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Property Tax Competition: A Quantitative Assessment

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Abstract

We develop a model of property taxation and characterize equilibria under three alternative taxation regimes often used in the public finance literature: decentralized taxation, centralized taxation, and "rent seeking" regimes. We show that decentralized taxation results in inefficiently high tax rates, whereas centralized taxation yields a common optimal tax rate, and tax rates in the rent-seeking regime can be either inefficiently high or low. We quantify the effects of switching from the observed tax system to the three regimes for Japan and Germany. The decentralized or rent-seeking regime best describes the Japanese tax system, whereas the centralized regime does so for Germany. We also quantify the welfare effects of regime changes.

Keywords: property taxes, tax competition, efficiency

JEL classification: H71, H72, R13, R51

1 Introduction

Property taxes are globally one of the main sources of local governments' tax revenue. In fact, local governments in OECD countries, on average, obtained 46% of their 2015 tax revenue from property taxes.[1] Property tax is imposed on properties used for corporate activities and residential housing, which implies its significant impacts on firm distribution across local jurisdictions. Moreover, property tax revenue is mainly spent on improving residents' welfare, implying that property taxes also significantly affect the distribution of population among jurisdictions. This paper develops a framework that enables us to assess the impacts of property taxation by local governments on the geographical distribution of economic activities and on social welfare both qualitatively and quantitatively.

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[1]Revenue Statistics-OECD countries: Comparative tables, in OECD. Stat (`stats.oecd.org/index.aspx?DataSetCode=REV`, last accessed on August 20, 2018)

Given the high mobility of agents across jurisdictions, local governments' policies inevitably cause various fiscal externalities, which are extensively researched in the tax competition literature (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wilson, 1999; Wilson and Wildasin, 2004; Cremer and Pestieau, 2004). Equilibrium efficiency properties under fiscal externalities are crucially dependent on policy regimes, including the objective function and strategic behavior of local governments. The most popular setting assumes decentralized taxation by benevolent local governments seeking to maximize residents' welfare, which results in an inefficient outcome. Here, cooperation between governments can improve welfare and can even attain an efficient outcome if conducted appropriately. Another popular setting assumes that the local governments maximize the welfare of a particular interest group, i.e., rent seekers, resulting in a different type of inefficiency. To study the efficiency properties of property tax competition, it is therefore important to pin down a policy regime that approximates the actual regime. This paper provides a framework wherein we can quantitatively investigate how close the actual property tax system is to a particular regime.

To that end, we consider three alternative taxation regimes: a decentralized taxation regime in which each local government can freely choose its property tax rate to maximize the welfare of its residents, a centralized taxation regime in which a national government chooses property tax rates to maximize national welfare, and a "rent seeking" regime in which local governments choose their property tax rates to maximize the welfare of landowners. We call the last regime the homevoter regime. We choose these regimes because they are often employed in tax competition models.

We build a multi-municipality model in which property taxation affects the distribution of households and productive activities. The model economy has a final good and structure (or housing) service, both of which are consumed by residents consisting of mobile workers and immobile landowners. The final good is produced using labor and structure services whereas structures are produced using capital and land. Each local government imposes a property tax on structure services (those consumed by residents and those used by firms) and provides public goods to its residents. Thus, property taxation decreases the final goods production. Moreover, it is a source of public goods provision, implying its significant impacts on the distribution of population across municipalities. We assume heterogeneity among municipalities in factors such as total factor productivity in final goods and structure service production, amenities, and land endowment.

We first theoretically characterize the equilibrium (in)efficiency in the three regimes. We show that equilibrium property tax rates are inefficiently high under decentralized taxation.

Centralized taxation, which coincides with welfare optimal taxation in our model, yields a common tax rate for all municipalities. Lastly, equilibrium property tax rates can be inefficiently high or low under the homevoter regime. Property taxation results in two opposing externalities. First, property taxation by a municipality increases public goods provision and equilibrium utility, which spills over to other municipalities via workers' geographical arbitrage, resulting in a positive externality. Second, it attracts mobile workers from other municipalities, reducing landowner welfare in those municipalities by decreasing tax bases and production, thereby resulting in a negative externality.

Under decentralized taxation, the latter dominates the former, and the equilibrium tax rates become inefficiently high. Under the homevoter regime, the governments aim to maximize landowners' welfare only. This is in contrast to the decentralized regime where they aim to maximize the sum of workers' and landowners' welfare. Because public good provision financed by property taxation attracts mobile workers, governments have a lower incentive to tax property under the homevoter regime than under the decentralized taxation. This yields the possibility of inefficiently low tax rates in equilibrium under the homevoter regime. The centralized taxation results in an optimal tax rate that is common to all municipalities. Moreover, we find that at the optimum, a marginal change in a common tax rate has no effect on population distribution.

Using Japanese and German data, we then numerically evaluate which regime can best represent the actual property tax system. We focus on Japan and Germany because the two countries have distinct institutional regimes in determining property tax rates. For example, the Japanese central government appears to limit local discretion, so statutory tax rates are nearly uniform across municipalities. In contrast, the German municipalities do not explicitly coordinate the property tax rates with each other; thus, the local-level tax rates can vary substantially. Surprisingly, our quantitative analysis reveals that the Japanese system is best described by the decentralized taxation or homevoter regime, whereas the German system is best described by the centralized taxation regime. This indicates that Japanese local governments have implicit discretion and use various tax exemptions and deductions, and German local governments implicitly coordinate on property taxation. We also show that both country could suffer significant welfare losses as a result of inefficiency in the observed tax rates.

Our paper contributes to the extensive literature on tax competition. Early research focused on theoretically identifying the welfare loss from capital tax competition (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wilson, 1999). In the baseline tax competition models wherein households are assumed to be immobile, capital taxation causes capital flight to other municipalities, resulting in a positive externality and yields inefficiently low tax rates in equilibrium. However, as shown by subsequent studies such as Myers (1990), Krelove (1992) and Burbidge and Myers (1994), household mobility can internalize this traditional fiscal externality by "incentive equivalence" and another type of fiscal externality can arise depending on the type of household heterogeneity. In our model, workers are mobile but landowners are not, and the landowner's immobility is the source of the negative externality.

The quantification of welfare losses from tax competition is a much newer strand in the literature. Some recent papers have analyzed taxes in the tradition of quantitative spatial economics, as we do in this paper. Quantitative models have the advantage of linking theory with data and therefore enable us to conduct counterfactual policy analysis, including welfare analysis, which is difficult using only econometric tools. Desmet and Rossi-Hansberg (2013), Eeckhout and Guner (2017), Fajgelbaum, Morales, Suarez-Serrato, and Zidar (2019), and Oshiro and Sato (2021) analyzed the misallocation due to income taxes at the state and local levels. In their models, however, tax rates are exogenous. By contrast, our primary focus is on the endogenous determination of local tax rates.

To the best of our knowledge, only a small number of contributions use quantitative models to study tax competition. Mendoza and Tesar (2005) is an early example. They study the capital income tax competition between European countries in a calibrated model and argue that the welfare loss from tax competition is small. Another recent contribution is Ossa (2018), who analyzes state subsidies for firms in the United States and studies the welfare effect of subsidy competition. He finds that observed subsidies are closer to cooperative than non-cooperative subsidies and that moving to a decentralized Nash equilibrium would lead to a loss of 1.1% of real income. Wang (2020) studies international corporate tax competition with multinational firms. He quantifies the welfare effects of tax competition, and finds that cooperation among the participating countries would raise welfare by 1%.

Our paper differs from these contributions in several respects. First, we focus on property taxation, which received little attention in the literature thus far. Second, we conduct both a theoretical characterization of equilibrium and a quantitative analysis, allowing us to bridge the gap between the extensive literature on tax competition theory and recent quantitative analyses. Third, and most importantly, we compare the observed tax rates with the equilibrium tax rates under three regimes that are often used in the local public finance literature, in particular, decentralized and centralized taxation, and the one that maximizes landowner welfare. Given the importance of property tax revenues in local tax revenues, we believe our findings significantly contribute to the literature on local public finance by providing a method to evaluate property taxation.

The remainder of the paper proceeds as follows. Section 2 provides the baseline framework. Section 3 characterizes the three tax regimes. Section 4 explains the institutional background and data. Section 5 conducts the quantitative analysis. Section 6 discusses the robustness of our results and Section 7 concludes the paper.

2 Baseline framework

Consider an economy consisting of M local jurisdictions, which we call municipalities. Municipality i ($i \in \{1, .., M\}$) has a population of endogenous size n_i. Each municipality's population consists of landowners and workers. Each landowner is endowed with F_i units of land. Letting H_i denote the (fixed) land supply in municipality i, there are H_i/F_i landowners and $n_{Wi} = n_i - H_i/F_i$ workers. We use subscripts L and W to represent variables relating to landowners and workers. We assume that landowners are immobile whereas workers are mobile between municipalities, implying that H_i/F_i is given exogenously and n_i is determined endogenously. The economy's total population is exogenously given by N, and hence, we have $\sum_{i=1}^{M} n_i = N$.

The utility function of an individual in municipality i is Cobb-Douglas:

$$u_i = \frac{\xi_i}{\mu^\mu (1-\mu)^{1-\mu}} d^\mu c^{1-\mu} g_i^\eta, \tag{1}$$

where $0 < \mu < 1$ and $0 < \eta < 1$. c and d denote the consumption of the numéraire and housing, respectively, and μ is the expenditure share of housing. g_i is the level of local public goods, which is financed by property taxation, and η indexes the preference for public relative to private goods. ξ_i is a positive constant representing a municipality specific amenity. Think for instance of nice weather, parks, cultural facilities, etc.

Let I_{ji} denote the income level of a type j ($j \in \{W, L\}$)) agent residing in municipality i. Since residents have to pay ad valorem property tax at rate τ_i on housing, the budget constraint is

$$I_{ji} = c_{ji} + (1+\tau_i) p_i d_{ji}, \qquad j \in \{W, L\},$$

where p_i represents the net-of-tax price of housing. Workers are endowed with one unit of labor each, which they supply inelastically to earn wage income, w_i. A landowner is endowed with F_i units of land as well as one unit of labor, which they supply inelastically to earn land rent, $r_i F_i$, and wage income, w_i. Hence, the income of a worker in municipality i is $I_{Wi} = w_i$ and that of a landowner is $I_{Li} = w_i + r_i F_i$. Utility maximization of a type j individual in municipality i

yields the demand functions

$$d_{ji} = \frac{\mu I_{ji}}{(1+\tau_i)p_i}, \tag{2}$$

$$c_{ji} = (1-\mu)I_{ji},$$

and the indirect utility

$$v_{ji} = \xi_i \frac{g_i^\eta I_{ji}}{[(1+\tau_i)p_i]^\mu}. \tag{3}$$

Land and capital are used as inputs for producing structures that can be used as housing or production facilities. We assume that structures are produced under constant returns to scale and perfect competition, and the production function of the structure service is given by the Cobb-Douglas function

$$x_i = B_i h_i^\gamma k_i^{1-\gamma}, \quad B_i > 0, \tag{4}$$

where x_i is structure service supply in jurisdiction i, h_i and k_i are land and capital inputs, respectively, and γ is a positive constant satisfying $0 < \gamma < 1$. B_i represents the total factor productivity (TFP) in structure service production, which can differ between municipalities. We follow Eeckhout and Guner (2017) in assuming that the numéraire can also be used as capital so that the capital rental rate is 1.[2] The profit function of a structure service firm is given by $p_i x_i - r_i h_i - k_i$.

Because land supply in municipality i is given exogenously by H_i, the land market clearing condition requires $h_i = H_i$. Using this condition together with the profit maximization of structure service firms yields the land rent and supply of structure service in municipality i:

$$r_i = \gamma \Gamma \left(B_i p_i\right)^{\frac{1}{\gamma}}, \tag{5}$$

$$x_i = \Gamma H_i B_i^{\frac{1}{\gamma}} p_i^{\frac{1-\gamma}{\gamma}}, \tag{6}$$

where Γ is defined as

$$\Gamma \equiv (1-\gamma)^{\frac{1-\gamma}{\gamma}}.$$

The numéraire is used as the final consumption goods and capital which can be traded freely across municipalities. We consider a representative firm in each municipality, which produces the numéraire under constant returns to scale and perfect competition. The numéraire production

[2]We can alternatively assume a world capital market, wherein the capital rental price is determined exogenously. This does not alter our results.

function is given by the Cobb-Douglas function

$$y_i = A_i n_i^{\varepsilon} l_i^{\alpha} m_i^{1-\alpha}, \quad A_i > 0,$$

where α and ε are positive constants satisfying $0 < \alpha < 1$ and $0 \leq \varepsilon < 1$. A_i represents the TFP in the final goods production, which can differ between municipalities. For instance, some municipalities have locations more convenient for production, or are inherently endowed with more productive land (Albouy et al., 2019). l_i and m_i denote inputs of labor and structure service, respectively. We allow for the existence of agglomeration economies in the final goods production, so if $\varepsilon > 0$ firms' output depends on aggregate labor supply in the municipality, n_i, and ε represents the agglomeration elasticity. A large literature in urban and regional economics has shown how such agglomeration economies emerge, for instance, from learning spillovers, input sharing, and worker-firm matching (Duranton and Puga, 2004).

The literature on systems of cities has discussed the determination of city size emanating from the trade-off between centripetal forces caused by agglomeration economies and centrifugal forces caused by limited land supply or congestion (Henderson, 1974; Kanemoto, 1980; Fujita, 1989; Abdel-Rahman and Anas, 2004). If $\varepsilon > 0$, our model also features agglomeration economies and limited land supply. We thus have a model of a system of cities involving property taxation.

We assume ε is sufficiently small to prevent all mobile workers from concentrating in one municipality. More specifically, we assume that ε is small enough to ensure Assumption 1 below. In our numerical analysis below, we start by assuming $\varepsilon = 0$ as the baseline case, and later extend it to the case of $\varepsilon > 0$ in Section 6.2.

Consumers as well as final good producers have to pay an ad valorem property tax at rate τ_i on structures. As we will see later in Section 5, this corresponds to the Japanese and German property tax systems. Therefore, the gross price of structure service to a final good producer in municipality i is $(1 + \tau_i)p_i$, and the profit function of a final good firm is given by $y_i - w_i l_i - (1 + \tau_i)p_i m_i$.

The first-order conditions for profit maximization in the final good sector are

$$\begin{aligned} w_i &= A_i \alpha n_i^{\varepsilon} \left(\frac{m_i}{l_i}\right)^{1-\alpha}, \\ (1 + \tau_i)p_i &= A_i(1 - \alpha) n_i^{\varepsilon} \left(\frac{l_i}{m_i}\right)^{\alpha}. \end{aligned} \tag{7}$$

We assume that the labor and structure service markets are local (i.e., there is no commuting between municipalities), implying that the wage rate, w_i, and the net price of structure service, p_i, can vary across municipalities. The labor supply in municipality i is n_i, and the supply of structure service is $\Gamma H_i B_i^{1/\gamma} p_i^{(1-\gamma)/\gamma}$. The labor market clearing condition is then $l_i = n_i$. The structure service market clearing condition is

$$x_i = m_i + \frac{H_i}{F_i} d_{Li} + \left(n_i - \frac{H_i}{F_i} \right) d_{Wi}, \tag{8}$$

where the first term on the right hand side is firms' demand, the second is landowners' demand, and the third term is workers' demand. This equation pins down the net price of structure service, p_i, once municipality i's population, n_i, and property tax rate, τ_i, are determined. We can solve (8) for p_i:

$$p_i = \Lambda_i^{\frac{\alpha\gamma}{\alpha+\gamma-\alpha\gamma}} n_i^{\frac{\gamma(\alpha+\varepsilon)}{\alpha+\gamma-\alpha\gamma}}, \tag{9}$$

where Λ_i is defined as

$$\Lambda_i \equiv \frac{\left(1 + \frac{\alpha\mu}{1-\alpha}\right) \left[\frac{A_i(1+\alpha)}{1+\tau_i}\right]^{\frac{1}{\alpha}}}{\Gamma H_i B_i^{\frac{1}{\gamma}} \left(1 - \frac{\mu\gamma}{1+\tau_i}\right)}.$$

For ease of exposition, technical details of proofs and derivations are relegated to Online Appendix A. From (9), we see that the net price of structure service, p_i, is increasing in the final goods production TFP, A_i, which increases the structure service demand, and decreasing in structure service production TFP, B_i, which increases the supply of structure service. Furthermore, p_i decreases with the tax rate, τ_i, and increases with the size of the municipality population, n_i, which drives up the demand for residential housing.

Workers are freely mobile between municipalities, so workers' utility must be equalized across municipalities: $v_{Wi} = v_a$, $\forall i$, where v_a is the common equilibrium utility level. For expositional simplicity, we assume an interior solution for the majority of our analysis, and refer to corner solutions only when necessary.

We assume that providing one unit of public goods requires one unit of the numéraire. Hence, the budget constraint of municipality i's government is $g_i = \tau_i p_i x_i = \tau_i \Gamma H_i B_i^{1/\gamma} p_i^{1/\gamma}$, which, combined with (35) and (36), gives

$$v_{Wi} = \alpha (1-\alpha)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} \xi_i \left(\Gamma H_i B_i^{\frac{1}{\gamma}} \right)^{\eta} \frac{\tau_i^{\eta}}{(1+\tau_i)^{\frac{\mu+(1-\alpha)}{\alpha}}} p_i^{\frac{\eta}{\gamma} - \mu - \frac{1-\alpha}{\alpha}} n_i^{\frac{\varepsilon}{\alpha}}. \tag{10}$$

Since p_i depends on n_i and τ_i (see (9)), we now know that v_{Wi} is a function of n_i and τ_i. The no-migration condition $v_{Wi} = v_a$ then determines the number of workers in municipality i, n_i, as a function of τ_i and v_a.

Before proceeding, it is useful to discuss some intermediate results on how taxation affects workers and landowners. First, for given population size, taxation in a particular municipality drives down the net structure service price and land rent, but increases the gross structure service price there. This hurts both workers and landowners. Second, taxation also affects wages. For a given population size, a higher tax rate decreases the demand for structure and labor due to complementarity between production inputs, which lowers wages. Third, whereas the net price and supply of structure service decrease with the tax rate, public goods supply increases. Fourth, property taxation leads to migration of mobile workers, which affects the endogenous prices in the model. A worker inflow raises the net structure service price and land rent, while it decreases the wage rate. Moreover, it enlarges the tax base and hence the public goods supply. This benefits residents due to scale economies in public goods consumption.

As a result of the above effects, property taxation increases the gross structure service price and public goods supply, and decreases the wage rate, whereas its effects on the net structure service price and land rent depend on which effect prevails.

We now analyze how population reacts to a change in the tax rate. We make the following two assumptions.

Assumption 1 ***(stability).*** *In equilibrium, the following inequality holds true:*

$$\frac{\partial v_{Wi}}{\partial n_i} < 0, \quad \forall i.$$

This assumption requires that for a given tax rate, a small perturbation in the population distribution results in a restoration of the migration equilibrium. In our framework, we have scale economies in public goods provision and agglomeration economies if $\varepsilon > 0$. This implies that workers' utility increases with the municipality population, whereas fixed land supply implies that utility decreases with the municipality population. We focus on stable equilibria wherein the congestion effect of fixed land supply dominates. This type of stability condition is standard in the urban economics literature (Henderson, 1974; Kanemoto, 1980; Fujita, 1989). Note that from (9) and (10), Assumption 1 holds true if and only if $(1 - \mu\gamma + \eta)(\alpha + \varepsilon)/(\alpha + \gamma - \alpha\gamma) < 1$, which we assume throughout the paper.

Assumption 2 ***(tax effect).*** *In equilibrium, the following inequality holds true:*

$$\frac{\partial v_{Wi}}{\partial \tau_i} > 0, \quad \forall i.$$

This assumption implies that for a given population distribution, a marginal increase in the property tax rate in municipality i increases workers' utility there. In particular, Assumption 2 holds if and only if

$$\begin{aligned}\epsilon_{uti} &\equiv \frac{1+\tau_i}{v_{wi}}\frac{\partial v_{Wi}}{\partial(1+\tau_i)} \\ &= \eta\frac{(1+\tau_i)}{\tau_i} - \frac{(1-\alpha)(1+\eta)+\alpha\mu}{\alpha+\gamma-\alpha\gamma} + \frac{1+\tau_i}{1+\tau_i-\mu\gamma}\left[1-\frac{\alpha(1+\eta-\mu\gamma)}{\alpha+\gamma-\alpha\gamma}\right] > 0. \end{aligned} \tag{11}$$

ϵ_{uti} represents the tax elasticity of worker utility. The first term captures the direct positive effect of taxation on worker utility by increasing public goods provision. The second term accounts for the indirect negative effects via the tax base and factor demand. A rise in the property tax rate decreases structure demand and thus suppresses the structure service price, lowering the tax base. It also lowers the wage rates by reducing labor demand. The third term describes the indirect positive effect caused by a decline in structure demand, which lowers the structure service price. Assumption 2 requires that in equilibrium, the positive effects dominate the negative ones. Note that the concavity of the utility function with respect to public goods consumption implies that this inequality holds true for a sufficiently low tax rate. In our quantitative simulations below, we check whether Assumptions 1 and 2 are satisfied.

Since the migration equilibrium is determined by the condition $v_{Wi} = v_a$, $\forall i$, together with the population constraint, $n_i + \sum_{j\neq i} n_j = N$, the two assumptions above imply that $\partial n_i/\partial\tau_i > 0, \partial n_j/\partial\tau_i < 0$, $\forall j \neq i$. When municipality i increases its tax rate, it instills a fiscal externality on other municipalities by attracting mobile workers from them. This out-migration positively impacts these municipalities by relaxing the restrictions of fixed land supply and increasing wages; it also has a negative impact by decreasing production and the tax base. However, municipality i neglects these externalities, which results in inefficient levels of property taxation in a decentralized equilibrium where each municipality sets its tax rate to maximize its residents' welfare. In the following sections, we analyze the efficiency of equilibrium in each of the three regimes we consider.

3 Property taxation

In the theoretical analysis, we characterize the following three regimes of property taxation. First, we consider decentralized taxation wherein each municipality's government chooses the property tax rate to maximize its residents' welfare. Second, we investigate centralized taxation wherein the national government sets property tax rates to maximize national social welfare. By definition, this coincides with the optimal taxation. Third, we assume that each municipality's government chooses the property tax rate to maximize the utility of landowners. This regime captures the effects of rent seeking. In the numerical analysis, we examine which of the three regimes can best approximate the observed allocation in Japan and Germany.

3.1 Decentralized taxation

We first look at the decentralized taxation regime. In each municipality, the local government decides on the level of local public goods provision, g_i, financed by property taxation at rate τ_i to maximize the municipality residents' welfare, W_i. Following Cremer and Pestieau (2004), we define the government's objective as the Benthamite welfare W_i, that is, the sum of landowners' and workers' utilities:

$$\begin{aligned} W_i &\equiv \frac{H_i}{F_i} v_{Li} + \left(n_i - \frac{H_i}{F_i} \right) v_{Wi} \\ &= n_i v_{Wi} \Delta_i, \end{aligned} \tag{12}$$

where

$$\Delta_i \equiv 1 + \frac{r_i H_i}{w_i n_i} = 1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{1 + \tau_i - \mu\gamma}. \tag{13}$$

Δ_i captures the income gap between landowners and workers, which depends only on the property tax rate.

We assume that each municipality government considers local market responses, implying its awareness that p_i is determined by (8) and hence depends on n_i and τ_i, and n_i is determined by $v_{Wi} = v_a$ and hence depends on τ_i and v_a. Therefore, when it changes the property tax rate, τ_i, it recognizes the corresponding changes in the number of workers, n_i. However, the government regards v_a as given because we assume a large number of municipalities.[3]

Hence, each government considers the following responses of the number of workers to a tax change:

$$\frac{\partial n_i^d}{\partial \tau_i} = -\frac{\frac{\partial v_{Wi}}{\partial \tau_i}}{\frac{\partial v_{Wi}}{\partial n_i}} > 0, \tag{14}$$

where the inequality follows from Assumptions 1 and 2. The superscript d is used for the reactions considered by municipality governments under decentralized taxation, which will usually differ from the overall reactions as we will see below.

Maximizing each government's welfare function with respect to τ_i gives the following first-order conditions:[4]

[3] We make this assumption for two reasons: first, it simplifies the analysis. Second, it is a good approximation to a model with strategic interaction with a large number of municipalities, which applies to our cases of local property taxation in Japan and Germany.

[4] We assume the second-order conditions are satisfied.

$$\frac{1}{n_i}\frac{\partial n_i^d}{\partial \tau_i}\Delta_i + \frac{\partial \Delta_i}{\partial \tau_i} = 0, \quad \forall i. \tag{15}$$

Together with the population constraint, $\sum_{i=1}^{M} n_i = n$, these $M+1$ equations determine the common utility level of workers, v_a, and the tax rates, τ_i, in the decentralized equilibrium.

We now give the conditions for an equilibrium.

Definition 1 *An equilibrium under decentralized taxation is given by* $(p_i, w_i, n_i, \tau_i, v_a)$, $\forall i$, *that satisfies the structure service market clearing conditions (8), the labor market clearing conditions* $l_i = n_i, \forall i$, *the no-migration conditions* $v_{Wi} = v_a, \forall i$, *welfare maximization by local governments (15), and the population constraint* $\sum_{i=1}^{M} n_i = N$.

We next examine the efficiency properties of the property tax rates, τ_i, chosen by local governments in the decentralized equilibrium, and determined by (15). As an efficiency criterion, we use total national welfare, which is the weighted sum of utilities of landowners and workers in all municipalities:

$$SW \equiv \sum_{i=1}^{M} W_i = \sum_{i=1}^{M} n_i v_{Wi} \Delta_i. \tag{16}$$

When deciding on τ_i, each municipality government disregards the effects of its choice on other municipalities. Changes in τ_i affect other municipalities' population levels, thus altering the strength of scale economies in public goods provision and agglomeration economies in these municipalities. Since local government i ignores these effects, its choice of τ_i results in externalities and inefficient taxation levels.

To characterize these externalities, we differentiate SW with respect to τ_i and evaluate the derivatives at the decentralized equilibrium. If they are positive, then we know that the equilibrium tax rates are inefficiently low, and conversely, if they are negative, the equilibrium tax rates are inefficiently high.[5] We assume that SW is continuously differentiable and concave in τ_i.

The equilibrium values of n_i, n_j and v_a are determined by the M no-migration conditions

$$v_{Wi} = v_a, \forall i, \tag{17}$$

along with the population constraint, $n_i + \sum_{j \neq i} n_j = n$. Differentiating these $M+1$ conditions and using (14) gives

[5] This is equivalent to comparing the equilibrium and optimal tax rates.

$$\frac{\partial n_i^o}{\partial \tau_i} = \frac{1}{\frac{\partial v_{wi}}{\partial n_i}} \frac{\partial v_a^o}{\partial \tau_i} + \frac{\partial n_i^d}{\partial \tau_i}, \tag{18}$$

$$\frac{\partial n_j^o}{\partial \tau_i} = \frac{1}{\frac{\partial v_{wj}}{\partial n_j}} \frac{\partial v_a^o}{\partial \tau_i} > 0, j \neq i, \tag{19}$$

$$\frac{\partial v_a^o}{\partial \tau_i} = \frac{\frac{\partial v_{Wi}/\partial \tau_i}{\partial v_{Wi}/\partial n_i}}{\sum_{l=1}^{M} \frac{1}{\partial v_{wl}/\partial n_l}} > 0. \tag{20}$$

where the last inequality follows from Assumptions 1 and 2. The superscript "o" is used to represent the overall reactions derived from the entire system of equations.

In summary, as previously argued, when a jurisdiction raises its property tax rate, it attracts mobile workers from other municipalities. As a result, the common utility level of mobile workers rises.

Define ϵ_{un} as the population elasticity of worker utility:

$$\begin{aligned}\epsilon_{un} &\equiv \frac{n_i}{v_{wi}} \frac{\partial v_{Wi}}{\partial n_i} \\ &= \frac{(1 - \mu\gamma + \eta)(\alpha + \varepsilon)}{\alpha + \gamma - \alpha\gamma} - 1 < 0,\end{aligned}$$

where again the last inequality follows from Assumption 1. Then, taking the derivative of SW with respect to τ_i and evaluating it at the equilibrium under decentralized taxation, we obtain from $\partial n_i^o/\partial \tau_i + \sum_{j \neq i} \partial n_j^o/\partial \tau_i = 0$ and (15) that

$$\left.\frac{\partial SW}{\partial \tau_i}\right|_{\text{d-equilibrium}} = \frac{\partial v_a^o}{\partial \tau_i} \frac{1 + \epsilon_{un}}{\epsilon_{un}} \sum_{l=1}^{M} n_l \Delta_l < 0. \tag{21}$$

From this, we have the following proposition.

Proposition 1 *Under decentralized taxation, the equilibrium property tax rates are inefficiently high.*

An increase in the property tax rate by municipality i's government increases its public goods provision and worker's utility there, attracting mobile workers from other municipalities. While the municipality i's government recognizes the effects on its own municipality, it ignores the externalities it imposes on other municipalities. It turns out that, although the increase in mobile worker utility benefits other municipalities as well, landowner utility in municipalities $j \neq i$ falls due to decreases in public goods supply and land rent. The net effect is a reduction in welfare, implying a negative fiscal externality. Therefore, the equilibrium tax rates under decentralized taxation are inefficiently high.

Such fiscal externality has been widely discussed in the tax competition literature (Wilson, 1986; Zodrow and Mieszkowski, 1986; Wilson, 1999; Wilson and Wildasin, 2004). The baseline tax competition models study capital taxation under household immobility, which causes capital flight to other municipalities and results in a positive fiscal externality. Subsequent studies (Myers, 1990; Krelove, 1992; Burbidge and Myers, 1994) have shown that household mobility can internalize this traditional externality but household heterogeneity may give rise to another type of fiscal externality. In our model, we assume mobile workers and immobile landowners, and this heterogeneity in mobility results in a negative externality.

3.2 Centralized taxation

We next characterize the centralized taxation regime. In this scenario, a national government sets property tax rates, τ_i, for all $i \in \{1, \ldots, M\}$, to maximize national social welfare. Since the national government can internalize the fiscal externality when choosing τ_i, this regime yields the first best policy.

After some tedious algebra (see Appendix A), the first-order condition for each tax rate τ_i is given by

$$\frac{\partial SW}{\partial \tau_i} \propto \frac{(1+\epsilon_{un}^{-1})\epsilon_{uti}}{1+\tau_i} \frac{\sum_l n_l \Delta_l}{\sum_l n_l} - \frac{\epsilon_{un}^{-1}\epsilon_{uti}}{1+\tau_i}\Delta_i + \frac{\partial \Delta_i}{\partial \tau_i} = 0. \tag{22}$$

In contrast to the decentralized regime, the central government takes into account all fiscal externalities between municipalities caused by migration and the equilibrium change in the common utility level, which is shown by the first term of the left-hand side of (22).

Note that (22) depends only on τ_i, parameters, and variables common to all municipalities (this follows from inspection of (11), and (13)). Hence, the solution to (22) must be the same across cities. This implies that the socially optimal tax rate is uniform across municipalities. Thus, the M-dimensional optimization that determines τ_i for all i can be reduced to a single-dimensional problem that determines a single optimal tax rate.

Using this important result, we can appeal to uniform taxation and set $\tau_i = \tau$ for all i in (22). Then, Δ_i and ϵ_{uti} are also location-independent: $\Delta_i = \Delta$ and $\epsilon_{uti} = \epsilon_{ut}$. The first-order condition can then be written as

$$\begin{aligned}\epsilon_{ut} &= -\frac{1+\tau}{\Delta}\frac{\partial \Delta}{\partial \tau} \\ &= \frac{\gamma[(1-\alpha)/\alpha + \mu](1+\tau)}{(1+\tau-\mu\gamma)\{1+\tau-\mu\gamma+\gamma[(1-\alpha)/\alpha+\mu]\}}.\end{aligned} \tag{23}$$

That is, the tax elasticity of worker utility should be equal to the tax elasticity of the income gap. The solution, denoted by $\tau = \tau^c > 0$ for all i is uniquely determined. The superscript c indicates variables that are related to the centralized regime.

We now state the conditions for the centralized equilibrium.

Definition 2 *An equilibrium under centralized taxation is given by* $(p_i, w_i, n_i, \tau_i, v_a)$, $\forall i$, *that satisfies the structure service market clearing conditions (8), the labor market clearing conditions* $l_i = n_i, \forall i$, *the no-migration conditions* $v_{Wi} = v_a, \forall i$, *the national government's welfare maximization (22), and the population constraint* $\sum_{i=1}^{M} n_i = N$.

Proposition 2 *Under centralized taxation, the optimal property tax rate is uniform across municipalities and is characterized by* (23). *Starting from the optimal tax rate, a marginal change in the common tax rate is neutral to the distribution of population, namely,*

$$\left.\frac{\partial n_i^c}{\partial \tau}\right|_{\tau=\tau^c} = 0, \quad \forall i.$$

(23) implies that the equilibrium tax rate is independent of not only the agglomeration parameter, ε, but also the heterogeneous characteristics like productivity, amenity, land areas, and the mass of landlords. In addition, (23) does not depend on n_i implying that the equilibrium tax rate for a corner solution, in which the no-migration condition does not hold and some municipalities have no workers, is also given by (23). In our framework, property taxation may distort the worker distribution through changes in worker utility. At the optimum, the central government should set a low tax rate in municipalities with a high tax elasticity of worker utility and a high tax rate in municipalities with a low tax elasticity of worker utility, which is in line with the inverse elasticity rule (Ramsey, 1927; Diamond and Mirrlees, 1971a,b). However, as shown in (23), all municipalities have the same tax elasticity of worker utility, implying that the optimal tax rate is common to all municipalities.

There have been long standing debates about fiscal centralization versus decentralization, as well as tax competition versus coordination in local public economics. Proponents of decentralization have argued that uniform centralized policies create welfare losses if preferences differ between jurisdictions, and that tax competition generates efficiency gains by preventing wasteful policies. In contrast, proponents of centralization and coordination point to the efficiency costs from decentralized policies stemming from cross-jurisdictional externalities. Interestingly, Proposition 2 shows that, even though nothing in our setup forced this result, uniform policies may sometimes be socially optimal even if jurisdictions differ.[6]

[6]Note, however, that this result would break down if households had different preferences over local public goods. In that case, Tiebout sorting would lead to optimal differentiated local public goods supply.

Proposition 2 is also informative on the tax rate to be set. Property taxation should be neutral to the spatial distribution of population at the optimum in the sense that a marginal change in the common tax rate causes no population movement. This condition can help to empirically assess the optimality of observed property tax rates.

Although the uniformity and neutrality of the optimal tax rates, of course, rely on specific functional forms and the Benthamite welfare function, ad valorem taxation, and constant returns to scale technologies, these specifications are widely used in economics. Hence, we believe that our results form an important point of reference in evaluating the efficiency of property taxation.

3.3 Homevoter governments

In the third scenario, we assume that local governments maximize the utility of landowners. For instance, Fischel (2001) argues that the political power of homeowners leads local governments to maximize their welfare. Following his work, we refer to governments that act in the best interests of homeowners (strictly speaking, landowners) as homevoter governments. Hence, each government maximizes v_{Li} with respect to τ_i, taking local market responses into account and regarding v_a as given. We can rewrite v_{Li} as

$$v_{Li} = v_{Wi}\left(1 + \gamma \frac{n_i F_i}{H_i} \frac{\mu + \frac{1-\alpha}{\alpha}}{1 + \tau_i - \mu\gamma}\right).$$

Compared to the decentralized case in which $\partial W_i / \partial \tau_i = 0$ holds, homevoter governments have an incentive to set lower tax rates because they need not attract mobile workers. This can be seen by differentiating v_{Li} with respect to τ_i:

$$\frac{H_i}{F_i}\frac{\partial v_{Li}}{\partial \tau_i} = \frac{\partial W_i}{\partial \tau_i} - v_a \underbrace{\left(\frac{\partial n_i^d}{\partial \tau_i}\right)}_{+} < \frac{\partial W_i}{\partial \tau_i}.$$

The first-order condition of government i's maximization of landowner welfare, $\partial v_{Li}/\partial \tau_i = 0$, gives

$$\frac{\partial n_i^d}{\partial \tau_i} - \frac{n_i}{1 + \tau_i - \mu\gamma} = 0. \tag{24}$$

Homevoter and benevolent governments have different incentives to tax because the former considers only landowners' welfare whereas the latter considers workers' and landowners' welfare. Since landowners earn land rents in addition to wages, the incentive of the homevoter government depends on the effect of property taxes on land rent. Land rent rises with the property tax rate as long as the rate is extremely low due to in-migration of mobile workers. However, it starts to fall once the rate exceeds a certain threshold value. Because the threshold value is lower than the optimal tax rate for workers, the homevoter government has a lower incentive to tax than the benevolent government.

We define the homevoter equilibrium as follows.

Definition 3 *An equilibrium under the homevoter regime is given by* $(p_i, w_i, n_i, \tau_i, v_a)$, $\forall i$, *that satisfes the structure service market clearing conditions (8), the labor market clearing conditions* $l_i = n_i, \forall i$, *the no-migration conditions* $v_{Wi} = v_a, \forall i$, *homevoter welfare maximization by local governments (24), and the population constraint* $\sum_{i=1}^{M} n_i = N$.

By evaluating $\partial SW/\partial \tau_i$ at the equilibrium under the homevoter regime and using (24), we obtain

$$\left.\frac{\partial SW}{\partial \tau_i}\right|_{\text{h-equilibrium}} = \frac{\partial n_i^d}{\partial \tau_i}\left(-\frac{\frac{1+\epsilon_{un}}{\epsilon_{un}}}{\sum_{l=1}^{M}\frac{1}{\partial v_{wl}/\partial n_l}}\sum_{l=1}^{M} n_l \Delta_l + v_a\right). \tag{25}$$

This results in the following proposition.

Proposition 3 *(i) Under the homevoter regime, the equilibrium property tax rates are inefficiently high iff*

$$\frac{\frac{1+\epsilon_{un}}{\epsilon_{un}}}{\sum_{l=1}^{M}\frac{1}{\partial v_{wl}/\partial n_l}}\sum_{l=1}^{M} n_l \Delta_l > v_a. \tag{26}$$

(ii) The equilibrium tax rates are lower under the homevoter regime than under the decentralized taxation regime.

Note first that again the direction of inefficiency is the same for all municipalities: the equilibrium tax rates are either inefficiently high or inefficiently low in all municipalities. This can be seen from (26), which shows that the direction of the inefficiency depends only on terms common to all jurisdictions. Note second that, in contrast to the decentralized regime, the homevoter regime may result in inefficiently low tax rates. This is due to the fact that homevoter governments have lower incentives to tax properties than benevolent governments, yielding the possibility of a race to the bottom under the homevoter regime.

4 Institutional background and data for quantitative analysis

In the following, we calibrate our model to Japanese and German data and examine which regime best represents the current property tax systems in the two countries. Before doing so, we now describe the institutional background and data we use.

4.1 Property taxation in Japan and Germany

Japan

In Japan, property tax is determined by municipalities (*Shi-Ku-Cho-Son*), the lowest tier in the Japanese administrative system, of which there were 1719 in 2015. The share of property tax revenue comprises around 45% of municipalities' total tax revenue in 2015. In principle, municipalities have the authority to set property tax rates, except for 23 special wards in Tokyo, where the Tokyo prefectural government administers property taxation. However, the national government sets a standard statutory tax rate of 1.4%, and around 91% of municipalities employ this standard tax rate. 9% of municipalities employ tax rates slightly higher than the standard tax rate.

Property taxes cover land, buildings, and depreciable business assets, i.e., tangible assets except for land and buildings. Each municipality appraises the values of taxable assets using a unified formula established by the Ministry of Internal Affairs and Communications, and levies property tax on the appraised values as of January 1 every year. The appraised values primarily are determined by consulting transaction prices. When computing the tax base for land, the officially appraised land prices (*Kouji-Chika*) are first discounted to 7/10 to determine the taxable land value. Then, the taxable land values are discounted to 1/6 for residential housing with a floor area smaller than $200m^2$, to 1/3 for residential housing with a floor area equal to or larger than $200m^2$, and to 7/10 for commercial sites.

During the past few decades, local governments have often introduced various reductions in effective property tax burdens. For instance, many of them have offered a special discount on property tax rates to encourage businesses to locate in their enterprise zones (*Kigyo-Ricchi-Sokushin-Jorei*). Moreover, since the 2012 tax reform, they can control property taxation by establishing exceptional tax treatments (*Waga-Machi-Tokurei*) if the exceptions are deemed reasonable. These reductions and special treatments can be the source of variation in effective tax rates across Japanese municipalities, even if most of them rely on the standard statutory rate.

Germany

In Germany, property tax is levied by the municipalities (*Gemeinden*), the lowest tier in the federal administrative system, of which there were 12227 in 2011. There are two property taxes, the real property tax A, levied on agricultural and forestry properties, and the real property tax B, levied on developed or developable land and buildings. We focus on the property tax B here.[7] Revenue from the property tax B amounts to 13% of total municipal tax revenue and 24% of 'own' municipal revenue, i.e., revenue from taxes for which the rates are determined by municipalities.

The property tax is levied annually on land and buildings. They are based on the appraised values (*Einheitswerte*) in 1964 for West Germany and 1935 for East Germany.[8] The tax rate applied to the value of land or buildings consists of the federally set basic tax rate (*Grundsteuer-messzahl*) and the local rate (*Hebesatz*), which is set by the municipality. The basic rate varies from 0.26% to 0.35% in West Germany and from 0.5% to 1% in East Germany, depending on the property type. The local rates vary from 45% to 960% across communities, with a weighted average of around 460%. The tax liability is determined as the product of the appraised value, the basic rate, and the local rate.

Note that the tax base is calculated using the current appraised values in Japan whereas it is based on the past appraised values in Germany. Hence, whereas it would be reasonable to employ our model of ad valorem tax for Japan, as described in the previous section, it might be more appropriate to use a model with specific taxes for Germany (since the tax is not, as it were, assessed on the real market value of properties). Although we first use the model of ad valorem tax in our quantitative analysis, we will address this issue as a robustness check in Section 6. It will turn out, however, that this distinction is quantitatively unimportant.

4.2 Data

In our quantitative analysis, we will conduct counterfactual policy analysis, wherein we compare the equilibria described above to the observed allocations in Japan and Germany. Here, we describe the data we use to obtain the observed equilibrium values for some key endogenous variables, which are required for conducting the counterfactuals.

Japan

The total number of municipalities covering all Japanese provincial units is 1719 in 2015. From these, we omit 7 small municipalities in Fukushima prefecture because most of their residents were temporarily evacuated because of the nuclear power plant accident caused by the Great East Japan Earthquake. Thus, we have 1712 municipalities with a total population of 127 million.

[7] The property tax A is relatively insignificant, as it amounts to less than 1% of municipal tax revenue.

[8] The Constitutional Court has recently ruled these assessments unconstitutional, so the German legislature had to reform the property tax. The current reform stipulates that the assessment values have to rely essentially on property area and land value, property type, rent, building area and building age, and that new assessments have to be completed every seven years.

We obtain municipality-level data for government budgets in 2015 from the *Record of Financial Closing by City* (Ministry of Internal Affairs and Communications, Japan).[9] It reports tax revenue from property tax, the special land-ownership tax, and the urban planning tax. For Tokyo's 23 special wards, where the Tokyo prefectural government collects their property taxes, we additionally use data from the *Annual Statistical Yearbook* provided by the Bureau of Taxation of Tokyo Prefectural Government. We obtain the public expenditure financed by property taxation, g_i, from these sources. Municipality-level population is drawn from the *2015 Population Census*. As a proxy for the mass of immobile landowners, H_i/F_i, we use the number of persons who live in owner-occupied housing in a given municipality, which is available from the *2015 Population Census*.

The *Record of Prices of Fixed Assets* (Ministry of Internal Affairs and Communications, Japan) gives detailed information on appraised values of fixed assets such as land, buildings and depreciating assets, and their breakdown by ownership. The Japanese government treats properties differently in property tax deductions depending on the ownership. In particular, there are special measures to adjust tax burdens for residential properties. Taxable values of residential land are discounted to 1/6 if a plot is not larger than $200m^2$ and to 1/3 otherwise. Taxable values of commercial land are discounted to 7/10.[10]

To capture these nationally determined discounts, we need to distinguish fixed assets according to the ownership type. Since the *Record of Prices of Fixed Assets* does not report property values by ownership type at the municipality level, we infer them from national data. To do so, we use the share of building values by ownership type and by construction type (wooden or not) at the national level. We multiply the building values by construction type at the municipality level by these shares to obtain the building values by ownership type and by construction type at the municipality level. We then sum them up to obtain the building values by ownership type at the municipality level.[11] We assume the same ownership share for land values as for building values. Moreover, appraised land values have been set to about 70% of the market values since 1994. We infer market land values by multiplying the appraised values by 10/7. We thus obtain the total market property values, $(p_i x_i)_{(stock)}$, residential property values, $(p_i d_i)_{(stock)}$, and corporate property values, $(p_i m_i)_{(stock)}$.

Our static model does not explicitly distinguish between stock and flow variables. In our

[9] All variables are measured in thousand Japanese yen.

[10] These discounts are uniformly applied for the whole country and substantially curtail the tax base. Indeed, the median land area for residential housing in Japan has been smaller than $200m^2$ for the last decade (*Housing and Land Survey*, the Ministry of Internal Affairs and Communications).

[11] At the national level, households own 98% of wooden buildings and 44% of non wooden ones. We then compute municipality-level building values owned by households by summing 98% of wooden building values and 44% of not-wooden values in each municipality.

quantitative analysis, we regard structure service and tax base as annual flow variables. In contrast, the property values described above are stock variables. To convert stock variables in our data into flow variables, we use an annual user cost or discount rate of 0.04, which is used for the cost-benefit analysis of Japanese public projects.[12] We assume that it is common to all municipalities.

Note that the structure service in our framework is regarded as a composite good of actual various fixed assets, and all tax revenues are generated from this composite structure service. Hence, the property tax rate is first constructed as a crude tax rate that is given by the ratio of total property tax revenues to total market property (i.e., structure service) values in the municipality. This tax rate measures the overall property tax burden for a representative agent there. In this way, we abstract from the complexity of the actual tax system in its treatment of tax credits, exemptions, deductions, and tax categories.

Therefore, the following two equations simultaneously give the crude property tax rate, τ_i, and the flow market property value, $p_i x_i$: $\tau_i = (\text{tax revenue})/(p_i x_i)$ and $(p_i x_i)_{(stock)} = (1 - \tau_i) p_i x_i / 0.04$, where tax revenue and the stock market value $(p_i x_i)_{(stock)}$ come from the data. In obtaining flow values by ownership, we similarly use the discounted cash flow formulae: $(p_i m_i) = [0.04/(1 - \tau_i)](p_i m_i)_{(stock)}$ and $(n_i p_i d_i) = [0.04/(1 - \tau_i)](n_i p_i d_i)_{(stock)}$, where the stock market values of the corporate and residential properties also come from observed data.

Note further that the obtained crude tax rate τ_i is flow-measured whereas the actual property tax rate is stock-measured. Here, we convert the flow tax rates into stock tax rates and report both. Moreover, we decompose the stock crude tax rates into a multiplier that converts market values into taxable values and effective tax rates set by the local government such that

$$\tau_{i(stock)} = \bar{\theta}_{i(stock)} \tau^*_{i(stock)},$$

where $\tau_{i(stock)}$ represents the estimated crude tax rate, $\bar{\theta}_{i(stock)}$ is a multiplier on the stock value which is the ratio of the sum of taxable values to the sum of market values, and $\tau^*_{i(stock)}$ is the estimated effective tax rate.[13] $\bar{\theta}_{i(stock)}$ captures the differences in the composition of properties, which, combined with the nationally set discount system, result in different multipliers among municipalities. Because differences in the crude tax rate $\tau_{i(stock)}$ include the compositional

[12]See *The Cost-Benefit Analysis Manual*, published by the Ministry of Land, Infrastructure, Transport and Tourism, Japan (`www.mlit.go.jp/road/zaigen/hyoka/manuan.html`, last accessed on June 14, 2022).

[13]More specifically, let $(p_i x_i)^{\omega}_{(stock)}$ be the market value of an asset of category ω in the stock term so that $(p_i x_i)_{(stock)} = \sum_{\omega} (p_i x_i)^{\omega}_{(stock)}$. The multiplier $\bar{\theta}_{i(stock)}$ and stock tax rate $\tau^*_{i(stock)}$ are determined by $\bar{\theta}_{i(stock)} = \sum_{\omega} \varrho^{\omega} (p_i x_i)^{\omega}_{(stock)} / (p_i x_i)_{(stock)}$ and $\tau^*_{i(stock)} = (\text{tax revenue})/[\bar{\theta}_{i(stock)} (p_i x_i)_{(stock)}]$. ϱ^{ω} is the appraisal adjustment rate for an asset of category ω, for example, 0.7/6 for small residential housing. Thus $\varrho^{\omega} (p_i x_i)^{\omega}_{(stock)}$ is the taxable value of category ω.

differences $\bar{\theta}_{i(stock)}$, we focus on $\tau^*_{i(stock)}$ rather than $\tau_{i(stock)}$ as a variable representing differences in property taxation among municipalities. Hereafter, we simply call $\tau^*_{i(stock)}$ the stock tax rate.

Applying the above procedure, we get an average stock tax rate $\tau^*_{i(stock)}$ of 1.4% and an average multiplier $\bar{\theta}_{i(stock)}$ of 67%, both of which are reasonable. We report all of the flow crude tax rate, τ_i, the stock crude tax rate, $\tau_{i(stock)}$, and the stock tax rate, $\tau^*_{i(stock)}$.

Table 4 in Appendix B shows the summary statistics of the observed equilibrium values for the Japanese data.

Germany

Data for Germany are obtained from the German Federal Statistical Office, *Regionaldatenbank Deutschland*.[14] This official site compiles data from the federal and state statistical offices and other sources (e.g., the share of migrants and the share of owner occupied dwellings are from the 2011 Census). The fiscal variables and population are for 2017, whereas the homeownership rate is for 2011. All variables are measured in Euros. We exclude municipalities in the former Eastern Germany because of the concern about structural differences between Western and Eastern German municipalities. After excluding a few municipalities that lacked appropriate records, the number of municipalities examined here is 8,353 with total population of 64 million. The share of landowners is set equal to the share of owner-occupied dwellings at the local level.

From *Regionaldatenbank Deutschland*, we obtain total revenues and local rates for the property tax B for all German municipalities. In our simulations, we set the federal base rate to 0.35% for simplicity, and proceed as follows. First, since we do not directly observe appraised property values, we impute them by dividing total property tax revenue by the tax rate (local tax rate $\times$ 0.0035), to get the local tax base.[15] Second, we need to infer market values, which again we do not observe directly. Instead, we assume that market values for Western German municipalities are on average five times the appraised values (e.g., Löffler and Siegloch, 2021).[16] Third, according to our model, we have to apportion the tax base to property values of firms and households. For better information, we set the proportion of the value of land and buildings owned by individuals and corporations to those at the national level. Because commercial buildings account for roughly two-thirds of all building values nationwide, we allocate two-thirds of total appraised value to firms and one-third to households.

As described above for Japan, we convert stock values into flow values by applying a user

[14] `www.regionalstatistik.de`

[15] Note here that the actual basic rate slightly varies across types of properties and locations. 0.35% is close to the average rate.

[16] This may underestimate the true market values for municipalities that have experienced hikes in market prices over the last few decades.

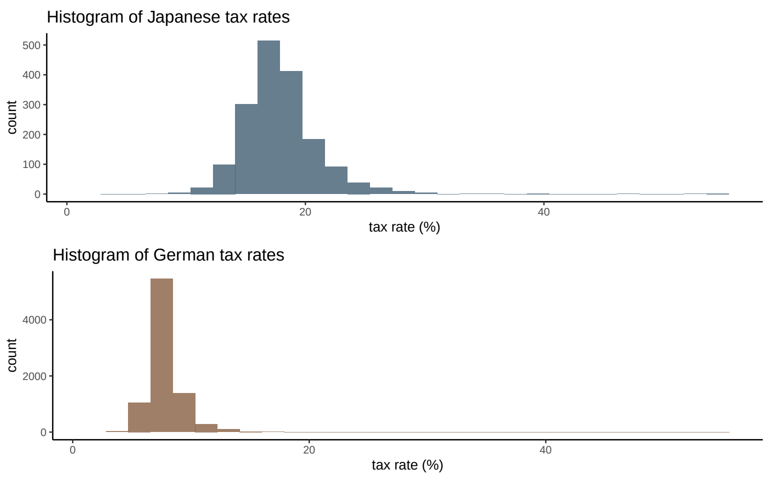
cost. Its figure for Germany is 0.03, which is the discount rate for the cost-benefit analysis of German public projects (OECD, 2018).

Table 5 in Appendix B shows the summary statistics of the observed equilibrium values for the German data.

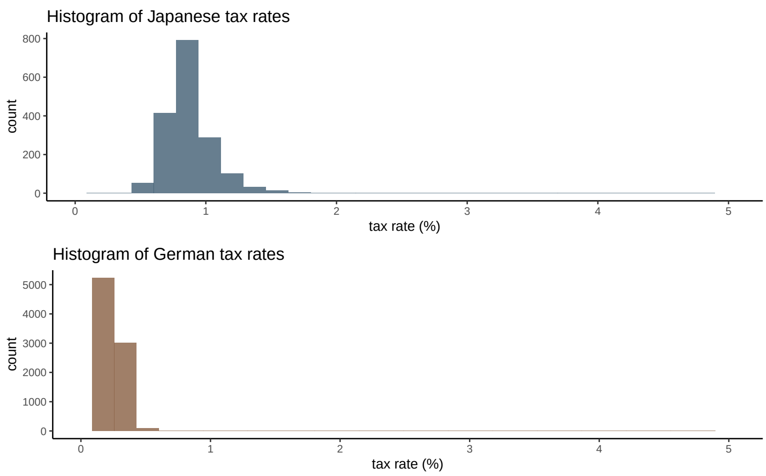
4.3 Tax rates in the observed equilibrium

Figure 1 compares distributions of the estimated τ_i, $\tau_{i(stock)}$, and $\tau^*_{i(stock)}$ in both countries. From the bottom of Figure 1, we know that Japanese municipalities set similar $\tau^*_{i(stock)}$s as a result of the nationally set standard statutory tax rate of 1.4%. German municipalities, by contrast, set a wide range of $\tau^*_{i(stock)}$s, which corresponds to their *Hebesatz*. In contrast, with respect to τ_i and $\tau_{i(stock)}$, shown in the top and middle of Figure 1, variation in the Japanese tax rates is larger than that in the German tax rates. The German property tax system as a whole, including state-level base rates, appears to be *de facto* harmonized.

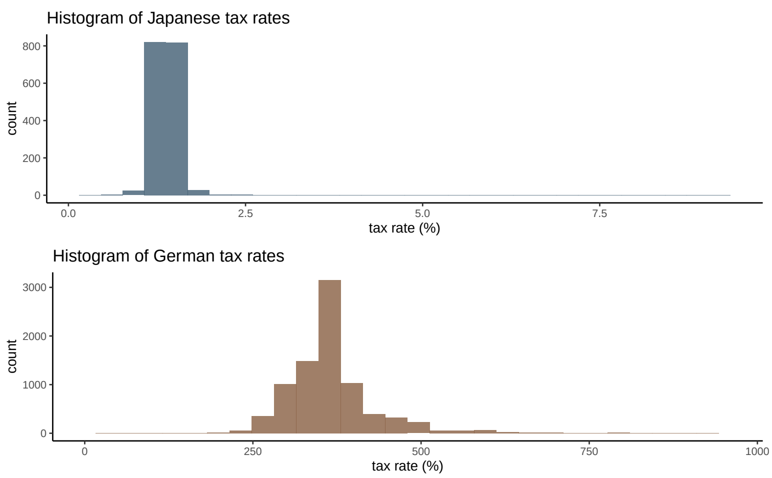
The variation in the Japanese crude tax rate comes from the variation in the multiplier $\bar{\theta}_{i(stock)}$, which reflects the composition of land-use patterns. The tax burden is likely to be significantly lower in a municipality with many small residential properties than in a municipality with many large commercial properties. One can argue that the Japanese system has determined a centralized standard rate without taking into account such heterogeneity in the land-use mix.



(a) Flow crude tax rates τ_i in Japan and Germany.



(b) Stock crude tax rates $\tau_{i(stock)}$ in Japan and Germany.



(c) Stock tax rates $\tau^*_{i(stock)}$ in Japan and Germany.

Figure 1: Tax rates in the observed equilibrium

5 Quantitative analysis

We now take our model to the data to conduct counterfactual simulations. We will start from the observed equilibrium and compute counterfactual equilibria representing the three different taxation regimes introduced in Section 3: decentralized taxation, centralized taxation, and homevoter regimes.

We follow the recent quantitative spatial economics literature and use Dekle et al. (2007)'s "exact hat algebra."[17] The basic idea is to conduct comparative statics numerically taking some observed equilibrium of the economy as a starting point. Consider then a counterfactual equilibrium wherein certain parameters or assumptions differ from those in the observed equilibrium. In our analysis, we first consider the observed equilibrium where the tax rates are those observed in the data and we won't specify the regime determining them. We then examine the counterfactual equilibria where tax rates are determined by one of the three regimes described above.

Of course, exogenous variables and equilibrium conditions that are unrelated to the changes are the same in both equilibria. Thus, we consider the observed values of endogenous variables, their counterfactual values, and the associated equilibrium conditions. Now take proportional changes of an endogenous variable and denote it by a "hat". Rewriting the ex-post, counterfactual equilibrium conditions using hat variables, we can often eliminate many parameters to pin down, and simplify the quantitative analysis. Here, in any counterfactual equilibrium, the structure service market clearing conditions, labor market clearing conditions, no-migration conditions, and population constraint continue to hold. Moreover, parameters are assumed to be fixed throughout.

For any variable z, we define $\hat{z}$ as $\hat{z} = z'/z$, where z' is the counterfactual value. Moreover, let $\lambda_i \equiv n_i/N$ be a municipality's population share, implying that $n_i = \lambda_i N$ and $\hat{\lambda}_i = \hat{n}_i$. Then, we can rewrite equilibrium conditions so far as those determine $\{p_i, w_i, \lambda_i, \tau_i, v_a\}, \forall i$ (equilibrium in levels) and those determine $\{\hat{p}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\tau}_i, \hat{v}_a\}$, $\forall i$ (equilibrium in changes, i.e., in hat forms). We here summarize these equilibrium conditions.[18]

Equilibrium in levels

An equilibrium in levels consists of an allocation $\{p_i, w_i, \lambda_i, \tau_i, v_a\}, \forall i$ satisfying the following five conditions.

[17] See Ossa (2016) for a survey of the exact hat algebra. See also Caliendo and Parro (2015), Redding (2016), Ossa (2018), Arkolakis et al. (2021), and Faber and Fally (2022) for recent analyses employing this method.

[18] See Appendix C for derivations of equations in the hat form.

(i) The structure service market clearing conditions:

$$p_i = \Lambda_i^{\frac{\alpha\gamma}{\alpha+\gamma-\alpha\gamma}} n_i^{\frac{\gamma(\alpha+\varepsilon)}{\alpha+\gamma-\alpha\gamma}}, \forall i. \tag{27}$$

(ii) The labor market clearing conditions:

$$l_i = n_i, \forall i. \tag{28}$$

(iii) The no-migration conditions:

$$\left(n_i - \frac{H_i}{F_i}\right)(v_{Wi} - v_a) = 0, \quad n_i - \frac{H_i}{F_i} \geq 0, \quad v_{Wi} - v_a \leq 0, \forall i. \tag{29}$$

(iv) In the observed equilibrium, the property tax rate is set to the estimated τ_i.[19] In the counterfactual equilibrium, it is determined by either the local government's maximization in the decentralized taxation regime (D), the national government's maximization in the centralized taxation regime (C), or the local government's maximization in the homevoter regime (H):

$$\begin{aligned}
&\text{(D)} \max_{\tau_i} W_i \text{ s.t. (27), (28), and (29) for all } i \text{ },\\
\text{or } &\text{(C)} \max_{\tau_i} SW \text{ s.t. (27), (28), and (29) for all } i \text{ },\\
\text{or } &\text{(H)} \max_{\tau_i} u_{Li} \text{ s.t. (27), (28), and (29) for all } i \text{ }.
\end{aligned}$$

(v) The population constraint:

$$\sum_{i=1}^{M} \lambda_i - 1 = 0. \tag{30}$$

In our analysis, we implicitly set some parameters so that the observed variables from the previous section (see Tables 4 and 5) are consistent with the above equilibrium conditions. However, we don't need to explicitly calibrate them if they don't appear in the equilibrium conditions in changes below. This is one advantage of using the exact hat algebra.

Equilibrium in changes

In deriving an equilibrium allocation in changes, it is convenient to define the gross property tax rate t_i as $t_i \equiv 1 + \tau_i$ and write equilibrium conditions in terms of t_i. Once we know the value of t_i, we can directly backout τ_i. An equilibrium allocation in changes consists of an allocation $\{\hat{p}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\tau}_i, \hat{v}_a\}$, $\forall i$, satisfying the following five conditions.

[19] Here τ_i is the flow crude tax rate estimated in the previous section.

(i) The structure service market clearing condition:

$$\hat{p}_i = \left(\hat{t}_i^{1-\frac{1}{\alpha}} \frac{t_i - \gamma\mu}{\hat{t}_i t_i - \gamma\mu} \right)^{\frac{\alpha\gamma}{\alpha+\gamma-\alpha\gamma}} \hat{\lambda}_i^{\frac{\gamma(\alpha+\varepsilon)}{\alpha+\gamma-\alpha\gamma}}, \forall i. \tag{31}$$

(ii) The labor market clearing conditions:

$$\hat{l}_i = \hat{\lambda}_i, \forall i. \tag{32}$$

(iii) The no-migration conditions:

$$\left(\hat{\lambda}_i n_i - \frac{H_i}{F_i} \right) (\hat{v}_{Wi} - \hat{v}_a) = 0, \quad \hat{\lambda}_i n_i - \frac{H_i}{F_i} \geq 0, \quad \hat{v}_{Wi} - \hat{v}_a \leq 0, \forall i. \tag{33}$$

(iv) Either the local government's maximization in the decentralized tax regime (D'), the national government's maximization in the centralized tax regime (C'), or the local government's maximization in the homevoter regime (H'):

$$\begin{aligned} &\text{(D')} \ \max_{\tau_i'} \hat{W}_i \text{ s.t. (31), (32), and (33) for all } i, \\ \text{or } &\text{(C')} \ \max_{\tau_i'} S\hat{W} \text{ s.t. (31), (32), and (33), for all } i, \\ \text{or } &\text{(H')} \ \max_{\tau_i'} \hat{v}_{Li} \text{ s.t. (31), (32), and (33) for all } i. \end{aligned}$$

(v) The population constraint:

$$\sum_{i=1}^{M} \lambda_i \hat{\lambda}_i - 1 = 0. \tag{34}$$

We need a smaller number of parameters to solve numerically for the equilibrium in changes than to solve for the equilibrium in levels. In particular, we do not need to know the values of productivity and amenity parameters, A_i, B_i, and ξ_i. Here, the parameters to be pinned down are α, ε, γ, μ, Γ, η, and N. With these values and the observed equilibrium values of some key endogenous variables such as n_i and t_i in hand, we can numerically solve for the equilibrium in changes, from which we can compute the counterfactual equilibrium values when necessary. The choice of parameters is described below.

Welfare measure

To quantify the welfare impacts of counterfactuals, we compute the equivalent variation (EV).[20] Note that EV relative to income corresponds to the net utility change:

$$\frac{EV_{ji}}{I_{ji}} = \hat{v}_{ji} - 1.$$

Hence, the percentage change in utility can be interpreted as the percentage of income required to maintain agents' utility at the counterfactual level. In our analysis, we report this relative EV. We also report the municipality-wide relative EV averaged by ex ante population share:

$$\frac{\overline{EV_i}}{I_i} = \left(1 - \frac{H_i}{n_i F_i}\right) \frac{EV_{Wi}}{I_{Wi}} + \frac{H_i}{n_i F_i} \frac{EV_{Li}}{I_{Li}}.$$

5.1 Algorithm

We perform a grid search over $\tau_i' \in (0, 1)$ values to find a solution for governments' optimization.[21] We here focus on equilibria that satisfy the stability Assumption 1. We do not impose Assumption 2 ex ante, and then check to see if it holds true after obtaining a solution.

In computing the equilibria where tax rates are determined by local governments (i.e., decentralized taxation and homevoter regimes), we first guess an equilibrium value for the common utility, $\hat{v}_a$. We can then find a stable equilibrium $\hat{p}_i$, $\hat{w}_i$, and $\hat{\lambda}_i$ that satisfy (31), (32), and (33) for a given tax rate τ_i' on the grid. We next find the tax rate that maximizes the government's objective function, which yields $\hat{t}_i$ to determine $\hat{p}_i$, $\hat{w}_i$, and $\hat{\lambda}_i$. Finally, we check whether the population constraint is satisfied, i.e., $\sum \hat{\lambda}_i \lambda_i = 1$. If this equality holds true, we conclude that the allocation $\{\hat{p}_i, \hat{w}_i, \hat{\lambda}_i, \hat{\tau}_i, \hat{v}_a\}$ is a solution to the equilibrium system. Otherwise, we perturb the initial guess and run the process again. We repeat this process until the system converges.[22]

For the case of centralized taxation, it is difficult to solve the M-dimensional optimization problem directly. Instead, we first compute the optimal tax rate based on the first-order condition (23). Then, we find the other endogenous variables that satisfy (31), (32), (33), and (34).

[20] We have also computed the compensating variations (CV), which give very similar results to those obtained from the EV.

[21] We numerically confirmed that the local governments' objective functions are concave on the relevant parameter space.

[22] The criterion for convergence is $|\sum \hat{\lambda}_i \lambda_i - 1| < 10^{-6}$.

5.2 Parameters

We set the parameter values as follows (see Table 1). We set the labor share in the final goods production, α, to 0.60 following Karabarbounis and Neiman (2014). The land share in structure service production, γ, is set to 0.25, following Ahlfeldt et al. (2015) (see also Combes, Duranton, and Gobillon, 2018; Epple, Gordon, and Sieg, 2010). We assume that the share of housing in consumption expenditure, μ, is 0.251 for Japan and 0.235 for Germany. The values are taken from the *OECD Affordable Housing Database* and are in line with the estimates in Davis and Ortalo-Magné (2011).[23] In the baseline analysis, we set the agglomeration elasticity ε to zero. In Section 6.2, we examine the robustness of our findings to the assumption of a positive value of ε.

We calibrate the taste parameter, η, which governs the substitutability of private and public goods, by targeting the elasticity of land rent to public infrastructure investment, reported in Haughwout (2002) as 0.11-0.23. We derive counterfactual changes in the land rent, r_i, for each municipality caused by a one standard deviation increase in g_i (37 billion yen for Japan and 90 million Euro for Germany) while holding τ_i fixed, and obtain the average elasticity of land rent to the public good provision. We then choose η so that the elasticity is 0.11-0.23, which yields η values ranging from 0.086 to 0.132 for Japan and 0.093 to 0.137 for Germany. In our simulations, for each country, we use the lower bound value ($\eta = 0.086$ for Japan and $\eta = 0.093$ for Germany) and higher bound value ($\eta = 0.132$ for Japan and $\eta = 0.137$ for Germany).

Table 1: List of parameters.

parameter	description	value
α	Labor share in numéraire production	0.60
ε	Elasticity of agglomeration econ in numéraire production	0.00
γ	Land share in structure production	0.25
μ	Housing share in consumption expenditure	0.251 for Japan
		0.235 for Germany
Γ	Bundle of parameters	0.39
η	Preference for local public goods	0.086 or 0.132 for Japan
		0.093 or 0.137 for Germany
N	Total population	127.1×10^6 for Japan
		64.1×10^6 for Germany
	Time discount rate	0.04 for Japan
		0.03 for Germany

We do not needt to explicitly determine the productivity in production, A_i and B_i, amenity

[23]To ensure positive land rents at equilibrium, we need the restriction $(1-\alpha)/(\alpha\mu) > (p_i m_i)/(p_i d_i n_i)$, which is satisfied given our parametrization.

values, ξ_i, and initial reservation utility, v_a, since we do not need those values for the counteractual simulations. If municipalities were homogeneous in these aspects, the equilibrium allocation would be symmetric. The source of variation among municipalities in the counterfactual exercises is heterogeneity in amenities, which is implicitly embedded in observed equilibrium variables such as n_i and τ_i.

5.3 Results

We now run counterfactual simulations for the three regimes described in Section 3: decentralized taxation, centralized taxation, and homevoter regime. As described above, we use the exact hat algebra to compute changes in endogenous variables relative to the observed equilibrium allocation (the 'benchmark'). Table 2 summarizes the counterfactual results.[24]

[24] Appendix D reports the details of the counterfactual simulation results.

Table 2: Summary of counterfactual results

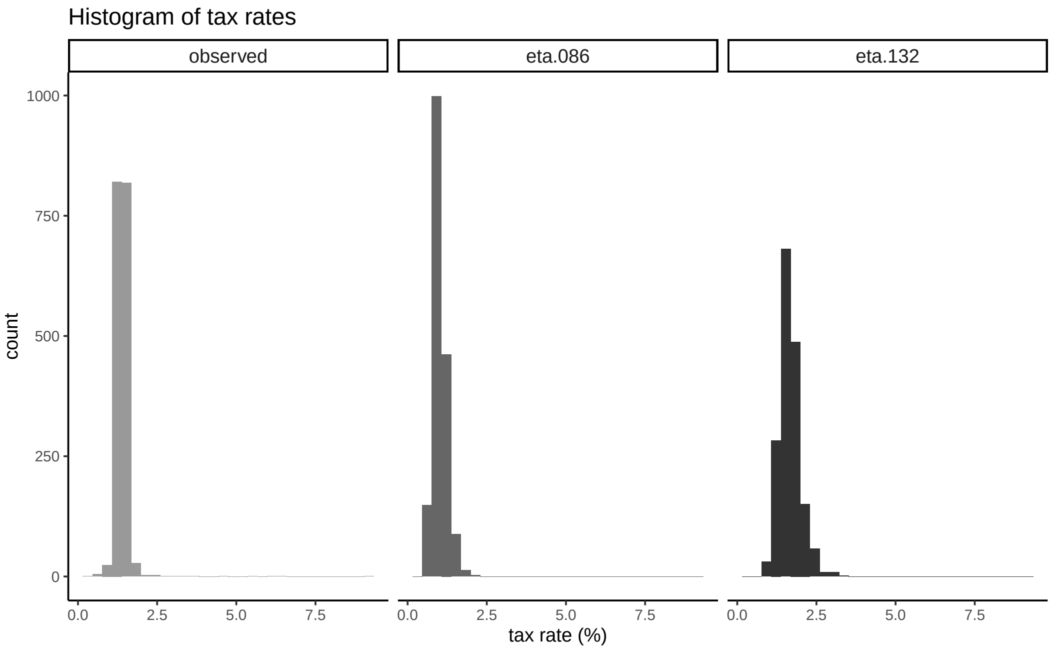
outcome	Japan $\eta = .086$	Japan $\eta = .132$	Germany $\eta = .093$	Germany $\eta = .137$
[Observed tax rate]				
Stock tax rate, $\tau^*_{i(stock)}$ (%)	1.42		367.1	
A: Decentralized taxation regime				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.02	1.66	736.7	1149.0
Change in municipality population (%)	2.0	-1.0	5.1	16.7
Change in structure service price (%)	1.9	-0.9	-1.2	-1.3
Change in wage rate (%)	1.4	-0.6	-3.2	-6.6
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.1	0.4	1.1	3.8
Landowner's relative EV, EV_{Li}/I_{Li} (%)	1.9	-0.6	0.6	4.6
Average relative EV, $\bar{EV}/I_i$ (%)	1.6	-0.4	0.8	4.4
B: Centralized taxation regime				
Stock tax rate $\tau^{*\prime}_{i(stock)}$ (%)	0.78	1.26	564.8	869.7
Change in municipality population (%)	2.0	-1.0	5.1	16.7
Change in structure service price (%)	2.8	0.4	-0.2	0.0
Change in wage rate (%)	2.5	0.9	-2.1	-5.2
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	-0.1	0.1	0.8	3.5
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.6	0.3	1.1	5.4
Average relative EV, $\bar{EV}/I_i$ (%)	2.1	0.2	1.0	4.7
C: Homevoter regime				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.86	1.46	622.9	1017.4
Change in municipality population (%)	2.0	-1.0	5.1	16.7
Change in structure service price (%)	2.5	-0.3	-0.6	-0.7
Change in wage rate (%)	2.1	0.1	-2.5	-5.9
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.0	0.3	0.9	3.7
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.4	-0.1	1.0	5.0
Average relative EV, $\bar{EV}/I_i$ (%)	1.9	0.0	1.0	4.6

Note: This table presents means for selected variables.

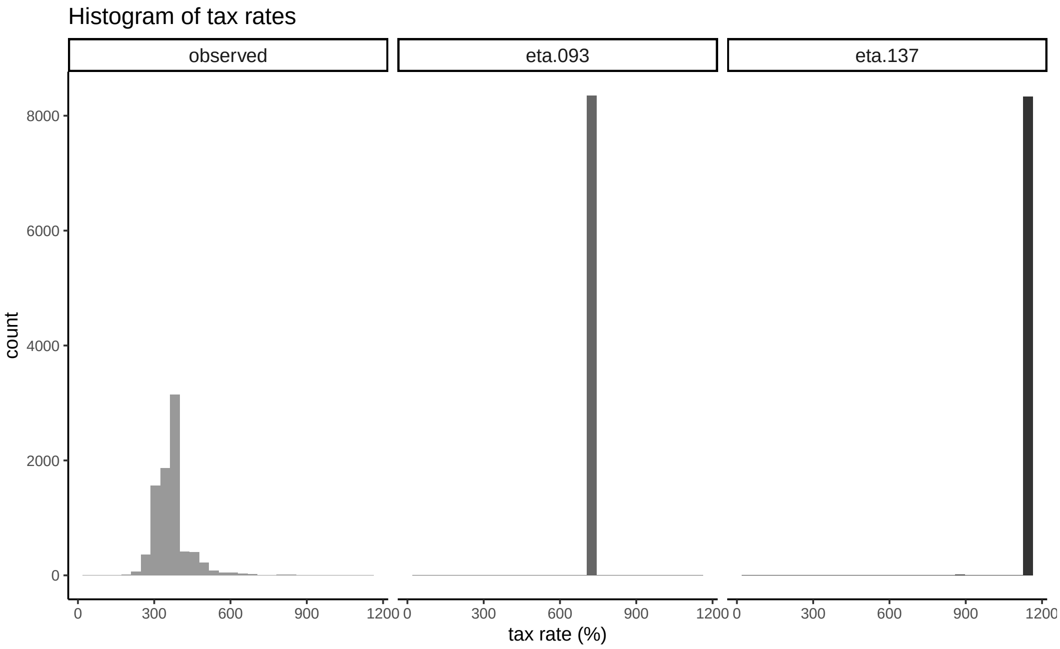
5.3.1 Decentralized taxation

In our first counterfactual exercise, we explore how the introduction of non-cooperative taxation by decentralized local governments affects tax rates and welfare. Panel A of Table 2 shows the mean and standard deviation of the tax rate, $\tau^*_{i(stock)}$, as well as changes in key endogenous variables.[25] Figure 2 shows a histogram of the equilibrium tax rates.

[25] We assume that the multiplier $\bar{\theta}_{i(stock)}$ stays exogenous to the counterfactual changes. Counterfactual stock tax rate is then given by $\tau^{*\prime}_{i(stock)} = \tau^{\prime}_{i(stock)}/\bar{\theta}_{i(stock)}$.



(a) Japan



(b) Germany

Figure 2: Histogram of the stock tax rates, $\tau^*_{i(stock)}$, in the decentralized regime.

As shown in Table 2 and Figure 2, the results of introducing decentralized taxation surprisingly differ for Japan and Germany, leading to different welfare implications.

In Japan, the average decentralized tax rate is 1.02-1.66% and hence lower than the average observed tax rate for a low η but higher than the average observed rate for a high η.[26] Property taxation primarily decreases the structure service price by suppressing housing demand and

[26]Here and below, the bounds of numerical ranges refer to the low and high η cases.

decreases land rents by suppressing factor demand in the structure service sector. These effects are partly offset by the indirect effect that it strengthens the scale economies in public goods consumption and attracts mobile workers, which increases the structure service price, other things equal.

Table 2 and Figure 2a show that under a low η, local governments have less incentive to tax property and set lower tax rates than observed. The structure service prices then rise by an average of 1.9%. These price changes and tax reductions benefit landowners. Higher structure service prices increase the labor demand in the numéraire sector which raises the wage rate, making workers better off, too. The resulting relative EV is 0.1% for workers, 1.9% for landowners, and 1.6% on average. A high η implies a strong preference for public goods consumption, which leads governments to levy higher tax rates than the observed ones. The structure service prices then falls by an average of 0.9%, resulting in a lower cost of living and higher welfare for workers but a lower income and lower welfare for landowners. In this case, the relative EV is 0.4% for workers, −0.6% for landowners, and −0.4% on average.

Table 2 and Figure 2b show that the average decentralized tax rate for Germany is 736.7%-1149.0% and hence higher than the observed one. Raising tax rates increases public goods supply, while increasing the cost of living and decreasing wages and land rent. For both values of η, we find that the positive effect on public goods supply outweighs the negative effects. The welfare gain measured in terms of relative EV is 1.1%-3.8% for workers, 0.6%-4.6% for landowners, and 0.8%-4.4% on average. The welfare gain rises with η, whereas it falls in Japan.

The standard deviation of tax rates declines for both countries. It changes from 0.3% to 0.2-0.3% in Japan, and from 70% to 4-12% in Germany. Interestingly, decentralization by itself does not increase the variation in tax rates across municipalities.

5.3.2 Centralized taxation

Our next counterfactual scenario is the centralized taxation regime, where the central government chooses a common tax rate satisfying (23) for all municipalities. Since this is the welfare optimal tax rate, average welfare must rise relative to the observed state. Note that here we first obtain the optimal crude tax $\tau^{c\prime}$ and then convert it to $\tau^{*\prime}_{i(stock)}$ using the multiplier $\bar{\theta}_{i(stock)}$. Hence, the optimal $\tau^{*\prime}_{i(stock)}$ can be different between municipalities.

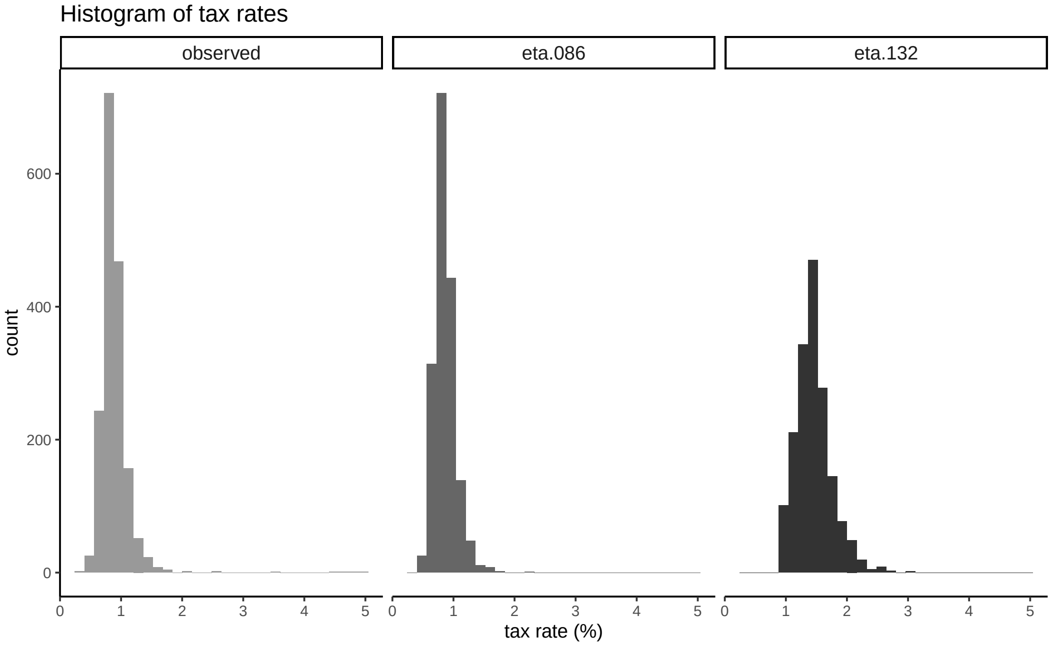
Panel B of Table 2 reports the counterfactual simulation results. In Japan, the optimal tax rate is 0.78%-1.26%. It is lower than the average observed tax rate, implying that the observed tax rates are inefficiently high.

For a low η, the welfare gain measured in the relative EV is −0.1% for workers, 2.6% for landowners, and 2.1% on average. For a high η, it is 0.1% for workers, 0.3% for landowners, and 0.2% on average. Depending on the value of η, moving from the observed state to centralized taxation can harm workers even though there is a positive effect on aggregate social welfare. Intuitively, for a low η, the gain from changes in wages and the cost of living is too small to offset the losses from a decrease in public goods provision.

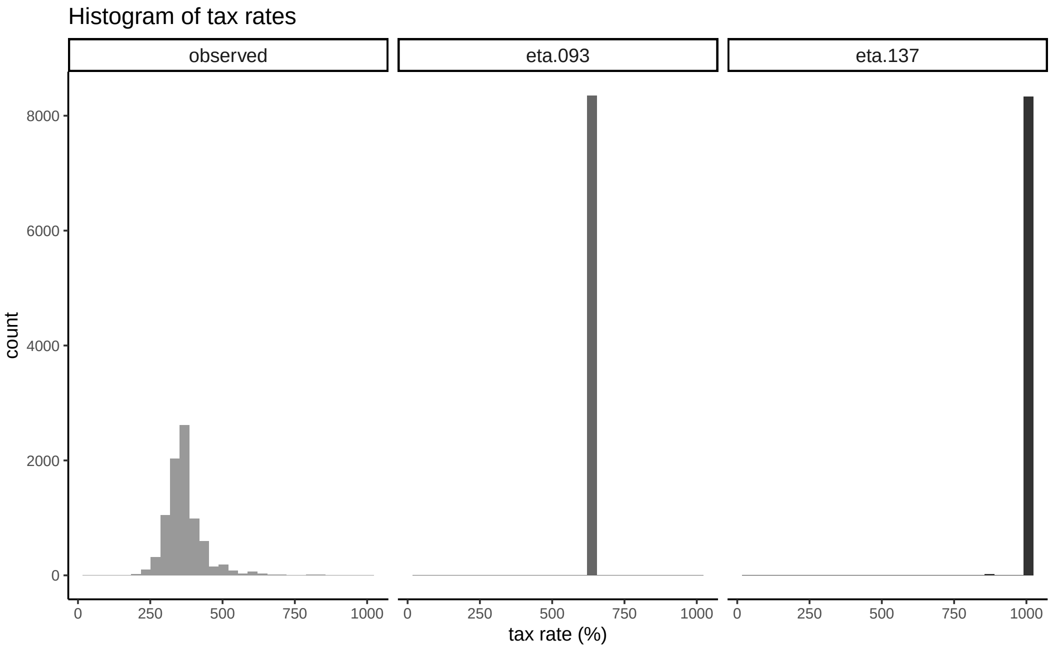
For Germany, the optimal tax rate is 564.8%-869.7%, which is higher than the average observed tax rate. As a result, in comparison to the observed state, public goods supply increases, while wages fall and the cost of living rises. The net effect is an increase in social welfare, which is higher for a larger η. The welfare gain measured in terms of the relative EV is 0.8%-3.5% for workers, 1.1%-5.4% for landowners, and 1.0%-4.7% on average.

5.3.3 Homevoter

Finally, we present a counterfactual simulation for the homevoter regime in which local governments maximize the welfare of landowners. Panel C of Table 2 and Figure 3 report the simulation results for the homevoter equilibrium.



(a) Japan



(b) Germany

Figure 3: Histogram of the tax rates, $\tau^*_{i(stock)}$, in the homevoter regime.

Proposition 3 implies that the equilibrium tax rates in the homevoter regime are lower than those in the decentralized taxation regime because the concern for lowered land rents mitigates the governments' incentives to raise property taxes. As a result, the effects on structure service prices and wages are also smaller and workers gain less while landowners gain more in the homevoter regime than in the decentralized regime.

For Japan, the average homevoter tax rate is 0.86%-1.46% and hence lower than the decentralized and observed tax rates but still higher than the optimal tax rate. For a low η, the relative EV is 0.0% for workers, 2.4% for landowners, and 1.9% on average. For a high η, it is 0.3% for workers, −0.1% for landowners, and 0.0% on average.

For Germany, the average homevoter tax rate is 622.9%-1017.4%, and again between those in the centralized and decentralized regimes. The relative EV is 0.9%-3.7% for workers, 1.0%-5.0% for landowners, and 1.0%-4.6% on average.

5.4 Discussion

Thus far, we have examined the effects of introducing three regimes on tax rates and welfare. Here, we discuss which regime can best describe the observed state. Because we are uncertain about the governments' objective in reality, governments' behavior may or may not be well approximated by a particular regime. If it were, it would yield a similar allocation to the observed equilibrium allocation. Otherwise, its allocation would deviate significantly from the observed one. We focus on the resulting tax rates to evaluate how well each regime approximates the observed tax rates. More specifically, we compute the sum of squared differences between the observed and equilibrium tax rates to measure the distance between the observed and counterfactual tax rate distributions.

Table 3: Distance between the distribution of τ_i and that of τ_i'.

		Japan		Germany	
measure	regime	$\eta = .086$	$\eta = .132$	$\eta = .093$	$\eta = .137$
Sum of $(\tau^{*\prime}_{i(stock)} - \tau^*_{i(stock)})^2$	decentralized	0.047	0.044	118,221	515,064
	centralized	0.090	0.032	36,747	215,097
	homevoter	0.072	0.029	58,735	357,453

Note: Tax rates are stock-measured effective rates. Even if we use other tax rates (i.e., τ_i or $\tau_{i(stock)}$), our conclusion is unaltered.

Table 3 shows the results. For Japan, the decentralized or homevoter regime gives the smallest sum of squares, while for Germany, the centralized regime gives the smallest sum of squares. Of course, we cannot conclude that the observed state is exactly described by one of the three regimes. Still, in terms of tax rates, we can say that, depending on the value of the preference for public goods, η, the decentralized taxation or homevoter regime is the closest to the observed state for Japan, and the centralized uniform taxation regime is the closest to the observed state for Germany. Hence, when discussing property taxation, our model predicts that the decentralized taxation or homevoter model best approximates the current situation of Japan. In contrast, we should refer to the centralized taxation (i.e.,, tax harmonization) model for Germany. This is somewhat surprising given that taxation appears to be more centralized in Japan than in Germany. One possible explanation is that Japanese local governments aggressively offer special discounts and exemptions on property tax rates in order to attract firms and workers. Concerning Germany, the fact that observed tax rates are much lower than the counterfactual rates is a reason why it appears closer to the centralized regime. As mentioned in Section 4.2, there is a sizable gap between the market and assessed property values. This erodes the tax base and may cause a severe downward pressure on the estimated crude tax rates. It may be interesting to speculate on the effects of the current reform of the German property tax system. While the details differ somewhat between states, the initial impetus for the reform was the outdated assessment values. According to our model, allowing local governments to adjust their assessed values might lead to higher tax rates that are closer to the counterfactual ones.

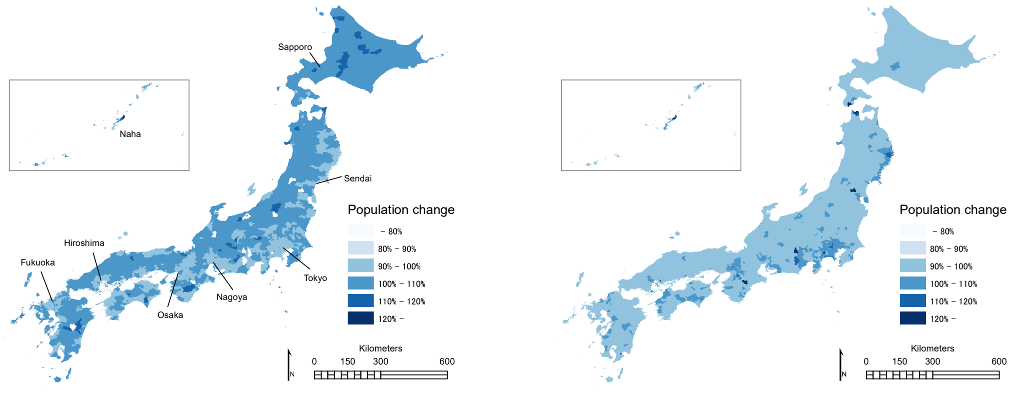
A few comments are in order. First, the standard deviations of the counterfactual tax rates are often smaller than the observed rates. This difference could be driven by the assumption of homogeneous parameters across cities. Relaxing parameter homogeneity might lead to counterfactual tax rates with more variation.

Second, the homevoter regime frequently produces a sizable welfare gain relative to the status quo. In recent policy debates in Japan, there has been concern about an increasing number of plots whose owners are unknown. In fact, the Ministry of Land, Infrastructure, Transportation and Tourism reported in 2016 that approximately 20% of land parcels in land registers had incorrect information on owners' addresses, and owners remained unidentified despite intensive scrutiny for 0.41% of land parcels.[27] The absence of landowners can cause another problem by inducing local governments to behave more in favor of mobile workers, i.e., the observed state may become closer to the welfare-inferior decentralized regime.

Third, although changes in tax rates lead to population relocation across Japanese municipalities, the population flows responding to the tax changes are fairly limited. Table 2 reports the population changes caused by the change from the observed equilibrium to the equilibrium under each regime, and Figure 4 shows their spatial distribution for the centralized taxation regime for Japan as an example.[28]

[27] `www.mlit.go.jp/common/001201306.pdf`, last accessed on June 28, 2022

[28] Figures for the other regimes are provided in Appendix D. The magnitudes of changes are somewhat different between regimes, but the direction of changes is similar.

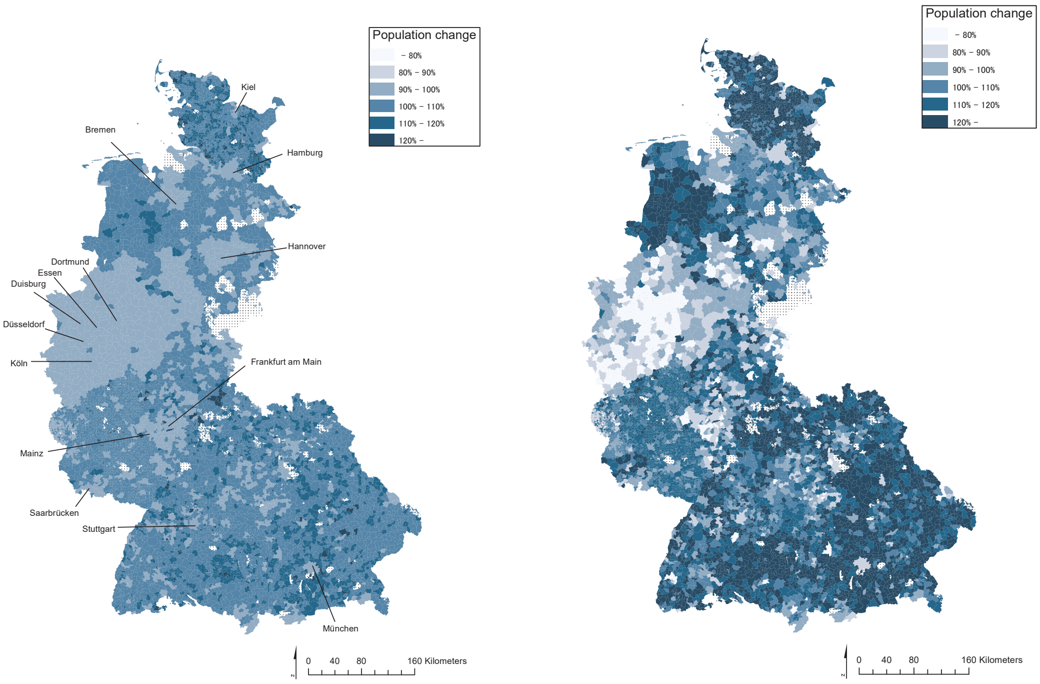


(a) $\eta = .086$ (b) $\eta = .132$

Figure 4: $\hat{n}_i$ in the centralized regime for Japan.

For a small η, the tax rates in the centralized regime are the most different from the observed tax rates among the three regimes, implying that the transition from the observed state to the centralized taxation equilibrium will be accompanied by the largest change in tax rates. Nonetheless, Figure 4a shows that population changes are mostly small. Further investigation uncovers that central areas including the largest three cities in Japan (Tokyo, Osaka, and Nagoya) lose and peripheral areas gain population. This implies that central areas over-provide public goods to attract mobile workers compared with peripheral areas in the observed state.

Finally, Figure 5 shows the spatial distribution of population changes for the decentralized taxation regime in Germany, which yields the tax structure that is most different from the observed state among the three regimes. The middle areas lose population whereas northern and southern areas gain population. Noting that the observed state in Germany is best described by the centralized (welfare optimal) regime, the change to the decentralized regime yields inefficient changes. Figure 5 implies that decentralization induces northern and southern areas to over-provide public goods to attract mobile workers compared to middle areas.



(a) $\eta = .093$ (b) $\eta = .137$

Figure 5: $\hat{n}_i$ in the decentralized regime for Germany.
Note: The thinly stippled areas are the out-of-the-sample regions.

6 Robustness checks

In this section, we discuss the robustness of our main findings. We consider four alternative scenarios by employing different time discount rates, introducing agglomeration economies, using a different share of landowners, and considering specific taxation. We also refer to the availability of other sources of fiscal revenues. We relegate the detailed results of the robustness checks to Appendix E and briefly explain them here.

6.1 Time discount rate

Our baseline specification had values of 0.04 for the time discount rate for Japan and 0.03 for Germany. Below, we examine how that choice affects the ability of each regime to approximate the observed state.

Here, we replace the value of the discount rate between the two countries: we set 0.03 for Japan and 0.04 for Germany. The calibration of η yields $\eta = 0.086$ or 0.132 for Japan and $\eta = 0.093$ or 0.137 for Germany. Table 18 reports the results for three counterfactual scenarios under the swapped discount rates. Using a lower discount rate for Japan decreases the counterfactual tax rates. As a result, the decentralized regime is the closest to the observed state for both low and high η. This again implies that the centralized regime is never the closest to the observed state. Using a higher discount rate for Germany increases the counterfactual tax rates. Hence, we again find that the centralized taxation regime yields the closest tax rates to the observed ones. However, the disparity between the centralized and the observed tax rates becomes even larger.

6.2 Agglomeration economies

In the baseline analysis, we assume no agglomeration economies (i.e., $\varepsilon = 0$). However, the existence of agglomeration economies has been well documented both theoretically and empirically in urban economics (Duranton and Puga, 2004; Combes and Gobillon, 2015). Hence, we check the effect of assuming agglomeration economies on our main results.

We set the agglomeration parameter, ε, to 0.04, following the extensive literature on agglomeration economies (Combes and Gobillon, 2015).

The results are presented in Table 20 of Appendix E. The quantitative results are very similar to those in the baseline specification except for population changes, which are somewhat larger than the baseline specification. The intuitive explanation is that larger wage responses amplify population movements due to agglomeration economies.

6.3 Share of landowners

In the baseline analysis, we use the homeownership rate as the share of landowners, $H_i/(n_iF_i)$, in the observed equilibrium. Because only a small proportion of municipalities are in corner solutions with respect to population, i.e., $\hat{\lambda}_i = H_i/(n_iF_i)$, our preceding analysis is not sensitive to the choice of the homeowner share. Nonetheless, because it determines the proportion of mobile households in the total population as well as the utility of landowners, their share can potentially play a significant role in determining the homevoter government's behavior and population distribution. This section conducts a robustness check against an alternative value for the share of landowners.

Here, we show the results for an economy in which all municipalities have an identical share of landowners in the observed equilibrium that is set to 30% or 70% for all municipalities. Tables 21 and 22 in Appendix E present the results. As can be seen from the tables, the results are almost the same as our baseline results. Although landowners' utility is affected by the choice of the landowner share, the qualitative and quantitative properties of the equilibrium remain largely unchanged.

6.4 Specific taxation

In this subsection we ask whether our results are sensitive to the assumption of specific rather than ad valorem taxes. This is especially relevant to Germany because the appraised values in Germany were determined more than 50 years ago and are not systematically associated with current market values. To incorporate this institutional feature, we introduce a specific tax for Germany.

Consider a specific tax of T_i per unit and an appraised price P_i that is historically determined and thus assumed to be exogenous. Households' expenditure on housing service is $(p_i + T_i P_i)d_i$, instead of $(1 + \tau_i)p_i d_i$ where τ_i is the ad valorem rate. Hence, the marginal effect of a change in the tax rate does not necessarily interact with the market price p_i. We provide theoretical details of the specific tax case in Appendix F. The parameters we use are the same as for the ad valorem model. We convert the observed stock values of properties into flow values by using the following asset pricing equation: $(stock \quad value) = (p_i x_i - T_i P_i x_i)/0.03$.

Table 23 in Appendix E gives the counterfactual results under specific taxation.[29] While the levels of equilibrium tax rates are far from the ad valorem rates, the responses of the endogenous variables to the counterfactual changes are fairly similar to those in the ad valorem tax model.

6.5 Other sources of fiscal revenues

Finally, we briefly discuss the availability of other fiscal revenues. In general, local governments have revenue sources other than property taxation, e.g., local income taxation and inter-regional or inter-governmental fiscal transfer. If another revenue source is also distortionary as in the case of income taxation, then the governments would choose the income and property tax rates so that they minimize the resulting dead weight losses. If another revenue source is not distortionary as in the case of lump-sum transfer from the national government, the governments would finance their expenditures by the lump-sum transfer as much as possible, and minimize the dependence on the property taxation.

Hence, introduction of other fiscal revenues may alter the previous results both qualitatively and quantitatively. First, the uniformity of the centralized tax rates may no longer hold because the tax elasticity of utility can be affected by the city-specific availability of other fiscal revenues. Second, by introducing other fiscal revenues, decentralized tax rates are likely to be significantly lower. Put differently, the validity of our numerical results depends on the local governments' necessity of financing public expenditure by property taxation.

[29] We can not prove the uniformity of the centralized tax rates under a specific tax. We nevertheless report as centralized regime the case where the national government sets a common property tax rate across cities as in the ad valorem taxation.

However, it is clearly beyond the scope of this paper to consider all these possibilities. Given that property tax revenues account for a sizable portion of local governments' total tax revenues in many countries, we believe our analysis is a reasonable first step in evaluating the property taxation.

7 Concluding remarks

In this paper, we developed a model of property taxation and characterized equilibria under three alternative taxation regimes: decentralized taxation, centralized (welfare optimal) taxation, and homevoter regime. We show that decentralized taxation yields inefficiently high tax rates; the optimal tax rate is common to all municipalities and its marginal change is neutral to population distribution; and the tax rate can be inefficiently high or low in the homevoter regime.

Using the exact hat algebra, we calibrated our model to the Japanese and German data, and conducted counterfactual exercises wherein the taxation regime switches from the observed one to either one of the three regimes. We found that the Japanese tax system is best described by the decentralized or homevoter taxation regime whereas the German tax system is best described by the centralized taxation regime. This implies that the observed state in Japan is far from optimal whereas that in Germany is close to optimal. At the same time, the equivalent variation obtained in the centralized regime seems to be much higher in Germany than in Japan. This is due in part to obsolete property appraisal that erodes the tax base of German municipalities. Our model would in principle be amenable to study the results of the recently implemented property tax reform in Germany. We think our paper provides an important step in discussing property tax reforms, as it allows us both to assess which regime best fits observed taxes in a particular country and to assess the potential effects on the distribution of population and welfare that would stem from particular reform proposals. Future research might build on these results to delve further into the analysis of existing and proposed property tax systems.

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Online Appendices (not for publication)

In Appendices, we define the gross tax rate t_i as $t_i \equiv 1 + \tau_i$ in order to simplify the exposition.

A Derivation of the endogenous variables

Structure service price p_i

We plug the labor market clearing condition $l_i = n_i$ into (7) to obtain

$$\begin{aligned} w_i &= A_i \alpha n_i^{\varepsilon+\alpha-1} m_i^{1-\alpha}, \\ t_i p_i &= A_i(1-\alpha)\left(\frac{n_i^{\frac{\alpha+\varepsilon}{\alpha}}}{m_i}\right)^{\alpha}. \end{aligned} \tag{35}$$

The second equation of (35) yields the firm's demand for structure service

$$m_i = \left[\frac{A_i(1-\alpha)}{t_i p_i}\right]^{\frac{1}{\alpha}} n_i^{\frac{\alpha+\varepsilon}{\alpha}}. \tag{36}$$

The structure service market clearing condition (8), combined with (2), yields

$$p_i x_i = p_i m_i + \frac{\mu}{t_i}\left(w_i n_i + r_i H_i\right). \tag{37}$$

From (5) and (35), we know that

$$\begin{aligned} r_i H_i &= \gamma p_i x_i, \\ w_i &= \frac{\alpha y_i}{n_i} = \frac{\alpha}{1-\alpha}\frac{t_i p_i m_i}{n_i}. \end{aligned} \tag{38}$$

This implies that (37) can be written as

$$\left(1 - \frac{\mu\gamma}{t_i}\right) p_i x_i = \left(1 + \frac{\alpha\mu}{1-\alpha}\right) p_i m_i.$$

Using (6) and (36), we can solve it for p_i to obtain (9).

Government's objective functions

In the decentralized taxation regime, the government's objective function is given by

$$
\begin{aligned}
W_i &= \left(n_i - \frac{H_i}{F_i}\right) v_{Wi} + \frac{H_i}{F_i} v_{Li} \\
&= \frac{\xi_i g_i^{\eta}}{(t_i p_i)^{\mu}} \left(n_i w_i + r H_i\right).
\end{aligned}
$$

From (38), we know that

$$
\frac{rH_i}{n_i w_i} = \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \mu\gamma}, \tag{39}
$$

implying that W_i can be written as

$$
W_i = n_i v_{Wi} \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \mu\gamma}\right).
$$

In the homevoter regime, the government's objective function is given by v_{Li}, which combined with (39), can be rewritten v_{Li} as

$$
v_{Li} = v_{Wi} \left(1 + \gamma \frac{n_i F_i}{H_i} \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \mu\gamma}\right).
$$

Characterization of the decentralized regime

Taking the derivative of SW with respect to τ_i, we obtain

$$
\begin{aligned}
\frac{\partial SW}{\partial \tau_i} &= v_a \frac{\partial n_i^o}{\partial \tau_i} \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \mu\gamma}\right) + \frac{\partial v_a^o}{\partial \tau_i} \sum_{l=1}^{M} n_l \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_l - \mu\gamma}\right) \\
&\quad - \gamma v_a n_i \frac{\mu + \frac{1-\alpha}{\alpha}}{(t_i - \mu\gamma)^2} + v_a \sum_{j \neq i} \frac{\partial n_j^o}{\partial \tau_i} \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_j - \mu\gamma}\right) \\
&= \frac{\partial v_a^o}{\partial \tau_i} \frac{1 + \epsilon_{un}}{\epsilon_{un}} \sum_{l=1}^{M} n_l \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_l - \mu\gamma}\right) \\
&\quad + v_a \left[\frac{\partial n_i^d}{\partial \tau_i} \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \mu\gamma}\right) - \gamma n_i \frac{\mu + \frac{1-\alpha}{\alpha}}{(t_i - \mu\gamma)^2}\right],
\end{aligned}
$$

where ϵ_{un} represents the population elasticity of worker utility:

$$
\epsilon_{un} \equiv \frac{n_i}{v_{Wi}} \frac{\partial v_{Wi}}{\partial n_i} = \frac{(1 - \mu\gamma + \eta)(\alpha + \varepsilon)}{\alpha + \gamma - \alpha\gamma} - 1 < 0.
$$

Evaluating $\partial SW/\partial\tau_i$ at the equilibrium under decentralized taxation, we obtain from $\partial n_i^o/\partial\tau_i + \sum_{j\neq i}\partial n_j^o/\partial\tau_i = 0$ and (15) that

$$\left.\frac{\partial SW}{\partial\tau_i}\right|_{\text{d-equilibrium}} = \frac{\partial v_a^o}{\partial\tau_i}\frac{1+\epsilon_{un}}{\epsilon_{un}}\sum_{l=1}^{M} n_l\left(1+\gamma\frac{\mu+\frac{1-\alpha}{\alpha}}{t_l-\mu\gamma}\right).$$

Characterization of the centralized regime

Here, we first represents various tax impacts in elasticity forms, and then compute the first-order condition for an interior solution wherein the no-migration condition holds with equality. We then prove the latter part of Proposition 2 that states the neutrality of taxation. Last, we remark on the corner solution.

Let $\lambda_i^W(t_i, v_a)$ be the population share of municipality i that satisfies both (27) and (29) (i.e.,, within-municipality equilibrium conditions) and write it as a function of the gross tax rate $t_i(\equiv 1+\tau_i)$ and the common utility level v_a. By definition, $v_{Wi}(\lambda_i^W(t_i, v_a), t_i) = v_a$ holds true for an interior solution. Let next $v_a^o(\boldsymbol{t})$ be the equilibrium common utility level that satisfies the population constraint $\sum_j \lambda_j^W(t_i, v_a) = 1$ and write it as a function of the M-length vector of tax rates $\boldsymbol{t}$. We denote $\lambda_i^o(\boldsymbol{t}) \equiv \lambda_i^W(t_i, v_a^o(\boldsymbol{t}))$ for expositional simplicity.

From the no-migration condition $v_{Wj} = v_a$, we obtain

$$\frac{\partial v_{Wj}(\lambda_j^W, t_j)}{\partial\lambda_j}\frac{\partial\lambda_j^W(t_j, v_a)}{\partial v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i} = \frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i},$$

which implies

$$\left(\frac{\lambda_j}{v_{Wj}}\frac{\partial v_{Wj}(\lambda_j^W, t_j)}{\partial\lambda_j}\right)\left(\frac{v_{Wj}}{\lambda_j}\frac{\partial\lambda_j^W(t_j, v_a)}{\partial v_a}\right) = \epsilon_{un}\epsilon_{nu} = 1, \tag{40}$$

where $\epsilon_{nu} \equiv (v_{Wj}/\lambda_j)(\partial\lambda_j^W/\partial v_a)$.

From the no-migration condition $v_{Wi} = v_a$, conditioning on v_a, we can see that

$$\frac{\partial v_{Wi}}{\partial\lambda_i}\frac{\partial\lambda_i^W}{\partial t_i} + \frac{\partial v_{Wi}}{\partial t_i} = 0.$$

We can arrange it to obtain

$$\frac{v_{Wi}}{t_i}\left[\frac{\lambda_i}{v_{Wi}}\frac{\partial v_{Wi}}{\partial\lambda_i}\frac{t_i}{\lambda_i}\frac{\partial\lambda_i^W}{\partial t_i} + \frac{t_i}{v_{Wi}}\frac{\partial v_{Wi}}{\partial t_i}\right] = 0,$$

which yields

$$\frac{v_{Wi}}{t_i}\left[\epsilon_{un}\epsilon_{nti} + \epsilon_{uti}\right] = 0,$$

where $\epsilon_{nti} \equiv (t_i/\lambda_i)(\partial\lambda_i^W/\partial t_i)$. Hence, we obtain

$$\epsilon_{un}\epsilon_{nti} + \epsilon_{uti} = 0. \tag{41}$$

The population constraint yields

$$\frac{\partial\lambda_i^o(\boldsymbol{t})}{\partial t_i} + \sum_{j\neq i}\frac{\partial\lambda_j^o(\boldsymbol{t})}{\partial t_i} = 0. \tag{42}$$

Differentiating the definition $v_{Wi}(\lambda_i^o(\boldsymbol{t}), t_i) = v_a^o(\boldsymbol{t})$ and substituting (40), (41), and (42), we get

$$\frac{t_i}{\lambda_i}\epsilon_{un}\left(-\sum_{j\neq i}\frac{\partial\lambda_j^o(\boldsymbol{t})}{\partial t_i}\right) + \epsilon_{uti} = \frac{t_i}{v_{Wi}}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}. \tag{43}$$

From the definition, $\partial\lambda_j^o(\boldsymbol{t})/\partial t_i = [\partial\lambda_j^W(t_j, v_a^o(\boldsymbol{t}))/\partial v_a][\partial v_a^o(\boldsymbol{t})/\partial t_i]$. Rewriting this as

$$\begin{aligned}\frac{t_i}{\lambda_j}\frac{\partial\lambda_j^o(\boldsymbol{t})}{\partial t_i} &= \frac{v_{Wj}}{\lambda_j}\frac{\partial\lambda_j^W(t_j, v_a^o(\boldsymbol{t}))}{\partial v_a}\left(\frac{t_i}{v_{Wj}}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}\right)\\ &= \epsilon_{nu}\left(\frac{t_i}{v_a^o}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}\right).\end{aligned}$$

Combining this with (43), we obtain

$$\frac{t_i}{\lambda_i}\epsilon_{un}\left(-\sum_{j\neq i}\epsilon_{nu}\frac{\lambda_j}{t_i}\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}\right) + \epsilon_{uti} = \frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}.$$

Rearranging this equation, we can see that

$$\epsilon_{uti} = \frac{\sum_{j\neq i}\lambda_j}{\lambda_i}\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}.$$

Thus, we obtain the tax elasticity of the common utility as the population share (over other municipalities' share) times the tax elasticity of worker utility of each municipality:

$$\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i} = \left[\frac{\lambda_i}{\sum_{j\neq i}\lambda_j}\right]\epsilon_{uti}.$$

Next consider a maximization of $SW = N\sum_j v_{Wj}\lambda_j\Delta_j$. The first-order condition (FOC) with respect to τ_i is

$$\begin{aligned}
0 = \frac{\partial}{\partial \tau_i}\frac{SW}{N} &= \sum_j \left[\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}\lambda_j + v_{Wj}\frac{\partial \lambda_j^o(\boldsymbol{t})}{\partial t_i}\right]\Delta_j + v_{Wi}\lambda_i\frac{\partial \Delta_i}{\partial t_i} \\
&= \frac{v_a}{t_i}\sum_j \left[\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i} + \frac{t_i}{\lambda_j}\frac{\partial \lambda_j^o(\boldsymbol{t})}{\partial t_i}\right]\lambda_j\Delta_j + v_{Wi}\lambda_i\frac{\partial \Delta_i}{\partial t_i}, \\
&= \frac{v_a}{t_i}\sum_j \left[\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i} + \epsilon_{nu}\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t_i}\right]\lambda_j\Delta_j + \frac{v_a}{t_i}\epsilon_{nti}\lambda_i\Delta_i + v_{Wi}\lambda_i\frac{\partial \Delta_i}{\partial t_i}, \\
&= \frac{v_a}{t_i}(1+\epsilon_{nu})\epsilon_{uti}\frac{\lambda_i}{\sum_{j\neq i}\lambda_j}\sum_j \lambda_j\Delta_j + \frac{v_a}{t_i}\epsilon_{nti}\lambda_i\Delta_i + v_{Wi}\lambda_i\frac{\partial \Delta_i}{\partial t_i}.
\end{aligned}$$

The no-migration condition $v_{Wi} = v_a$ implies that the last line is proportional to $v_a\lambda_i$. Eliminating $v_a\lambda_i$, the FOC is then rewritten as

$$\frac{(1+\epsilon_{nu})\epsilon_{uti}}{t_i}\frac{\sum_j \lambda_j\Delta_j}{\sum_{j\neq i}\lambda_j} + \frac{1}{t_i}\epsilon_{nti}\Delta_i + \frac{\partial \Delta_i}{\partial t_i} = 0.$$

This, combined with (41), yields (22). Note here that it is independent of municipality-specific characteristics except the gross tax rate t_i. So the solution is common to all municipalities as long as there is no multiplicity of solutions.

Set $t_i = t$ for all i in an interior solution. Then, Δ_i is also location-independent: $\Delta_i = \Delta$. The FOC becomes

$$\epsilon_{uti} = \frac{t}{\Delta}\frac{\partial \Delta}{\partial t}.$$

The tax elasticity of worker utility (the left-hand side) is equal to the tax elasticity of the income gap (the right-hand side). This yields (23).

Next we show the location neutrality. From the definition $\lambda_i^o(\boldsymbol{t}) = \lambda_i^W(t_i, v_a^o(\boldsymbol{t}))$, we have

$$\frac{\partial \lambda_i^o(\boldsymbol{t})}{\partial t} = \frac{\partial \lambda_i^W(t_i, v_a^o(\boldsymbol{t}))}{\partial t_i} + \frac{\partial \lambda_i^W(t_i, v_a^o(\boldsymbol{t}))}{\partial v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t},$$

which yields

$$\frac{t_i}{\lambda_i}\frac{\partial \lambda_i^o(\boldsymbol{t})}{\partial t} = \epsilon_{nti} + \epsilon_{nu}\frac{t_i}{v_a}\frac{\partial v_a^o(\boldsymbol{t})}{\partial t}. \tag{44}$$

Because $t_i = t$ for all i, the right-hand side of (44) is location independent (ϵ_{nti} is independent of i). Thus, the tax elasticity of the population share is common to all municipalities:

$$\frac{t}{\lambda_i}\frac{\partial \lambda_i^o(\boldsymbol{t})}{\partial t} = \frac{t}{\lambda_j}\frac{\partial \lambda_j^o(\boldsymbol{t})}{\partial t}, \quad \forall i, j.$$

Since population shares are non-negative, the sign of $\partial \lambda_i^o(\boldsymbol{t})/\partial t$ is equal to the sign of $\partial \lambda_j^o(\boldsymbol{t})/\partial t$.

Moreover, the total population constraint $\sum_j \lambda_j = 1$ implies

$$\sum_j \frac{\partial \lambda_j^o(\boldsymbol{t})}{\partial t} = 0.$$

Hence, we obtain $\partial \lambda_i^o(\boldsymbol{t})/\partial t = 0$ for all municipalities in interior centralized equilibrium. In other words, at the margin, a common tax shock does not change the population distribution in the optimum.

Finally, consider a corner solution in which no worker resides in municipality i: $n_i = H_i/F_i$ and $v_{Wi} \leq v_a$. Such a municipality is isolated from the urban system and any marginal shock does not affect λ_i. Maximization of social welfare with respect to tax rate in municipality i requires to maximize landlords' utility: $\max_{t_i} SW \Leftrightarrow \max_{t_i} v_{Li}$ because municipality i has no mobile workers. Since $n_i = H_i/F_i$, the landlords' utility is $v_{Li} = v_{Wi}\Delta_i$. Then, the first-order condition $\partial v_{Li}/\partial \tau_i = 0$ is given by

$$\epsilon_{uti}\Delta_i + t_i \frac{\partial \Delta_i}{\partial t} = 0,$$

which is exactly same as the FOC for the interior solution. Thus, we can conclude that the equilibrium tax rate in the centralized regime (and hence the optimal tax rate) is given by the same FOC.

B Observed equilibrium

Table 4: Summary statistics of the observed equilibrium values for Japan

variable	description	mean	S.D.	median
τ_i	Flow crude tax rate (%)	18.0	3.6	17.5
$\tau_{i(stock)}$	Stock crude tax rate (%)	0.89	0.28	0.85
$\tau^*_{i(stock)}$	Stock tax rate (%)	1.42	0.33	1.38
$p_i x_i/n_i$	Per capita value of structure service	387.4	314.6	335.5
$p_i d_i$	Per capita value of residential structure service	222.8	104.6	203.9
$p_i m_i/n_i$	Per capita value of corporate structure service	164.5	271.7	203.9
$p_i m_i/(p_i x_i)$	Share of corporate structure service	38.8	12.1	36.1
n_i	Municipality population	74237	288599	24033
g_i	Public expenditure	5.85×10^6	37.3×10^6	1.44×10^6
$H_i/(n_i F_i)$	Share of landowners in population (%)	80.1	10.3	81.5
w_i	Wage rate	294.2	502.3	210.0
$r_i H_i/n_i$	Land rent income per capita	96.8	78.7	83.9

Note: This table reports means, standard deviations and median for selected variables. The unit of housing values is thousand yen. The unit of population is person. Wage rate and land rent are computed by the formulas held in equilibrium: $w_i = [(1-\alpha)/\alpha]t_i(p_i m_i)/n_i$ and $r_i H_i/n_i = \gamma p_i x_i/n_i$.

Table 5: Summary statistics of the observed equilibrium values for Germany.

variable	description	mean	S.D.	median
τ_i	Flow crude tax rate (%)	7.9	1.3	7.8
$\tau_{i(stock)}$	Stock crude tax rate (%)	0.26	0.05	0.26
	Stock crude tax rate (specific tax) (%)	367.1	70.0	365.0
$\tau^*_{i(stock)}$	Stock tax rate (%)	367.1	70.0	365.0
$p_i x_i/n_i$	Per capita value of structure service	1447.5	1131.2	1387.7
$p_i d_i$	Per capita value of residential structure service	482.5	377.1	462.6
$p_i m_i/n_i$	Per capita value of corporate structure service	965.0	754.1	925.1
$p_i m_i/(p_i x_i)$	Share of corporate structure service (%)	66.7	0.0	66.7
n_i	Municipality population	7669.2	36534.8	1939
g_i	Public expenditure	1.32×10^6	8.96×10^6	0.20×10^6
$H_i/(n_i F_i)$	Share of immobile landlords in population (%)	63.6	10.9	64.7
w_i	Wage rate	1561.3	1212.9	1496.0
$r_i H_i/n_i$	Land rent income per capita	361.9	282.8	346.9

Note: This table reports means, standard deviations and median for selected variables. The unit of housing values is EUR. The unit of population is person. Wage rate and land rent are computed by the formulas held in equilibrium: $w_i = [(1-\alpha)/\alpha]t_i(p_i m_i)/n_i$ and $r_i H_i/n_i = \gamma p_i x_i/n_i$.

C Derivation of equations in the hat form

The prices that clear the structure service markets yield price changes, i.e., the price in the hat form, as:

$$\hat{p}_i = \left(\hat{t}_i^{1-\frac{1}{\alpha}} \frac{t_i - \gamma\mu}{\hat{t}_i t_i - \gamma\mu}\right)^{\frac{\alpha\gamma}{\alpha+\gamma-\alpha\gamma}} \hat{\lambda}_i^{\frac{\gamma(\alpha+\varepsilon)}{\alpha+\gamma-\alpha\gamma}}.$$

The supply and demand for the structure service in the hat form are

$$\hat{p}_i \hat{x}_i = \hat{p}_i^{\frac{1}{\gamma}},$$
$$\hat{p}_i \hat{m}_i = \hat{t}_i^{-\frac{1}{\alpha}} \hat{p}_i^{1-\frac{1}{\alpha}} \hat{\lambda}_i^{\frac{\varepsilon}{\alpha}+1}.$$

The factor prices in the hat form are

$$\hat{w}_i = (\hat{t}_i \hat{p}_i)^{1-\frac{1}{\alpha}} \hat{\lambda}_i^{\frac{\varepsilon}{\alpha}}, \tag{45}$$
$$\hat{r}_i = \hat{p}_i^{\frac{1}{\gamma}}. \tag{46}$$

We assume that in all municipalities the utility of workers is equalized at the observed equilibrium: $v_{Wi} = v_a$ for all i. The counterfactual spatial equilibrium that allows both interior and corner solutions is characterized by the following Kuhn-Tucker conditions in the hat form:

$$\left(\hat{\lambda}_i - \frac{H_i}{n_i F_i}\right)(\hat{v}_{Wi} - \hat{v}_a) = 0, \quad \hat{\lambda}_i - \frac{H_i}{n_i F_i} \geq 0, \quad \hat{v}_{Wi} - \hat{v}_a \leq 0.$$

The hat form of the indirect utility of the workers (3) is a function of $\hat{\lambda}_i$, $\hat{t}_i$, and $\hat{v}_a$ as follows:

$$\hat{v}_{Wi} = \hat{w}_i (\hat{t}_i \hat{p}_i)^{-\mu} \hat{g}_i^{\eta} = \hat{\tau}_i^{\eta} \hat{t}_i^{-\frac{(1+\eta)(1-\alpha)+\alpha\mu}{\alpha+\gamma-\alpha\gamma}} \left(\frac{t_i - \mu\gamma}{\hat{t}_i t_i - \mu\gamma}\right)^{\frac{\alpha(1-\gamma\mu+\eta)}{\alpha+\gamma-\alpha\gamma}-1} \hat{\lambda}_i^{\frac{(\alpha+\varepsilon)(1-\gamma\mu+\eta)}{\alpha+\gamma-\alpha\gamma}-1}. \tag{47}$$

We can solve the no-migration condition $\hat{v}_a = \hat{v}_{Wi}$ to obtain an explicit solution of the changes in the municipality's population share, $\hat{\lambda}_i$, as a function of τ_i' and $\hat{v}_a$. Note that ξ_i does not appear in $\hat{v}_{Wi}$, which simplifies our numerical exercises. In practice, the non-negative constraint on population binds at 15 or less municipalities.

The welfare function of a local government in the hat form is

$$\hat{W}_i = \hat{v}_{Wi} \hat{\lambda}_i \left[\frac{1 + \gamma \frac{(1-\alpha)/\alpha+\mu}{\hat{t}_i t_i - \gamma\mu}}{1 + \gamma \frac{(1-\alpha)/\alpha+\mu}{t_i - \gamma\mu}}\right], \tag{48}$$

which from (47), becomes a function of $\hat{\lambda}_i$, $\hat{\tau}_i$, $\hat{v}_a$ and the parameters. $\hat{SW}$ is given by

$$\hat{SW} = \frac{\sum_i \lambda_i'(v_{Wi}' + v_{Li}')}{SW} = \sum_i \frac{\lambda_i(v_{Wi} + v_{Li})}{SW} \hat{W}_i, \tag{49}$$

where $\lambda_i(v_{Wi} + v_{Li})/SW = \lambda_i(v_{Wi} + v_{Li})/[\sum_j \lambda_j(v_{Wj} + v_{Lj})] \in [0, 1]$ represents a weight of each municipality's welfare in the social welfare that can be computed under ex ante utility equalization $v_{Wi} = v_a$. In fact, we obtain

$$\lambda_i(v_{Wi} + v_{Li}) = \frac{v_a}{N} n_i \left(1 + \gamma \frac{\mu + \frac{1-\alpha}{\alpha}}{t_i - \gamma\mu}\right),$$

implying that v_a/N is canceled out in $\hat{SW}$. Hence, the welfare weight is computable from the data on population, tax rates, and parameters.

The objective function of homevoter governments is the utility of landowners in the hat form:

$$\hat{v}_{Li} = \frac{1 + \hat{\lambda}_i \gamma \frac{(1-\alpha)/\alpha + \mu}{\hat{t}_i t_i - \gamma\mu} \frac{n_i F_i}{H_i}}{1 + \gamma \frac{(1-\alpha)/\alpha + \mu}{t_i - \gamma\mu} \frac{n_i F_i}{H_i}} \hat{v}_{Wi}. \tag{50}$$

This is evaluated using data on the share of landowners in the municipality population $H_i/(n_i F_i)$.

We quantify the welfare impacts of counterfactuals for agents using the equivalent variation along the change. Equivalent variation of a worker living in municipality i, denoted by EV_{Wi}, is given by $EV_{Wi} = (\hat{v}_{Wi} - 1)w_i$. Similarly, those of a landowner living in municipality i, EV_{Li}, is given by $EV_{Li} = EV_{Wi} + \gamma(p_i x_i)(F_i/H_i)(\hat{t}_i^{-\mu} \hat{p}_i^{1/\gamma - \mu} \hat{g}_i^{\eta} - 1)$.

D Counterfactual results

Table 6: Decentralized equilibrium in Japan when $\eta = .086$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	13.4	0.4	13.4
Stock crude tax rate (%)	0.62	0.03	0.62
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1.02	0.21	1.00
Municipality population	74238	286154	24354
Changes in municipality population (%)	2.0	6.5	0.5
Changes in structure service price (%)	1.9	2.2	1.4
Changes in wage rate (%)	1.4	0.7	1.4
Changes in public goods (%)	-18.0	8.9	-19.0
Changes in land rent (%)	8.0	11.2	5.9
Changes in workers' common utility (%)	0.1	—	—
Changes in landlord' utility (%)	1.4	1.9	1.0
Municipality's welfare change (%)	2.9	7.1	1.3
Worker's EV	0.4	0.8	0.3
Landowner's EV	9.4	26.4	4.8
Average EV	7.6	20.7	4.0
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.1	0.0	0.1
Landowner's relative EV, EV_{Li}/I_{Li} (%)	1.9	2.5	1.6
Average relative EV, $\bar{EV}/I_i$ (%)	1.6	2.1	1.3
Changes in social welfare (%)	0.6	—	—

Note: This table presents means and standard deviations for selected variables. Changes in variable x report percent change rates, $100(\hat{x} - 1)$. The unit of EVs is thousands of yen per capita. The unit of population is person.

Table 7: Decentralized equilibrium in Japan when $\eta = .132$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	20.1	0.4	20.1
Stock crude tax rate (%)	1.00	0.03	1.01
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1.66	0.34	1.63
Municipality population	74237	295014	23838
Changes in municipality population (%)	-1.0	6.7	-2.6
Changes in structure service price (%)	-0.9	1.7	-1.3
Changes in wage rate (%)	-0.6	18.4	8.5
Changes in public goods (%)	10.7	18.4	8.5
Changes in land rent (%)	-3.4	9.0	-5.1
Changes in workers' common utility (%)	0.4	—	—
Changes in landlord' utility (%)	-0.2	1.5	-0.5
Municipality's welfare change (%)	-1.0	6.9	-2.5
Worker's EV	1.1	1.9	0.8
Landowner's EV	-1.2	14.4	-2.9
Average EV	-0.7	12.3	-2.2
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.4	0.0	0.4
Landowner's relative EV, EV_{Li}/I_{Li} (%)	-0.6	2.2	-1.0
Average relative EV, $\bar{EV}/I_i$ (%)	-0.4	1.9	-0.8
Changes in social welfare (%)	-0.2	—	—

Note: See Table 6.

Table 8: Centralized equilibrium in Japan when $\eta = .086$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	10.6	0.00	10.6
Stock crude tax rate (%)	0.47	0.00	0.47
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	0.78	0.16	0.77
Municipality population	74237	286148	24353
Changes in municipality population (%)	2.0	7.1	0.5
Changes in structure service price (%)	2.8	2.4	2.4
Changes in wage rate (%)	2.5	0.7	2.5
Changes in public goods (%)	-32.6	7.3	-33.4
Changes in land rent (%)	12.2	13.3	9.8
Changes in workers' common utility (%)	-0.1	—	—
Changes in landlord' utility (%)	1.8	2.1	1.4
Municipality's welfare change (%)	3.2	7.9	1.5
Worker's EV	-0.3	0.5	-0.2
Landowner's EV	12.1	29.4	6.9
Average EV	9.5	23.0	5.5
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	-0.1	0.0	-0.1
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.6	2.8	2.2
Average relative EV, $\bar{EV}/I_i$ (%)	2.1	2.4	1.8
Changes in social welfare (%)	0.8	—	—

Note: See Table 6.

Table 9: Centralized equilibrium in Japan when $\eta = .132$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	16.0	0.00	16.0
Stock crude tax rate (%)	0.76	0.00	0.76
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1.26	0.25	1.23
Municipality population	74237	295010	23838
Changes in municipality population (%)	-1.0	6.7	-2.6
Changes in structure service price (%)	0.4	1.7	0.0
Changes in wage rate (%)	0.9	1.5	1.0
Changes in public goods (%)	-7.1	15.3	-9.1
Changes in land rent (%)	1.7	9.9	-0.1
Changes in workers' common utility (%)	0.1	—	—
Changes in landlord' utility (%)	0.2	1.6	-0.1
Municipality's welfare change (%)	-0.7	7.0	-2.3
Worker's EV	0.2	0.3	0.1
Landowner's EV	2.0	15.7	-0.7
Average EV	1.6	13.3	-0.5
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.1	0.0	0.1
Landowner's relative EV, EV_{Li}/I_{Li} (%)	0.3	2.3	-0.2
Average relative EV, $\bar{EV}/I_i$ (%)	0.2	2.0	-0.2
Changes in social welfare (%)	0.1	—	—

Note: See Table 6.

Table 10: Homevoter equilibrium in Japan when $\eta = .086$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	11.5	0.5	11.5
Stock crude tax rate (%)	0.52	0.03	0.52
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	0.86	0.18	0.84
Municipality population	74237	286137	24353
Changes in municipality population (%)	2.0	6.6	0.5
Changes in structure service price (%)	2.5	2.2	2.1
Changes in wage rate (%)	2.1	0.7	2.2
Changes in public goods (%)	-27.9	8.0	-28.8
Changes in land rent (%)	10.8	11.3	8.6
Changes in workers' common utility (%)	0.0	—	—
Changes in landlord' utility (%)	1.7	1.9	1.3
Municipality's welfare change (%)	3.1	7.2	1.5
Worker's EV	0.1	0.1	0.0
Landowner's EV	11.3	27.6	6.4
Average EV	9.0	21.4	5.2
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.0	0.0	0.0
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.4	2.5	2.1
Average relative EV, $\bar{EV}/I_i$ (%)	1.9	2.0	1.7
Changes in social welfare (%)	0.8	—	—

Note: See Table 6.

Table 11: Homevoter equilibrium in Japan when $\eta = .132$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	18.1	0.4	18.1
Stock crude tax rate (%)	0.88	0.02	0.88
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1.46	0.29	1.43
Municipality population	74237	295019	23839
Changes in municipality population (%)	-1.0	6.8	-2.6
Changes in structure service price (%)	-0.3	1.7	-0.7
Changes in wage rate (%)	0.1	1.5	0.2
Changes in public goods (%)	2.1	17.0	0.1
Changes in land rent (%)	-0.9	9.1	-2.7
Changes in workers' common utility (%)	0.3	—	—
Changes in landlord' utility (%)	0.0	1.5	-0.3
Municipality's welfare change (%)	-0.8	7.0	-2.3
Worker's EV	0.8	1.5	0.6
Landowner's EV	0.6	14.7	-1.6
Average EV	0.7	12.6	-1.2
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.3	0.0	0.3
Landowner's relative EV, EV_{Li}/I_{Li} (%)	-0.1	2.2	-0.6
Average relative EV, $\bar{EV}/I_i$ (%)	0.0	1.9	-0.4
Changes in social welfare (%)	0.0	—	—

Note: See Table 6.

Table 12: Decentralized equilibrium in Germany when $\eta = .093$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	14.7	0.1	14.7
Stock crude tax rate (%)	0.52	0.00	0.52
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	736.7	3.8	736.8
Municipality population	7669	34470	2052
Changes in municipality population (%)	5.1	7.1	3.9
Changes in structure service price (%)	-1.2	0.9	-1.4
Changes in wage rate (%)	-3.2	1.3	-3.1
Changes in public goods (%)	83.6	55.5	76.4
Changes in land rent (%)	-4.8	3.9	-5.6
Changes in workers' common utility (%)	1.1	—	—
Changes in landlord' utility (%)	0.6	1.6	0.4
Municipality's welfare change (%)	5.0	6.8	3.8
Worker's EV	16.5	12.8	15.8
Landowner's EV	13.3	51.1	7.5
Average EV	15.2	30.4	10.2
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	1.1	0.0	1.1
Landowner's relative EV, EV_{Li}/I_{Li} (%)	0.6	1.6	0.4
Average relative EV, $\bar{EV}/I_i$ (%)	0.8	0.9	0.6
Changes in social welfare (%)	0.2	—	—

Note: This table presents means and standard deviations for selected variables. Changes in variable x report percent change rates, $100(\hat{x} - 1)$. The unit of EVs is EUR. The unit of population is person.

Table 13: Decentralized equilibrium in Germany when $\eta = .137$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	21.1	0.2	21.1
Stock crude tax rate (%)	0.80	0.01	0.80
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1149.0	12.2	1149.6
Municipality population	7669	30045	2288
Changes in municipality population (%)	16.7	33.5	12.5
Changes in structure service price (%)	-1.3	3.3	-1.8
Changes in wage rate (%)	-6.6	2.7	-6.4
Changes in public goods (%)	172.7	270.7	151.0
Changes in land rent (%)	-4.3	18.0	-6.8
Changes in workers' common utility (%)	3.8	—	—
Changes in landlord' utility (%)	4.6	8.3	3.7
Municipality's welfare change (%)	18.6	33.6	14.3
Worker's EV	59.2	46.0	56.7
Landowner's EV	101.5	266.5	72.3
Average EV	86.4	156.0	67.6
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	3.8	0.0	3.8
Landowner's relative EV, EV_{Li}/I_{Li} (%)	4.6	8.6	3.7
Average relative EV, $\bar{EV}/I_i$ (%)	4.4	4.8	3.7
Changes in social welfare (%)	1.9	—	—

Note: See Table 12.

Table 14: Centralized equilibrium in Germany when $\eta = .093$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	11.6	0.0	11.6
Stock crude tax rate (%)	0.40	0.00	0.40
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	564.8	0.0	564.8
Municipality population	7669	34470	2052
Changes in municipality population (%)	5.1	7.1	3.9
Changes in structure service price (%)	-0.2	0.9	-0.5
Changes in wage rate (%)	-2.1	1.3	-2.0
Changes in public goods (%)	51.6	45.8	45.7
Changes in land rent (%)	-0.9	4.0	-1.8
Changes in workers' common utility (%)	0.8	—	—
Changes in landlord' utility (%)	1.1	1.6	0.8
Municipality's welfare change (%)	5.2	6.9	4.0
Worker's EV	12.4	9.6	11.9
Landowner's EV	25.1	52.1	16.2
Average EV	20.4	33.6	15.1
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.8	0.0	0.8
Landowner's relative EV, EV_{Li}/I_{Li} (%)	1.1	1.7	0.8
Average relative EV, $\bar{EV}/I_i$ (%)	1.0	1.0	0.8
Changes in social welfare (%)	0.4	—	—

Note: See Table 12.

Table 15: Centralized equilibrium in Germany when $\eta = .137$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	16.9	0.0	16.9
Stock crude tax rate (%)	0.61	0.00	0.61
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	869.7	0.0	869.7
Municipality population	7669	30045	2288
Changes in municipality population (%)	16.7	33.5	12.5
Changes in structure service price (%)	0.0	3.4	-0.4
Changes in wage rate (%)	-5.2	2.8	-4.9
Changes in public goods (%)	129.4	227.6	111.1
Changes in land rent (%)	0.9	19.0	-1.8
Changes in workers' common utility (%)	3.5	—	—
Changes in landlord' utility (%)	5.3	8.6	4.4
Municipality's welfare change (%)	19.0	33.7	14.6
Worker's EV	54.0	41.9	51.7
Landowner's EV	119.5	281.6	87.2
Average EV	94.9	164.1	75.3
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	3.5	0.0	3.5
Landowner's relative EV, EV_{Li}/I_{Li} (%)	5.4	8.9	4.4
Average relative EV, $\bar{EV}/I_i$ (%)	4.7	5.0	4.1
Changes in social welfare (%)	2.2	—	—

Note: See Table 12.

Table 16: Homevoter equilibrium in Germany when $\eta = .93$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	12.7	0.0	12.7
Stock crude tax rate (%)	0.44	0.00	0.44
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	622.9	1.3	622.9
Municipality population	7669	34470	2052
Changes in municipality population (%)	5.1	7.1	3.9
Changes in structure service price (%)	-0.6	0.9	-0.8
Changes in wage rate (%)	-2.5	1.3	-2.4
Changes in public goods (%)	63.0	49.3	56.6
Changes in land rent (%)	-2.3	4.0	-3.2
Changes in workers' common utility (%)	0.9	—	—
Changes in landlord' utility (%)	1.0	1.6	0.8
Municipality's welfare change (%)	5.2	6.9	4.0
Worker's EV	14.6	11.4	14.0
Landowner's EV	22.0	54.8	14.3
Average EV	19.6	32.9	14.3
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.9	0.0	0.9
Landowner's relative EV, EV_{Li}/I_{Li} (%)	1.0	1.6	0.7
Average relative EV, $\bar{EV}/\bar{I_i}$ (%)	1.0	1.0	0.8
Changes in social welfare (%)	0.4	—	—

Note: See Table 12.

Table 17: Homevoter equilibrium in Germany when $\eta = .137$.

variable	mean	S.D.	median
Flow crude tax rate, τ_i' (%)	19.2	0.1	19.2
Stock crude tax rate (%)	0.71	0.00	0.71
Stock tax rate, $\tau_{i(stock)}^{*\prime}$ (%)	1017.4	7.1	1017.7
Municipality population	7669	30045	2288
Changes in municipality population (%)	16.7	33.5	12.5
Changes in structure service price (%)	-0.7	3.3	-1.2
Changes in wage rate (%)	-5.9	2.7	-5.7
Changes in public goods (%)	153.5	251.6	133.3
Changes in land rent (%)	-1.9	18.4	-4.6
Changes in workers' common utility (%)	3.7	—	—
Changes in landlord' utility (%)	5.0	8.5	4.0
Municipality's welfare change (%)	18.8	33.6	14.5
Worker's EV	58.0	45.1	55.6
Landowner's EV	111.2	273.9	80.4
Average EV	91.7	160.4	72.5
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	3.7	0.0	3.7
Landowner's relative EV, EV_{Li}/I_{Li} (%)	5.0	8.7	4.1
Average relative EV, $\bar{EV}/\bar{I_i}$ (%)	4.6	4.9	3.9
Changes in social welfare (%)	2.1	—	—

Note: See Table 12.

Population changes

(a) $\eta = .086$ (b) $\eta = .132$

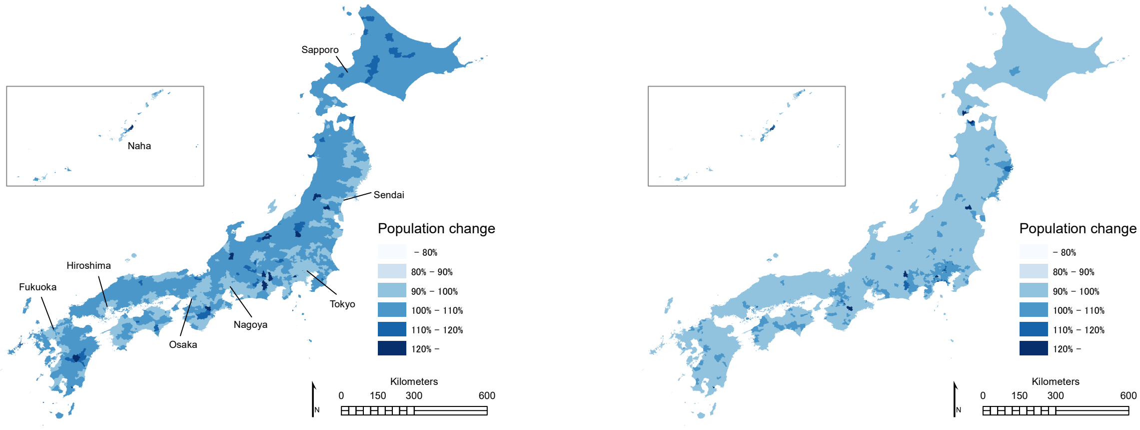


Figure 6: $\hat{n}_i$ in the decentralized regime for Japan. $\eta = .086$ in (a) and $\eta = .132$ in (b).

(a) $\eta = .086$ (b) $\eta = .132$

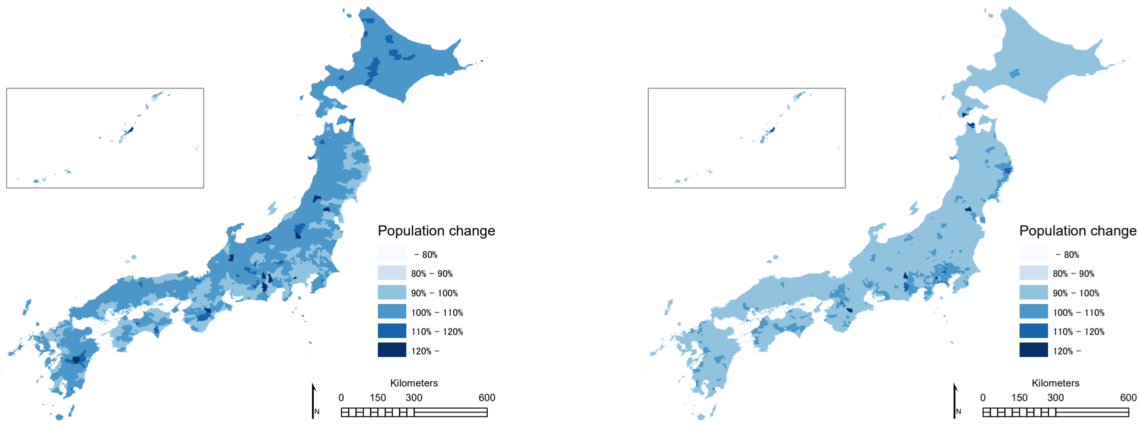
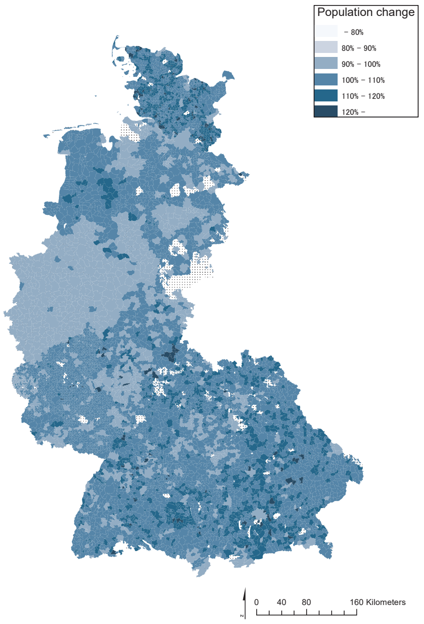


Figure 7: $\hat{n}_i$ in the homevoter regime for Japan. $\eta = .086$ in (a) and $\eta = .132$ in (b).
Note: The thinly stippled areas are the out-of-the-sample regions.

(a) $\eta = .093$



(b) $\eta = .137$

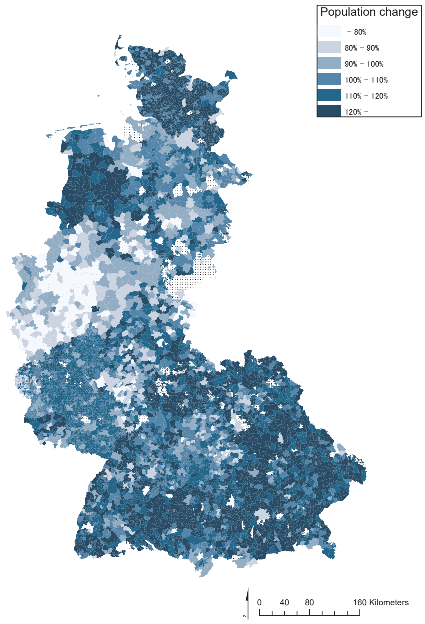
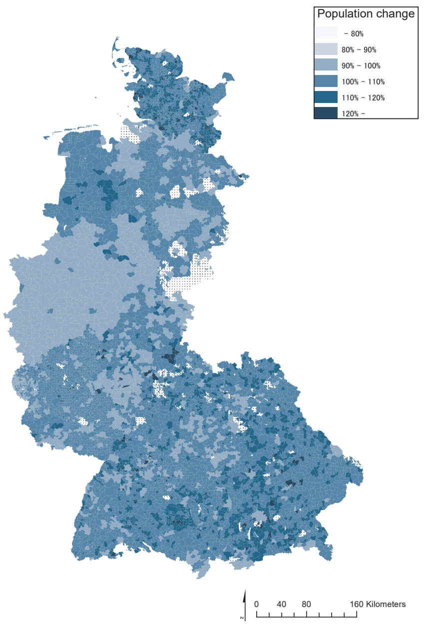


Figure 8: $\hat{n}_i$ in the centralized regime for Germany. $\eta = .093$ in (la) and $\eta = .137$ in (b). *Note*: The thinly stippled areas are the out-of-the-sample regions.

(a) $\eta = .093$



(b) $\eta = .137$

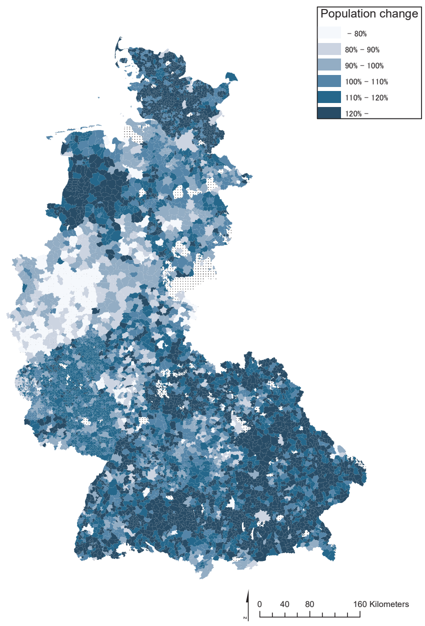


Figure 9: $\hat{n}_i$ in the homevoter regime for Germany. $\eta = .093$ in (a) and $\eta = .137$ in (b).

E Results of the robustness checks

This appendix provide tables reporting detailed results of the robustness checks.

Time discount rate

Table 18: Time discount rates is set to 0.03 for Japan.

variable	$\eta = .086$	$\eta = .132$
[Observed tax rate]		
Stock tax rate, $\tau^{*}_{i(stock)}$ (%)	1.42	
[Decentralized taxation regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.77	1.25
Changes in municipality population (%)	3.5	1.6
Changes in structure service price (%)	3.6	1.1
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.7	0.1
Landowner's relative EV, EV_{Li}/I_{Li} (%)	4.2	1.0
Average relative EV, $\bar{EV}/I_i$ (%)	3.5	0.8
[Centralized taxation regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.59	0.94
Changes in municipality population (%)	3.6	1.7
Changes in structure service price (%)	4.6	2.4
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.4	-0.3
Landowner's relative EV, EV_{Li}/I_{Li} (%)	5.0	2.0
Average relative EV, $\bar{EV}/I_i$ (%)	4.0	1.5
[Homevoter regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.65	1.10
Changes in municipality population (%)	3.5	1.6
Changes in structure service price (%)	4.3	1.7
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	0.6	0.0
Landowner's relative EV, EV_{Li}/I_{Li} (%)	4.7	1.5
Average relative EV, $\bar{EV}/I_i$ (%)	3.9	1.2

Table 19: Time discount rates is set to 0.04 for Germany.

variable	$\eta = .093$	$\eta = .137$
[Observed tax rate]		
Stock tax rate, $\tau^{*}_{i(stock)}$ (%)	367.1	
[Decentralized taxation regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	982.3	1531.2
Changes in municipality population (%)	7.2	20.4
Changes in structure service price (%)	-1.4	-1.3
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	2.1	6.0
Landowner's relative EV, EV_{Li}/I_{Li} (%)	1.7	7.3
Average relative EV, $\bar{EV}/\bar{I_i}$ (%)	1.9	6.9
[Centralized taxation regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	753.1	1159.6
Changes in municipality population (%)	7.2	20.4
Changes in structure service price (%)	-0.5	0.0
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	1.9	5.7
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.3	8.1
Average relative EV, $\bar{EV}/\bar{I_i}$ (%)	2.1	7.3
[Homevoter regime]		
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	830.5	1356.1
Changes in municipality population (%)	7.2	20.4
Changes in structure service price (%)	-0.8	-0.7
Worker's relative EV, EV_{Wi}/I_{Wi} (%)	2.0	6.0
Landowner's relative EV, EV_{Li}/I_{Li} (%)	2.1	7.7
Average relative EV, $\bar{EV}/\bar{I_i}$ (%)	2.1	7.1

Agglomeration economies

Table 20: Introducing agglomeration economies, $\varepsilon = .04$.

variable	Japan $\eta = .086$	Japan $\eta = .132$	Germany $\eta = .093$	Germany $\eta = .137$
[Observed tax rate]				
Stock tax rate, $\tau^{*}_{i(stock)}$ (%)	1.42		367.1	
[Decentralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.04	1.62	752.1	870.7
Changes in municipality population (%)	4.1	-0.5	10.8	1929.2
Changes in structure service price (%)	2.2	-1.3	-0.2	-12.3
Average relative EV, $E\bar{V}/I_i$ (%)	2.0	-0.1	1.7	303.1
[Centralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.78	1.26	564.8	869.7
Changes in municipality population (%)	4.3	-0.4	10.8	1929.5
Changes in structure service price (%)	3.3	-0.2	0.9	-12.3
Average relative EV, $E\bar{V}/I_i$ (%)	2.6	0.5	2.0	314.9
[Homevoter regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.95	1.55	690.1	870.6
Changes in municipality population (%)	4.1	-0.4	10.8	1929.2
Changes in structure service price (%)	2.6	-1.2	0.2	-12.3
Average relative EV, $E\bar{V}/I_i$ (%)	2.2	0.1	1.9	304.2

The share of landowners

Table 21: Alternative landowner share (small η).

variable	Japan ($\eta = .086$) $H/nF = .7$	Japan ($\eta = .086$) $H/nF = .3$	Germany ($\eta = .093$) $H/nF = .7$	Germany ($\eta = .093$) $H/nF = .3$
[Observed tax rate]				
Stock tax rate, $\tau^{*}_{i(stock)}$ (%)	1.42		367.1	
[Decentralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.02	1.02	736.8	736.8
Changes in municipality population (%)	2.0	2.0	5.1	5.1
Changes in structure service price (%)	1.9	1.9	-1.2	-1.2
Average relative EV, $\bar{EV}/I_i$ (%)	1.5	1.1	0.8	0.9
[Centralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.78	0.78	564.8	564.8
Changes in municipality population (%)	2.0	2.0	5.1	5.1
Changes in structure service price (%)	2.8	2.8	-0.2	-0.2
Average relative EV, $\bar{EV}/I_i$ (%)	2.0	1.3	1.0	1.0
[Homevoter regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	0.86	0.86	622.9	622.9
Changes in municipality population (%)	2.0	2.0	5.1	5.1
Changes in structure service price (%)	2.5	2.5	-0.6	-0.6
Average relative EV, $\bar{EV}/I_i$ (%)	1.9	1.3	1.0	1.0

Table 22: Alternative landowner share (large η).

variable	Japan ($\eta = .132$) $H/nF = .7$	Japan ($\eta = .132$) $H/nF = .3$	Germany ($\eta = .137$) $H/nF = .7$	Germany ($\eta = .137$) $H/nF = .3$
[Observed tax rate]				
Stock tax rate, $\tau^{*}_{i(stock)}$ (%)	1.42		367.1	
[Decentralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.66	1.66	1147.7	1149.6
Changes in municipality population (%)	-1.0	-1.0	16.6	16.7
Changes in structure service price (%)	-0.9	-0.9	-1.3	-1.3
Average relative EV, $E\bar{V}/I_i$ (%)	-0.4	-0.1	4.4	4.2
[Centralized taxation regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.26	1.26	869.7	869.7
Changes in municipality population (%)	-1.0	-1.0	16.6	16.7
Changes in structure service price (%)	0.4	0.4	0.0	0.0
Average relative EV, $E\bar{V}/I_i$ (%)	0.2	0.2	4.7	4.4
[Homevoter regime]				
Stock tax rate, $\tau^{*\prime}_{i(stock)}$ (%)	1.46	1.46	1016.7	1017.7
Changes in municipality population (%)	-1.0	-1.0	16.6	16.7
Changes in structure service price (%)	-0.3	-0.3	-0.7	-0.7
Average relative EV, $E\bar{V}/I_i$ (%)	0.0	0.1	4.6	4.4

Specific taxation

Table 23: Introducing specific taxes in Germany.

variable	$\eta = .093$	$\eta = .137$
[Observed tax rate]		
Stock tax rate, T_i (%)	367.1	
[Decentralized taxation regime]		
Specific tax rate, T_i' (%)	676.2	972.4
Changes in municipality population (%)	5.1	16.7
Changes in structure service price (%)	-1.2	-1.3
Average relative EV, $E\bar{V}/I_i$ (%)	0.8	4.4
[Centralized uniform taxation regime]		
Specific tax rate, T_i' (%)	561.1	793.4
Changes in municipality population (%)	5.3	16.3
Changes in structure service price (%)	-0.4	-0.1
Average relative EV, $E\bar{V}/I_i$ (%)	1.0	4.7
[Homevoter regime]		
Specific tax rate, T_i' (%)	586.7	884.8
Changes in municipality population (%)	5.1	16.7
Changes in structure service price (%)	-0.6	-0.7
Average relative EV, $E\bar{V}/I_i$ (%)	1.0	4.6

F Specific taxation model

This section replaces ad valorem taxation with specific taxation. First, we examine its effects on efficiency results. This is feasible by fixing the appraised values of land in the baseline framework shown in the main text. Second, we lays down equations rewritten in the fat form.

The housing demand of households and the indirect utility are

$$d_i = \mu(p_i + T_i P_i)^{-1} I_{ji} \text{ and } u_i = \xi_i g_i^{\eta}(p_i + T_i P_i)^{-\mu} I_{ji}.$$

The factor demand of the numéraire sector is

$$m_i \left[\frac{(1-\alpha)A_i}{p_i + T_i P_i}\right]^{1/\alpha} n_i^{(\alpha+\varepsilon)/\alpha}.$$

The market clearing condition for structure service is no longer solved explicitly.

$$(1-\gamma\mu)p_i x_i + g_i = (1 + \frac{\alpha\mu}{1-\alpha})(p_i + T_i P_i) m_i.$$

The market clearing condition determines p_i, from which we know that $\partial p_i/\partial T_i < 0$ and $\partial p_i/\partial n_i > 0$.

We need to slightly modify Assumption 1 because p_i can not be explicitly solved.

Assumption 1' (stability). In equilibrium, the following inequalities holds true:

$$-1 < \epsilon_i < 0,$$

where ϵ_i is defined as

$$\epsilon_i \equiv \frac{n_i}{v_{Wi}} \frac{\partial v_{Wi}}{\partial n_i}.$$

Here we define

$$\Phi_i \equiv 1 + \frac{r_i H_i}{w_i n_i} = 1 + \frac{\gamma(1-\alpha+\alpha\mu)}{\alpha(1-\mu\gamma T_i P_i/p_i)}.$$

Then, the Benthamite welfare is given by

$$W_i = n_i v_{Wi} \Phi_i.$$

The structure-market clearing condition becomes

$$x_i = m_i + \frac{\mu}{p_i + T_i P_i}(w_i n_i + r_i H_i),$$

which, combined with (38) under the specific taxation, yields

$$\frac{(x_i - m_i)}{\mu[\alpha/(1-\alpha)]m_i} = 1 + \frac{r_i H_i}{w_i n_i}.$$

Plugging this into W_i, we obtain

$$W_i = n_i v_{Wi} \left(\frac{1-\alpha}{\alpha\mu} \frac{x_i - m_i}{m_i} \right).$$

Social welfare SW can be written as

$$SW = \sum_{l=1}^{M} W_l = \sum_{l=1}^{M} n_l v_{Wl} \Phi_l.$$

The first-order condition under the decentralized taxation is given by

$$\frac{\partial n_i}{\partial T_i} \Phi_i + n_i \frac{\partial \Phi_i}{\partial T_i} = 0.$$

Evaluating $\partial SW/\partial T_i$ under $v_{Wi} = v_a, \forall i$, we obtain

$$\left.\frac{\partial SW}{\partial T_i}\right|_{\text{d-equilibrium}} = \frac{\partial v_a^o}{\partial T_i}\sum_{l=1}^{M} n_l \Phi_l \frac{1+\epsilon_l}{\epsilon_l}.$$

Hence, the equilibrium property tax rate is inefficiently high if and only if

$$\sum_{l=1}^{M} n_l \Phi_l \frac{1+\epsilon_l}{\epsilon_l} < 0,$$

which is satisfied from Assumption 1'. Hence, we obtain similar properties to the case of ad valorem tax (Proposition 1).

Under the homevoter regime, equilibrium tax rate is lower than that under the decentralized taxation, which increases the possibility of inefficiently low tax rate. Moreover, the sign of $\partial SW/\partial T_i$ depends on municipalities, implying that some municipalities have inefficiently high tax rates whereas others have inefficiently low tax rates. The centralized taxation again yields the optimal tax rate. The conditions that characterize the sign of $\partial SW/\partial T_i$ become complicated under the homevoter regime and the centralized taxation, which are not interesting for readers. Hence, we omit them here.

Next, we provide key equations under specific taxation in the hat form. The market clearing condition in the hat form is given by

$$\frac{(1-\gamma\mu)p_i x_i \hat{p}_i \hat{x}_i + \hat{g}_i g_i}{(1-\gamma\mu)p_i x_i + g_i} = \left(p_i \hat{+} T_i P_i\right)^{1-1/\alpha} \hat{\lambda}_i^{(\alpha+\varepsilon)/\alpha},$$

where

$$\hat{g}_i = \hat{P}_i \hat{T}_i \hat{x}_i = \hat{T}_i \hat{p}_i^{1/\gamma - 1}, \text{ and } \hat{m}_i = \left(p_i \hat{+} T_i P_i\right)^{-1/\alpha} \hat{\lambda}_i^{(\alpha+\varepsilon)/\alpha}.$$

The tax-inclusive price is computable with data on $p_i x_i, g_i$ and T_i as follows.

$$(p_i \hat{+} T_i P_i) = \frac{\hat{p}_i p_i x_i + (P_i \hat{T}_i) g_i}{p_i x_i + T_i P_i x_i} = \frac{\hat{p}_i p_i x_i + \hat{T}_i g_i}{p_i x_i + g_i}.$$

The utility of a worker in the hat form is

$$\hat{v}_{Wi} = \hat{g}_i^{\eta} \left(p_i \hat{+} T_i P_i\right)^{1-\mu-1/\alpha} \hat{\lambda}_i^{\varepsilon/\alpha}.$$

The each municipality's welfare in the hat form is

$$\begin{aligned}
\hat{W}_i &= \hat{\lambda}_i \hat{v}_{Wi} \frac{1}{\hat{m}_i} \frac{x_i' - m_i'}{x_i - m_i} \\
&= \hat{\lambda}_i \hat{v}_{Wi} \frac{\hat{p}_i^{(1-\gamma)/\gamma} \widehat{(p_i + T_i P_i)}^{1/\alpha} \hat{\lambda}_i^{-(\alpha+\varepsilon)/\alpha} - p_i m_i/(p_i x_i)}{1 - p_i m_i/(p_i x_i)}.
\end{aligned}$$

The social welfare in the hat form is given by

$$\hat{SW} = \sum_i \frac{W_i}{SW} \hat{W}_i.$$

Maximizing SW' is equivalent to maximizing $\hat{SW}$.

The landowners' utility in the hat form is given by

$$\hat{v}_{Li} = \hat{g}_i^{\eta} \widehat{(p_i + T_i P_i)}^{-\mu} \frac{\hat{w}_i + \hat{r}_i r_i F_i / w_i}{1 + r_i F_i / w_i}.$$

The factor prices in the hat form are

$$\hat{w}_i = \widehat{(p_i + T_i P_i)}^{1-1/\alpha} \hat{\lambda}_i^{\varepsilon/\alpha},$$

$$\hat{r}_i = \widehat{(p_i x_i)} = \hat{p}_i^{1/\gamma}.$$

These equations in the hat form in hand, we can define equilibrium similarly to the case of ad valorem taxation.