bar{r} \equiv r-g+\phi$.

A.6 Proofs of Lemmas 4 and 5 (the job destruction curve).

Lemma 4 (the upward-sloping (JD) curve): The (JD) curve is implicitly defined by the equation

$$1 = [b - (r - g)\, T]\, e^{\phi \bar{a}} + \beta \int_0^{\bar{a}} \lambda_w(a; \bar{a}, \lambda_f) e^{\phi \bar{a}} S(a; \bar{a}, \lambda_f) da. \tag{JD}$$

We show that the RHS of this expression is increasing in $\bar{a}$ and decreasing in λ_f, which implies that the (JD) curve is upward-sloping in $(\lambda_f, \bar{a})$ space.

(4a) The RHS of (JD) is increasing in $\bar{a}$: The first term is increasing in $\bar{a}$ under assumption A2b. Now take the derivative of the function to be integrated in the second term, and express it in terms of elasticities

$$\left\{ \frac{\partial \lambda_w}{\partial \bar{a}} \frac{\bar{a}}{\lambda_w} + \frac{\partial \tilde{S}}{\partial \bar{a}} \frac{\bar{a}}{\tilde{S}} \right\} \frac{\tilde{S} \lambda_w}{\bar{a}} \tag{34}$$

where $\tilde{S} = e^{\phi \bar{a}} S$. The elasticity of the density λ_w is given by

$$\frac{\partial \lambda_w}{\partial \bar{a}} \frac{\bar{a}}{\lambda_w} = -\frac{\sigma \bar{a} + \lambda_f \bar{a} e^{-\left(\sigma + \lambda_f\right) \bar{a}}}{\sigma \bar{a} + \lambda_f \int_0^{\bar{a}} e^{-\left(\sigma + \lambda_f\right) a} da}.$$

Thus the first term in (34) is negative, but its absolute value is less than one since $e^{-\left(\sigma + \lambda_f\right) a} > e^{-\left(\sigma + \lambda_f\right) \bar{a}}$ for $a \leq \bar{a}$. We will now show that the elasticity of the modified surplus function $\tilde{S}$ with respect to $\bar{a}$ is positive and greater than or equal to one. It will therefore follow that the integral in (JD) is increasing in $\bar{a}$.

We proceed in three steps. First, we show that the elasticity of $\tilde{S}$ with respect to $\bar{a}$ is increasing in a for a given $\bar{a}$. That is, if the elasticity is greater than one at $a = 0$, then it is greater than one for all a. Second, we show that for small enough $\bar{a}$ the elasticity is greater or equal to zero at $a = 0$; in particular, we show that $\lim_{\bar{a} \to 0} \left(\partial \tilde{S} / \partial \bar{a} \right) \left(\bar{a} / \tilde{S} \right) \geq 1$. Third, we show at $a = 0$ the elasticity is increasing in $\bar{a}$. The three steps together imply that the elasticity is greater or equal to one for all $a \leq \bar{a}$.

The elasticity of $\tilde{S}$ with respect to $\bar{a}$ is given by

$$\frac{\partial \tilde{S}}{\partial \bar{a}} \frac{\bar{a}}{\tilde{S}} = \frac{\phi \bar{a}}{1 - H\left(\bar{a} - a\right)} \text{ with } H\left(x\right) \equiv e^{-\phi x} \frac{\left(1 - e^{-\rho_1 x}\right) / \rho_1}{\left(1 - e^{-\rho_2 x}\right) / \rho_2}.$$

The sign of the derivative of the function H is given by

$$sign\left(H'\right) = \rho_2 \gamma e^{-\left(\rho_2 + \phi\right)} \left\{ \int_0^x e^{\phi y} dy - \int_0^x e^{\rho_2 y} dy \right\}.$$

Since $\rho_2 = \rho_1 + \phi$ and by assumption $\rho_1, \phi > 0$, the function H is decreasing in x. Therefore the elasticity is increasing in a.

The limit of the elasticity at $a = 0$ as $\bar{a}$ converges to zero is greater or equal to one. To see this note that for $\bar{a} \to 0$, the numerator and denominator converge to zero, and by l'Hôpital's rule

$$\lim_{\bar{a}\to 0} \frac{\bar{a}\partial\tilde{S}/\partial\bar{a}}{\tilde{S}} = \lim_{\bar{a}\to 0} \frac{\bar{a}\left(\partial^2\tilde{S}/\partial\bar{a}^2\right) + \partial\tilde{S}/\partial\bar{a}}{\partial\tilde{S}/\partial\bar{a}} = 1 + \lim_{\bar{a}\to 0} \frac{\left(\partial^2\tilde{S}/\partial\bar{a}^2\right)\bar{a}}{\partial\tilde{S}/\partial\bar{a}} \geq 1$$

since $\partial\tilde{S}/\partial\bar{a}$, $\partial^2\tilde{S}/\partial\bar{a}^2 \geq 0$.

Finally we need to show that at $a = 0$, the elasticity is increasing in $\bar{a}$, that is $G(\bar{a}) = \bar{a}/\left[1 - H(\bar{a})\right]$ is increasing in $\bar{a}$. First, multiply numerator and denominator by $\rho_1(1-e^{-\rho_2\bar{a}})$. This delivers

$$G(\bar{a}) = \frac{\rho_1\bar{a}(1-e^{-\rho_2\bar{a}})}{\rho_1(1-e^{-\rho_2\bar{a}}) - \rho_2(1-e^{-\rho_1\bar{a}})e^{-\phi\bar{a}}} = \bar{a}\frac{\rho_1(1-e^{-\rho_2\bar{a}})}{\rho_1 - \rho_2 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}}.$$

Notice that the denominator of this expression is positive: at $\bar{a} = 0$, it equals 0, and its derivative equals $\phi\rho_2(e^{-\phi\bar{a}} - e^{-\rho_2\bar{a}})$, which is positive because of the assumption that $\rho_2 = \rho_1 + \phi$ and $\phi, \rho_1 > 0$. For large $\bar{a}$, the expression is large: $\lim_{\bar{a}\to\infty} G(\bar{a}) = \lim_{\bar{a}\to\infty} \bar{a} = \infty$.

The derivative of G equals

$$\begin{aligned}
\frac{G'(\bar{a})}{\rho_1} &= \frac{1-e^{-\rho_2\bar{a}}}{\rho_1 - \rho_2 e^{-\phi\bar{a}} + \phi e^{-\sigma_2\bar{a}}} \\
&\quad -\bar{a}\frac{\rho_2 e^{-\rho_2\bar{a}}\left(\rho_1 - \rho_2 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right) - \left(1-e^{-\rho_2\bar{a}}\right)\phi\rho_2\left(e^{-\phi\bar{a}} - e^{-\rho_2\bar{a}}\right)}{\left(\rho_1 - \sigma_0 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right)^2} \\
&= \frac{1-e^{-\rho_2\bar{a}}}{\rho_1 - \rho_2 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}} \\
&\quad -\bar{a}\rho_2\frac{\rho_1 e^{-\rho_2\bar{a}} - \sigma_0 e^{-(\phi+\rho_2)\bar{a}} + \phi\left(e^{-2\rho_2\bar{a}} - e^{-\phi\bar{a}} + e^{-\rho_2\bar{a}} + e^{-(\phi+\rho_2)\bar{a}} - e^{-2\rho_2\bar{a}}\right)}{\left(\rho_1 - \sigma_0 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right)^2}.
\end{aligned}$$

Substituting for $\rho_1 = \rho_2 - \phi$ and simplifying yields

$$\frac{G'(\bar{a})}{\rho_1} = \frac{1-e^{-\rho_2\bar{a}}}{\rho_1 - \rho_2 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}} - \bar{a}\rho_2\frac{\rho_2\left(e^{-\rho_2\bar{a}} - e^{-(\phi+\rho_2)\bar{a}}\right) + \phi\left(e^{-(\phi+\rho_2)\bar{a}} - e^{-\phi\bar{a}}\right)}{\left(\rho_1 - \sigma_0 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right)^2}.$$

Thus, it is sufficient to study

$$\left(1-e^{-\rho_2\bar{a}}\right)\left(\rho_1 - \sigma_0 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right) - \bar{a}\sigma_0\left[\rho_2\left(e^{-\rho_2\bar{a}} - e^{-(\phi+\sigma_0)\bar{a}}\right) + \phi\left(e^{-(\phi+\rho_2)\bar{a}} - e^{-\phi\bar{a}}\right)\right].$$

This expression equals

$$\left(1-e^{-\rho_2\bar{a}}\right)\left(\rho_1 - \sigma_0 e^{-\phi\bar{a}} + \phi e^{-\rho_2\bar{a}}\right) - \bar{a}\sigma_0\left[\rho_2 e^{-\rho_2\bar{a}}\left(1-e^{-\phi\bar{a}}\right) - \phi e^{-\phi\bar{a}}\left(1-e^{-\rho_2\bar{a}}\right)\right].$$

Factorizing, we are left with

$$\left(1-e^{-\rho_2 \bar{a}}\right)\left(\rho_1-\sigma_0 e^{-\phi \bar{a}}+\phi e^{-\rho_2 \bar{a}}-\bar{a} \sigma_0^2 e^{-\rho_2 \bar{a}} \frac{1-e^{-\phi \bar{a}}}{1-e^{-\rho_2 \bar{a}}}+\bar{a} \phi \rho_2 e^{-\phi \bar{a}}\right).$$

The left factor is larger than zero (it starts at zero and increases). After substituting for $\rho_1=\rho_2-\gamma$, the right factor can be rewritten as

$$\rho_2\left(1-e^{-\phi \bar{a}}\right)-\phi\left(1-e^{-\rho_2 \bar{a}}\right)+\rho_2 \phi \bar{a}\left(e^{-\phi \bar{a}}-e^{-\rho_2 \bar{a}} \frac{1-e^{-\phi \bar{a}}}{1-e^{-\rho_2 \bar{a}}} \frac{\rho_2}{\phi}\right).$$

Utilizing a simple integral formula, this becomes

$$\rho_2 \phi\left(\int_0^{\bar{a}} e^{-\phi x} d x-\int_0^{\bar{a}} e^{-\rho_2 x} d x\right)+\rho_2 \phi \bar{a}\left(e^{-\phi \bar{a}}-e^{-\rho_2 \bar{a}} \frac{\int_0^{\bar{a}} e^{-\phi x} d x}{\int_0^{\bar{a}} e^{-\rho_2 x} d x}\right).$$

The first of these two terms is positive, since $\rho_2>\gamma$ by assumption. If we can show that the second term is positive, we are done. That term has two sub-terms; we will prove that their ratio exceeds one. The ratio reads

$$\frac{e^{-\phi \bar{a}} \int_0^{\bar{a}} e^{-\rho_2 x} d x}{e^{-\sigma_0 \bar{a}} \int_0^{\bar{a}} e^{-\phi x} d x}=\frac{\int_0^{\bar{a}} e^{-(\phi+\rho_2) x} e^{-(\bar{a}-x) \phi} d x}{\int_0^{\bar{a}} e^{-(\phi+\rho_2) x} e^{-(\bar{a}-x) \rho_2} d x}.$$

But since the weighting function $e^{-(\bar{a}-x) \phi}$ is everywhere above the weighting function $e^{-(\bar{a}-x) \rho_2}$, again because $\rho_2=\rho_1+\phi$ (and $\bar{a}>x$), and the rest of the integrand is positive, the ratio indeed must exceed 1.

(4b) The RHS of (JD) is decreasing in λ_f for a Cobb-Douglas matching function with $\alpha>1/2$: We rewrite equation (JD) as

$$1=[b-(r-g) T] e^{\phi \bar{a}}+\beta\left[m(\theta, 1) \lambda_f\right] \int_0^{\bar{a}}\left[\frac{\lambda_w(a ; \bar{a}, \lambda_f) / \lambda_f}{m(\theta, 1)}\right] \tilde{S}(a ; \bar{a}, \lambda_f) d a.$$

It is immediate that the two terms under the integral, the modified density λ_w and the modified surplus function $\tilde{S}$, are decreasing in λ_f. Assuming a Cobb-Douglas matching function and substituting for θ, the term pre-multiplying the integral becomes

$$A \lambda_f^{(1-2 \alpha) /(1-\alpha)},$$

which is decreasing in λ_f for $\alpha>1/2$.

Lemma 5 (The asymptote of the (JD) curve): For λ_f large, the density λ_w converges to a uniform density on $[0, \bar{a}]$, $\lim_{\lambda_f \rightarrow \infty} \lambda_w(a)=\dfrac{1}{\bar{a}+1/\sigma}$, and the surplus function converges to zero, $\lim_{\lambda_f \rightarrow \infty} S(a ; \bar{a}, \lambda_f)=0$. Therefore equation (JD) converges to

$$1=[b-(r-g) T] e^{\phi \bar{a}^{\max}} \Rightarrow \bar{a}^{\max}=-\ln [b-(r-g) T] / \phi.$$

A.7 Proofs of Lemma 6, 7, 8, and 9 (comparative statics).

Lemma 6 (b): Obvious from inspection of (JC) and (JD).

Lemma 7 (γ): The RHS of (JC) is increasing in the rate of embodied technical change γ, because the function $f(\gamma) = e^{\phi(\gamma)a} S(a; \bar{a}, \lambda_f, \gamma)$ is increasing in γ. The derivative of f with respect to γ is

$$\frac{\partial f}{\partial \gamma} = \left[x - \left(1 - e^{-\rho_1 x}\right)/\rho_1\right] \omega e^{ga - \phi\bar{a}}/\rho_1$$

with $x = \bar{a} - a$. Notice that $\partial f/\partial\gamma = 0$ at $x = 0$, and that the term in brackets is increasing in x, and therefore $\partial f/\partial\gamma > 0$ for $a \in [0, \bar{a}]$. Since the RHS of (JC) is increasing in γ and $\bar{a}$, the (JC) curve shifts downward as γ increases.

The RHS of (JD) is increasing in the rate of embodied technical change γ because the function $\tilde{S} \equiv e^{\phi(\gamma)\bar{a}} S(a; \bar{a}, \lambda_f, \gamma)$ is increasing in γ. The derivative of $\tilde{S}$ with respect to γ is

$$\frac{\partial \tilde{S}}{\partial \gamma} = \omega \left\{ x e^{\phi x} \left(1 - e^{-\rho_2 x}\right)/\rho_2 + \left[e^{-\rho_1 x}(1 + \rho_1 x) - 1\right]/\rho_1^2 \right\}$$

with $x = \bar{a} - a$. Notice that $\partial\tilde{S}/\partial\gamma = 0$ at $x = 0$ and that $\partial\tilde{S}/\partial\gamma$ is increasing in x, i.e.,

$$\frac{\partial^2 \tilde{S}}{\partial\gamma\partial x} = \omega(1 + \phi x) e^{\phi x}\left(1 - e^{-\rho_2 x}\right)/\rho_2 \geq 0 \text{ for } x \geq 0.$$

Therefore $\partial\tilde{S}/\partial\gamma \geq 0$ for all $x \geq 0$. Since $\tilde{S}$ is increasing in γ for all $a \in [0, \bar{a}]$, the RHS of (JD) is increasing in γ. Since the RHS of (JD) is increasing in γ and $\bar{a}$, the (JD) curve shifts downward as γ increases.

Lemma 8 (r): Obvious from inspection of (JC) and (JD).

Lemma 9 (A): When the frictions disappear, both meeting probabilities tend to infinity. We proved above that as $\lambda_f \to \infty$, the job creation condition converges to the competitive condition (3). Consider now the wage equation (23). It is easy to compute that as $\lambda_f \to \infty$, the term $\lambda_f (1 - \beta) S(a)$ converges to $e^{-\phi a} - e^{-\phi\bar{a}}$, implying $w(a) = (1-\beta)(r-g)(U - T) + \beta e^{-\phi\bar{a}} = e^{-\phi\bar{a}}$, where the last equality follows from condition (15). Thus, the job destruction condition converges to the competitive one as well, which proves the Lemma.

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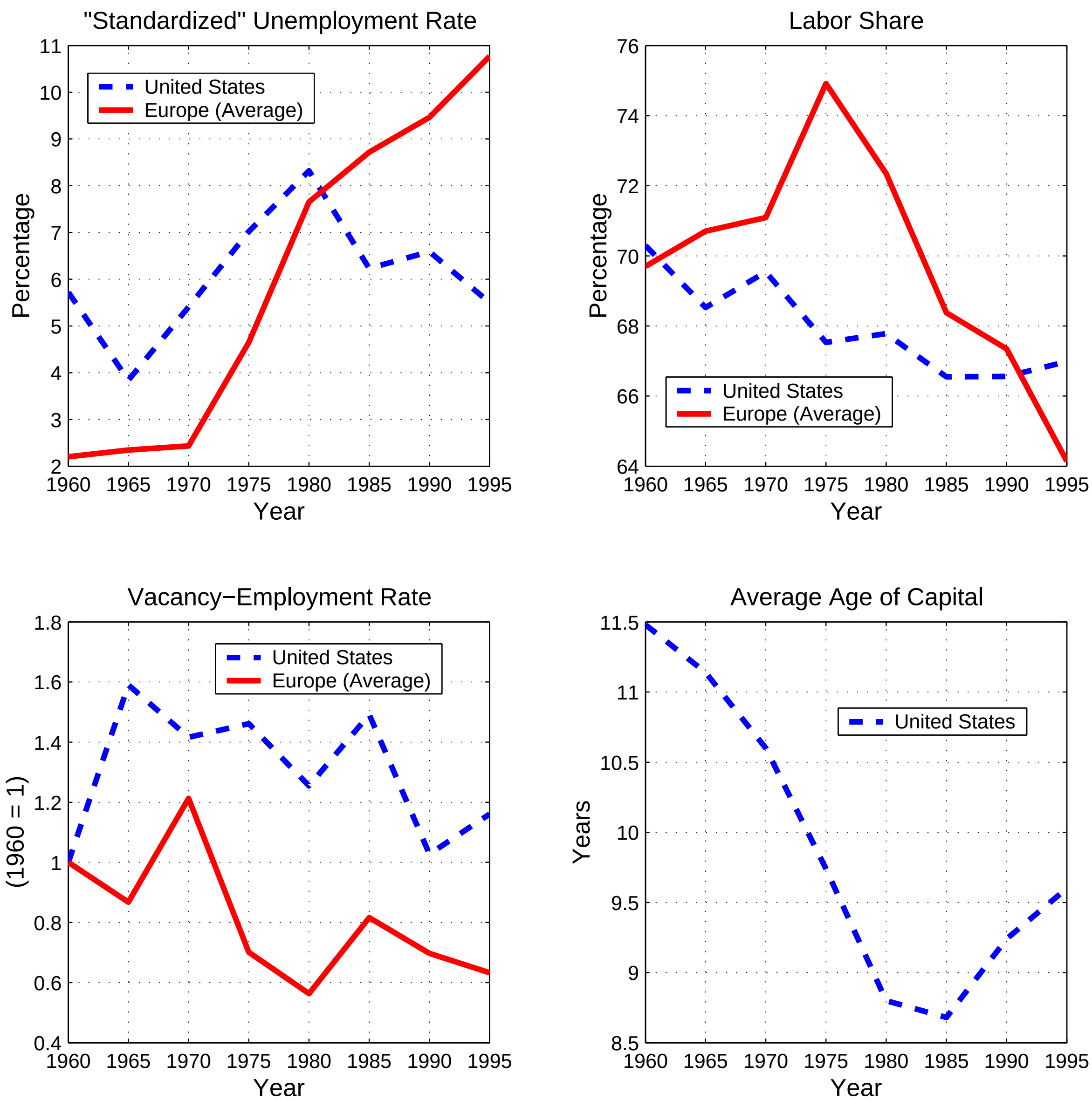

Figure 1: The first three panels depict the OECD standardized definition of the unemployment rate, the labor share of output, the vacancy-employment ratio for the U.S. and for an average of 15 European countries. The fourth panel plots the evolution of the age of total capital (equipment and structures) in the U.S. economy. See the Appendix for a detailed description of the data sources and the methodology.

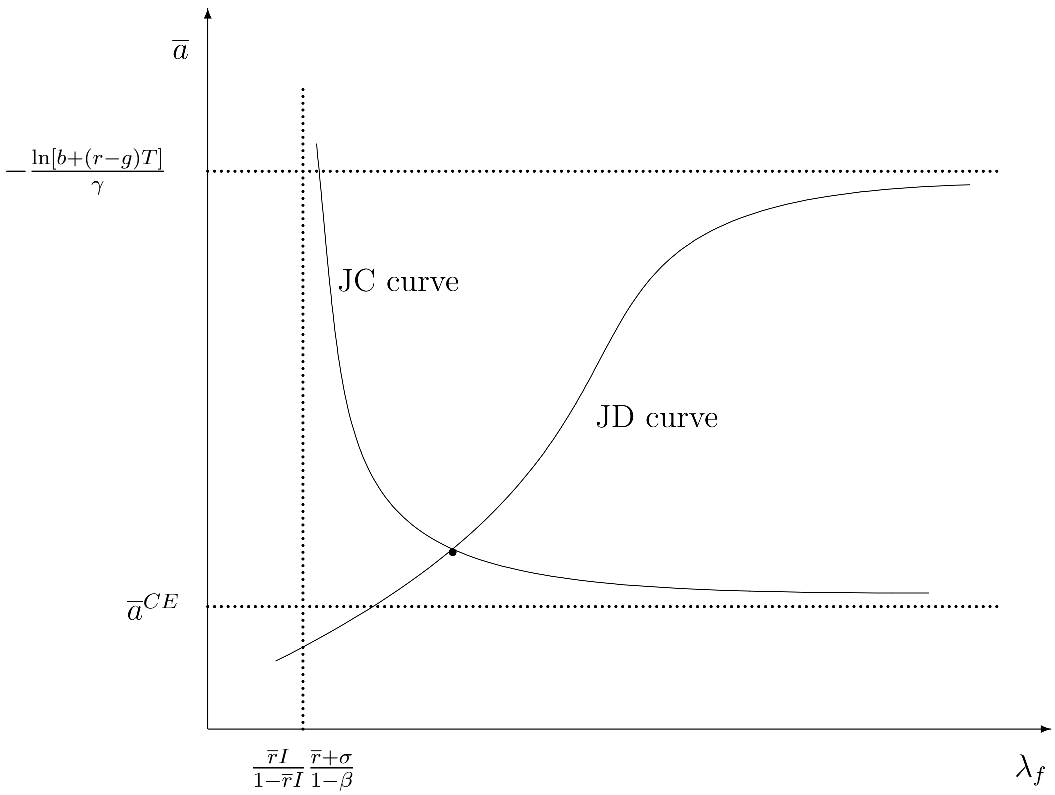

Figure 2: Job Creation and Job Destruction conditions, plotted in the $(\lambda_f, \overline{a})$ space.

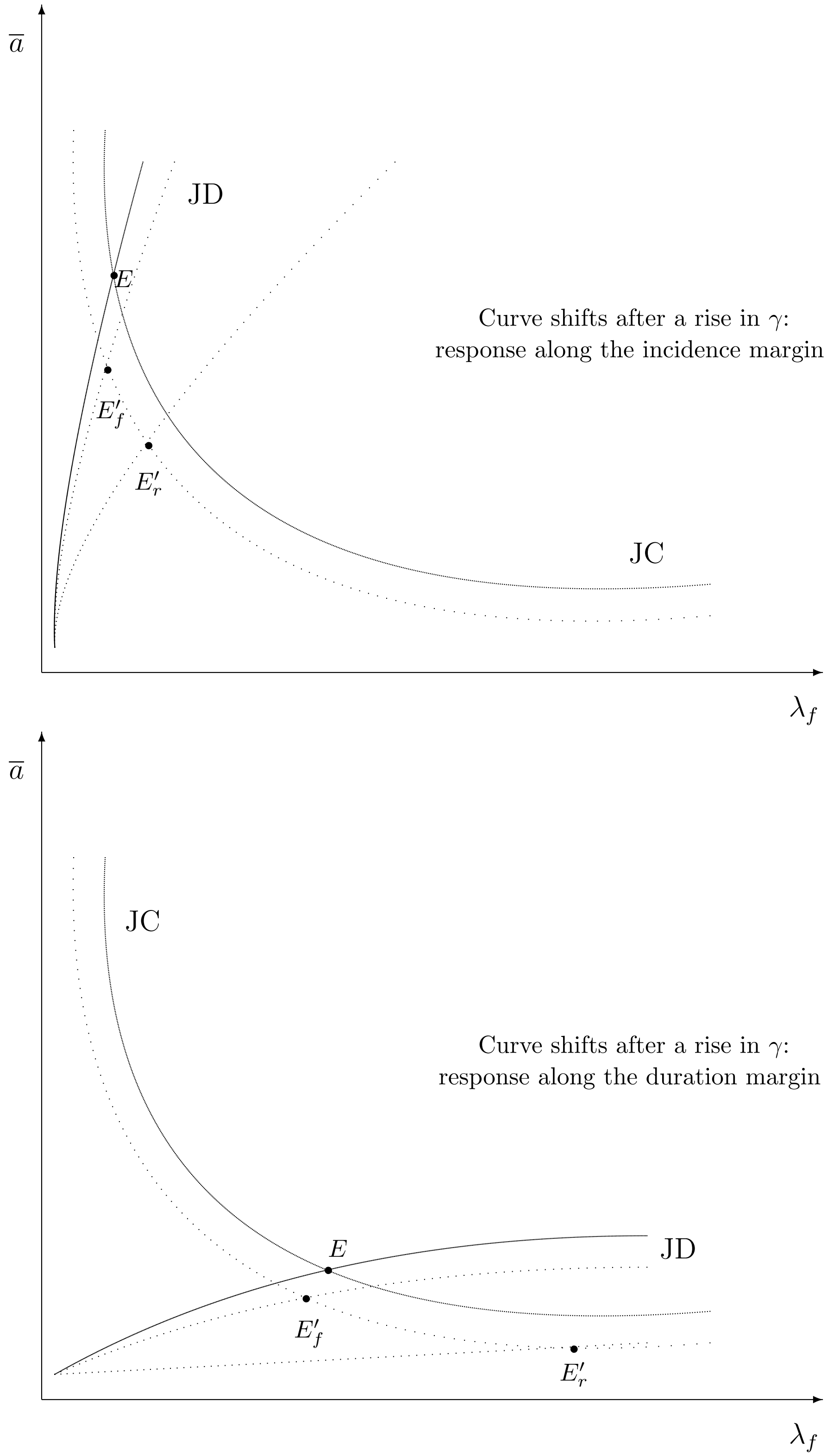

Figure 3: Qualitative comparative statics with respect to γ. The label E refers to the initial (pre-shock) equilibrium, the label E'_f to the final equilibrium in the flexible economy calibrated to the U.S., and the label E'_r to the final equilibrium in the rigid economy calibrated to Europe. The two economies differ only in the pair of policies (b, T) and have the same initial equilibrium.

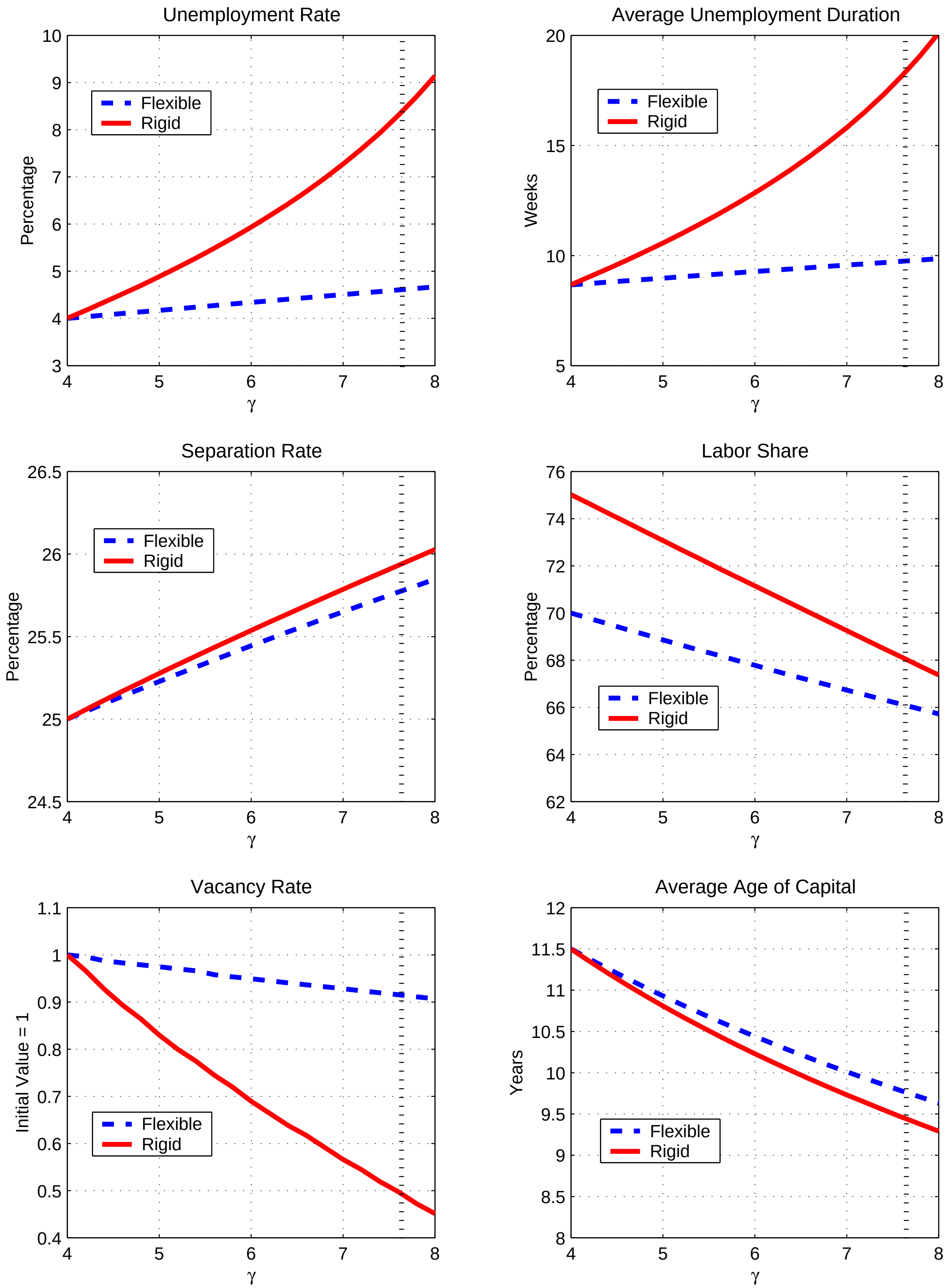

Figure 4: Experiment where the common shock is a rise in the rate of embodied technical change γ and the economies differ only in the level of policies, i.e., welfare benefits b and firing taxes T. The flexible economy represents the U.S., the rigid economy represents Europe. The vertical dotted line indicates the new long-run level of embodied technical change.