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Self-fulfilling Lockdowns in a Simple SIR-Macro Model

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Abstract

This study aims to show that a self-fulfilling (extrinsic) lockdown can occur even in economies where lockdowns are not necessary to keep the number of infected people below a threshold level. We conducted our analysis using a simple epidemiological SIR model modified by Eichenbaum et al. (2020). We derive various extrinsic lockdowns by solving a control problem explicitly. A key mechanism that leads to extrinsic lockdowns is that an anticipated lockdown increases the consumption of susceptible people before the lockdown and raises the number of infected people. Consequently, the anticipated lockdown becomes a self-fulfilling prophecy in the following period. The welfare effect of extrinsic lockdowns is in marked contrast to intrinsic lockdowns. When extrinsic lockdowns are expected to occur, the increased number of infections increases the probability of infection and deteriorates the welfare of susceptible people. The results suggest that an appropriate policy announcement might be key to ruling out undesirable self-fulfilling expectations.

JEL codes: C0, E2, E6, E7.

Keywords: self-fulfilling prophecy, pandemic, lockdown, expectations

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1. Introduction

As the COVID-19 pandemic spreads worldwide, governments are struggling with managing the pandemic. The discussion on desirable policy responses grew as quickly as the virus spread globally. This has motivated numerous studies on how economic policy should respond to this unprecedented shock.[1] When the number of infections increases in the pandemic, lockdowns that strictly restrict people's consumption are a powerful tool to avoid explosive spread of infections. Several studies have explored the optimal lockdown policy for a planner who wants to control the fatalities of a pandemic while minimizing the economic losses of the lockdown (see, for example, Acemoglu et al. (2020); Alvarez et al. (2020), Jones et al. (2020); Kaplan et al. (2020); Gonzalez-Eiras and Niepelt (2020); Garriga et al. (2020) for theoretical contributions and Greenstone and Nigam (2020) for empirical studies). However, they rarely explore whether an announcement of lockdowns can become a self-fulfilling prophecy when people's consumption behavior changes before the lockdowns.

This study aims to show that a self-fulfilling lockdown can occur even in economies where there is no need for a lockdown to keep the number of infected people below a threshold level. We conduct our analysis within a simple epidemiological SIR ("Susceptible-Infected-Recovered") model modified by Eichenbaum et al. (2020) (hereafter, ERT).[2] We study the dynamics of the pandemic, for susceptible, infected, recovered, diseased as functions of some exogenously chosen diffusion parameters. Individuals participating in market activities alter their consumption and work patterns, being aware of the resulting infection and death risks but without considering the externality of their behavior on the infection risks of others. The following analysis departs from ERT in two crucial dimensions. First, we assume that the government announces the rule of its lockdown policy in advance. According to this rule, the government makes a credible commitment to implement its lockdown if and only if the equilibrium number of infections exceeds a predetermined threshold value. It also makes a credible commitment to restrict people's consumption below a preannounced level. Second, we assume that all consumers are aware the government's rule and faithfully follow the restrictions. Thus, their consumption is

[1] Early contributions include McKibbin and Fernando (2020), Toda (2020), Faria-e Castro (2020), Guerrieri et al. (2020), Atkeson (2020), Farboodi et al. (2020), and Barrero et al. (2020).

[2] A tremendous number of studies extended the model of ERT to analyze consumer behavior in the pandemic. They include Krueger et al. (2020), Barnett et al. (2020), Alexandre (2021), and La Torre et al. (2021).

restricted to the preannounced level during the lockdown. Unless a lockdown is implemented, consumers decide their consumption and labor supply to maximize their expected lifetime utility.

In the previous debate, it was seldom recognized that lockdowns are not entirely exogenous. However, their timing depends on the number of infected people, the soundness of the medical care system, and several economic and social factors. To illuminate the feedback loops between medical and economic factors, we develop a minimal economic model of pandemics in which lockdowns are endogenously determined. The novel aspect of our analysis is that once lockdowns are endogenously determined, a self-fulfilling lockdown can occur even in economies where we need no lockdown intrinsically. We derive various extrinsic lockdowns by explicitly solving a control problem, where consumers maximize their expected lifetime utility by considering the dynamic nature of the problem. A key mechanism that derives an extrinsic lockdown is that an anticipated lockdown increases the consumption of susceptible people before the lockdown and raises the number of infected people. Consequently, the anticipated lockdown becomes a self-fulfilling prophecy in the following period.

The phenomenon that an anticipated lockdown increased consumption has been observed in various countries during the COVID-19 pandemic. For example, Figure 1 depicts the movement trends before and after the second lockdowns in France, Greater London, U.K., Ireland, Italy, and Ontario, Canada.[3] The data is based on the movement trends of "retail and recreation" in Google Community Mobility Reports. We normalized the data so that it was zero seven days before lockdown. The figure shows that the movement trends had been approximately 20 points higher for a few days before dramatic declines due to the lockdowns. This indicates that the anticipated lockdown led to a temporal but substantial increase in consumption before its implementation.

The welfare effect of extrinsic lockdowns is in marked contrast with that of intrinsic lockdowns in that they reduce the expected lifetime utility of susceptible people. In the SIR-macro models, each susceptible person decides their consumption without considering that the infection adversely impacts the infection probability of other susceptible persons. Intrinsic lockdowns thus positively impact social welfare by reducing excess consumption in the economy. However, the argument no longer holds for extrinsic lockdowns. When extrinsic lockdowns are expected to

[3] The second lockdown started from October 30, 2020 in France, November 5 2020 in Greater London, U.K., October 21, 2020 in Ireland, December 24, 2020 in Italy, and December 26, 2020 in Ontario, Canada.

occur, the increased number of infections increases the probability of infection and deteriorates the welfare of susceptible people.

This study aims to provide intuitive explanations on why self-fulfilling lockdowns occur, and does not explore how the number of infections changes in a specific economy. Thus, the following analysis uses a tractable model that is too simple to track the number of infected people in the real world. However, it has an important implication for the appropriate control of expectations that can help reduce the speed of infection and hence the size of the health shock. In literature, studies such as Chang and Velasco (2020) and Moser and Yared (2020) pointed out that when announcing its lockdown policy, the government needs to consider how forward-looking economic agents will react to the decision. This study has a common implication with these studies in that the behavior of forward-looking economic agents may affect the effectiveness of the lockdown policy. However, our results have a novel policy implication in that the preannounced policy rule may lead to undesirable self-fulfilling lockdowns. An appropriate policy announcement might be the key to ruling out undesirable self-fulfilling expectations. Needless to say, our results do not deny the role of lockdowns in controlling the fatalities of a pandemic. Intrinsic lockdowns have a potent mitigation mechanism during an epidemic, even in our model. However, to the extent that consumers' behavior is forward-looking, it is important to note that inappropriate policy announcements sometimes have various unintended impacts on social welfare when the government may implement extrinsic lockdowns.

2. The SIR-macro model

This section describes our SIR-macro model based on ERT. The population is divided into four groups: susceptible, infected, recovered, and diseased. The fractions of people in these four groups at time t are denoted by S_t, I_t, R_t, and D_t, respectively. The number of newly infected individuals is denoted by T_t.

Susceptible people can become infected when they meet infected people. They can come in contact with infected people when consuming and working outside of the home and through ways not directly related to consuming or working. The total number of newly infected people at time t is given by:

$$T_t = \pi_1(S_tC^s_t)(I_tC^i_t) + \pi_2(S_tN^s_t)(I_tN^i_t) + \pi_3S_tI_t, \qquad (1)$$

Here, C^s_t and C^i_t are consumption expenditures at time t by each susceptible and infected person, respectively, and N^s_t and N^i_t are hours worked outside of the home at time t by each susceptible and infected person, respectively. Parameters π_1, π_2, and π_3 reflect the probability of becoming infected due to consumption, working interactions, and other activities, respectively.

The number of susceptible people at time t+1 is equal to the number of susceptible people at time t minus the number of susceptible people that got infected at time t, that is, $S_{t+1} = S_t - T_t$. In contrast, the number of infected people at time t+1 is equal to the number of infected people at time t plus the number of newly infected people (T_t) minus the number of infected people that recovered (π_rI_t) or died (π_dI_t) at time t, where π_r is the exogenous recovery probability from the infection, and π_d is the exogenous mortality rate of infected people. Defining $\alpha \equiv 1 - \pi_r - \pi_d$, the number of infected people at time t+1 thus evolves as follows:

$$I_{t+1} = \alpha I_t + T_t. \qquad (2)$$

In the economy, all agents are identical except for their health status: susceptible (s), infected (i), and recovered (r). At time t, a type-j agent (j = s, i, r) maximizes the following expected lifetime utility:

$$U_t^j \equiv \sum_{k=0}^{\infty} \beta^k E_t\, u(c_t^j, n_t^j), \qquad (3)$$

subject to the budget constraint:

$$c_t^j = w\, \phi^j n_t^j. \qquad (4)$$

Here, E_t is the conditional expectation operator based on information at time t. c_t^j and n_t^j denote the type-j agent's consumption and hours worked outside of the home at time t, respectively. ϕ^j is the parameter governing labor productivity, which is equal to one for susceptible and recovered people ($\phi^s = \phi^r = 1$) but less than one for infected people ($\phi^i \equiv \phi < 1$). $\beta \in (0,1)$ is the discount factor, and w is the constant wage rate. For simplicity, the consumption goods are considered perishable, so that no savings exist in the budget constraint.

The model does not explicitly include heterogeneous consumption goods, which should be essential when each agent is restricted to consume outside of the home. In reality, several types of consumption, such as consumption of home production, involve smaller amounts of contact with other people. For analytical simplicity, we abstract from heterogeneous consumption and assume that consumption of home production is implicitly reflected as a constant term or hours worked in the utility function.

Without any lockdown, the maximization problem of a recovered person and an infected person is simple under a constant wage rate with no savings. The lifetime utility of a recovered person is $U_t^r = \sum_{k=0}^{\infty} \beta^k u(c_t^r, n_t^r)$. The first-order conditions and budget constraint then lead to $w\, u_1(c_t^r, n_t^r) = u_2(c_t^r, n_t^r)$ and $c_t^r = w\, n_t^r$. This implies that c_t^r, n_t^r, and U_t^r are constant over time, where $U_t^r \equiv U^r = u(c^r, n^r)/(1-\beta)$. Similarly, since the cost of death is the foregone utility of life, the expected lifetime utility of an infected person is $U_t^i = u(c_t^i, n_t^i) + \beta(\alpha U_{t+1}^i + \pi_r U^r)$. The first-order conditions and budget constraint thus lead to $\phi w\, u_1(c_t^i, n_t^i) = u_2(c_t^i, n_t^i)$ and $c_t^i = \phi w\, n_t^i$. Since U^r is constant, c_t^i, n_t^i, and U_t^i are constant in equilibrium. We define that $c_t^i \equiv c^i$, $n_t^i \equiv n^i$, and $U_t^i \equiv U^i = [u(c^i, n^i) + \beta \pi_r U^r]/(1-\alpha\beta)$ in the following analysis.

In contrast, the expected lifetime utility of a susceptible person at time t is:

$$U_t^s = u(c_t^s, n_t^s) + \beta[(1-\tau_t)U_{t+1}^s + \tau_t U^i]. \tag{5}$$

Here, the variable τ_t denotes the probability that a susceptible person becomes infected at time t. Since $\tau_t = \pi_1 c_t^s (I_t c^i{}_t) + \pi_2 n_t^s (I_t n^i{}_t) + \pi_3 I_t$, $c_t^s = w n_t^s$, and $c^i = w n^i$, it holds that:

$$\tau_t = (\Pi\, c^i c_t^s + \pi_3) I_t, \tag{6}$$

where $\Pi \equiv \pi_1 + \pi_2/(\phi w^2)$.

A susceptible person maximizes the expected lifetime utility (5) subject to the budget constraint $c_t^s = w n_t^s$ and infection probability (6). There is no way for a susceptible person to pool the risk associated with infection. Thus, the first-order conditions lead to:

$$u_1(c_t^s, c_t^s/w)+(1/w)\, u_2(c_t^s, c_t^s/w) = \beta\Pi c^i I_t(U_{t+1}^s - U^i). \tag{7}$$

Since $u_{11}+(2/w)u_{12}+(1/w^2)\,u_{22} < 0$, the above equation implies that c_t^s is decreasing in $I_t(U_{t+1}^s - U^i)$. Thus, we can define a function f such that $c_t^s \equiv f(I_t(U_{t+1}^s - U^i))$ and $f' < 0$. For example, suppose that $u(c_t^s, n_t^s) = A + (c_t^s)^{1-a}/(1-a) - b\, n_t^s$, where $a > 0$ and $b > 0$. Then, it holds that $f(I_t(U_{t+1}^s - U^i)) = 1/[\{\beta\Pi\, c^i I_t(U_{t+1}^s - U^i) + (b/w)\}]^{1/a}$. This indicates that an increase in Π, c^i, or I_t reduces the consumption of susceptible persons by increasing the infection probability τ_t. It also indicates that an increase in $\beta\,(U_{t+1}^s - U^i)$ reduces the consumption of susceptible persons by increasing the disutility from infection. It is worth noting that the model's intertemporal substitution of consumption exists, even if the consumption goods are perishable. This is because an increase in consumption raises the probability of infection and reduces the expected utility for susceptible people. Intertemporal substitution is the main source of self-fulfilling lockdowns in the following analysis.

3. Equilibrium when the expected lifetime utilities at time T+1 are exogenously given

When the dynamics have both backward and forward properties, we can derive their solution given the terminal and initial conditions. However, because of the nonlinearity in the dynamics, it is challenging to derive a general solution of the dynamic path for general functional forms. Thus, this section investigates the existence of a self-fulfilling lockdown at time T when the expected lifetime utilities of susceptible and infected persons at time T+1 are exogenously given. Since the numbers of susceptible and infected persons at time T-1 are predetermined, the environment is a special case in which we can derive the environment under which a self-fulfilling lockdown exists for general functional forms. The following analysis assumes that the government implements a "lockdown" at time T if and only if I_T exceeds the threshold value $\bar{I}$. We can then derive an intuitive environment under which a self-fulfilling lockdown can occur in our model.

First, we investigate how I_T is determined in equilibrium when there is no lockdown. The analysis denotes the expected lifetime utility of a susceptible person at $t = T$+1 by U^S when there is no lockdown. Since $U_t^i = U^i$ and $c_t^i = c^i$ for all t when there is no lockdown, it holds that:

$$U_T^S = U_T^S(I_T)$$

$$\equiv u(f(I_T(U^S - U^i)), f(I_T(U^S - U^i))/w) - \beta[\Pi c^i f(I_T(U^S - U^i)) + \pi_3] I_T(U^S - U^i) + \beta U^S. \qquad (8)$$

Noting that $T_{T-1} = (\Pi c^i c_{T-1}^S + \pi_3) S_{T-1} I_{T-1}$, Equation (2) leads to

$$I_T = \alpha I_{T-1} + [\Pi c^i f(I_{T-1}(U_T^S(I_T) - U^i)) + \pi_3] S_{T-1} I_{T-1}. \qquad (9)$$

Given I_{T-1}, S_{T-1}, U^S, and U^i, Equation (9) determines the equilibrium value of I_T when there is no lockdown. Since $0 < [\Pi c^i f(I_{T-1}(U_T^S(I_T) - U^i)) + \pi_3] S_{T-1} I_{T-1} < S_{T-1}$ when $I_{T-1} > 0$, the right-hand side of Equation (9) is greater than I_T as $I_T \to +0$ and is smaller than I_T as $I_T \to +\infty$. Due to the continuity, the intermediate value theorem suggests an equilibrium value of I_T that satisfies Equation (9). We can also show that given I_{T-1}, S_{T-1}, U^S, and U^i, the equilibrium value of I_T exists uniquely for reasonable conditions (see Appendix).

In the following analysis, we assume that the equilibrium value of I_T exists uniquely. Then, the unique equilibrium value of I_T satisfies the following proposition:

Proposition 1: Denote the unique equilibrium value of I_T by I_T^* when there is no lockdown. Then, it holds *that* $I_T^* < \bar{I}$ if and only if:

$$\bar{I} > \alpha I_{T-1} + [\Pi c^i f(I_{T-1}(U_T^S(\bar{I}) - U^i)) + \pi_3] S_{T-1} I_{T-1}. \qquad (10)$$

Proof: When $I_{T-1} > 0$, it holds that $I_T < \alpha I_{T-1} + [\Pi c^i f(I_{T-1}(U_T^S(I_T) - U^i)) + \pi_3] S_{T-1} I_{T-1}$ as $I_T \to +0$. The continuity thus implies that $\partial[\{\Pi c^i f(I_{T-1}(U_T^S(I_T) - U^i)) + \pi_3\} S_{T-1} I_{T-1}]/\partial I_T < 1$ at $I_T = I_T^*$. To the extent that I_T^* exists uniquely, this implies that $I_T > \alpha I_{T-1} + [\Pi c^i f(I_{T-1}(U_T^S(I_T) - U^i)) + \pi_3] S_{T-1} I_{T-1}$ if and only if $I_T > I_T^*$. This derives the proposition. [Q.E.D.]

When the equilibrium value of I_T exceeds a threshold value $\bar{I}$, lockdowns that strictly restrict people's consumption are a powerful tool to avoid the explosive spread of infections. In contrast, when the equilibrium value of I_T is less than $\bar{I}$, lockdowns that hurt economic activities may not be a desirable policy tool. Proposition 1 suggests that the government needs no lockdown at time

T when the inequality (10) holds.

However, when the government rule of the lockdown is common knowledge among consumers, I_T can exceed the threshold value $\bar{I}$ even if the inequality (10) holds. The following analysis assumes that the government has a credible policy rule by which it decides the implementation of a "lockdown" at time T when $I_T > \bar{I}$ and restricts the consumption of any person not to exceed $\bar{c}$, that is, $c_T^j \leq \bar{c}$ for j = s, i, and r. We also assume that all consumers know the government rule in advance and follow the restriction faithfully under the lockdown. We can then derive the following proposition.

<u>Proposition 2:</u> Denote the expected lifetime utility of a susceptible person at $t = T$+1 by V^S when a lockdown occurs at $t = T$. Denote also the consumption of an infected person at $t = T$ by $c_T{}^i$ ($\equiv \min[c^i, \bar{c}]$) when a lockdown occurs at $t = T$. Then, given I_{T-1}, S_{T-1}, V^S, and U^i, a lockdown occurs at time T if and only if:

$$\bar{I} < \alpha I_{T-1} + [\Pi\, c^i f(I_{T-1}(V_T^s(\bar{I}) - U_T{}^i)) + \pi_3]\, S_{T-1} I_{T-1}, \tag{11}$$

where $V_T^s(I_T) \equiv u(\bar{c}, \bar{c}/w) - \beta[\Pi\, c_T{}^i \bar{c} + \pi_3]\, I_T(V^S - U^i) + \beta V^S$ and $U_T{}^i \equiv u(c_T{}^i, c_T{}^i/w) + \beta[\alpha U^i + \pi_r U^r]$.

<u>Proof</u>: When susceptible people anticipate that the government will implement a lockdown at time T, Equation (9) is rewritten as $I_T = \alpha I_{T-1} + [\Pi\, c^i f(I_{T-1}(V_T^s(I_T) - U_T{}^i)) + \pi_3]\, S_{T-1} I_{T-1}$. Since $\partial[\{\Pi\, c^i f(I_{T-1}(V_T^s(I_T) - U_T{}^i)) + \pi_3\}\, S_{T-1} I_{T-1}]/\partial I_T < 1$ at $I_T = I_T{}^*$, it holds that $I_T < \alpha I_{T-1} + [\Pi\, c^i f(I_{T-1}(V_T^s(I_T) - U_T{}^i) + \pi_3]\, S_{T-1} I_{T-1}$ if and only if $I_T < I_T{}^*$. Since the lockdown occurs if and only if $\bar{I} < I_T{}^*$, this derives the proposition. [Q.E.D.]

It is worth noting that the inequality (11) in Proposition 2 can hold even when inequality (10) in Proposition 1 holds. This indicates that the lockdown occurs even in an economy where the government does not need to implement a lockdown. We call it a "self-fulfilling lockdown" when the equilibrium value of I_T is smaller than $\bar{I}$ without any lockdown but exceeds $\bar{I}$ with the lockdown. Intuitively, when susceptible people anticipate a lockdown at time T, they increase their consumption at time T -1. This increases the number of infected people and results in a selffulfilled lockdown at time *T*.

The above two propositions indicate that a self-fulfilling lockdown occurs at time *T* if and only if:

$$\Pi c^i f(I_{T-1}(U_T^s(\bar{I}) - U^i)) < (\bar{I} - \alpha I_{T-1})/(S_{T-1}I_{T-1}) - \pi_3 < \Pi\, c^i f(I_{T-1}(V_T^s(\bar{I}) - U_T^i]). \quad (12)$$

The above inequality can hold for several reasonable environments. For example, consider a specific utility function such that $u(c_t^s, n_t^s) = ln(c_t^s) - bn_t^s$, where $b = 0$ when $n_t^s < n^*$ but $b = +\infty$ when $n_t^s \geq n^*$. For simplicity, we assume that $\pi_3 = 0$ and $c^i \leq \bar{c}$. Since $c_T{}^i = c^i$ and $U_T{}^i = U^i$, it holds that $\Pi c^i f(I_{T-1}(U_T^s(\bar{I}) - U^i)) = 1/\{\beta I_{T-1}(U_T^s(\bar{I}) - U^i)\}$ and $\Pi c^i f(I_{T-1}(V_T^s(\bar{I}) - U^i)) = 1/\{\beta I_{T-1}(V_T^s(\bar{I}) - U^i)\}$ when $n_t^s < n^*$. Since $n_t^s < n^*$ when n^* is large enough, this implies that the inequalities in (12) hold if and only if $V_T^s(\bar{I}) < S_{T-1}/\{\beta(\bar{I} - \alpha I_{T-1})\} + U^i < U_T^s(\bar{I})$ when n^* is large enough, where $U_T^s(\bar{I}) = \ln[1/\{\beta\Pi c^i\bar{I}(U^S - U^i)\}] - 1 + \beta U^S$ and $V_T^s(\bar{I}) = \ln(\bar{c}) - \beta\Pi c^i\bar{c}\ \bar{I}(V^S - U^i) + \beta V^S$. The inequality indicates that when n^* is sufficiently large, self-fulfilling lockdown tends to occur when the gap between $U_T^s(\bar{I})$ and $V_T^s(\bar{I})$ is sufficiently large. In other words, a self-fulfilling lockdown can occur when the lockdown leads to a substantial reduction in the expected lifetime utility of susceptible people at $t = T$. For instance, since $\partial V_T^s(\bar{I})/\partial\bar{c} = 1/\bar{c} - \beta\Pi c^i\bar{I}(V^S - U^i) = 1/\bar{c} - 1/f(\bar{I}(V^S - U^i)) > 0$ and $\partial U_T^s(\bar{I})/\partial\bar{c} = 0$, the inequality tends to hold if $\bar{c}$ is small enough. This suggests that a self-fulfilling lockdown tends to occur when the lockdown imposes a substantial restriction on consumption.

4. Equilibrium dynamics with a specific utility function

In the last section, we show the existence of a self-fulfilling lockdown when the expected lifetime utilities at time *T*+1 are given. This section explores the environment in which a self-fulfilling lockdown exists in a general dynamic framework and investigates how the self-fulfilling lockdown will change the equilibrium dynamic path. To derive the equilibrium dynamic path, we use the specific utility function of a type-*j* agent ($j = s, i, r$) such that $u(c_t^j, n_t^j) = A + a\ ln(c_t^j) - bn_t^s$, where $A > 0$, $a > 0$, and $b > 0$. Each consumer maximizes his or her expected lifetime utility, knowing that the government will decide a "lockdown" and restrict the consumption not to exceed

$\bar{c}$ at time t when I_t exceeds the threshold value $\bar{I}$.

For some horizon H, we guess sequences for $\{c_t^s\}_{t=1}^{H-1}$ which satisfies a sequence of constraints in the model. In practice, given I_1, S_1, and U_H^s, we solve the model for $H = 150$ weeks. We assume that either a highly effective treatment or a vaccine is developed at $t = H$ so that the expected lifetime utility of susceptible people converges to the steady state value $U_H^s \equiv U^r$ at $t = H$. We iterate backward from the post-pandemic steady-state value of U_H^s and forward from the initial values of I_1 and S_1 to compute the equilibrium sequence of $\{c_t^s\}_{t=1}^{H-1}$. Without any lockdown, infected and recovered persons have constant lifetime utility of $[A + a\ ln(c^r) - (b/w)\ c^r]/(1\text{-}\beta)$ and $[A + a\ ln(c^i) - \{b/(\phi w)\}\ c^i]/(1\text{-}\beta)$, respectively, where $c^r \equiv aw/b$ and $c^i \equiv a\varphi w/b$. In contrast, when a lockdown occurs at time t, the expected lifetime utility is recursively determined by $U_t^i = A + a\ ln(c_t^i) - (b/w)\ c_t^i + \beta\ [\alpha U_{t+1}^i + \pi_r U_{t+1}^r]$ and $U_t^r = A + a\ ln(\bar{c}) - (b/w)\ \bar{c} + \beta\ U_{t+1}^r$, where $c_t^i \equiv \min[c^i,\ \bar{c}]$.

In the following computations, we set $\beta = 0.995$, $A = 2$, $a = 1$, $b = 0.25$, $\phi = 0.00025$, $w = 1$, $\Pi = 0.005$, $\pi_3 = 10^{-6}$, $\pi_r = 0.39$, and $\pi_d = 0.01$ for the parameters. We also set $S_1 = 100{,}000$ and $I_1 = 100$ as the initial conditions. We then compute the equilibrium sequence of $\{c_t^s\}_{t=1}^{H-1}$ with and without a lockdown. The equilibrium sequence without any lockdown is that of $\{c_t^s\}_{t=1}^{H-1}$ when the government imposes no restriction on the consumption of any agent for all t. The equilibrium sequence with the lockdown(s) is, in contrast, that of $\{c_t^s\}_{t=1}^{H-1}$ when the government restricts consumption c_t^j ($j = s, i, r$) to no more than $\bar{c}$ when I_t exceeds the threshold level $\bar{I}$ at time t.

Figure 2 depicts the dynamic path of I_t and S_t when the government does not implement a lockdown. It shows that the number of infected people increases until $t = 9$ and declines thereafter. Consequently, the number of susceptible people declines sharply until around $t = 20$ and becomes almost stable after $t = 50$, when approximately 81.5% of the population has been infected. Correspondingly, the number of susceptible people declined dramatically until $t = 20$ but became almost stable after $t = 50$, when only 18.5% of the population remained uninfected. It is worth noting that when the government does not implement a lockdown, the maximum value of I_t is less than 22,000. This implies that the economy does not need an intrinsic lockdown to the extent that the threshold level $\bar{I}$ is greater than 22,000.

However, when people know that the government will implement a lockdown when I_t exceeds the threshold level $\bar{I}$, the economy can have self-fulfilling lockdowns even if $\bar{I}$ is greater than 22,000. For example, suppose that $\bar{I} = 27{,}000$, which is much higher than the maximum value of

I_t with no lockdown. We also suppose that $\bar{c}$ = 0.005. Then, we can show that a self-fulfilling lockdown occurs at t = 7, 8, 9, or 10. Figure 3 depicts the dynamic path of I_t when a lockdown occurs at t = 7, 8, 9, or 10, respectively. Each dynamic path shows that I_t exceeds 27,500, 32,400, 30,600, and 27,000 at t = 7, 8, 9, and 10, respectively, all of which are much higher than the maximum value of I_t with no lockdown. Each lockdown is self-fulfilling because the equilibrium value of I_T is smaller than $\bar{I}$ without any lockdown, but exceeds $\bar{I}$ with the lockdown. This indicates that even in an economy where no lockdown occurs intrinsically, a lockdown can occur extrinsically and dramatically increase the number of infected people.

The extrinsic lockdown occurs because susceptible people who anticipate a lockdown in the next period dramatically increase their consumption. Figure 4 depicts the dynamic path of c_t^s when a self-fulfilling lockdown occurs at t = 7, 8, 9, or 10, respectively. Each dynamic path indicates that the consumption of susceptible people increased dramatically at t = 6, 7, 8, or 9 and fell to almost zero at t = 7, 8, 9, or 10, respectively. When susceptible people increase their consumption dramatically, the number of infected people will increase dramatically in the next period. Consequently, a self-fulfilling lockdown would occur endogenously, even in an economy where a lockdown does not occur intrinsically. It is worth noting that a self-fulfilling lockdown can occur at any period between 7 and 10. It is also worth noting that the dynamic path without a lockdown remains one of the equilibria. This implies that the equilibrium dynamic path of I_t is highly unpredictable when a self-fulfilling lockdown can occur because it depends on the changeable expectations of susceptible people.

The self-fulfilling lockdown increases the total number of accumulated infections. For example, compared with an economy without a lockdown, the total number of accumulated infections would increase by approximately 0.24% at t = 150 when the self-fulfilling lockdown occurs at t = 7. This result is in marked contrast with that of the intrinsic lockdown. In the SIR-macro models, each susceptible person decides the consumption without considering that the infection negatively impacts the infection probability of other susceptible persons. This causes excess consumption in the economy when there is no lockdown. The intrinsic lockdown would reduce excess consumption. Previous studies have thus concluded that lockdowns are a powerful tool to avoid the explosive growth of infections. However, the above result suggests that the conclusion no longer holds for self-fulfilling lockdowns. It is worth noting that each self-fulfilling lockdown reduces the expected lifetime utility of susceptible people. Figure 5 shows the expected lifetime utility of susceptible people at $t = 1$ with and without lockdowns. This indicates that the expected lifetime utility with an extrinsic lockdown is always lower than that without the lockdown, although it varies depending on when the lockdown occurs. This is because the increased number of accumulated infections increase the probability of infections. The increased probability of infections deteriorates the welfare of susceptible people in an economy with an extrinsic lockdown.

5. Robustness analysis I

The previous section showed the existence of a self-fulfilling lockdown assuming that $\bar{I}$ = 27,000 and $\bar{c}$ = 0.005. However, as we showed in Section 3, the existence of a self-fulfilling lockdown depends on the strength of the lockdown. This section explores the robustness of our results when using alternative values of $\bar{I}$ and $\bar{c}$.

We first explore how our results change when we use lower values for threshold level $\bar{I}$, given that $\bar{c}$ = 0.005. Varieties of self-fulfilling lockdowns would occur when the threshold level $\bar{I}$ is lower than 27,000. For example, suppose that $\bar{I}$ = 22,000, which is slightly higher than the maximum value of I_t with no lockdown. We can then show that a self-fulfilling lockdown occurs at $t = 11$ in addition to at $t = 7$, 8, 9, or 10. Interestingly, we can also show that multiple self-fulfilling lockdowns occur at $t = 7$ and 9 or $t = 8$ and 10. Figure 6 depicts the dynamic path of I_t when new self-fulfilling lockdowns occur. Unlike those in the previous section, the single self-fulfilling lockdown at $t = 11$ delays the peak of infection. However, as in the previous section, it increases the number of accumulated infections at $t = 150$. Consequently, compared with those without the lockdown, the expected lifetime utility of susceptible people at $t = 1$ is lower. Similarly, multiple self-fulfilling lockdowns increase the number of accumulated infections at $t = 150$. The number of accumulated infections is not necessarily larger than that of the other self-fulfilling lockdown(s). However, when multiple self-fulfilling lockdowns occur, the expected lifetime utility of susceptible people at $t = 1$ is lower not only than those without the lockdown but also those with any self-fulfilling lockdown.

A single self-fulfilling lockdown may have somewhat different features from these lockdowns when $\bar{I}$ is lower than the maximum value of I_t with no lockdown. For example, suppose that $\bar{I}$ = 19,400, which is slightly lower than the maximum value of I_t with no lockdown. We can then show that a new self-fulfilling lockdown occurs at t = 12 (Figure 7). The single self-fulfilling lockdown at t = 12 not only brings the peak of infection backward but also makes the maximum value of I_t smaller than the maximum value with no lockdown. In particular, the self-fulfilling lockdown decreases the number of accumulated infections at t = 150, smaller than that without the lockdown. However, as the other single self-fulfilling lockdowns did, it reduces the expected lifetime utility of susceptible people at t = 1 lower than that without the lockdown.

We next explore how our result changes when we use alternative values for the restricted consumption level $\bar{c}$. Specifically, we explore robustness of our result when $\bar{c}$ = 0.05 or 0.001 given that $\bar{I}$ = 25,000. Figure 8 depicts the dynamic path of I_t when a self-fulfilling lockdown occurs at t = 8 when $\bar{c}$ = 0.05, 0.005, or 0.001, respectively. The number of infections at t = 8 increases to approximately 38,000 when $\bar{c}$ = 0.001, whereas it decreases to less than 25,500 when $\bar{c}$ = 0.05. This implies that the more severely the government restricts consumption, the more the self-fulfilling lockdown increases the number of infections. The expected lifetime utility of susceptible people at t = 1 decreases as the number of infections increases. Consequently, the expected lifetime utility of susceptible people at t = 1 decreases as the government restricts consumption more severely.

The choice of $\bar{c}$ also affects the existence of self-fulfilling lockdowns. For example, given that $\bar{I}$ = 27,000, a self-fulfilling lockdown did not occur at t = 11 when $\bar{c}$ = 0.005. However, a self-fulfilling lockdown occurs at t = 11when $\bar{c}$ = 0.001. This implies that the more severely the government restricts consumption, the more likely a self-fulfilling lockdown occurs. In contrast, given that $\bar{I}$ = 26,000, a self-fulfilling lockdown occurred at t = 8 and t = 10 when $\bar{c}$ = 0.005. However, these self-fulfilling lockdowns no longer exist when $\bar{c}$ = 0.05. This implies that the less severely the government restricts consumption, the less likely a self-fulfilling lockdown occurs.

6. Robustness analysis II

Previous sections investigated the equilibrium dynamic path by setting the value of Π to 0.005. Since $\tau_t = (\Pi c^i c_t^s + \pi_3)I_t$, the number of infected persons increased relatively quickly and peaked around t = 9 in the simulations. This section explores the robustness of our results when setting the value of Π to be 0.001. When Π = 0.001, a susceptible person became infected with a relatively low probability. Thus, the number of infected persons increases relatively gradually, and the total number of infected populations is relatively small in the new simulations.

Figure 9 depicts the dynamic path of I_t and S_t when the government never implements a lockdown, respectively. Except for the change in the value of Π from 0.005 to 0.001, the new simulation used the same parameters and initial conditions as those in the previous sections. This shows that the number of infected people I_t increases relatively gradually until $t = 18$ and declines thereafter. Susceptible people become almost stable after $t = 100$, when approximately 61% of the population has been infected. The maximum value of I_t is less than 6,800, which is very low compared with those in the previous sections. This implies that when the threshold level $\bar{I}$ is greater than 6,800, the economy needs no lockdown intrinsically when $\Pi = 0.001$.

Since the number of people to be infected is relatively small when $\Pi = 0.001$, the model would not have a self-fulfilling lockdown if the government sets the threshold value $\bar{I}$ reasonably high. However, the model can have a self-fulfilling lockdown if the government sets the threshold value $\bar{I}$ relatively low. That is, the model can have lockdowns even if $\bar{I}$ is greater than 6,800. For example, suppose that $\bar{I}$ = 7,400, which is much higher than the maximum value of I_t with no lockdown. We also assume that $\bar{c}$ = 0.005. Then, we can show that a self-fulfilling lockdown occurs at t = 17, 18, or 19. Figure 10 depicts the dynamic path of I_t when a lockdown occurs at t = 17, 18, or 19, respectively. Each dynamic path shows that I_t exceeds 7,400 at t = 17, 18, or 19. This is self-fulfilling because the equilibrium value of I_T is smaller than $\bar{I}$ without any lockdown, but exceeds $\bar{I}$ with the lockdown. In other words, even in an economy where the number of people to be infected is relatively small, a lockdown can occur extrinsically and increases the number of infected people dramatically when the government sets the threshold value relatively low.

7. Concluding remarks

This study showed that a self-fulfilling lockdown could occur even in economies where the government does not need to implement a lockdown to keep the number of infected people below a threshold level. Our analysis was based on a simple but standard macroeconomic model, where agents consume and work, combined with a standard epidemiological SIR model. Unlike intrinsic lockdowns, a self-fulfilling lockdown has the feature that it increases the total number of accumulated infections and reduces the expected lifetime utility of susceptible people. This is in marked contrast to that of the intrinsic lockdowns. This occurs because the increased number of accumulated infections raises the probability of infections under self-fulfilling lockdowns.

The future course for our analysis is to explore whether a self-fulfilling lockdown can occur in an empirically reasonable environment. The phenomenon that an anticipated lockdown increased consumption has been observed in various countries during the COVID-19 pandemic. However, it is unclear whether the increase in consumption has been large enough to derive a self-fulfilling lockdown. It is important to investigate whether a self-fulfilling lockdown can occur within an empirically reasonable framework.

Appendix

The purpose of this Appendix is to show that the equilibrium value of I_T exists uniquely for reasonable environments in Section 3. Specifically, we investigate the sufficient conditions by which I_T that satisfies Equation (9) exists uniquely.

Define $f_{T-1} \equiv f(I_{T-1}(U_T^s - U^i))$ and $f_T \equiv f(I_T(U^S - U^i))$. Then, because $u_1 + (1/w)u_2 = \beta \Pi I_t(U_{t+1}^S - U_{t+1}^i)$, it holds that $\partial U_T^s/\partial I_T = [\{u_1 + (1/w)u_2\} f_T' - \beta \Pi I_T(U^S - U^i) f_T' - \beta(\Pi f_T + \pi_3)](U^S - U^i) = -\beta(\Pi f_T + \pi_3)(U^S - U^i) < 0$. Since $f_{T-1}' < 0$, we can thus show that $\partial f_{T-1}/\partial I_T = I_{T-1} f_{T-1}' (\partial U_T^s/\partial I_T) > 0$. We can also show that $\partial^2 f_{T-1}/\partial I_T^2 = I_{T-1}\{I_{T-1} f_{T-1}'' (\partial U_T^s/\partial I_T)^2 + f_{T-1}' (\partial^2 U_T^s/\partial I_T^2)\} = I_{T-1}\{I_{T-1} f_{T-1}'' (\partial U_T^s/\partial I_T)^2 - \beta \Pi I_T f_{T-1}' f_T' (U^S - U^i)^2\} = \beta I_{T-1}(U^S - U^i)^2\{\beta I_{T-1} f_{T-1}'' (\Pi f_T + \pi_3)^2 - \Pi I_T f_{T-1}' f_T'\}$. In general, the sign of $\partial^2 f_{T-1}/\partial I_T^2$ is ambiguous. However, since $\Pi f_T + \pi_3 < 1/I_{T-1}$, it holds that $\beta I_{T-1} f_{T-1}'' (\Pi f_T + \pi_3)^2 < \beta f_{T-1}''/I_{T-1}$. The term $\beta I_{T-1} f_{T-1}'' (\Pi f_T + \pi_3)^2$ is thus negligible when $\beta f_{T-1}''/I_{T-1}$ is sufficiently small around $I_T = I_T^*$. Since $\Pi I_T f_{T-1}' f_T' > 0$, this indicates that $\partial^2 f_{T-1}/\partial I_T^2 < 0$ when $\beta f_{T-1}''/I_{T-1}$ is sufficiently small around $I_T = I_T^*$. When $\partial^2 f_{T-1}/\partial I_T^2 < 0$, it holds that $\partial f_{T-1}/\partial I_T < 1$ for all $I_T > I_T^*$ because $\partial f_{T-1}/\partial I_T < 1$ at $I_T = I_T^*$. This implies that the equilibrium value of I_T exists uniquely for reasonable environments where $\beta f_{T-1}''/I_{T-1}$ is sufficiently small around $I_T = I_T^*$.

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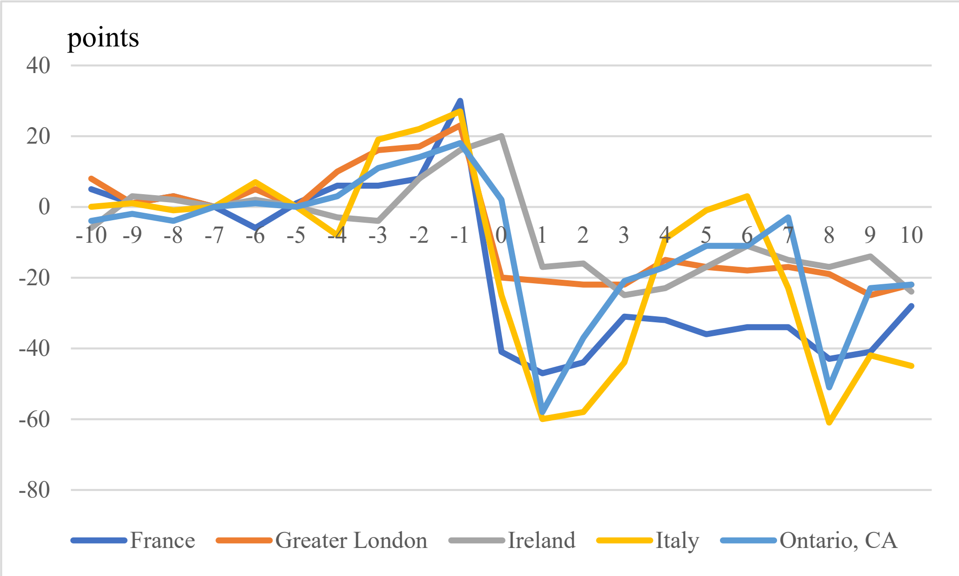
Figure 1. Movement trends before and after the second lockdowns



Source: “retail and recreation” in Google Community Mobility Reports.

Figure 2. The dynamic path of I_t and S_t when there is no lockdown

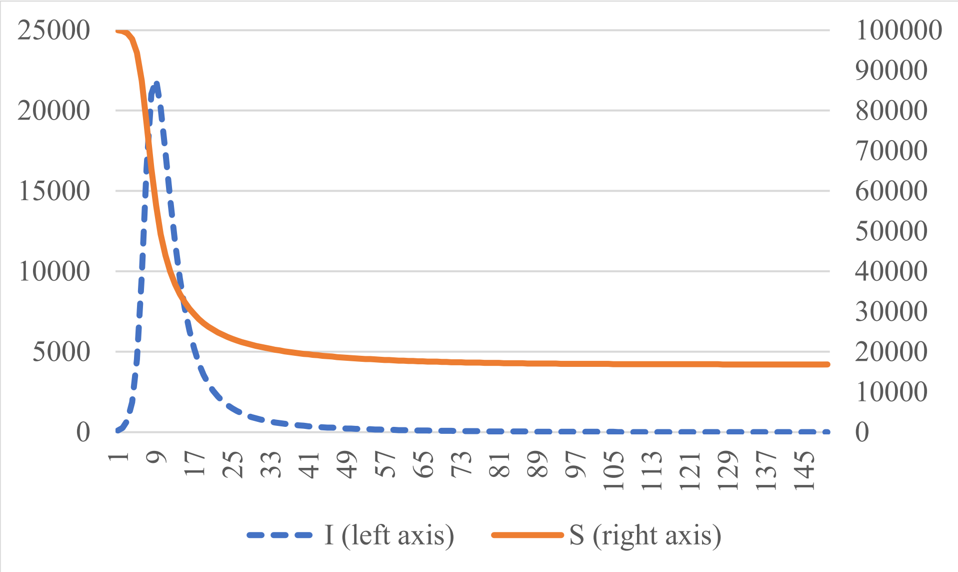


Figure 3. The dynamic path of I_t when a lockdown occurs at t = 7, 8, 9, or 10.

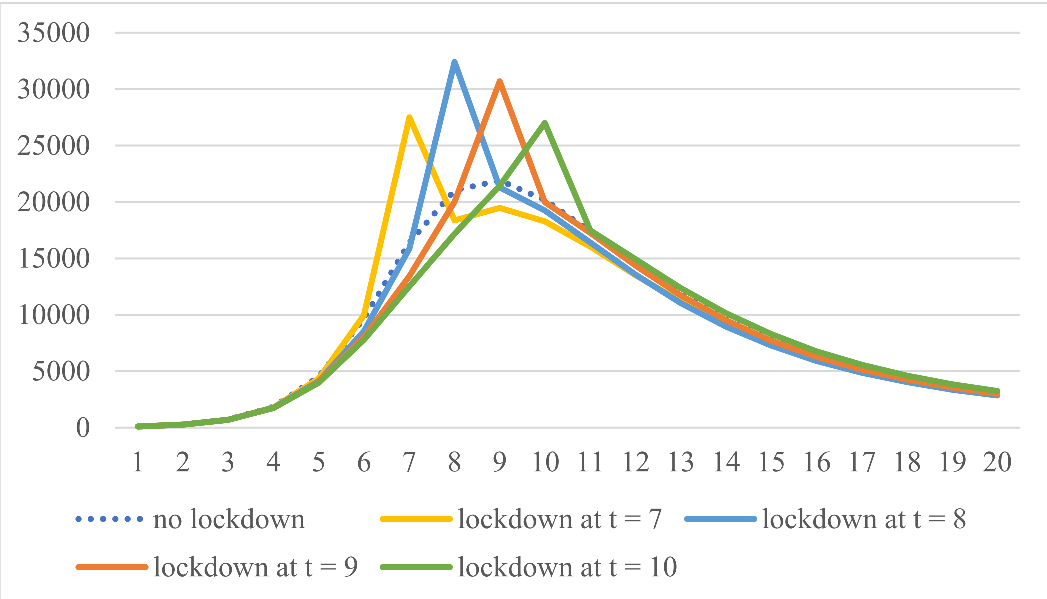


Figure 4. The dynamic path of c_t^s when a self-fulfilling lockdown occurs

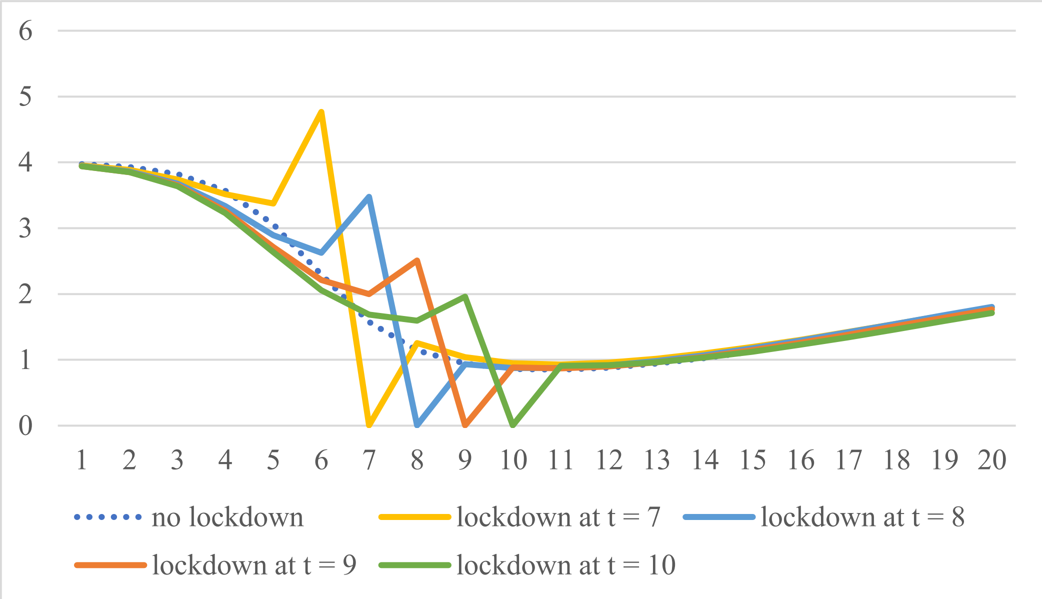


Figure 5. The expected lifetime utility of susceptible people at $t = 1$ with and without lockdowns

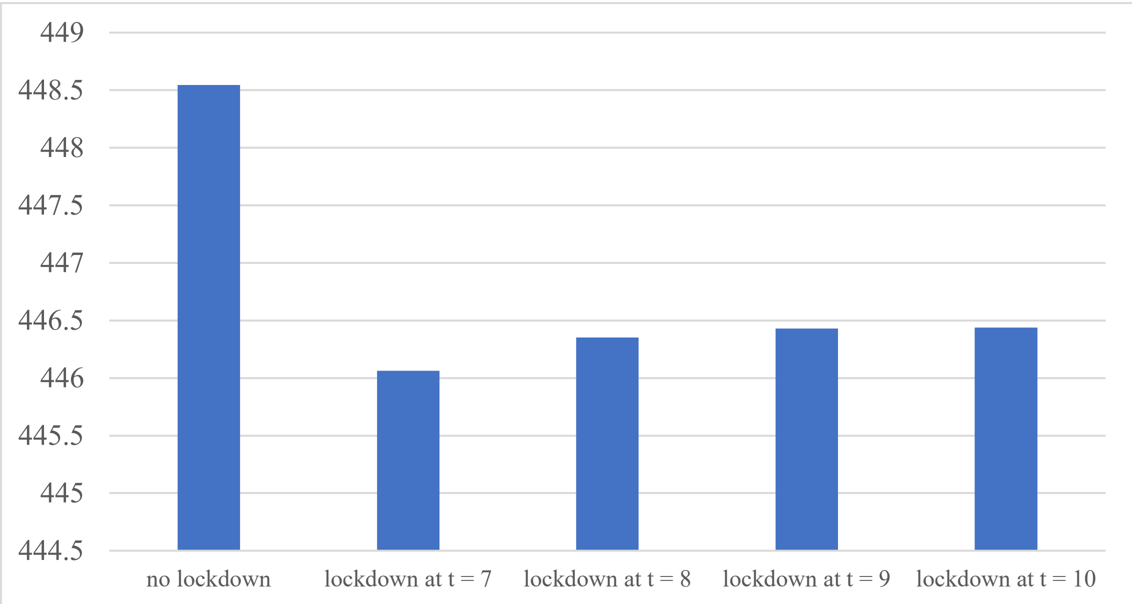


Figure 6. The dynamic path of I_t when the threshold level is lower

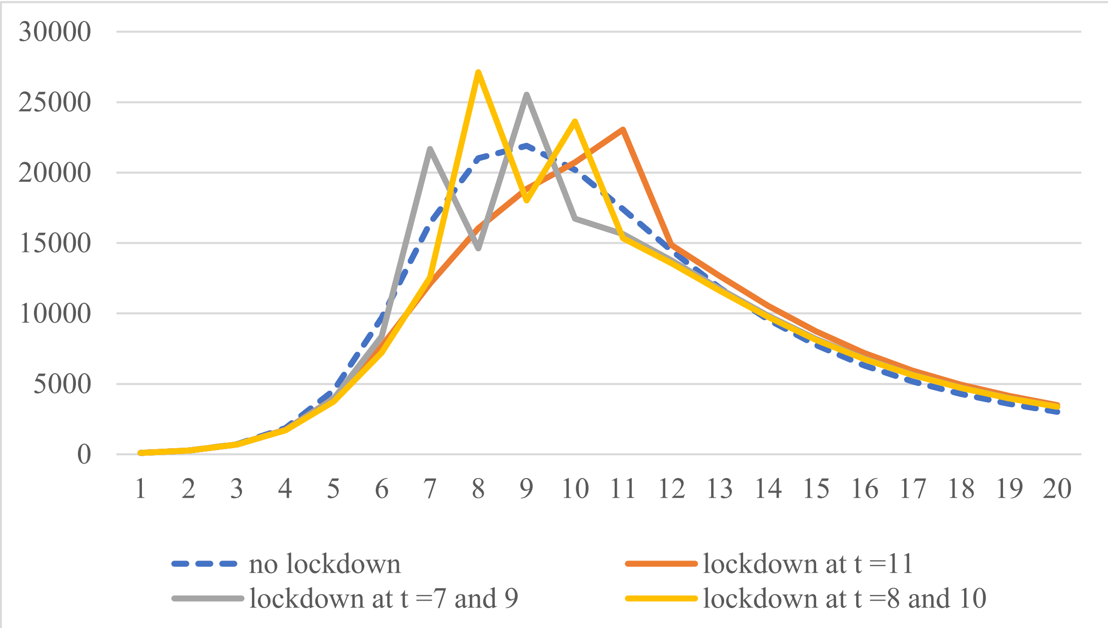


Figure 7. The dynamic path of I_t when the threshold level is much lower

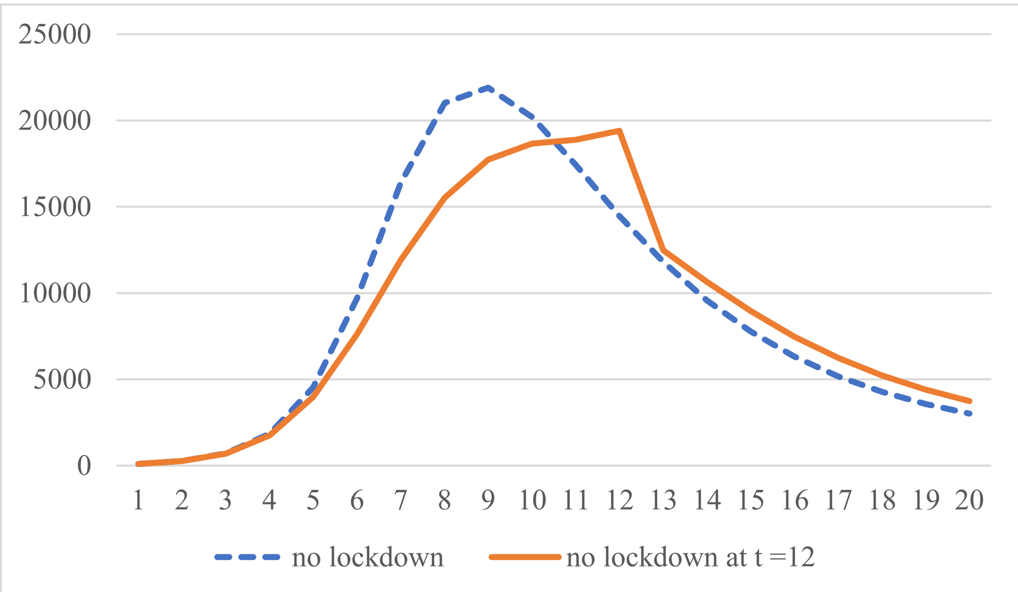


Figure 8. The dynamic path of I_t for alternative values of $\bar{c}$

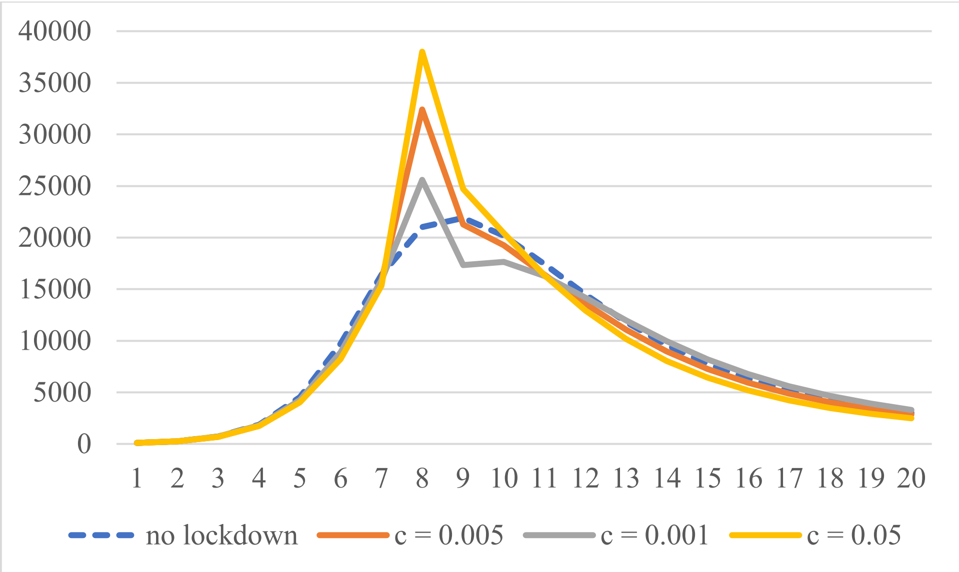


Figure 9. The dynamic path of I_t and S_t for lower value of Π

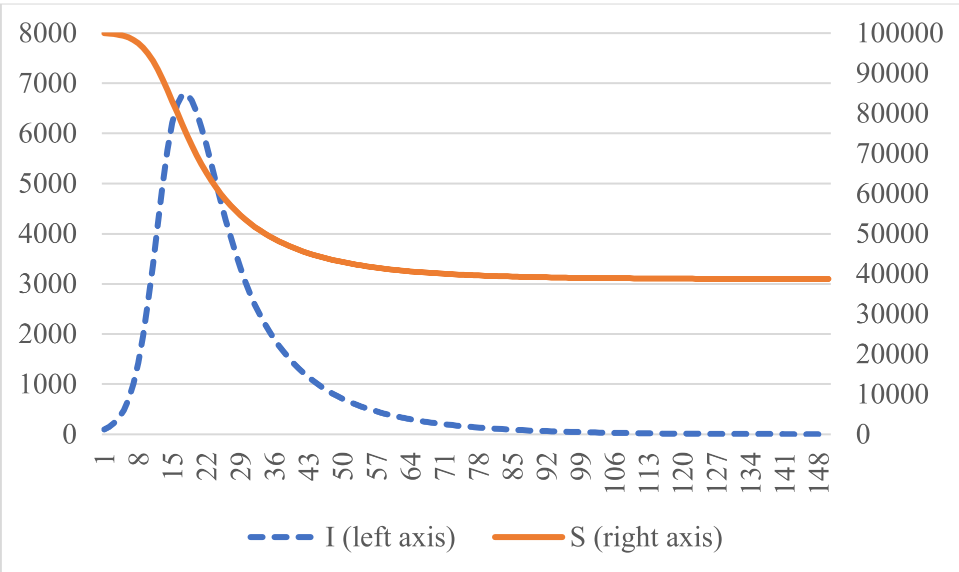


Figure 10. The dynamic path of I_t when a lockdown occurs at t = 17, 18, or 19.