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between Safety and Consumptions**

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A theory of public goods under complementarity between safety and consumption

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Abstract

We explore the theoretical properties of public good provision under the complementarity between safety and private/public good consumption. The presence of the mobile worker generates fiscal externality, making the equilibrium inefficient. The direction of inefficiency is determined by three factors: the characteristics of the utility function, the difference in income between the immobile and mobile workers, and the immobile worker's marginal utility of hosting another mobile worker. We show that the complementarity between safety and private good consumption plays a crucial role in determining the impacts of the third factor whereas the complementarity between safety and public good does not.

Keywords: complementarity, public security, tax competition, efficiency

JEL classification: H21, H24, H41, R51

1 Introduction

It is no doubt that safety crucially determines the value of life. This is partly reflected by the effects of crime on property values. In fact, since the seminal work by Thaler (1978), many studies showed that crimes have a significant negative impact on property prices (see Gibbons, 2004; Delgado and Wences, 2020; and Kim and Lee, 2018, for recent works). This implies that safety increases the value of housing and land consumption. Put differently, there exists complementarity between safety and private goods consumption.

Recent studies also show that safety is complement to public good consumption. Bowes and Ihlanfeldt (2001) found that crime is associated with lower values of rail stations close to downtown, especially, where the station has parking, in Atlanta. Troy and Grove (2008) found

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crime is associated with lower values of parks in Baltimore. More recently, Albouy et al. (2020), by using detailed crime and housing data in Chicago, New York, and Philadelphia, showed that local crime rates decrease the value of park proximity.¹

We aim to conduct a baseline analysis on the theoretical properties of public good provision under the complementarity between safety and private/public good consumption. For this purpose, we develop a model of public good provision that involves such complementarity. We then derive the condition of optimal public good provision, i.e., the Samuelson condition under the complementarity. Given this modified Samuelson condition, we examine how such complementarity affects the policy decisions by local governments. Here, the reason to focus on the local government is that the crime risk is highly local: One street difference really matters. This implies that the right spatial scale to focus on should be municipalities.

Moreover, given high people mobility between municipalities, we should notice the effects of population concentration on crime risk. In fact, places with high population density are often associated with high crime rates (Glaeser and Sacerdote, 1999; O’Flaherty and Sethi, 2015). Thus, we consider the effects of governments’ decision on population distributions, which in turn, affects the safety and values of private and public goods consumption.

As is shown by the literature of tax competition, which dates back at least to Zodrow and Mieszkowski (1986) and Wilson (1986), local governments’ decision can cause various externalities in the presence of factor (labor or capital) mobility (see Wilson, 1999; Zodrow, 2003; Wilson and Wildasin, 2004; and Cremer and Pestieau, 2004 for surveys). We provide detailed conditions under which and how the complementarity affects these externalities.

Although our framework is quite simple, it provides the rich and brand-new insights. Our results predict that the tax rate is inefficient in most cases, and whether it is too high or low depends on three factors: the characteristics of the utility function, the difference in income between the immobile and mobile workers, and the immobile worker’s marginal utility of hosting another mobile worker. Especially, we show that the complementarity between safety and private good consumption plays a crucial role in determining the impacts of the third factor whereas the complementarity between safety and public good consumption does not. This result holds true in a quite general setting, and implies that considering complementarity between safety and private good is more important than that between security and public good, in the presence of fiscal externality.

We also illustrate the model numerically to grasp the characteristics of the equilibrium and

¹On the contrary, Anderson and West (2006) found crime is associated with higher values of open space in Minneapolis. This suggests the possibility that safety and public good are substitutes. We can slightly modify our framework to deal with such a possibility.

optimum. The numerical examples imply that asymmetry in the immobile worker's income is more crucial for shaping the gap between the equilibrium and optimum. Asymmetry in the population of the immobile worker does not affect the equilibrium and optimum tax rates so much, but the distribution of the mobile worker is highly influenced by this asymmetry, because it directly affects the mobile worker's utility through public safety and congestion.

The remainder of the paper proceeds as follows. Section 2 provides the baseline framework, and derive the modified Samuelson condition and the equilibrium conditions. Section 3 characterizes efficiency properties. Section 4 illustrates the model numerically. Section 5 concludes.

2 The Model

We first provide the very basic structure of the model and derive the condition of optimal public good provision, i.e., the Samuelson condition, in our framework. We then examine how complementarity between public security and private/public good affects the properties of tax competition between local jurisdictions.

2.1 General Framework

Consider a jurisdiction that consists of two types (Type L and Type M) of workers. The population sizes of Type L and Type M workers are l and m , respectively. The jurisdiction is closed so that l and m are positive constants. The income levels of Type L and Type M workers are y_L and y_M , respectively, and they are fixed. The utility function of Type $J \in \{L, M\}$ worker is given by

$$U_J = w_c(S)u(c_J) + w_g(S)v(g),$$

where $u(c_J)$, $v(g)$, $w_c(S)$ and $w_g(S)$ are twice continuously differentiable, increasing, and strictly concave functions with respect to its arguments. g and c_J are the levels of public and private (the numéraire) goods consumption, respectively. S represents the public security level of the jurisdiction, and $w_c(S)$ and $w_g(S)$ are positive for any S . We here employ a partially additively separable function in order to focus on the effects of complementarity between c_J and S , and between g and S , one by one. In this expression, $w_c(S)$ and $w_g(S)$ represent the degree of complementarity between security and private good and that between security and public good, respectively. Here, we have in mind that people feel better from their private consumption as well as public services in a more secure place.

We assume that S depends on the number of two types of workers: $S = S(l, m)$, where $S(l, m)$ is a twice continuously differentiable function with respect to its arguments. We further

assume that S is decreasing in its arguments: $\partial S/\partial l < 0$ and $\partial S/\partial m < 0$. This assumption implies that the public security level decreases as the jurisdiction has a larger population. This reflects the empirical evidences that the crime rate is higher in a jurisdiction with larger population than in the one with smaller population.²

We assume that one unit of the numéraire can be transformed into one unit of the public good. Then, the resource constraint in this economy is $ly_L + my_M = lc_L + mc_M + g$, from which we obtain the following proposition:

Proposition 1 *In the Pareto-optimal allocation, the following Samuelson condition is satisfied:*

$$l \frac{w_g(S)v'(g)}{w_c(S)u'(c_L)} + m \frac{w_g(S)v'(g)}{w_c(S)u'(c_M)} = 1. \quad (1)$$

The left hand side of (1) represents the sum of the marginal benefit of public good consumption over two types of workers, and its right hand side is the marginal rate of transformation between the public good and the numéraire. We know that the left-hand side of (1) is increasing in S if and only if the elasticity of the degree of complementarity is larger for public good than for private good:

$$\frac{Sw'_g(S)}{w_g(S)} > \frac{Sw'_c(S)}{w_c(S)}.$$

If this holds true, then because of decreasing marginal utility of public good consumption, a higher security increases the optimal public good provision. This is because this inequality implies a stronger complementarity between security and public good than between security and private good, and it is more beneficial for workers to increase public good consumption for a higher security level.

2.2 Multiple Jurisdictions

We next introduce tax competition among multiple jurisdictions. Now suppose that there exist two jurisdictions, 1 and 2, and only Type M workers are mobile between the jurisdictions. Hence, l_i ($i = 1, 2$) are fixed, whereas m_i can vary satisfying that $\sum_{i=1}^2 m_i = 2m$.

The utility function of Type J worker living in jurisdiction i is given by

$$U_{Ji} = w_c(S_i)u(c_{Ji}) + w_g(S_i)v(g_i) - D(l_i, m_i).$$

Here, following a large number of theoretical studies in urban economics, we introduce congestion costs, $D(l_i, m_i)$, where D is a strictly increasing function with respect to its arguments. The

²See for instance O'Flaherty and Sethi (2015) Table 23.10, which shows that the elasticities of crime rates with respect to the police jurisdiction population are positive in the United States for the year 2012.

existence of congestion costs is important to the determination of population size of mobile workers.³ In the absence of congestion costs, all mobile workers are more likely to concentrate on one jurisdiction because public good provision exhibits the increasing returns to scale. Although changes in the public security level associated with an increase in mobile workers generate negative externalities because a large population worsen the public security level, introducing $D(l_i, m_i)$ makes it easier to guarantee the stability of the distribution of mobile worker. Because the optimal public good provision does not depend on $D(l_i, m_i)$, the Samuelson condition is still given by (1).

Each local government finances the cost of public good provision by imposing ad valorem tax on workers' income. Let $\tau_i \in [0, 1]$ denote the income tax rate, and then the private good consumption of Type L and Type M workers are $c_{Li} = (1 - \tau_i)y_{Li}$ and $c_{Mi} = (1 - \tau_i)y_M$, respectively. Here, y_{Li} is not necessarily identical across jurisdictions, whereas y_M is identical, implying that mobile workers' pre-tax income does not depend on a jurisdiction where they live. Because one unit of the numéraire can be transformed into one unit of the public good, the level of public good provision is given by the government's budget constraint, $g_i = (ly_L + m_i y_M)\tau_i$.

Each local government maximizes the utility of immobile workers by choosing income tax rate, τ_i . The government of jurisdiction i expects that the number of mobile workers, m_i , will change in response to its policy changes satisfying that

$$w_c(S_i)u(c_{Ji}) + w_g(S_i)v(g_i) - D(l_i, m_i) = w_c(S_k)u(c_{Jk}) + w_g(S_k)v(g_k) - D(l_k, m_k), \quad (i \neq k). \quad (2)$$

By plugging $g_i = (l_i y_{Li} + m_i y_M)\tau_i$ and $c_{Mi} = (1 - \tau_i)y_M$ into (2) and differentiating both sides with respect to τ_i , we can derive changes in m_i caused by a change in τ_i as

$$\frac{\partial m_i}{\partial \tau_i} = \frac{1}{\mu_{Mi} + \mu_{Mk}} [y_M w_c(S_i)u'(c_{Mi}) - (l_i y_{Li} + m_i y_M)w_g(S_i)v'(g_i)],$$

where μ_{Mi} is the mobile worker's marginal utility of hosting another mobile worker and is given by

$$\mu_{Mi} \equiv \frac{\partial U_{Mi}}{\partial m_i} = w'_c(S_i)u(c_{Mi}) \frac{\partial S_i}{\partial m_i} + w'_g(S_i)v(g_i) \frac{\partial S_i}{\partial m_i} + \tau_i y_M w_g(S_i)v'(g_i) - \frac{\partial D(l_i, m_i)}{\partial m_i}.$$

We assume that $\mu_{Mi} < 0$ is satisfied for all i , so that the distribution of mobile worker is stable. Here, the stability requires that a small perturbation causes utility changes that yield incentives to restore the original allocation. Hence, an increase in mobile workers in a particular jurisdiction needs to decrease the mobile worker's utility there, i.e., $\mu_{Mi} < 0$.

An increase in tax rate then results in the following changes in immobile worker's utility:

³See, for example, Kanemoto (1980) for the role of congestion in determining the size of cities.

$$\begin{aligned}
\frac{\partial U_{Li}}{\partial \tau_i} &= -y_{Li}w_c(S_i)u'(c_{Li}) + (l_i y_{Li} + m_i y_M)w_g(S_i)v'(g_i) + \mu_{Li} \frac{\partial m_i}{\partial \tau_i} \\
&= -y_{Li}w_c(S_i)u'(c_{Li}) + (l_i y_{Li} + m_i y_M)w_g(S_i)v'(g_i) \\
&\quad + \frac{\mu_{Li}}{\mu_{Mi} + \mu_{Mk}} [y_M w_c(S_i)u'(c_{Mi}) - (l_i y_{Li} + m_i y_M)w_g(S_i)v'(g_i)],
\end{aligned} \tag{3}$$

where μ_{Li} is the immobile worker's marginal utility of hosting another mobile worker, defined as

$$\mu_{Li} \equiv \frac{\partial U_{Li}}{\partial m_i} = w'_c(S_i)u(c_{Li}) \frac{\partial S_i}{\partial m_i} + w'_g(S_i)v(g_i) \frac{\partial S_i}{\partial m_i} + \tau_i y_M w_g(S_i)v'(g_i) - \frac{\partial D(l_i, m_i)}{\partial m_i}.$$

Note that μ_{Li} is not necessarily negative, because Type L workers are immobile, and the stability condition is not required. In the next section, we will use (3) in evaluating the efficiency of the equilibrium tax rate. The first term of (3) represents utility changes associated to private good consumption decreases caused by income taxation, its second term represents the utility increases caused by increased public good provision, and its last term represents the effects of changes in the number of mobile workers caused by income taxation.

The equilibrium tax rate is determined by the first-order condition of the local government maximization, $\partial U_{Li}/\partial \tau_i = 0$ for all i , and we assume interior solutions.⁴ Equilibrium is given by a 3-tuple (m_i, τ_i, g_i) that is determined by the mobile worker's arbitrage (2), the local government optimization $\partial U_{Li}/\partial \tau_i = 0$, and the local government's budget constraint $g_i = (l_i y_{Li} + m_i y_M)\tau_i$.

3 Efficiency Properties

Now we examine whether the complementarity between security and private/public good affects the efficiency of equilibrium of the tax competition. In order to see whether the equilibrium tax rate is efficient, we evaluate (3) at the Pareto-optimal tax rate, which satisfies (1). If it is positive, the equilibrium tax rate is higher than the optimal tax rate, and if it is negative, the opposite holds true. By plugging (1) into (3) and rearranging the equation, we obtain

$$\left. \frac{\partial U_{Li}}{\partial \tau_i} \right|_{\text{optimal}} = \Delta_i \Phi_i w_g(S_i)v'(g_i), \tag{4}$$

where Δ_i and Φ_i are defined as

$$\Delta_i \equiv y_M u'(c_{Mi}) - y_{Li} u'(c_{Li}) \quad , \quad \Phi_i \equiv \frac{m_i}{u'(c_{Mi})} + \frac{\mu_{Li}}{\mu_{Mi} + \mu_{Mk}} \frac{l_i}{u'(c_{Li})}.$$

⁴We assume the second-order conditions are satisfied.

The derivation of (4) is provided in Appendix A. Since $w_g(S_i)v'(g_i)$ is positive by definition, the sign of (4) is determined by those of Δ_i and Φ_i . Notice that all endogenous variables included in (4) are those satisfying the Samuelson condition given by (1).

First, we focus on Δ_i . To investigate the sign of Δ_i , we consider the following derivative:

$$\frac{\partial}{\partial y_{Ji}} (y_{Ji}u'(c_{Ji})) = u'(c_{Ji}) + c_{Ji}u''(c_{Ji}) = (1 - r_{Ji})u'(c_{Ji}),$$

where $r_{Ji} \equiv -c_{Ji}u''(c_{Ji})/u'(c_{Ji})$ measures the relative risk aversion, or the elasticity of marginal utility from consumption of private goods. For simplicity, we here assume that r_{Ji} is a positive constant, and we denote it by r .⁵ Then, we obtain the following lemma.

Lemma 1 *The sign of Δ_i is determined by r , y_{Li} and y_M as follows:*

$$(a) \text{ if } r < 1, \text{ then } y_{Li} \lesseqgtr y_M \iff \Delta_i \gtrless 0.$$

$$(b) \text{ if } r = 1, \text{ then } \Delta_i = 0 \text{ for any } y_{Li} \text{ and } y_M;$$

$$(c) \text{ if } r > 1, \text{ then } y_{Li} \lesseqgtr y_M \iff \Delta_i \gtrless 0;$$

Lemma 1 implies that, in jurisdiction i , the equilibrium tax rate always equals the optimal one either when there exists no income difference, i.e., $y_{Li} = y_M$, or when the coefficient of the relative risk aversion is equal to one, i.e., $r = 1$, which holds true if we employ a log utility function.

Next, we focus on Φ_i . Notice that

$$\mu_{Li} - \mu_{Mi} = w'_c(S_i) (u(c_{Li}) - u(c_{Mi})) \frac{\partial S_i}{\partial m_i}. \quad (5)$$

Since we assume that $w'_c(S_i) > 0$ and $\partial S_i / \partial m_i < 0$, we obtain

$$y_{Li} \lesseqgtr y_M \iff \mu_{Li} \gtrless \mu_{Mi}.$$

Recall the stability condition $\mu_{Mi} < 0$. Then, if $y_{Li} \geq y_M$, we have $\mu_{Li} \leq \mu_{Mi} < 0$, implying that $\mu_{Li}/(\mu_{Mi} + \mu_{Mk})$ is always positive, and thus $\Phi_i > 0$ holds. However, if $y_{Li} < y_M$, it is possible that μ_{Li} is positive, which means the sign of Φ_i is not necessarily positive. Φ_i is negative if and only if $\mu_{Li} > \tilde{\mu}_i$ holds, where $\tilde{\mu}_i$ is given by

$$\tilde{\mu}_i \equiv \frac{m_i u'(c_{Li})}{l_i u'(c_{Mi})} |\mu_{Mi} + \mu_{Mk}|.$$

We summarize these results in the following lemma.

Lemma 2 *The sign of Φ is determined by y_{Li} , y_M and the relationship among μ_{Li} , μ_{Mi} , l_i , m_i , c_{Li} and c_{Mi} as follows:*

⁵This is possible when we employ an isoelastic function for $u(c_{Ji})$.

- (a) if $y_{Li} \geq y_M$, then $\Phi_i > 0$;
(b) if $y_{Li} < y_M$, then $\mu_{Li} \begin{matrix} \leq \\ \geq \end{matrix} \tilde{\mu}_i \iff \Phi_i \begin{matrix} \geq \\ \leq \end{matrix} 0$.

Combining Lemma 1 and 2, the sign of (4) can be summarized as in Table 1.

[Table 1 around here]

Hence, we obtain the following proposition:

Proposition 2

- (a) *Equilibrium is efficient if the coefficient of relative risk aversion is equal to one (i.e., $r = 1$) or if there exists no income difference (i.e., $y_{Li} = y_M$).*
- (b) *Suppose $r < 1$. Then, the equilibrium tax rate is inefficiently low if the immobile worker's income is higher than the mobile worker's income (i.e., $y_{Li} > y_M$). If the opposite holds true (i.e., $y_{Li} < y_M$), a large value of immobile worker's marginal utility of hosting another mobile worker (i.e., $\mu_{Li} > \tilde{\mu}_i$) yields inefficiently low tax rate whereas a small value of that (i.e., $\mu_{Li} < \tilde{\mu}_i$) yields inefficiently high tax rate.*
- (c) *Suppose $r > 1$. Then, the equilibrium tax rate is inefficiently high if the immobile worker's income is higher than the mobile worker's income (i.e., $y_{Li} > y_M$). If the opposite holds true (i.e., $y_{Li} < y_M$), a large value of immobile worker's marginal utility of hosting another mobile worker (i.e., $\mu_{Li} > \tilde{\mu}_i$) yields inefficiently high tax rate whereas a small value of that (i.e., $\mu_{Li} < \tilde{\mu}_i$) yields inefficiently low tax rate.*

The equilibrium is optimal only in exceptional cases (i.e., $r = 1$ or $y_{Li} = y_M$). In most cases, equilibrium is inefficient, and the direction of inefficiency depends on income difference, $y_{Li} - y_M$, the coefficient of relative risk aversion, r , and the immobile worker's marginal utility of hosting another mobile worker, μ_{Li} .

The relative risk aversion represents the rate at which marginal utility from private good decreases in response to an increase in the amount of consumption. Thus, the larger r , the smaller utility obtained at a certain amount of consumption, because the marginal utility decreases relatively fast when r is large. Since income taxation affects the amount of consumption, this coefficient is concerned with the efficiency of the equilibrium.

Hosting another mobile worker affects the immobile worker's utility through three different channels. First, it worsens the security and decreases the immobile worker's utility. Second, it

increases the tax revenue and her utility. Third, it worsens the congestion and decreases her utility.

If the immobile worker's income is higher than the mobile worker's income (i.e., $y_{Li} > y_M$), the negative effects of hosting another mobile worker dominate its positive effect, implying that $\mu_{Li} < 0$, and r determines the direction of inefficiency.

When r is low (i.e., $r < 1$), the marginal utility does not decrease so fast, and the utility derived from consumption of private goods is highly affected by changes in tax rate. Thus, the government tries to enhance the utility of immobile workers through lowering tax rates. However, this tax cut tends to be excessive because low tax rates decrease the provision of public goods and have an effect of reducing the inflow of mobile workers. Then, the government's behavior results in an inefficiently low tax.

When r is high (i.e., $r < 1$), the marginal utility decreases relatively fast, and the utility derived from consumption of private goods is not so much affected by changes in tax rate. Thus, the government aims to enhance the utility of immobile workers by raising tax rates and increasing the provision of public goods. However, this tax hike tends to be excessive because high tax rates also have an effect of reducing the inflow of mobile workers through decreasing their consumption levels. As a result, tax rate becomes inefficiently high.

If the immobile worker's income is lower than the mobile worker's income (i.e., $y_{Li} < y_M$), things become more complicated. In addition to r , μ_{Li} now matters. In this case, we face a possibility that μ_{Li} is positive and hosting another mobile worker improves the immobile worker's utility. The reason for which the possibility of μ_{Li} becoming positive arises is attributed to the mobile worker's high income. Since the immobile worker's income is relatively low, the mobile worker may be regarded as the source of public good provision to improve the immobile worker's utility, as long as the inflow of mobile workers does not damage the immobile worker's utility through an increase in congestion and a deterioration in public security. This is why things become complex when $y_{Li} < y_M$.

We first consider the case in which μ_{Li} is sufficiently small (i.e., $\mu_{Li} < \tilde{\mu}_i$) and even negative. In this case, hosting another mobile worker is likely to be undesirable for immobile workers.

When r is low (i.e., $r < 1$), a low tax rate can cause an increase in the inflow of mobile workers, because the utility that the mobile worker derives from consumption of private goods is highly affected by changes in tax rate. To prevent this unpleasant inflow, the government tries to raise the tax rate, resulting in an inefficiently high tax.

When r is high (i.e., $r < 1$), in contrast, a high tax rate can induce an increase in the inflow of mobile workers, because the mobile worker's utility does not so much affected by changes in

tax rates, and the amount of public goods becomes a more important factor that determines the inflow and outflow of mobile workers. To prevent the inflow, the government aims to set a lower tax. As a result, tax rate becomes inefficiently low.

Considering the case in which μ_{Li} is sufficiently large (i.e., $\mu_{Li} > \tilde{\mu}_i$), however, opposite results arise because the inflow of mobile workers gets to be desirable for immobile workers. To increase the inflow of mobile workers, the government tries to reduce (resp. raise) the tax rate, when r is low (resp. high), which leads to an inefficiently low (resp. high) tax.

3.1 Effects of Complementarity

From Proposition 2, we know that equilibrium tax rate is inefficient for most cases. We then examine how the inefficiency properties depends on the complementarity between security and private/public good.

No Complementarity between Security and Private Good

We start from the case wherein there is no complementarity between security and private good. Here, we set $w'_c(S_i) = 0$, implying that $w_c(S_i)$ is a constant.

In this case, by (5), we obtain $\mu_{Li} = \mu_{Mi} < 0$, implying that the sign of $\mu_{Li}/(\mu_{Mi} + \mu_{Mk})$ is always positive, and so is the sign of Φ_i . Thus, the sign of (4) is determined only by that of Δ_i , and Lemma 1 allows us to obtain the properties of inefficiency under no complementarity between security and private good. The result is summarized in Table 2.

[Table 2 around here]

Lemma 3 *Suppose there exists no complementarity between security and private good.*

- (a) *Equilibrium is efficient if the coefficient of relative risk aversion is equal to one (i.e., $r = 1$) or if there exists no income difference (i.e., $y_{Li} = y_M$).*
- (b) *Suppose $r < 1$. Then, the equilibrium tax rate is inefficiently low if the immobile worker's income is higher than the mobile worker's income (i.e., $y_{Li} > y_M$). If the opposite holds true (i.e., $y_{Li} < y_M$), the equilibrium tax rate is inefficiently high.*
- (c) *Suppose $r > 1$. Then, the equilibrium tax rate is inefficiently high if the immobile worker's income is higher than the mobile worker's income (i.e., $y_{Li} > y_M$). If the opposite holds true (i.e., $y_{Li} < y_M$), the equilibrium tax rate is inefficiently low.*

No Complementarity between Security and Public Good

We can also investigate the role of complementarity between security and public good. We here let $w'_g(S_i) = 0$, keeping $w'_c(S_i)$ positive.

Since Δ_i in (4) is not affected by this change, the sign of Δ_i is still determined as in Lemma 1. Moreover, we know that, by (5), $\mu_{Li} \neq \mu_{Mi}$ still holds as long as $y_{Li} \neq y_{Mi}$ is satisfied, implying that the sign of Φ_i is determined in the same way as Lemma 2. Therefore, the properties of inefficiency are also same as those given in Proposition 2, which are summarized in Table 1.

Lemma 4 *Suppose there exists no complementarity between security and public good. Then the efficiency properties are the same as those given in Proposition 2.*

No Complementarity between Security and Both Goods

Finally, we suppose there exists no complementarity between security and two goods. We thus let $w'_c(S_i) = w'_g(S_i) = 0$. In this case, we again obtain $\mu_{Li} = \mu_{Mi}$, implying that the sign of (4) is determined only by that of Δ_i . Hence, the results become the same as those obtained in Lemma 3, which are summarized in Table 2.

Lemma 5 *Suppose there exists no complementarity between security and two goods. Then the efficiency properties are the same as those given in Lemma 3.*

From Lemmas 3-5, we obtain the following proposition.

Proposition 3 *The complementarity between security and public good has no effect on the (in)efficiency properties of equilibrium of tax competition. In contrast, the complementarity between security and private good might reverse them.*

The results of Lemmas 3-5 imply that it is more important to care about complementarity between security and private good in the presence of fiscal externality, because it can lead to a reversed property of inefficiency.

4 Numerical Illustrations

To illustrate the model, we specify the functional forms and compute the equilibrium and optimal tax rates numerically.

The utilities obtained from private and public goods are represented by constant-relative-risk-aversion functions, given by

$$u(c_{J_i}) = \begin{cases} \frac{c_{J_i}^{1-\sigma} - 1}{1-\sigma} & (\sigma \neq 1) \\ \ln c_{J_i} & (\sigma = 1) \end{cases}, \text{ and } v(g_i) = \begin{cases} \frac{g_i^{1-\theta} - 1}{1-\theta} & (\theta \neq 1) \\ \ln g_i & (\theta = 1) \end{cases},$$

respectively, where σ and θ are positive constants that measure the relative risk aversion.

The level of public security, S_i , is decreasing in l_i and m_i , and assumed to be given by $S_i = (l_i + m_i)^{-\gamma}$. $\gamma > 0$ can be interpreted as the elasticity of the public security level with respect to the total population in jurisdiction i . We also assume that both $w_c(S_i)$ and $w_g(S_i)$ are in the form of power function: $w_c(S_i) = S_i^\alpha$ and $w_g(S_i) = S_i^\beta$, where $\alpha, \beta \in (0, 1)$.

The congestion costs, $D(l_i, m_i)$, is specified by a linear function: $D(l_i, m_i) = f(l_i + m_i)$, where $f > 0$ represents the marginal increase in congestion with respect to the total population in jurisdiction i .

4.1 Symmetric Jurisdictions

First, we consider the case with symmetric jurisdictions, in which $l_1 = l_2 = l$ and $y_{L1} = y_{L2} = y_L$ hold. Because of symmetry, the number of mobile workers living in each jurisdiction and the tax rate set by each government are both identical across jurisdictions. That is, $m_1 = m_2 = m$ and $\tau_1 = \tau_2 = \tau$ hold at both the equilibrium and optimum.

[Figure 1 around here]

Figure 1 illustrates the relationship between the risk aversion, σ , and the equilibrium and optimal tax rates.⁶ The solid line represents the equilibrium tax rate, while the dashed line does the optimal one. Because we here let $y_L > y_M$, the equilibrium tax rate is below the optimal one when $\sigma < 1$. The two curves intersect at $\sigma = 1$, at which the utility from private goods is given by a log utility function. When $\sigma > 1$, the equilibrium tax rate exceeds the optimal one.

[Figure 2 around here]

Figure 2 illustrates the relationship between the immobile worker's income, y_L , and the equilibrium and optimal tax rates.⁷ Again, The solid and dashed lines represents the equilibrium

⁶The values of parameters included in the model are given as follows: $l = 5$, $m = 4$, $y_L = 4$, $y_M = 3$, $\theta = 1.4$, $\gamma = 1$, $\alpha = 0.6$, $\beta = 0.4$, and $f = 0.8$.

⁷The values of parameters included in the model are given as follows: $l = 4.5$, $m = 2$, $y_M = 2.7$, $\sigma = 1.2$, $\theta = 0.6$, $\gamma = 1$, $\alpha = 0.2$, $\beta = 0.7$, and $f = 0.02$.

and optimal tax rates, respectively. There are two intersections in this figure. Because we let $y_M = 2.7$, the two curves intersect at $y_L = 2.7$. When y_L is higher than this level, the equilibrium tax rate exceeds the optimal one. When y_L is smaller than this level, the equilibrium tax rate is below the optimal one. However, we have the other intersection around $y_L = 2$. Recall that Proposition 2 mentions the possibility of a reversed property of inefficiency under $y_L < y_M$, depending on the immobile worker's marginal utility of hosting another mobile worker. Thus, this intersection is the point at which $\mu_L = \tilde{\mu}$ is satisfied. If y_L is smaller than this level, hosting another mobile worker becomes beneficial enough to reverse the relationship between the equilibrium and optimal tax rates, and the equilibrium tax rate again exceeds the optimal one.

4.2 Asymmetric Jurisdictions

Next, we let l_i and y_{Li} vary across jurisdictions. In this case, of course, the distribution of mobile workers is no longer symmetric, and the tax rate in each jurisdiction is also different.

Asymmetry in the Immobile Worker's Income

We first fix l_1 and l_2 at the same level, and focus on how asymmetry in the mobile worker's income affects the equilibrium and optimum.

[Figure 3 around here]

Figure 3 illustrates how the gap between the equilibrium and optimal tax rates in each jurisdiction is affected by the difference in y_{L1} and y_{L2} . The vertical axis represents the difference between the equilibrium and optimal tax rates. Thus, this value is positive (resp. negative) if the equilibrium tax rate is inefficiently high (resp. low). The gap in jurisdiction 1 is given by the solid line, while that in jurisdiction 2 is represented by the dashed line. We here fix $y_{L2} = 3$, and set y_{L1} for the horizontal axis.⁸

As implied by the theoretical results, the equilibrium tax rate is equal to the optimal one in jurisdiction 1 at $y_{L1} = y_M = 4$. Since we now let $\sigma < 1$, the equilibrium tax rate in jurisdiction 1 is inefficiently low when $y_{L1} > y_M$. Although the theoretical results predict the possibility of the equilibrium tax rate being lower than the optimal one even when $y_{L1} < y_M$ holds, this example does not exhibit such a case and the equilibrium tax rate in jurisdiction 1 is inefficiently

⁸The other values of parameters included in the model are given as follows: $l_1 = l_2 = 8$, $m = 5$, $y_M = 4$, $\sigma = 0.8$, $\theta = 1.2$, $\gamma = 1$, $\alpha = 0.3$, $\beta = 0.4$, and $f = 2$.

high under $y_{L1} < y_M$. This example also implies that, even if the immobile worker's income in one jurisdiction changes, the gap between the equilibrium and optimum in the other jurisdiction is not affected so much.

Appendix B provides the values of the equilibrium and optimal tax rates in each jurisdiction and the share of mobile worker living in jurisdiction 1 at the equilibrium and optimum, for several pairs of y_{L1} and y_{L2} . The numerical results imply that the distribution of the mobile worker is affected little, at both the equilibrium and optimum.

Although we here let $\alpha < \beta$, Appendix B also provides the numerical results with $\alpha > \beta$. The result implies that the equilibrium and optimal tax rates becomes higher, the gap between the equilibrium and optimum is not so different from that in the case with $\alpha > \beta$.

Asymmetry in the Population of the Immobile Worker

we next fix y_{L1} and y_{L2} at the same level, and focus on how asymmetry in the population of the immobile worker affects the equilibrium and optimum.

[Figure 4 around here]

Figure 4 illustrates how the gap between the equilibrium and optimal tax rates in each jurisdiction is affected by the difference in l_1 and l_2 . Again, the vertical axis represents the difference between the equilibrium and optimal tax rates, and the solid and dashed lines are the gaps in jurisdiction 1 and 2, respectively. We here fix $l_2 = 8$, and set l_1 for the horizontal axis.⁹

The equilibrium tax rates are excessive for any l_1 in the figure. Since we here let $y_{L1} = y_{L2} < y_M$, this implies that the immobile worker's marginal utility of hosting another mobile worker is sufficiently low in both jurisdictions.

The gap tends to be smaller, compared with the case in which we focus on asymmetry in the mobile worker's income. This implies that asymmetry in the mobile worker's income is more crucial for shaping the gap between the equilibrium and optimum, than asymmetry in the population of the immobile worker.

Appendix C provides the values of the equilibrium and optimal tax rates in each jurisdiction and the share of mobile worker living in jurisdiction 1 at the equilibrium and optimum, for several pairs of l_1 and l_2 . According to the numerical results, the distribution of the mobile worker is affected so much by changes in the population of the immobile worker, in sharp contrast to

⁹The other values of parameters included in the model are given as follows: $y_{L1} = y_{L2} = 3$, $m = 5$, $y_M = 4$, $\sigma = 0.8$, $\theta = 1.2$, $\gamma = 1$, $\alpha = 0.3$, $\beta = 0.4$, and $f = 2$.

the case in which we focus on asymmetry in the mobile worker's income. The reason for this difference is that the number of residents directly affects the immobile worker's utility through public security and congestion.

Although we now let $\alpha < \beta$, Appendix C also includes the case with $\alpha > \beta$. The result is qualitatively the same, but the equilibrium and optimal tax rates tend to be higher, compared with the case with $\alpha < \beta$.

5 Concluding Remarks

We constructed a model of public good provision under the complementarity between safety and private/public good consumption. In the presence of fiscal externality, the equilibrium is inefficient in most cases, and the direction of inefficiency is determined by three factors: the characteristics of the utility function, the difference in income between the immobile and mobile workers, and the immobile worker's marginal utility of hosting another mobile worker. The third factor becomes relevant only when complementarity between security and private good is taken into account, implying that it is more important to consider complementarity between security and private good in the presence of fiscal externality, because a reversed property of inefficiency can arise due to this type of complementarity.

We also conducted numerical analyses. The results imply that asymmetry in the mobile worker's income is more crucial for shaping the gap between the equilibrium and optimum, but it does not affect so much the distribution of the mobile worker. In contrast, asymmetry in the population of the immobile worker has a large impact on the distribution of the mobile worker, although it is less crucial to the equilibrium and optimal tax rates.

Our theoretical and numerical results give an implication for empirical researches. When we investigate the relationship between safety and public good, it is also important to account for income differences and complementarity between safety and private good.

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Appendices

Appendix A: Derivation of (4)

(3) can be rewritten as follows:

$$\begin{aligned} \frac{\partial U_{Li}}{\partial \tau_i} = & -y_{Li}w_c(S_i)u'(c_{Li}) \left(1 - l_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Li})} \right) + m_i y_M w_g(S_i)v'(g_i) \\ & + \frac{\mu_{Li}}{\mu_{Mi} + \mu_{Mk}} \left[y_M w_c(S_i)u'(c_{Mi}) \left(1 - m_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Mi})} \right) - l_i y_L w_g(S_i)v'(g_i) \right]. \end{aligned}$$

Note that (1) gives us

$$1 - l_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Li})} = m_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Mi})}, \text{ and } 1 - m_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Mi})} = l_i \frac{w_g(S_i)v'(g_i)}{w_c(S_i)u'(c_{Li})}.$$

Plugging these into the above equation and arranging it, we obtain (4).

Appendix B: Asymmetry in the Mobile Worker's Income and The Numerical Results

Table 3 provides the numerical values of the equilibrium and optimal tax rates in each jurisdiction and the share of mobile worker living in jurisdiction 1 at the equilibrium and optimum, for several pairs of y_{L1} and y_{L2} , where $\alpha = 0.3$ and $\beta = 0.4$. Table 4 considers the case with $\alpha = 0.4$ and $\beta = 0.3$. The other parameters are given as follows: $l_1 = l_2 = 8$, $m = 5$, $y_M = 4$, $\sigma = 0.8$, $\theta = 1.2$, $\gamma = 1$, and $f = 2$.

Each cell in the table contains six things. In the top row, we report the equilibrium tax rates in jurisdictions 1 (on the left side) and jurisdiction 2 (on the right side). In the middle row, we report the optimal tax rates in jurisdictions 1 (on the left side) and jurisdiction 2 (on the right side). In the bottom row, we report the share of the mobile worker living in jurisdiction 1 at the equilibrium (on the left side) and the optimum (on the right side).

[Table 3 around here]

[Table 4 around here]

Appendix C: Asymmetry in the Population of the Mobile Worker and The Numerical Results

Table 5 provides the numerical values of the equilibrium and optimal tax rates in each jurisdiction and the share of mobile worker living in jurisdiction 1 at the equilibrium and optimum, for several pairs of l_1 and l_2 , where $\alpha = 0.3$ and $\beta = 0.4$. Table 6 considers the case with $\alpha = 0.4$ and $\beta = 0.3$. The other parameters are given as follows: $y_{L1} = y_{L2} = 3$, $m = 5$, $y_M = 4$, $\sigma = 0.8$, $\theta = 1.2$, $\gamma = 1$, and $f = 2$. How to see the table is the same as in Appendix B.

[Table 5 around here]

[Table 6 around here]

Table 1: **Equilibrium vs. optimum.**

Sign of (4)	$y_{Li} = y_M$	$y_{Li} > y_M$	$y_{Li} < y_M$
$r = 1$	0	0	0
$r < 1$	0	-	- if $\mu_{Li} > \tilde{\mu}_i$
			0 if $\mu_{Li} = \tilde{\mu}_i$
			+ if $\mu_{Li} < \tilde{\mu}_i$
$r > 1$	0	+	+ if $\mu_{Li} > \tilde{\mu}_i$
			0 if $\mu_{Li} = \tilde{\mu}_i$
			- if $\mu_{Li} < \tilde{\mu}_i$

Table 2: **Equilibrium vs. optimum under $w'_c(S) = 0$.**

Sign of (4)	$y_{Li} = y_M$	$y_{Li} > y_M$	$y_{Li} < y_M$
$r = 1$	0	0	0
$r < 1$	0	-	+
$r > 1$	0	+	-

Table 3: **The numerical results with asymmetric y_{Li} and $\alpha < \beta$.**

1	(0.4343, 0.4343)						
	(0.3092, 0.3092)						
	(0.5000, 0.5000)						
	(0.3421, 0.4340)	(0.3421, 0.3421)					
	(0.2947, 0.3093)	(0.2948, 0.2948)					
	(0.5022, 0.5015)	(0.5000, 0.5000)					
	(0.2974, 0.4338)	(0.2975, 0.3421)	(0.2975, 0.2975)				
3	(0.2810, 0.3094)	(0.2810, 0.2949)	(0.2811, 0.2811)				
	(0.5034, 0.5026)	(0.5012, 0.5011)	(0.5000, 0.5000)				
	(0.2690, 0.4336)	(0.2691, 0.3420)	(0.2691, 0.2975)	(0.2692, 0.2692)			
4	(0.2690, 0.3095)	(0.2691, 0.2949)	(0.2691, 0.2812)	(0.2692, 0.2692)			
	(0.5043, 0.5035)	(0.5022, 0.5020)	(0.5009, 0.5009)	(0.5000, 0.5000)			
	(0.2486, 0.4335)	(0.2487, 0.3420)	(0.2488, 0.2976)	(0.2488, 0.2692)	(0.2489, 0.2489)		
5	(0.2587, 0.3096)	(0.2588, 0.2950)	(0.2588, 0.2812)	(0.2588, 0.2692)	(0.2589, 0.2589)		
	(0.5050, 0.5042)	(0.5029, 0.5027)	(0.5016, 0.5016)	(0.5007, 0.5007)	(0.5000, 0.5000)		
	(0.2330, 0.4335)	(0.2331, 0.3420)	(0.2331, 0.2976)	(0.2332, 0.2692)	(0.2332, 0.2489)	(0.2333, 0.2333)	
6	(0.2497, 0.3096)	(0.2498, 0.2950)	(0.2498, 0.2812)	(0.2498, 0.2692)	(0.2498, 0.2589)	(0.2499, 0.2499)	
	(0.5056, 0.5048)	(0.5035, 0.5034)	(0.5022, 0.5023)	(0.5013, 0.5014)	(0.5006, 0.5006)	(0.5000, 0.5000)	
	(0.2204, 0.4334)	(0.2205, 0.3420)	(0.2206, 0.2976)	(0.2207, 0.2693)	(0.2207, 0.2489)	(0.2207, 0.2333)	(0.2208, 0.2208)
7	(0.2418, 0.3097)	(0.2419, 0.2951)	(0.2419, 0.2813)	(0.2419, 0.2693)	(0.2419, 0.2589)	(0.2420, 0.2499)	(0.2420, 0.2420)
	(0.5062, 0.5054)	(0.5040, 0.5039)	(0.5028, 0.5028)	(0.5018, 0.5019)	(0.5011, 0.5012)	(0.5005, 0.5005)	(0.5000, 0.5000)
	1	2	3	4	5	6	7
				y_{L2}			

Table 4: The numerical results with asymmetric y_{Li} and $\alpha > \beta$.

y_{L1}	1	(0.5618, 0.5618)							
		(0.4213, 0.4213)							
		(0.5000, 0.5000)							
	2		(0.4601, 0.5612)	(0.4599, 0.4599)					
			(0.4039, 0.4214)	(0.4040, 0.4040)					
			(0.5025, 0.5018)	(0.5000, 0.5000)					
	3		(0.4074, 0.5609)	(0.4074, 0.4598)	(0.4073, 0.4073)				
			(0.3873, 0.4215)	(0.3874, 0.4041)	(0.3874, 0.3874)				
			(0.5040, 0.5031)	(0.5015, 0.5013)	(0.5000, 0.5000)				
	4		(0.3727, 0.5606)	(0.3727, 0.4597)	(0.3727, 0.4073)	(0.3728, 0.3728)			
			(0.3727, 0.4216)	(0.3727, 0.4041)	(0.3727, 0.3875)	(0.3728, 0.3728)			
			(0.5051, 0.5042)	(0.5026, 0.5024)	(0.5011, 0.5011)	(0.5000, 0.5000)			
	5		(0.3472, 0.5605)	(0.3473, 0.4597)	(0.3474, 0.4073)	(0.3474, 0.3728)	(0.3474, 0.3474)		
			(0.3599, 0.4217)	(0.3599, 0.4042)	(0.3599, 0.3875)	(0.3599, 0.3728)	(0.3600, 0.3600)		
			(0.5060, 0.5051)	(0.5035, 0.5033)	(0.5020, 0.5020)	(0.5009, 0.5009)	(0.5000, 0.5000)		
	6		(0.3274, 0.5603)	(0.3275, 0.4596)	(0.3276, 0.4073)	(0.3276, 0.3728)	(0.3277, 0.3475)	(0.3277, 0.3277)	
			(0.3487, 0.4217)	(0.3487, 0.4042)	(0.3487, 0.3875)	(0.3487, 0.3728)	(0.3487, 0.3600)	(0.3487, 0.3487)	
			(0.5067, 0.5059)	(0.5042, 0.5041)	(0.5027, 0.5027)	(0.5016, 0.5017)	(0.5007, 0.5008)	(0.5000, 0.5000)	
	7		(0.3113, 0.5602)	(0.3114, 0.4596)	(0.3115, 0.4073)	(0.3116, 0.3728)	(0.3116, 0.3475)	(0.3116, 0.3277)	(0.3117, 0.3117)
			(0.3388, 0.4218)	(0.3388, 0.4043)	(0.3388, 0.3875)	(0.3387, 0.3728)	(0.3387, 0.3600)	(0.3387, 0.3487)	(0.3387, 0.3387)
			(0.5073, 0.5065)	(0.5048, 0.5047)	(0.5034, 0.5034)	(0.5022, 0.5023)	(0.5014, 0.5014)	(0.5006, 0.5007)	(0.5000, 0.5000)
		1	2	3	4	5	6	7	
		y_{L2}							

Table 5: The numerical results with asymmetric l_i and $\alpha < \beta$.

5	(0.3116, 0.3116)						
	(0.2934, 0.2934)						
6	(0.5000, 0.5000)						
	(0.3096, 0.3085)	(0.3065, 0.3065)					
	(0.2922, 0.2901)	(0.2890, 0.2890)					
7	(0.4498, 0.4498)	(0.5000, 0.5000)					
	(0.3076, 0.3055)	(0.3046, 0.3036)	(0.3018, 0.3018)				
	(0.2910, 0.2870)	(0.2879, 0.2859)	(0.2849, 0.2849)				
8	(0.3997, 0.3997)	(0.4499, 0.4499)	(0.5000, 0.5000)				
	(0.3056, 0.3026)	(0.3028, 0.3009)	(0.3001, 0.2992)	(0.2975, 0.2975)			
	(0.2899, 0.2841)	(0.2868, 0.2831)	(0.2839, 0.2821)	(0.2811, 0.2811)			
9	(0.3496, 0.3496)	(0.3997, 0.3997)	(0.4499, 0.4499)	(0.5000, 0.5000)			
	(0.3038, 0.2999)	(0.3010, 0.2983)	(0.2984, 0.2966)	(0.2959, 0.2951)	(0.2936, 0.2936)		
	(0.2887, 0.2813)	(0.2857, 0.2804)	(0.2829, 0.2794)	(0.2802, 0.2785)	(0.2776, 0.2776)		
10	(0.2994, 0.2994)	(0.3496, 0.3496)	(0.3998, 0.3998)	(0.4499, 0.4499)	(0.5000, 0.5000)		
	(0.3020, 0.2974)	(0.2993, 0.2958)	(0.2968, 0.2942)	(0.2944, 0.2927)	(0.2921, 0.2913)	(0.2899, 0.2899)	
	(0.2876, 0.2787)	(0.2847, 0.2778)	(0.2819, 0.2769)	(0.2792, 0.2760)	(0.2767, 0.2752)	(0.2743, 0.2743)	
11	(0.2494, 0.2493)	(0.2995, 0.2995)	(0.3497, 0.3496)	(0.3998, 0.3998)	(0.4499, 0.4499)	(0.5000, 0.5000)	
	(0.3003, 0.2950)	(0.2977, 0.2934)	(0.2952, 0.2920)	(0.2929, 0.2905)	(0.2907, 0.2891)	(0.2885, 0.2878)	(0.2865, 0.2865)
	(0.2866, 0.2762)	(0.2837, 0.2754)	(0.2809, 0.2745)	(0.2783, 0.2737)	(0.2759, 0.2729)	(0.2735, 0.2721)	(0.2713, 0.2713)
	(0.1993, 0.1993)	(0.2494, 0.2494)	(0.2996, 0.2996)	(0.3497, 0.3497)	(0.3998, 0.3998)	(0.4499, 0.4499)	(0.5000, 0.5000)
	5	6	7	8	9	10	11
				l_2			

Table 6: The numerical results with asymmetric l_i and $\alpha > \beta$.

5	(0.4119, 0.4119)						
	(0.3903, 0.3903)						
	(0.5000, 0.5000)						
6	(0.4117, 0.4104)	(0.4103, 0.4103)					
	(0.3911, 0.3886)	(0.3894, 0.3894)					
	(0.4498, 0.4498)	(0.5000, 0.5000)					
7	(0.4115, 0.4091)	(0.4101, 0.4089)	(0.4088, 0.4088)				
	(0.3918, 0.3870)	(0.3900, 0.3877)	(0.3884, 0.3884)				
	(0.3996, 0.3996)	(0.4498, 0.4498)	(0.5000, 0.5000)				
8	(0.4113, 0.4078)	(0.4099, 0.4076)	(0.4086, 0.4075)	(0.4073, 0.4073)			
	(0.3925, 0.3855)	(0.3907, 0.3862)	(0.3890, 0.3868)	(0.3874, 0.3874)			
	(0.3495, 0.3495)	(0.3997, 0.3997)	(0.4498, 0.4498)	(0.5000, 0.5000)			
9	(0.4111, 0.4065)	(0.4097, 0.4064)	(0.4084, 0.4063)	(0.4072, 0.4061)	(0.4060, 0.4060)		
	(0.3930, 0.3841)	(0.3912, 0.3847)	(0.3896, 0.3854)	(0.3880, 0.3860)	(0.3865, 0.3865)		
	(0.2993, 0.2993)	(0.3495, 0.3495)	(0.3997, 0.3997)	(0.4499, 0.4499)	(0.5000, 0.5000)		
10	(0.4109, 0.4054)	(0.4095, 0.4053)	(0.4082, 0.4052)	(0.4070, 0.4050)	(0.4058, 0.4049)	(0.4047, 0.4047)	
	(0.3936, 0.3827)	(0.3918, 0.3834)	(0.3901, 0.3840)	(0.3885, 0.3846)	(0.3870, 0.3851)	(0.3856, 0.3856)	
	(0.2492, 0.2492)	(0.2994, 0.2994)	(0.3496, 0.3496)	(0.3997, 0.3997)	(0.4499, 0.4499)	(0.5000, 0.5000)	
11	(0.4106, 0.4043)	(0.4093, 0.4042)	(0.4080, 0.4041)	(0.4068, 0.4039)	(0.4057, 0.4038)	(0.4046, 0.4037)	(0.4035, 0.4035)
	(0.3941, 0.3815)	(0.3922, 0.3821)	(0.3905, 0.3827)	(0.3890, 0.3832)	(0.3875, 0.3838)	(0.3860, 0.3842)	(0.3847, 0.3847)
	(0.1991, 0.1991)	(0.2493, 0.2493)	(0.2995, 0.2995)	(0.3496, 0.3496)	(0.3998, 0.3998)	(0.4499, 0.4499)	(0.5000, 0.5000)
	5	6	7	8	9	10	11
				l_2			

Figure 1

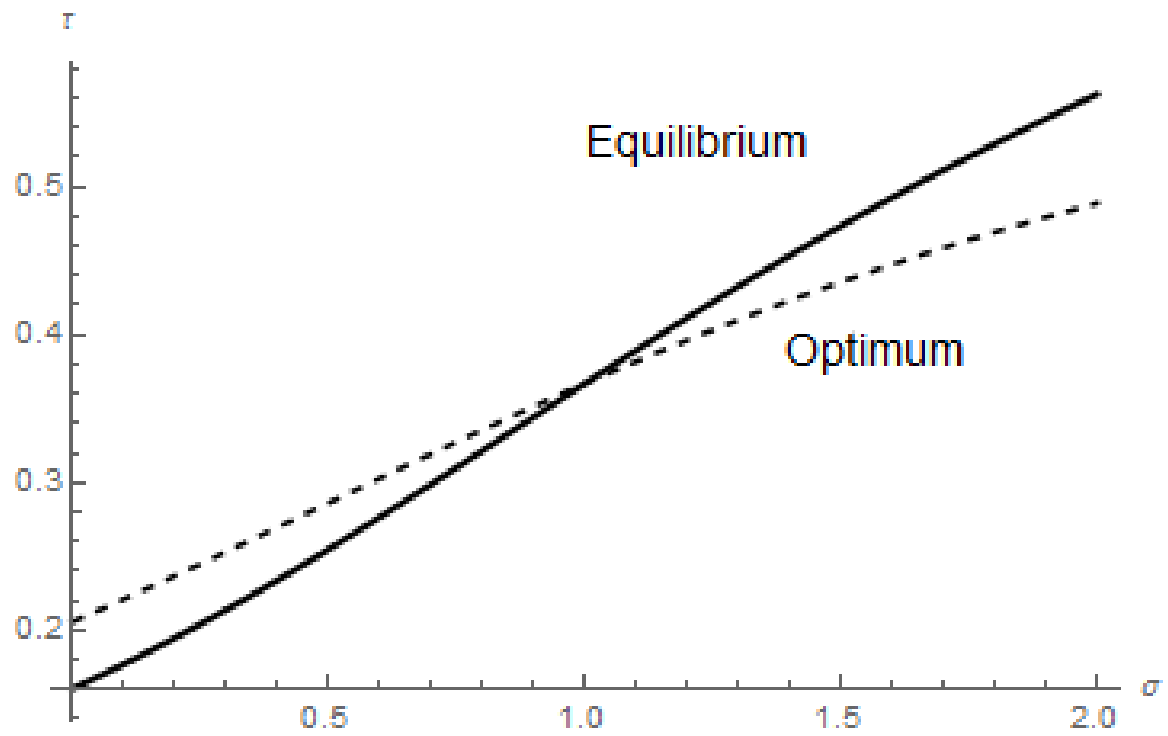


Figure 2

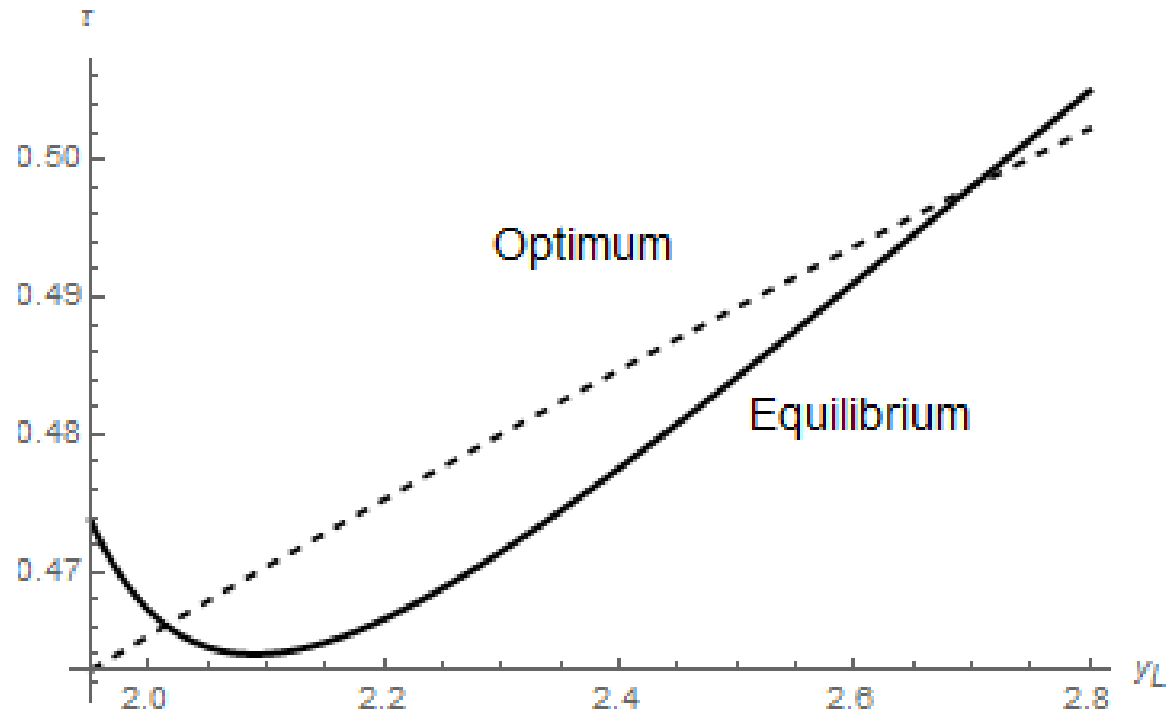


Figure 3

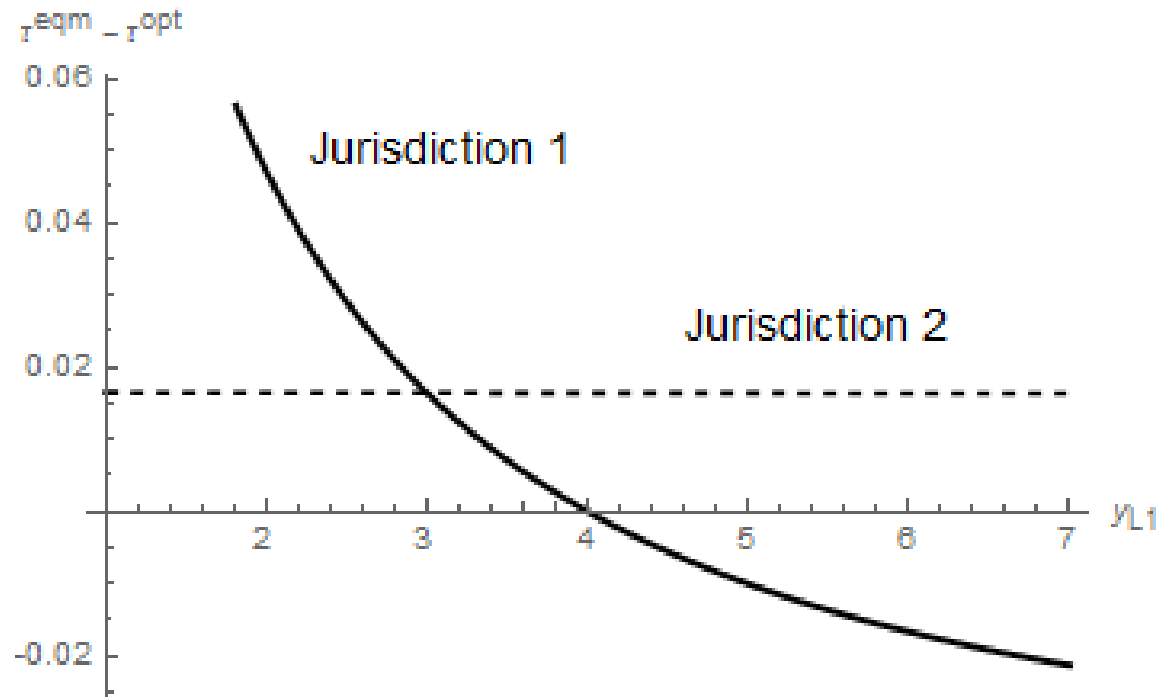


Figure 4

