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Assimilation Patterns in Cities

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Abstract

We develop a model in which ethnic minorities can either assimilate to the majority's norm or reject it by trading off higher productivity and wages with a greater social distance to their culture of origin. We show that "oppositional" minorities reside in more segregated areas, have worse outcomes (in terms of income) but are not necessary worse off in terms of welfare than assimilated minorities who live in less segregated areas. We find that a policy that reduces transportation cost decreases rather than increases assimilation in cities. We also find that when there are more productivity spillovers between the two groups, ethnic minorities are more likely not to assimilate and to reject the majority's norm. Finally, we show that ethnic minorities tend to assimilate more in bigger and more expensive cities.

Keywords: Identity, agglomeration, cities, ethnic minorities, welfare.

JEL Classification: J15, R14, Z13.

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1 Introduction

In 2017, in Michigan, an Indian-American emergency-room doctor who belongs to the Dawoodi Bohra community, a Shiite Muslim sect, was charged with performing female genital mutilation on several young girls. In Minnesota, a black police officer, the first Somali-American cop in his precinct, shot an unarmed Australian woman. Both incidents were immediately seized upon by the far right as examples of the inability – or refusal – of Muslims to assimilate. Assimilation of immigrants is indeed a hot debate in the United States but also in Europe. For some, assimilation is based on pragmatic considerations, like achieving some fluency in the dominant language or some educational or economic success. For others, it involves relinquishing all ties to the old country. For yet others, the whole idea of assimilation is wrongheaded, and integration is seen as the better model.

Since, both in the United States and in Europe, ethnic minorities live disproportionately in cities, assimilation or the lack of it can only be understood within a spatial framework. What are the costs and benefits of assimilation? Does residential location affect the assimilation process of ethnic minorities? Do people who assimilate to the majority's norm tend to reside in the same areas as the majority group? Is segregation good or bad? Are ethnic minorities better off by assimilating to the majority's values?

In this paper, we investigate these issues by studying how ethnic minorities assimilate or reject the majority's norm and how this impacts on their residential location, housing prices and the size of the city. Surprisingly, at least from a theoretical viewpoint, there is very little research on the relationship between the urban space and the assimilation choices of ethnic minorities.

We develop a model in which ethnic minorities can either assimilate to the majority's norm or reject it. If they assimilate, their productivity and thus their wage will be "pooled" with that of the majority group and they will therefore obtain a higher income than "oppositional" minorities who reject the majority's norm. This, in particular, implies that their economic status (their relative income with respect to that of the society) will be higher. There is also a social cost of assimilation since they need to distance themselves from their culture of origin. However, the higher is the fraction of minorities who assimilate, the lower is the cost of the perceived distance between assimilation and values from the minority group. This means that there are complementarities in assimilation choices since the higher is the fraction of assimilated minorities, the greater are the benefits from assimilation. We assume that all individuals are ex ante heterogenous in terms of the weight α they put on the importance of income in their utility function. This implies that individuals will very high α will tend to assimilate to the majority's norm while, those with very low α will tend to reject the majority's norm.

We show that three types of equilibria may emerge: An Assimilation Social Identity Equilibrium (ASIE), in which all minority individuals choose to totally assimilate to the majority group, an Oppositional Social Identity Equilibrium (OSIE), in which all minority individuals totally reject the social norm of the majority group, and a Mixed Social Identity Equilibrium (MSIE), in which a fraction of minority individuals assimilate while the other fraction choose to be "oppositional". We give the exact conditions under which each equilibrium is unique and stable but also when there are multiple equilibria. We show, in particular, that the fraction of ethnic minorities in the population has to be large enough while the productivity spillover effect has to be low enough for a MSIE to emerge.

In the second part of the paper, we explicitly model the city and, therefore, the location choices of all individuals. The city is assumed to be monocentric and landlords are absentee. Each individual consume a non-spatial good and decide the size of their housing, the price to pay for it and her location in the city. To keep the model tractable, we now assume that all individuals are ex ante identical in terms of α, the weight they put on the importance of income in their utility function. As a result, a MSIE cannot emerge anymore because it is not stable. We establish under which conditions either an Assimilation Social Identity Urban Equilibrium (ASIUE) or an Oppositional Social Identity Urban Equilibrium (OSIUE) emerges or when both can coexist (multiple equilibria). We show that, in the former equilibrium, all ethnic minorities assimilate and live together with the majority group while, in the latter, all ethnic minorities reject the majority's norm and segregate themselves at the vicinity of the CBD while the majority individuals reside at the periphery of the city. We find that "oppositional" minorities reside in more segregated areas, have worse outcomes (in terms of income) but are not necessary worse off (in terms of welfare) than assimilated minorities who live in less segregated areas.

Moreover, we find that a policy that reduces commuting costs or increases the supply of land makes the Oppositional Social Identity Urban Equilibrium (OSIUE) more likely to emerge. In other words, these two policies decrease assimilation in cities. We also find that more productivity spillovers between the majority and minority groups makes ethnic minorities more likely to reject the majority's norm while an increase in the fraction of minorities in the population makes multiple more likely to emerge. Finally, we show that ethnic minorities tend to assimilate more in bigger and more expensive cities.

There is a growing literature trying to understand the process of assimilation of ethnic minorities. Different studies have shown distinct significant influences on the assimilation process for immigrants: the quality of immigrant cohorts (Borjas, 1985), country of origin (e.g. Beenstock et al., 2010; Borjas, 1987, 1992; Chiswick and Miller, 2011), ethnic concentration (e.g. Edin et al., 2003; Lazear, 1999) and personal English skill (e.g. Chiswick and Miller, 1995, 1996; Dustmann and Fabbri, 2003; McManus et al., 1983).

There is also an important literature that studies the concept of oppositional cultures among ethnic minorities. In this literature, as in our model, ethnic groups may "choose" to adopt what are termed "oppositional" identities, that is, some actively reject the dominant ethnic (e.g., white) behavioral norms (they are oppositional) while others totally assimilate to it (see, in particular, Ainsworth-Darnell and Downey, 1998).[1] From a theoretical perspective, researchers have put forward the role of cultural identity (Akerlof and Kranton, 2010) in the assimilation patterns of ethnic minorities (see, e.g. Bisin et al., 2011a,b, 2016; Panebianco, 2014; Verdier and Zenou, 2017, 2018) and show how oppositional identities can emerge as an equilibrium outcome (Akerlof, 1997; Austen-Smith and Fryer, 2005; Selod and Zenou, 2006; Battu et al., 2007; Bisin et al., 2011a; De Marti and Zenou, 2017, Eguia, 2017).[2] Finally, some recent papers have highlighted the role of cultural leaders and/or social networks as an important aspect of the identity choices and integration of ethnic minorities in Europe and the United States (Hauk and Mueller, 2015; Carvalho and Koyama, 2016; Prummer and Siedlarek, 2017; Verdier and Zenou, 2017, 2018).

[1] Studies in the United States (but also in Europe for ethnic minorities) have found, for example, that African American students in poor areas may be ambivalent about learning standard English and performing well at school because this may be regarded as "acting white" and adopting mainstream identities (Fordham and Ogbu, 1986; Wilson, 1987; Delpit, 1995; Ogbu, 1997; Battu and Zenou, 2010; Fryer and Torelli, 2010; Bisin et al., 2011b; Patacchini and Zenou, 2016).

[2] In a series of papers, Zimmermann et al. (2007), Constant and Zimmermann (2008), Constant et al. (2009) have proposed a new measure of the ethnic identity of migrants by modeling its determinants and explores its explanatory power for various types of their economic performance. They have proposed the ethnosizer, a measure of the intensity of a person's ethnic identity, which is constructed from information on language, culture, societal interaction, history of migration, and ethnic self-identification.

Compared to this literature, our contribution is to put forward the role of the urban structure on the assimilation choices of ethnic minorities. In particular, we are able to show why segregation is detrimental in terms of economic outcomes for minorities, how bigger and more expensive cities affect the assimilation choices of minorities and how a transportation policy pushes them not to assimilate.

The rest of the paper unfolds as follows. In the next section, we develop the baseline model and provide the conditions under which each equilibrium emerges. In Section 3, we introduce the city structure and show how space affects the assimilation choices of ethnic minorities. Finally, Section 4 concludes. All proofs can be found in Appendix A while, in Appendix B, we determine the equilibrium values of the endogenous variables in any urban equilibrium.

2 Baseline model

2.1 Social groups

Consider a city with a continuum of individuals of size 1.[3] Among them a percentage μ are members of group m and a percentage $1-\mu$ are members of group c. We assume that $\mu < 1/2$, implying that the group m is the *minority group* and the group c is the *majority group*. If we think of ethnicity, then the group m is the ethnic minority group while group c corresponds to the native group.

Thus, there are two social groups, m and c, which are "categories" that individuals learn to recognize while growing up. Each individual is inherently a member of group m or c. These groups are given and we focus on the assimilation decision (identification process) of the ethnic minority group m, i.e., whether or not they want to assimilate to the majority group c. Quite naturally, we assume that the majority group c is sufficiently large so that they always identify with their own group and we do not deal with their identification decision. In contrast, each minority individual can either choose to identify with her own group m (i.e., rejection of the majority's norm) or to the majority group c (i.e., assimilation). In equilibrium, two different groups of ethnic minorities will emerge: those who choose to *assimilate* to the majority group's identity, referred to as *assimilated*

[3]We explicitly model the city and the location choices of all agents in Section 3.

ethnic minorities, and those who choose to *reject* the majority group's identity, referred to as *oppositional ethnic minorities.*

2.2 Production and wages

In the city, the numéraire good is produced by only using labor. The production of this good exhibits constant returns to scale at the firm level but involves agglomeration economies at the city level. Agglomeration economies are positive external effects of population concentration that arise from various factors such as spillovers among people and firms, labor pooling, and love of variety in consumption and production (see Duranton and Puga, 2004, for an overview). Much of them require intensive communication among individuals in the city. When urbanites identify themselves with different social groups, then agglomeration economies are relatively weak since individuals from a certain group do not fully socialize with individuals belonging to other social groups. Hence, the effects of agglomeration economies would be weaker with the lack of interaction of people from different social groups.

To capture this idea, we assume that the productivity of a minority individual identifying herself with groups m (oppositional) and c (assimilated) is respectively given by:

$$y_m(\lambda) = f\left(\left(1-\lambda\right)\mu + \varepsilon\left(\lambda\mu + 1 - \mu\right)\right), \tag{1}$$
$$y_c(\lambda) = f\left(\lambda\mu + 1 - \mu + \varepsilon\left(1-\lambda\right)\mu\right),$$

where y_J represents the output of a minority individual when she identifies with group J $(J = m, c)$, λ is the (endogenous) share of minority individuals identifying with the majority group, i.e., the ones who choose to assimilate to the majority's norm (group c) and $\varepsilon \in [0,1]$ is a constant. Because the majority individuals always identify themselves with the majority group, the total mass of people identifying with the majority group is given by $\lambda\mu + 1 - \mu$, while the total mass of people identifying with the minority group m (thus rejecting the majority's norm) is given by $(1-\lambda)\mu$. We assume that $f(\cdot)$ is twice continuously differentiable, $f'(\cdot) > 0$, and $f''(\cdot) < 0$.

The general idea behind (1) is that it is easier for assimilated ethnic minorities to interact and communicate with individuals from the majority group than oppositional ethnic minorities and this is reflected in terms of their productivity and wages. Indeed, by interacting less with the majority group, oppositional minorities may have difficulties in inter-ethnic relationships due to language barriers (see e.g. Lazear, 1999; De Marti and Zenou, 2017) or more generally to different social norms and cultures.[4]

In particular, in (1), ε can be interpreted as the degree of agglomeration economies due to the interaction of individuals from different social groups or the *inter-group productivity spillover effects.* If $\varepsilon = 0$, agglomeration economies hardly spread to different social groups and the production of each individual is only affected by the population she identifies with. In that case, $y_m(\lambda) = f((1-\lambda)\mu)$ and $y_c(\lambda) = f(\lambda\mu + 1 - \mu)$ and, because $\mu < 1/2$, $y_m(\lambda) < y_c(\lambda)$, $\forall \lambda \in [0, 1]$. If $\varepsilon = 1$, agglomeration economies are equally effective across different social groups and $y_m(\lambda) = y_c(\lambda)$. When $0 < \varepsilon < 1$, independently of the value of ε, $y_m(\lambda) < y_c(\lambda)$, $\forall \lambda \in [0, 1]$. Moreover, the higher is ε, the lower is the wage difference $y_c(\lambda) - y_m(\lambda)$.[5] We have the following result:

Lemma 1

(*i*) *The income of the majority group is always higher than that of the minority group, i.e.* $y_c(\lambda) \geq y_m(\lambda) \forall \lambda \in [0, 1]$ *and* $\forall \varepsilon \in [0, 1]$ *Suppose* $\varepsilon \neq 1$ *Then, when* λ*, the fraction of ethnic minorities identifying with the majority group, increases,* $y_c(\lambda)$ *increases and is concave in* λ *while* $y_m(\lambda)$ *decreases and is concave in* λ*. Moreover, the income ratio* $y_c(\lambda)/y_m(\lambda)$*increases with* λ*.*

(*ii*) *When* ε*, the degree of agglomeration effect, increases, both* $y_c(\lambda)$ *and* $y_m(\lambda)$ *increase and are concave in* ε*. Moreover, the income ratio* $y_c(\lambda)/y_m(\lambda)$ *decreases with* ε*.*

This lemma is important because it provides us with some important properties of the incomes, which will be useful for the equilibrium characterization. First, in (*i*), we look at the effect of an increase of λ (an endogenous variable) on the incomes of both groups. Figure 1(a) depicts the shape of these three curves. When more ethnic minorities choose to assimilate, the income of group c (which includes both the majority group and assimilated

[4]For example, Meng (2005) shows that intermarried (i.e., more assimilated) immigrants earn significantly higher incomes than endogamously married immigrants, even after human capital endowments and endogeneity of intermarriage are taken into account. Similarly, Biavaschi et al. (2017) find that migrants who Americanized their names, which embodies an intention to assimilate among low-skilled migrants, experienced larger occupational upgrading than those who did not.

[5]Indeed, since $f'(\cdot) > 0$, $f''(\cdot) < 0$ and $\mu < 1/2$, it is easily verified that $\partial [y_c(\lambda) - y_m(\lambda)] / \partial\varepsilon < 0$.

minorities) increases while the income of the oppositional ethnic minorities decreases. This implies that the income ratio between these two groups increases with λ. This captures the fact that there are positive (negative) externalities in production so that the higher is the fraction of assimilated minorities, the higher (lower) is the productivity of an assimilated (oppositional) minority. In other words, there are increasing (decreasing) returns to scale in production of the assimilation (oppositional) process of ethnic minorities.

Second, in *(ii)*, we analyze the effect of ε on incomes. Figure 1(b) depicts the shape of these three curves. When ε increases, there are more productive interactions between group c (majority and assimilated minorities) and group m (oppositional minorities). However, since μ, the fraction of ethnic minorities in the population, is less than 1/2, $y_m(\lambda) < y_c(\lambda)$ because the agglomeration effects are always stronger for the majority group. Interestingly, Lemma 1 shows that both groups benefit from an increase in ε because there are more interactions between the two groups and, therefore, their productivity increases. In other words, more interaction is always better and translates here by an increase in income of both groups. Finally, an increase in ε reduces y_c/y_m because the productivity spillover effects benefit more the "oppositional" group m than the assimilated group c. In the following, we focus on the case when $0 < \varepsilon < 1$.

[*Insert Figures* 1(*a*) *and* 1(*b*) *here*]

Each individual is assumed to be endowed with one unit of labor, which she supplies inelastically. Hence, when identifying with group J, an individual receives a wage income of y_J, which constitutes the first part of her utility function. The other parts, which we describe now, are defined in terms of social identity.

2.3 Social identity

Following Shayo (2009) and Sambanis and Shayo (2013), we assume that three main factors affect the social identity and thus the socialization process in terms of assimilation and rejection of each ethnic minority. First, each individual is aware of the different social groups or categories (i.e., groups m and c) that exist in the society. Second, each individual i has an attribute or a quality q_i and she wants to minimize the *perceived distance* between q_i and that of each social group. Third, each individual cares about the *relative status* of each social group so that higher status implies higher utility.

2.3.1 Perceived distance

The concept of perceived distance and its adoption to the process of identification originated in the literature of categorization in cognitive psychology (Nosofsky, 1986; Turner et al., 1987). It has also been modeled by economists where the perceived distance is between the action of each agent and that of her social norm and usually negatively affects her utility (Akerlof, 1997; Shayo, 2009; Patacchini and Zenou, 2012; Sambanis and Shayo, 2013; Liu et al., 2014; Boucher, 2016; Ushchev and Zenou, 2018).

Each individual i is born with an attribute or a quality q_i, which depends on the group i she is associated to ($i = m, c$). Ethnic minorities are born with q_m and the individuals from the majority group are born with q_c. Since we focus on the choice of the minority group, we write all the attributes as a single binary variable: $q_m = 1$ and $q_c = 0$. The *social norm* of each group $J = m, c$ is determined by the "typical" attribute of the group J, which is given by $\overline{q}_J$, the average attribute of the group. Since q_i is a binary variable, $\overline{q}_m$ is equal to 1 while $\overline{q}_c$ is determined by the share of minority individuals who choose to identify themselves with group c, that is:

$$\overline{q}_J = \begin{cases} 1 & \text{if } J = m \\ \frac{\lambda\mu}{\lambda\mu+1-\mu} & \text{if } J = c \end{cases} .$$

where $\lambda\mu/(\lambda\mu + 1 - \mu)$ is the fraction of ethnic minorities among all individuals choosing to identify themselves with group c, i.e., those who assimilate to the majority group's identity. The perceived distance between each minority individual's attribute and the social norm of group J is then given by: $D_J(\lambda) = d\left(|q_m - \overline{q}_J|\right)$, where $d(\cdot)$ is an increasing function of $|q_m - \overline{q}_J|$. We also assume that: $d(0) = 1$, $d(1) = \overline{d} > 1$, and $d'(1) = 0$, which, in particular, implies that there is a maximum perceived distance at $\overline{d}$. Figure 2 depicts such a function for $J = c$:[6]

[*Insert Figure* 2 *here*]

[6] In Figure 2, the perceived distance function is equal to: $d(x) = 3 - 2(x - 1)^2$, which satisfies all our assumptions, i.e., $d(0) = 1$, $d(1) = \overline{d} = 3$, and $d'(1) = 0$.

Hence, $D_J(\lambda)$ can be written as

$$D_J(\lambda) = \begin{cases} d(0) = 1 & \text{if } J = m \\ d\left(\frac{1-\mu}{\lambda\mu+1-\mu}\right) & \text{if } J = c \end{cases}. \tag{2}$$

This formulation thus assumes that, if an ethnic minority chooses to reject the majority's norm and thus lives in accordance to her own culture, her perceived distance is the lowest and equal to $d(0) = 1$. On the contrary, if an ethnic minority chooses to assimilate to the majority's norm, there is a perceived distance between her norm and that of her group, which is always greater than $d(0) = 1$ and which is increasing with $(1-\mu)/(\lambda\mu+1-\mu)$, the fraction of individuals from the majority group among all individuals adopting the social norm of the majority group. In particular, the higher is μ, the fraction of ethnic minorities in the population, the higher is $D_c(\lambda)$, the perceived distance for assimilated ethnic minorities.

2.3.2 Group status

The last part of the utility function includes a component related to the *status* of the identified group as well as the perceptions of similarity to other group members. The status of the group is determined through comparisons to other groups (Tajfel and Turner, 1986). In our framework, the utility obtained from the group status is determined by the difference between $\overline{y}_J(\lambda) := y_J(\lambda)$, the average income of group J and $\overline{y}(\lambda) := (1-\lambda)\mu y_m(\lambda) + (\lambda\mu + 1 - \mu)\, y_c(\lambda)$, the average income of the population. Thus, an individual obtains a higher utility as her group members obtain higher incomes compared to the population (city) average level.

2.4 Utility function

Let us put the three parts of the utility function together. The utility function of an individual belonging to group m and identifying herself with group J is then equal to:

$$U_J(\lambda) = \alpha \ln \underbrace{y_J(\lambda)}_{\text{individual income}} - \delta \ln \underbrace{D_J(\lambda)}_{\text{perceived distance}} + \sigma \ln \underbrace{\frac{\overline{y}_J(\lambda)}{\overline{y}(\lambda)}}_{\text{relative status of group } J} \tag{3}$$

The first term of (3) represents the utility from own income, the second term captures the disutility from deviating from the social norm of the group (the perceived distance between each individual and the identified group) and the last term is the payoff from the relative status of the identified group. Moreover, α represents the weight put by each individual on her own income. We assume that α differs among minority individuals and is distributed over $[\underline{\alpha}, \overline{\alpha}]$; its cumulative distribution function (cdf) by $G(\alpha)$ and its density function is given by $g(\alpha)$. Thus, when an ethnic minority m considers to assimilate to the majority's norm, she will trade off a higher income, a higher perceived distance, which negatively affects her utility, and a higher status since $\overline{y}_c(\lambda) > \overline{y}_m(\lambda)$. This choice will also be affected by her α, i.e., the weight she put on her income in her utility function. Clearly, ethnic minorities with low (high) α will be less (more) likely to assimilate.

2.5 Equilibrium

Let us determine the equilibrium, which is referred here to as a *Social Identity Equilibrium*. Different equilibria can emerge.

Definition 1

(*i*) *An Assimilation Social Identity Equilibrium (ASIE) is when all minority individuals choose to totally assimilate to the majority group, i.e., all choose the identity of group c and $\lambda = 1$.*

(*ii*) *An Oppositional Social Identity Equilibrium (OSIE) is when all minority individuals totally reject the social norm of the majority group, i.e., all choose the identity of group m and $\lambda = 0$.*

(*iii*) *A Mixed Social Identity Equilibrium (MSIE) is when a fraction of minority individuals choose to identify themselves to group m while the other fraction choose to identify themselves to group c, i.e., $0 < \lambda < 1$.*

We are looking here at a (pure-strategy) Nash equilibrium where the strategy of each player is her identity choice. Hence, it is sufficient to check whether each individual decision is consistent with the social environment. In other words, an ethnic minority identifies herself with group c, i.e., assimilates to the majority's norm, if and only if

$U_c(\lambda) > U_m(\lambda)$ and with group m, i.e., rejects the majority's norm, if and only if $U_m(\lambda) \geq U_c(\lambda)$. From (3), the condition $U_c(\lambda) > U_m(\lambda)$ can be written as:[7]

$$(\alpha + \sigma) \ln \frac{y_c(\lambda)}{y_m(\lambda)} > \delta \ln \frac{D_c(\lambda)}{D_m(\lambda)}. \tag{4}$$

We see here clearly the trade off she faces: by assimilating, she improves her relative income ($y_c(\lambda)/y_m(\lambda) > 1$) but also increases her relative (cultural) distance ($D_c(\lambda)/D_m(\lambda) > 1$). Because the left-hand side (LHS) of (4) is increasing in α while its right-hand side (RHS) is independent of α, any minority individual with a larger α is more likely to assimilate to the majority group than the one with a smaller α.[8] Define $\Gamma(\lambda; \alpha)$ as

$$\Gamma(\lambda; \alpha) \equiv (\alpha + \sigma) \ln \frac{y_c(\lambda)}{y_m(\lambda)} - \delta \ln \frac{D_c(\lambda)}{D_m(\lambda)}. \tag{5}$$

Proposition 1 *For a given λ, a minority individual will choose to identify herself with the group c and thus assimilates to the majority group if and only if $\Gamma(\lambda; \alpha) > 0$, and will identify herself with her own group, i.e., reject the majority's social identity, if and only if $\Gamma(\lambda; \alpha) \leq 0$*

In order to characterize the different possible equilibria, we need to determine the unique endogenous variable of this model, that is λ. For that, we differentiate $\Gamma(\lambda; \alpha)$ with respect to λ.

Lemma 2 *The higher is λ, the fraction of ethnic minorities choosing to assimilate to the majority's norm, the higher is $\Gamma(\lambda; \alpha)$, i.e. $\partial\Gamma(\lambda; \alpha)/\partial\lambda > 0$. Moreover, $\lim_{\mu \to 0} \partial\Gamma(\lambda; \alpha)/\partial\lambda = 0$.*

The first result implies that the higher is λ, the fraction of ethnic minorities who choose to assimilate to the majority's norm, the more likely ethnic minorities will assimilate to the majority's norm. In other words, there are *complementarities* in assimilation choices since someone is more likely to assimilate the higher is the fraction of individuals in the population that assimilate. This is because, when λ, the fraction of ethnic minorities who assimilate, increases, the relative income of assimilation, $y_c(\lambda)/y_m(\lambda)$, increases

[7]Observe that, in (4), $\overline{y}(\lambda)$, the average income of the population, disappears because it appears of both sides on the inequality.

[8]All our results would be qualitatively the same if, instead of α, the heterogeneity of the ethnic minorities would have been in terms of δ or σ.

and $D_c(\lambda) = d\left(|q_m - \overline{q}_c|\right)$, the perceived distance between minority and majority groups decreases. The second result, $\lim_{\mu \to 0} \partial\Gamma(\lambda;\alpha)/\partial\lambda = 0$, means that λ has no impact on the decision to assimilate when the fraction of ethnic minorities becomes zero. This result hinges on the assumption that $d'(1) = 0$, i.e., the perceived distance reaches its lowest value when the perceived distance of assimilating is maximal.

This result is related to the cultural transmission literature (Bisin and Verdier, 2000, 2001), which shows that the higher is the fraction of children adopting a certain trait, the higher is the effort of their parents in transmitting this trait. Here, we have something that has the same flavor since the higher is λ, the fraction of individuals adopting the $c-$trait, the higher is the fraction of individuals adopting the $c-$trait. In the cultural transmission literature, this is referred to as *cultural complementarity*. Empirically, Bisin et al. (2016) have confirmed this positive relationship between λ and $\Gamma(\lambda;\alpha)$.

Because we readily know that $\partial\Gamma(\lambda;\alpha)/\partial\alpha > 0$, Proposition 1 implies that for a given λ, there exists a threshold value of α, denoted by $\widetilde{\alpha}$, such that a minority individual with α larger (smaller) than $\widetilde{\alpha}$ chooses to assimilate (not to assimilate) to the majority's norm. Figure 3 represents the assimilation decision.

[*Insert Figure* 3 *here*]

Hence, the share of minority individuals who assimilate to the majority's norm is given by $1 - G(\widetilde{\alpha})$, which, in turn, determines λ. From (5), we have:

$$\begin{aligned} \Gamma(0;\alpha) &= (\alpha + \sigma)\ln\frac{f\left(1 - \mu + \varepsilon\mu\right)}{f\left(\mu + \varepsilon\left(1 - \mu\right)\right)} - \delta\ln\overline{d}, \\ \Gamma(1;\alpha) &= (\alpha + \sigma)\ln\frac{f(1)}{f(\varepsilon)} - \delta\ln d\left(1 - \mu\right). \end{aligned}$$

We can therefore summarize the equilibrium conditions as follows:

Definition 2

(i) An Assimilation Social Identity Equilibrium (ASIE) is a 3-tuple $(\alpha^*, \lambda^*, \Gamma)$ *that satisfies* $\alpha^* = \underline{\alpha}$, $\lambda^* = 1$, *and* $\Gamma(1;\underline{\alpha}) > 0$.

(ii) An Oppositional Social Identity Equilibrium (OSIE) is a 3-tuple $(\alpha^*, \lambda^*, \Gamma)$ *that satisfies* $\alpha^* = \overline{\alpha}$, $\lambda^* = 0$, *and* $\Gamma(0;\overline{\alpha}) < 0$.

(*iii*) *A Mixed Social Identity Equilibrium (MSIE) is a 3-tuple* $(\alpha^*, \lambda^*, \Gamma)$ *that satisfies* $\lambda^* = 1 - G(\alpha^*)$ *and* $\Gamma(\lambda^*; \alpha^*) = 0$.

Moreover, we impose a stability condition in the sense that a small perturbation yields incentives that restore the economy to the original equilibrium. From the above definitions, the ASIE and OSIE are stable. However, in order for a MSIE to be stable, we need another condition. Because we readily know that both $\Gamma(\lambda; \widetilde{\alpha}) = 0$ and $\lambda = 1 - G(\widetilde{\alpha})$ are downward sloping in the $\lambda - \widetilde{\alpha}$ plane and $\Gamma(\lambda; \widetilde{\alpha}) = 0$ determines $\widetilde{\alpha}$ for a given λ whereas $\lambda = 1 - G(\widetilde{\alpha})$ determines λ once $\widetilde{\alpha}$ is given, MSIE is stable if and only if $\lambda = 1 - G(\widetilde{\alpha})$ is steeper than $\Gamma(\lambda; \widetilde{\alpha}) = 0$ at the intersection of these two curves. Moreover, such an intersection is unique if $\lambda = 1 - G(\widetilde{\alpha})$ is globally steeper than $\Gamma(\lambda; \widetilde{\alpha}) = 0$, that is

$$-\frac{1}{g(\widetilde{\alpha})} < -\frac{\partial\Gamma(\lambda; \widetilde{\alpha})/\partial\lambda}{\partial\Gamma(\lambda; \widetilde{\alpha})/\partial\widetilde{\alpha}}, \qquad \forall(\lambda, \widetilde{\alpha}) \in [0, 1] \times [\underline{\alpha}, \overline{\alpha}], \tag{6}$$

where the left-hand side is the slope of $\lambda = 1 - G(\widetilde{\alpha})$ while the right-hand side is the slope of $\Gamma(\lambda; \widetilde{\alpha}) = 0$ in the $\lambda - \widetilde{\alpha}$ plane. Note here that if the two curves have multiple intersections, there exist multiple (stable) equilibria.

Given this result, in Figure 4, we are now able to describe all the possible equilibria. Figure 4(a) displays the case where condition (6) holds true so that there always exists a unique stable equilibrium. The two solid downward sloping curves are $\Gamma(\lambda; \widetilde{\alpha}) = 0$ and $\lambda = 1 - G(\widetilde{\alpha})$ when $\Gamma(1; \underline{\alpha}) \leq 0 \leq \Gamma(0; \overline{\alpha})$, where the steeper one represents $\lambda = 1 - G(\widetilde{\alpha})$. We have positive $\Gamma(\lambda; \widetilde{\alpha})$ in a region above $\Gamma(\lambda; \widetilde{\alpha}) = 0$ and negative $\Gamma(\lambda; \widetilde{\alpha})$ in a region below $\Gamma(\lambda; \widetilde{\alpha}) = 0$. The intersection of the two curves, (λ^*, α^*) is a MSIE.

Now suppose the economy is hit by a shock and λ changes from λ^* to λ' (or λ''). Then, this leads to the fact that $\widetilde{\alpha}$ is determined by $\Gamma(\lambda; \widetilde{\alpha}) = 0$, which, in turn, pins down λ via $\lambda = 1 - G(\widetilde{\alpha})$. Such movements are represented by arrows in Figure 4(a). We can confirm that a perturbation induces changes that restore the original equilibrium. In Figure 4(a), we also describe the two other equilibria, ASIE and OSIE. The upper dashed curve represents the case of $\Gamma(0; \overline{\alpha}) < 0$, which results in an OSIE whereas the lower dashed line describes the case of $\Gamma(1; \underline{\alpha}) > 0$, which yields a ASIE.

Figure 4(b) depicts the case where condition (6) does not hold. Again, the two solid downward sloping curves are $\Gamma(\lambda; \widetilde{\alpha}) = 0$ and $\lambda = 1 - G(\widetilde{\alpha})$ when $\Gamma(0; \underline{\alpha}) \leq 0 \leq \Gamma(1; \overline{\alpha})$. The upper dashed-line curve represents the case when $\Gamma(1; \overline{\alpha}) < 0$, which results in an

OSIE, whereas the lower dashed-line curve depicts the case when $\Gamma(0;\underline{\alpha}) > 0$, which yields a ASIE. If $\Gamma(0;\underline{\alpha}) \leq 0 \leq \Gamma(1;\overline{\alpha})$, there might exist multiple (stable) equilibria. In the figure, (λ^*, α^*) and ASIE are (stable) equilibria.

[*Insert Figure* 4 *here*]

The following proposition summarizes these findings.

Proposition 2

(*i*) *Suppose (6) holds true.*

(*ia*) *If* $\Gamma(1;\underline{\alpha}) > 0$, *there exists a unique stable Assimilation Social Identity Equilibrium (ASIE) where all minority individuals totally assimilate to the majority group.*

(*ib*) *If* $\Gamma(0;\overline{\alpha}) < 0$, *there exists a unique stable Oppositional Social Identity Equilibrium (OSIE) where all minority individuals identify themselves with their own group and reject the majority's norm.*

(*ic*) *If* $\Gamma(1;\underline{\alpha}) \leq 0 \leq \Gamma(0;\overline{\alpha})$, *there exists a unique stable Mixed Social Identity Equilibrium (MSIE) in which the ethnic minorities with* $\alpha > \widetilde{\alpha}$ *assimilate to the majority group's norm whereas the ethnic minorities with* $\alpha < \widetilde{\alpha}$ *adopt the identity norm of their own group.*

(*ii*) *Suppose (6) does not hold true.*

(*iia*) *If* $\Gamma(0;\underline{\alpha}) > 0$, *there exists a unique stable Assimilation Social Identity Equilibrium (ASIE) where all minority individuals totally assimilate to the majority group.*

(*iib*) *If* $\Gamma(1;\overline{\alpha}) < 0$, *there exists a unique stable Oppositional Social Identity Equilibrium (OSIE) where all minority individuals identify themselves with their own group and reject the majority's norm.*

(*iic*) *If* $\Gamma(0;\underline{\alpha}) \leq 0 \leq \Gamma(1;\overline{\alpha})$, *there exist multiple equilibria.*

This proposition provides the conditions under which each possible equilibrium can arise. In particular, we show under which conditions *oppositional cultures* among ethnic minorities can emerge, i.e., they may "choose" to adopt "oppositional" identities, that is, some actively reject the dominant majority behavioral norms while others totally assimilate to it. The novel aspect of this proposition is that these conditions crucially depend on five key parameters: α, σ, δ, μ and ε. In the next proposition, we focus on the impact of μ and ε on the emergence of each of these equilibria.

Proposition 3

(*i*) *When μ, the fraction of ethnic minorities in the population, is sufficiently small, only an Oppositional Social Identity Equilibrium (OSIE) or an Assimilation Social Identity Equilibrium can (ASIE) emerge. As μ becomes larger, a Mixed Social Identity Equilibrium (MSIE) can also arise.*

(*ii*) *When ε, the productivity spillover effect, is sufficiently small, all types of equilibrium can emerge. As ε increases, a unique Oppositional Social Identity Equilibrium (OSIE) is more likely to exist.*

The first result shows the importance of the size of the minority group (μ) on the assimilation process of ethnic minorities. When μ is very small, then either all minorities assimilate or they reject the majority's norm. However, as μ increases, more individuals assimilate (higher λ) and because there are positive spillovers between λ and the productivity of group c, a mixed equilibrium is more likely to emerge.

To understand the second result about ε, remember that when deciding their identity choice, ethnic minorities trade off the income gain of assimilation against its cultural cost in terms of perceived distance. When ε is very small, there is no much interaction between the two groups, but there is an important income gain of assimilation since $y_c(\varepsilon = 0) > y_m(\varepsilon = 0)$ but still a cost in terms of cultural distance. As a result, the ethnic minorities assimilate or reject the majority's norm depending on the income gains from assimilation and their α. When ε increases, this is not anymore true since the income ratio y_c/y_m decreases (see Lemma 1 and Figure 1(b)) but the perceived distance remains constant as it is not affected by ε. As a result, the ethnic minorities are more likely not to assimilate and thus to reject the majority's norm. At the limit, when $\varepsilon \to 1$, the income between the two groups becomes the same, i.e., $y_m = y_c$, and, thus, there is no benefit from assimilating to the majority's norm and, as a result, all ethnic minorities become "oppositional".

3 City structure and identity choices

3.1 The city

So far, we referred to the "city" in an abstract way. In this section, we formally model the city and its structure and analyze its impact on the assimilation process of ethnic minorities. Consider a linear monocentric city where all jobs are located in the unique Central Business District (CBD) and where housing areas are spread over the right-hand side of the CBD (located at zero).[9] We assume that each location is endowed with H units of land and that landlords are absentee.

All residents in the city must commute to the CBD in order to work and obtain a wage income, $y_{iJ}(\lambda)$, where i denotes the group the individual i belongs to ($i = m, c$) and J represents her identity choice ($J = m, c$). Because we only consider the assimilation choice of the minority group m, there are two incomes $y_{mm}(\lambda)$ and $y_{mc}(\lambda)$ for the minority group m and only one income $y_{cc}(\lambda)$ for the majority group c. From (1), we know that $y_{mc}(\lambda) = y_{cc}(\lambda) = \overline{y}_c(\lambda) = y_c(\lambda)$ and $y_{mm}(\lambda) = \overline{y}_m(\lambda) = y_m(\lambda)$. Remember that

$$y_c(\lambda) \geq y_m(\lambda), \, \forall \lambda \in [0, 1] \text{ and } \forall \varepsilon \in [0, 1] \, , \tag{7}$$

where the equality holds true if and only if $\varepsilon = 1$. In the following, we again focus on the case when $0 < \varepsilon < 1$.

3.2 Utility

In order to keep the analysis tractable, we assume that all individuals are ex ante identical in terms of α, which value is normalized to one. Now, if condition (4) holds for one individual, i.e., $\Gamma(\lambda) > 0$, then it holds for all individuals in the city, which implies that all minority individuals will choose to assimilate, i.e., $\lambda = 1$. Thus, if (4) holds

[9] For the detailed literature on the monocentric city models, see Fujita (1989) and Zenou (2009) among others.

true at $\lambda = 1$, i.e., $\Gamma(1) > 0$, there always exists an Assimilation Social Identity Urban Equilibrium (ASIUE) where all minority individuals totally assimilate to the majority group in the city.[10] Similarly, if the opposite is true at $\lambda = 0$, i.e., $\Gamma(0) < 0$, there always exists an Oppositional Social Identity Urban Equilibrium (OSIUE) where all minority individuals identify themselves with the majority group. Finally, a Mixed Social Identity Urban Equilibrium (MSIUE) will exist when $\Gamma(1) > 0$ and $\Gamma(0) < 0$ but it is unstable.[11] So we will not study it. Instead, when $\Gamma(1) > 0$ and $\Gamma(0) < 0$, we obtain multiple equilibria, i.e., for the same set of parameters, there exist an ASIUE and an OSIUE and both of them are stable. Thus, we can only focus on these two stable equilibria, ASIUE and OSIUE, which enables us to introduce the city structure while keeping the model tractable.

Let us extend the baseline model by introducing the monocentric city structure. Because of consumption and commuting costs, there is a budget constraint for each individual iJ given by

$$y_{iJ}(\lambda) - tx = z_{iJ} + R(x)h_{iJ} \tag{8}$$

where z_{iJ} is a non-spatial composite good taken as the numéraire (whose price is normalized to 1), h_{iJ} is the housing consumption of each individual iJ in the city, $R(x)$ is the price of housing at each location x from the CBD, and t is the commuting cost per unit of distance.

The (indirect) utility function (3) can now be written as a direct utility function that incorporates the non-spatial and the housing consumption. We have:

$$U_{iJ}(\lambda) = A + a\ln z_{iJ} + (1-a)\ln h_{iJ} - \delta \ln D_{iJ}(\lambda) + \sigma \ln \frac{\overline{y}_J(\lambda)}{\overline{y}(\lambda)}, \tag{9}$$

where $0 < a < 1$ is a constant and determines the weight put on the non-spatial composite good. We normalize $A := -[a\ln a + (1-a)\ln(1-a)]$ in order to simplify the exposition. Each individual iJ chooses h_{iJ} and z_{iJ} that maximize $U_{iJ}(\lambda)$ under the budget constraint

[10] We now add "Urban" in the definition of each equilibrium because we focus on the impact of the location of each agent on her assimilation choice.

[11] Indeed, since all ethnic minorities are identical ex ante in terms of α, then, at the MSIUE, a slight increase (decrease) in λ will push all ethnic minorities to assimilate to (to reject) the majority's norm and to converge to the ASIUE (OSIUE).

(8). We obtain the following demand functions:

$$z_{iJ}(x,\lambda) = a(y_{iJ}(\lambda) - tx) \qquad \text{and} \qquad h_{iJ}(x,\lambda) = (1-a)\frac{(y_{iJ}(\lambda) - tx)}{R(x)} \tag{10}$$

We see, in particular, that, for a given income, if $R'(x) < 0$, i.e., housing prices decrease with the distance to the CBD, then individuals consume more housing the farther away they reside from the CBD. Plugging these demand functions into the direct utility function (9), we obtain the following indirect utility function:[12]

$$V_{iJ}(x,\lambda) = \ln\left(y_{iJ}(\lambda) - tx\right) - (1-a)\ln R(x) - \delta \ln D_{iJ}(\lambda) + \sigma \ln \frac{\overline{y}_J(\lambda)}{\overline{y}(\lambda)}. \tag{11}$$

As it is standard in urban economics, city residents are assumed to relocate costlessly within the city. Therefore, there is no incentive for workers to relocate in equilibrium and all individuals of the same type should obtain the same (indirect) utility function. As a result, in equilibrium, all individuals of type mJ ($J = m, c$) enjoy the same utility level: $V_{mJ}(x,\lambda) = V_{mJ}(\lambda)$, and all individuals of type c obtain the same utility level equal to: $V_{cc}(x,\lambda) = V_{cc}(\lambda)$.

3.3 Urban equilibria

In order to determine the equilibrium location of all individuals in the city, we use the standard concept of *bid rents* (Fujita, 1989; Zenou, 2009), which is defined as the maximum housing price each individual is willing to pay at each location x in order to obtain her equilibrium utility level. From (11), we obtain the bid rent $\Phi_{iJ}(x,\lambda)$ of an individual iJ as follows:

$$\Phi_{iJ}(x,\lambda) = \exp\left[\frac{\ln\left(y_{iJ}(\lambda) - tx\right) - \delta \ln D_{iJ}(\lambda) + \sigma \ln\left(\overline{y}_J(\lambda)/\overline{y}(\lambda)\right) - V_{iJ}(\lambda)}{1-a}\right] \tag{12}$$

The bid rent $\Phi_{iJ}(x,\lambda)$ determines the location pattern in the city since absentee landlords will allocate land to the highest bidder at each location x. The market land rent $R(x)$ can then be written as:

$$R(x,\lambda) = \max\left[\Phi_{mm}(x,\lambda), \Phi_{mc}(x,\lambda), \Phi_{cc}(x,\lambda), \overline{R}\right], \tag{13}$$

[12] We defined A so that it cancels out the constant.

where $\overline{R}$ is the agricultural land rent outside the city, which we normalize to one without loss of generality. For a given λ (the fraction of minority individuals choosing to assimilate to the majority group), the three different equilibrium utility levels are determined by the bid rent equalization at the borders between the locations of the different types of agents. Moreover, the location of the edge of the city, $\bar{x}$, is determined by the population constraint condition:

$$\int_0^{\bar{x}} \frac{H}{h_{iJ}(x,\lambda)} dx = 1 \tag{14}$$

By denoting $\Gamma_{mon}(\lambda) \equiv V_{mc}(\lambda) - V_{mm}(\lambda)$,[13] we can summarize the equilibrium conditions as follows.

Definition 3

(*i*) *An Assimilation Social Identity Urban Equilibrium (ASIUE) is a 5-tuple* $(V_{iJ}^*, R^*(x), \bar{x}^*, \lambda^*, \Gamma_{mon}^*)$ *that satisfies (11), (13), (14),* $\lambda^* = 1$, *and* $\Gamma_{mon}^*(1) > 0$.

(*ii*) *An Oppositional Social Identity Urban Equilibrium (OSIUE) is a 5-tuple* $(V_{iJ}^*, R^*(x), \bar{x}^*, \lambda^*, \Gamma_{mon}^*)$ *that satisfies (11), (13), (14),* $\lambda^* = 0$, *and* $\Gamma_{mon}^*(0) < 0$.

Here, we do not define a Mixed Social Identity Urban Equilibrium (MSIUE) since, in footnote 11, we have shown that this equilibrium is never stable due to the fact that all ethnic minorities are ex ante identical in terms of α. In order to derive $\Gamma_{mon}(\lambda)$, we need to solve the land market equilibrium for a given λ. From (12), the slope of the bid rent with respect to the distance from the CBD, x, is

$$\frac{\partial \Phi_{iJ}(x,\lambda)}{\partial x} = -\frac{t\Phi_{iJ}(x,\lambda)}{(1-a)\,(y_{iJ}(\lambda) - tx)} < 0.$$

Indeed, we can see from (12) that the only variable that varies with distance is the commuting cost. Thus, individuals residing further away from the CBD need to be compensated in terms of housing prices. As a result, housing prices decrease with the distance x from the CBD.

Proposition 4 *In any urban equilibrium, assimilated ethnic minorities and individuals from the majority group have the same bid rent, which means that they will reside in the*

[13] The subscript "*mon*" refers to the "monocentric" city.

same area of the city. Moreover, "oppositional" ethnic minorities will have a different and steeper bid rent and therefore will reside closer to the CBD than the majority individuals or the assimilated minorities.

We show that, in any urban equilibrium, "oppositional" ethnic minorities (i.e., those who choose *not* to assimilate to the majority's norm) will reside close to the city center while assimilated ethnic minorities and majority individuals will reside in the same area of the city. Indeed, when an ethnic minority becomes assimilated, then, in terms of income, housing consumption, bid rent and thus location choice she is "identical" to someone from the majority group: their bid rents are exactly the same. As a result, they live together in the same area of the city. Moreover, since assimilated ethnic minorities and majority individuals have higher incomes than "oppositional" ethnic minorities (see (7)), they will consume more housing (see (10)) and thus will have flatter bid rents. As a result, they prefer to reside farther away from the center because the land is cheaper and they can live in larger houses.

This result corresponds to what is usually observed in American cities where poor African Americans tend to live close to city centers while middle to upper class African Americans as well as white workers reside at the periphery of the city (Fischer, 2003; Glaeser et al., 2008; Ross and Rosenthal, 2015). What is new here compared to the literature is that we give an explanation of these location patterns in terms of identity choices.

Let us now study the process of identity choices (assimilation to or rejection of the majority's norm). Let us derive $\Gamma_{mon}(\lambda) = V_{mc}(\lambda) - V_{mm}(\lambda)$. Since the urban structure is relatively simple with only two different areas in the city, in Appendix B, we are able derive the equilibrium values of all endogenous variables defined in Definition 3. Now, by using (B.2), we can write $\Gamma_{mon}(\lambda)$ as

$$\begin{aligned}\Gamma_{mon}(\lambda) &= V_{mc}(\lambda) - V_{mm}(\lambda) \\ &= \ln\left(\frac{y_c(\lambda) - t\tilde{x}(\lambda)}{y_m(\lambda) - t\tilde{x}(\lambda)}\right) + \sigma \ln \frac{y_c(\lambda)}{y_m(\lambda)} - \delta \ln D_{mc}(\lambda).\end{aligned}$$

where, as shown by (B.1), $\tilde{x}$ is a function of λ. As λ, the share of the minority individuals who assimilate increases, $y_c(\lambda)$ increases whereas $y_m(\lambda)$ decreases (see Lemma 1). We know from (B.1) that when λ increases, the residential area $\tilde{x}$ of the "oppositional" ethnic minorities becomes smaller, resulting in a decrease in total commuting costs, $t\tilde{x}$. As a result, the effect of an increase in λ on the relative net income $\ln\left(\frac{y_c(\lambda)-t\tilde{x}}{y_m(\lambda)-t\tilde{x}}\right)$ is ambiguous. Thus, we assume that the effect of an increase in λ on income is larger than that on commuting costs, and the net income of the minority individuals who do not assimilate, $y_m(\lambda) - t\tilde{x}$, is decreasing in λ.[14] Under this assumption, we obtain

$$\frac{\partial \Gamma_{mon}(\lambda)}{\partial \lambda} > 0,$$

which leads to the following proposition:

Proposition 5

(i) *If* $\Gamma_{mon}(0) > 0$, *there exists a unique stable Assimilation Social Identity Urban Equilibrium (ASIUE) where all minority individuals totally assimilate to the majority group. In that case, there is "urban integration" since both minority and majority individuals have the same bid rent and reside in the same areas of the city.*

(ii) *If* $\Gamma_{mon}(1) < 0$, *there exists a unique Oppositional Social Identity Urban Equilibrium (OSIUE) where all minority individuals identify themselves with their own group and reject the majority's norm. In that case, there is "urban segregation" since all ethnic minorities reside close to the CBD while all majority individuals reside at the periphery of the city.*

(iii) *If* $\Gamma_{mon}(0) \leq 0$ *and* $\Gamma_{mon}(1) \geq 0$, *there exist multiple equilibria where both the ASIUE and the OSIUE coexist.*

This proposition characterizes the different possible urban equilibria. However, in each equilibrium, $\Gamma_{mon}(\lambda)$ depends on the bid rents and thus on the urban configuration. Figure 5 depicts the two possible urban configurations.[15] In Figure 5(a), we have the Assimilation Social Identity Urban Equilibrium (ASIUE), where all individuals (majority and minority individuals) live together in the city between the CBD (located at $x = 0$) and $\bar{x}_A$, the edge

[14] Basically, we assume that:

$$\frac{y_c'(\lambda) - t\tilde{x}'(\lambda)}{y_c(\lambda) - t\tilde{x}(\lambda)} > \frac{y_m'(\lambda) - t\tilde{x}'(\lambda)}{y_m(\lambda) - t\tilde{x}(\lambda)}.$$

[15] The subscripts A and O refer, respectively, to the ASIUE and the OSIUE.

of the city. They all obtain the same utility level $V_{cc}^* = V_{mc}^*$ and all minorities assimilate to the majority's norm. In Figure 5(b), we have the Oppositional Social Identity Urban Equilibrium (OSIUE), where all ethnic minorities are "oppositional" since they reject the majority's norm and reside at the vicinity of the CBD while the majority individuals reside at the periphery of the city. It is a completely segregated urban equilibrium in which the two groups obtain different utility levels.

In Proposition 5, case (*iii*), the two equilibria can coexist and it is unclear which ethnic group obtains the highest utility level. Indeed, if we compare the equilibrium utility of "oppositional" minorities (Figure 5(b)) with that of assimilated minorities (Figure 5(a)), the former have a lower social or cultural distance with respect to their culture of origin but obtain a lower income than the latter. Moreover, the land rents are different and the total commuting costs are lower for the "oppositional" than the assimilated minorities. In other words, in this model, it is unclear if urban segregation is harmful or beneficial to ethnic minorities.

Cutler and Glaeser (1997) have empirically investigated this question and show that segregation is "bad" for ethnic minorities in the sense that blacks in more segregated areas have significantly worse outcomes (such as economic performance) than blacks in less segregated areas. Our model also shows that ethnic minorities in segregated areas (the OSIUE) perform worse in terms of outcomes such as income than in less segregated areas (the ASIUE). We show that this is due to the fact that, when they reject the majority's norm, they reside in segregated areas and are paid a lower income because of lower productivity due to the lack of interaction with the majority group. However, as discussed above, this does not imply that "oppositional" minorities have a lower utility. In Section 3.5 below, we will investigate in detail this issue.

[*Insert Figure* 5 *here*]

3.4 City structure and comparative statics results

Let us now investigate how the city structure affects these two different equilibria. In particular, we study the effect of the commuting cost t and housing supply H on $\Gamma_{mon}(1)$ and $\Gamma_{mon}(0)$.

Proposition 6 *The commuting cost t and the space available for housing H in the city do not affect the Assimilation Social Identity Urban Equilibrium (ASIUE) where all minority individuals totally assimilate to the majority group in the city (Figure 5(a)). On the contrary, a lower t or a higher H makes the Oppositional Social Identity Urban Equilibrium (OSIUE) more likely to emerge (Figure 5(b)).*

A lower commuting cost t increases the net income of workers whereas a higher H enables individuals to consume land at a more reasonable price. Both decrease the utility difference between the assimilated and "oppositional" minorities. This, in turn, decreases the minority's incentive to assimilate, making the OSIUE more likely to emerge. This is an interesting and counterintuitive result showing that a policy that reduces transportation cost decreases rather than increase assimilation in cities.

The following proposition provides some comparative statics results for the other parameters of the model.

Proposition 7 *A higher ε (spillover effects in production) or δ (weight on perceived distance) makes the OSIUE more likely to emerge whereas a higher σ (weight on relative income) makes the ASIUE more likely to emerge. Moreover, a higher μ (fraction of minorities in the population) makes multiple equilibria more likely to emerge. Finally, a (weight on non-spatial good) neither affects the ASIUE nor the OSIUE.*

The effects of ε, the productivity spillover effect and μ, the fraction of ethnic minorities in the population on equilibrium are similar to those shown in Proposition 3. When ε increases, the income ratio y_{mc}/y_{mm} decreases but the perceived distance remains constant as it is not affected by ε. As a result, ethnic minorities are more likely *not* to assimilate and to reject the majority's norm. As μ increases, more individuals assimilate (higher λ) and because there are positive spillovers between λ and the productivity of group c, multiple equilibria are more likely to emerge. A higher δ implies higher costs from perceived distance, making ethnic minorities less likely to assimilate. In contrast, a higher σ implies higher gains from belonging to a social group with high income, which raises the incentive to assimilate.

3.5 Assimilation versus non-assimilation

We know from Proposition 5 that, under some condition (see part (*iii*) of this proposition), there exist multiple equilibria where both the ASIUE and OSIUE coexist. We need, therefore, to better understand the differences between these two equilibria. As shown in Figure 5(a), in the Assimilation Social Identity Equilibrium (ASIUE), the minority and majority individuals live together in mixed areas whereas, in the Oppositional Social Identity Equilibrium (OSIUE), they reside in segregated areas (Figure 5(b)). These very different urban structures lead to distinct city sizes and land rents. Define the total land rent TLR in a city as the sum of all housing prices paid by the residents of the city times the supply of land H. We have the following result:

Proposition 8 *The city in the Assimilation Social Identity Equilibrium (ASIUE; Figure 5(a)) is larger, i.e., $\bar{x}_A > \bar{x}_O$, and the total land rent is higher, i.e., $TLR_A > TLR_O$, than in the city in the Oppositional Social Identity Equilibrium (OSIUE; Figure 5(b)). In other words, ethnic minorities tend to assimilate more in bigger and more expensive cities.*

We obtain these results because the total income of ethnic minorities and majority individuals are higher in the ASIUE than in the OSIUE. This is due, in particular, to the fact that assimilated minorities obtain a larger income than "oppositional" minorities. As a result, these individuals are able to pay higher housing prices, which increase total land rent in the ASIUE. Also, since land is a normal good, because of higher income, assimilated minorities consume more land, which increases the size of the city. Hence, the city size and the total land rents are larger in the ASIUE than in the OSIUE.

When there are multiple equilibria, we would now like to know under which equilibrium the ethnic minorities and the majority individuals are better off. Because this is in general ambiguous, we now resort to numerical analysis. We specify the productivity and perceived distance function, $f(\cdot)$ and $d(\cdot)$ as follows:

$$\begin{aligned} f(L) &= \theta L^{\beta}, \\ d(x) &= \overline{d} - (\overline{d} - 1)(x-1)^2. \end{aligned}$$

where $\theta > 0$ and $\beta > 0$ capture the baseline productivity level in the city and the degree of agglomeration economies, respectively. For the baseline case, we set the parameter values as follows: $a = 0.75$, $\beta = 0.15$, $\overline{d} = 3$, $\delta = 0.2$, $\varepsilon = 0.2$, $H = 10$, $\mu = 0.2$, $\sigma = 0.2$, $t = 0.1$, and $\theta = 3$. It is easily verified that, under these parameter values, we obtain multiple equilibria, i.e., $\Gamma_{mon}(0) = -0.067 \leq 0$ and $\Gamma_{mon}(1) = 0.075 \geq 0$ (Proposition 5, part (iii)).

Also, if we compare the equilibrium utilities, we find that, in the ASIUE, $V_{mc}(A) = 0.882$ and $V_{cc}(A) = 0.988$ while, in the OSIUE, $V_{mm}(O) = 0.922$ and $V_{cc}(O) = 1.075$. So, basically, in this example, both the majority individuals and ethnic minorities are better off in the segregated equilibrium OSIUE. This shows, in particular, that, even if they obtain a lower income, ethnic minorities can be better off by rejecting the majority's norm and spatially segregating themselves from the majority group because their cultural distance with their own group is quite small

Let us now perform some comparative statics exercises by changing a parameter within a range under which these inequalities hold true and examine how it affects the equilibrium utility difference between the two equilibria.

In the *upper panel* of Figure 6, we evaluate how a change of a given parameter affects $V_{mc}(A)/V_{mm}(O)$, which is the utility difference for ethnic minorities between assimilating in the ASIUE (where the utility is $V_{mc}(A)$ and $\lambda^* = 1$) and rejecting the majority's norm in the OSIUE (where the utility is $V_{mm}(O)$ and $\lambda^* = 0$). The dashed (yellow) line corresponds to a value of $V_{mc}(A)/V_{mm}(O)$ equal to 1 so that ethnic minorities are indifferent in terms of utility between the two equilibria. The solid (blue) curve represents the real value of $V_{mc}(A)/V_{mm}(O)$. Therefore, if the solid curve is above (below) the dashed line, then $V_{mc}(A) > V_{mm}(O)$ ($V_{mc}(A) < V_{mm}(O)$), and ethnic minorities in the ASIUE are better off (worse off) than in the OSIE. In the *lower panel*, we perform the same exercise, i.e., we evaluate the change of $V_{cc}(A)/V_{cc}(O)$, but for the majority individuals.

[*Insert Figure* 6 *here*]

In Figures 6(1) and 6(2), we consider changes in the commuting cost t and the space available for housing H in the city. First, as in the baseline model, both ethnic minorities and individuals from the majority group are always better off in the segregated equilibrium OSIE whatever the values of t and H. Second, a higher t always yields a lower utility for both minorities and majority individuals whereas a higher H raises utility for both of them. In Proposition 6, we showed that a lower t or a higher H makes the OSIE more likely to emerge. Hence, a decrease in t or an increase in H decreases the minorities's and majorities' utility in the OSIUE compared to that in the ASIUE although it increases the possibility of the OSIUE. These results suggest that investment in transportation infrastructure or land development might induce spatial and social segregation between ethnic minorities and individuals from the majority group despite the fact that it increases the desirability for integration.

Figures 6(3) and 6(4) perform the same exercises for ε, the productivity spillover parameter, and μ, the fraction of ethnic minorities in the population. A larger ε implies that there are more productive interactions between members of group c (i.e., majority and assimilated minorities) and group m (oppositional minorities). This reduces the productivity gains from assimilation of ethnic minorities and results in a lower relative utility for both ethnic groups. If we consider Figure 6(3), for low values of ε, ethnic minorities are better off by assimilating while, for higher value of ε, they are better off by rejecting the majority's norm. For the individuals from the majority group, they are always better off in the segregated equilibrium since spillover effects only affect the assimilation decision of the ethnic minorities.

When μ increases, the size of ethnic population becomes larger, which implies that ethnic minorities face smaller disutility from perceived distance. Moreover, productivity gains from minorities's assimilation becomes smaller for minorities and larger for majority individuals. Hence, an increase in μ decreases the minorities' relative utility. This is why for low value of μ, ethnic minorities are better off assimilating while the opposite is true when μ becomes larger.

Figures 6(5) and 6(6) look at the change in β, the degree of agglomeration economies and θ, the baseline productivity level in the city. A higher β or θ increases the benefits from minorities' assimilation and thus increases the relative utility for both ethnic groups. Also, as we know from (A.1) and (A.2), a higher β or θ makes the ASIUE more likely to emerge.

Figures 6(7), 6(8), and 6(9) study how a change in $\overline{d}$, the upper bound of the perceived distance, δ, the level of disutility for a given perceived distance, and σ, the level of utility for a given relative status of one's group, affects the utility difference between the two equilibria. When $\overline{d}$ or δ increases, the utility difference between the two equilibria increases because the benefits from assimilation is reduced. On the contrary, an increase in σ, increases the gains from social status for minorities but decreases them for the majority group, yielding a higher relative utility for minorities but a lower relative utility for the majority group.

Finally, Figure 6(10) displays the impact of a, the weight put on the non-spatial composite good, on relative utility. We see that a does not affect the utility difference so that all agents are better off under the segregated equilibrium OSIUE.

4 Concluding remarks

In this paper, we develop a model in which ethnic minorities may choose to adopt "oppositional" identities, that is, some actively reject the dominant ethnic norms while others totally assimilate to them. We show that three types of equilibria may emerge: An Assimilation Social Identity Equilibrium (ASIE), in which all minority individuals choose to totally assimilate to the majority group, an Oppositional Social Identity Equilibrium (OSIE), in which all minority individuals totally reject the social norm of the majority group, and a Mixed Social Identity Equilibrium (MSIE), in which a fraction of minority individuals assimilate while the other fraction choose to be "oppositional". We provide conditions under which each equilibrium exists and is unique and investigate the properties of each equilibrium.

We then extend this model by introducing the urban space where all individuals are embedded in. The benefits of assimilation are in terms of higher income while the costs are due to the higher perceived distance between this assimilation choice of ethnic minorities and the norms of their culture of origin. We show how residential location affects the assimilation process of ethnic minorities and why people who are "oppositional" tend to reside in segregated areas around the CBD away from the location of the majority group. We also demonstrate that segregation is bad in terms of economic outcomes but not necessary in terms of welfare.

As highlighted in the Introduction, many people blame immigrants for not assimilating to the majority's norm because they keep some of the values of their culture of origin. In this paper, we tried to fathom the way ethnic minorities assimilate or reject the majority's norm and how these choices affect or are affected by their residential location. This is a first stab at a very complex issue and we hope to see more research on this in the future.

References

[1] Ainsworth-Darnell, J.W. and D.B. Downey (1998), "Assessing the oppositional culture explanation for racial/ethnic differences in school performance," *American Sociological Review* 63, 536-553.

[2] Akerlof, G.A. (1997), "Social distance and social decisions," *Econometrica* 65, 1005-1027.

[3] Akerlof, G.A. and R.E. Kranton (2010), *Identity Economics: How Our Identities Shape Our Work, Wages, and Well-Being*, Princeton: Princeton University Press.

[4] Austen-Smith D. and R.D. Fryer, Jr (2005), "An economic analysis of 'acting white'," *Quarterly Journal of Economics* 120, 551-583.

[5] Battu, H., McDonald, M. and Y. Zenou (2007), "Oppositional identities and the labor market," *Journal Population Economics* 20, 643-667.

[6] Battu, H. and Y. Zenou (2010), "Oppositional identities and employment for ethnic minorities. Evidence for England," *Economic Journal* 120, 52-71.

[7] Beenstock, M., Chiswick, B.R. and A. Paltiel (2010), "Testing the immigrant assimilation hypothesis with longitudinal data," *Review of Economics of the Household* 8, 7–27.

[8] Biavaschi, C., Giulietti, C. and Z. Siddique (2017), "The economic payoff of name Americanization," *Journal of Labor Economics* 35, 1089–1116.

[9] Bisin, A., Patacchini, E., Verdier, T. and Y. Zenou (2011a), "Formation and persistence of oppositional identities," *European Economic Review* 55, 1046-1071.

[10] Bisin, A., Patacchini, E., Verdier, T. and Y. Zenou (2011b), "Ethnic identity and labor-market outcomes of immigrants in Europe," *Economic Policy* 26, 57-92.

[11] Bisin, A., Patacchini, E., Verdier, T. and Y. Zenou (2016), "Bend it like Beckham. Ethnic identity and integration," *European Economic Review* 90, 146-164.

[12] Borjas, G.J. (1987), "Self-selection and the earnings of immigrants," *American Economic Review* 77, 531–553.

[13] Borjas, G.J. (1992), "Ethnic capital and intergenerational mobility," *Quarterly Journal of Economics* 107, 123–150.

[14] Boucher, V. (2016), "Conformism and self-selection in social networks," *Journal of Public Economics* 136, 30-44.

[15] Carvalho, J.-P. and M. Koyama (2016), "Jewish emancipation and schism: Economic development and religious change," *Journal of Comparative Economics* 44, 562-584.

[16] Chiswick, B.R. and P.W. Miller (1995), "The endogeneity between language and earnings: International analyses," *Journal of Labor Economics* 13, 246-288.

[17] Chiswick, B.R. and P.W. Miller (1996), "Ethnic networks and language proficiency among immigrants," *Journal of Population Economics* 9, 19-35.

[18] Chiswick, B.R. and P.W. Miller (2011), "The 'negative' assimilation of immigrants: A special case," *Industrial and Labor Relations Review* 64, 502–525.

[19] Constant, A., Gataullina, L. and K.F. Zimmermann (2009), "Ethnosizing immigrants," *Journal of Economic Behavior and Organization* 69, 274-287.

[20] Constant, A. and K.F. Zimmermann (2008), "Measuring ethnic identity and its impact on economic behavior," *Journal of the European Economic Association* 6, 424-433.

[21] Cutler, D.M. and E.L. Glaeser (1997), "Are ghettos good or bad? " *Quarterly Journal of Economics* 112, 827–872.

[22] Delpit, L. (1995), *Other People's Children: Cultural Conflict in the Classroom*. The Free Press, New York.

[23] De Marti, J. and Y. Zenou (2017), "Segregation in friendship networks," *Scandinavian Journal of Economics* 119, 656-708.

[24] Duranton, G. and D. Puga (2004), "Micro-foundations of urban agglomeration economies," in: J.V. Henderson and J.-F. Thisse (Eds.), *Handbook of Regional and Urban Economics* 4.

[25] Dustmann, C. and F. Fabbri (2003), "Language proficiency and labour market performance of immigrants in the UK," *Economic Journal* 113, 695-717.

[26] Edin, P.A., Fredriksson, P. and O. Aslund (2003), "Ethnic enclaves and the economic success of immigrants – Evidence from a natural experiment," *Quarterly Journal of Economics* 118, 329–357.

[27] Eguia, J.X. (2017), "Discrimination and assimilation at school," *Journal of Public Economics* 156, 48–58.

[28] Fischer, M.J. (2003), "The relative importance of income and race in determining residential outcomes in U.S. urban areas, 1970-2000," *Urban Affairs Review* 38, 669–696.

[29] Fordham S. and J.U. Ogbu (1986), "Black student'school success: Coping with the burden of acting White," *The Urban Review* 18, 176-206.

[30] Fryer, R.G. Jr. and P. Torelli (2010), "An empirical analysis of 'acting white'," *Journal of Public Economics* 94, 380-396.

[31] Fujita, M. (1989), *Urban Economic Theory*, Cambridge: Cambridge University Press.

[32] Glaeser, E.L., Kahn, M.E. and J. Rappaport (2008), "Why do the poor live in cities? The role of public transportation," *Journal of Urban Economics* 63, 1–24.

[33] Hauk, E. and H. Mueller (2015), "Cultural leaders and the clash of civilizations," *Journal of Conflict Resolution* 59, 367-400.

[34] Lazear E. (1999), "Culture and language," *Journal of Political Economy* 107, S95–S126.

[35] Liu, X., Patacchini, E. and Y. Zenou (2014), "Endogenous peer effects: Local aggregate or local average?" *Journal of Economic Behavior and Organization* 103, 39-59.

[36] McManus, W., Gould, W. and F. Welch (1983), "Earnings of Hispanic men: The role of English language proficiency," *Journal of Labor Economics* 1, 101-103.

[37] Meng, G. (2005), "Intermarriage and the economic assimilation of immigrants," *Journal of Labor Economics* 23, 135-175.

[38] Nosofsky, R.M. (1986), "Attention, similarity and the identification-categorization relationship," *Journal of Experimental Psychology: General* 115, 39-57.

[39] Ogbu, J.U. (1997), "Beyond language: Ebonics, proper English and identity in a black American speech community," Unpublished manuscript, University of California at Berkeley, Department of Anthropology.

[40] Panebianco, F. (2014), "Socialization networks and the transmission of interethnic attitudes," *Journal of Economic Theory* 150, 583-610.

[41] Patacchini, E. and Y. Zenou (2012), "Juvenile delinquency and conformism," *Journal of Law, Economic, and Organization* 28, 1-31.

[42] Patacchini, E. and Y. Zenou (2016), "Racial identity and education in social networks," *Social Networks* 44, 85-94.

[43] Prummer, A., Siedlarek, J.-P. (2017), "Community leaders and the preservation of cultural traits," *Journal of Economic Theory* 168, 143-176.

[44] Ross, S.L. and S.S. Rosenthal (2015), "Change and persistence in the economic status of neighborhoods and cities," In: G. Duranton, V. Henderson and W. Strange (Eds.), *The Handbook of Regional and Urban Economics*, Vol 5 Amsterdam: Elsevier Science.

[45] Selod, H. and Y. Zenou (2006), "City-structure, job search, and labor discrimination. Theory and policy implications," *Economic Journal* 116, 1057–1087.

[46] Shayo, M. (2009), "A model of social identity with an application to political economy: Nation, class, and redistribution," *American Political Science Review* 103, 147-174.

[47] Sambanis, N. and M. Shayo (2013), "Social identification and ethnic conflict," *American Political Science Review* 107, 294-325.

[48] Tajfel, H. and J. Turner (1986), "The social identity theory of intergroup behavior," in: S. Worchel and W. Austin (Eds.), *Psychology of Intergroup Relations*, Chicago: Nelson Hall.

[49] Turner, J., Hogg, M., Oakes, P., Reicher, S. and M. Wetherell (1987), *Rediscovering the Social Group: A Self-Categorization Theory*, Oxford: Blackwell.

[50] Ushchev, P. and Y. Zenou (2018), "Social norms in networks," CEPR Discussion Paper No. 13250.

[51] Verdier, T. and Y. Zenou (2017), "The role of social networks in cultural assimilation," *Journal of Urban Economics* 97, 15-39.

[52] Verdier, T. and Y. Zenou (2018), "Cultural leaders and the dynamics of assimilation," *Journal of Economic Theory* 175, 374-414.

[53] Wilson, W.J. (1987), *The truly Disadvantaged: The Inner City, the Underclass, and Public Policy*, Chicago: University of Chicago Press.

[54] Zimmermann, L., Zimmermann, K.F. and A. Constant (2007), "Ethnic self-identification of first-generation immigrants," *International Migration Review* 41, 769-781.

[55] Zenou, Y. (2009), *Urban Labor Economics*, Cambridge: Cambridge University Press.

Appendix

A Proofs

Proof of Lemma 1: The incomes are defined by (1), which we reproduce here:

$$y_m(\lambda) = f\left((1-\lambda)\,\mu + \varepsilon\left(\lambda\mu + 1 - \mu\right)\right),$$
$$y_c(\lambda) = f\left(\lambda\mu + 1 - \mu + \varepsilon\left(1-\lambda\right)\mu\right).$$

Denote $N_c \equiv \lambda\mu + 1 - \mu + \varepsilon\,(1-\lambda)\,\mu$ and $N_m \equiv (1-\lambda)\,\mu + \varepsilon\,(\lambda\mu + 1 - \mu)$ and remember that $f'(.) > 0$, $f''\,(.) < 0$, $0 < \mu < 1/2$ and $0 \leq \varepsilon \leq 1$.

(i) It is easily verified that:

$$y_c(0) = f\left(1 - \mu\left(1-\varepsilon\right)\right) > f\left(\varepsilon + \mu\left(1-\varepsilon\right)\right) = y_m(0),$$

$$y_c(1) = f\left(1\right) > f\left(\varepsilon\right) = y_m(1).$$

For $\varepsilon \in (0,1)$**,** by differentiating (1), we obtain:

$$\frac{\partial y_m(\lambda)}{\partial \lambda} = -f'\left(N_m\right)\mu\left(1-\varepsilon\right) < 0 \text{ and } \frac{\partial^2 y_m(\lambda)}{\partial \lambda^2} = f''\left(N_m\right)\mu^2\left(1-\varepsilon\right)^2 < 0,$$

$$\frac{\partial y_c(\lambda)}{\partial \lambda} = f'\left(N_c\right)\mu\left(1-\varepsilon\right) > 0 \text{ and } \frac{\partial^2 y_c(\lambda)}{\partial \lambda^2} = f''\left(N_c\right)\mu^2\left(1-\varepsilon\right)^2 < 0.$$

Finally,

$$\frac{\partial\left(y_c(\lambda)/y_m(\lambda)\right)}{\partial \lambda} = \frac{f'\left(N_c\right)\mu\left(1-\varepsilon\right)y_m(\lambda) + f'\left(N_m\right)\mu\left(1-\varepsilon\right)y_c(\lambda)}{\left[y_m(\lambda)\right]^2} > 0.$$

(ii) It is easily verified that:

$$y_c(\varepsilon = 0) = f\left(\lambda\mu + 1 - \mu\right) > f\left((1-\lambda)\,\mu\right) = y_m(\varepsilon = 0),$$

$$y_c(\varepsilon = 1) = f\left(1\right) = y_m(\varepsilon = 1).$$

For $\varepsilon \in (0,1)$**,** by differentiating (1), we obtain:

$$y_m(\lambda) = f\left((1-\lambda)\mu + \varepsilon\left(\lambda\mu + 1 - \mu\right)\right),$$
$$y_c(\lambda) = f\left(\lambda\mu + 1 - \mu + \varepsilon\left(1-\lambda\right)\mu\right),$$

$$\frac{\partial y_m(\lambda)}{\partial \varepsilon} = f'\left(N_m\right)\left(\lambda\mu + 1 - \mu\right) > 0 \text{ and } \frac{\partial^2 y_m(\lambda)}{\partial \varepsilon^2} = f''\left(N_m\right)\left(\lambda\mu + 1 - \mu\right)^2 < 0,$$
$$\frac{\partial y_c(\lambda)}{\partial \varepsilon} = f'\left(N_c\right)\left(1-\lambda\right)\mu > 0 \text{ and } \frac{\partial^2 y_c(\lambda)}{\partial \varepsilon^2} = f''\left(N_c\right)\left(1-\lambda\right)^2\mu^2 < 0.$$

Finally,

$$\frac{\partial\left(y_c(\lambda)/y_m(\lambda)\right)}{\partial \varepsilon} = \frac{f'\left(N_c\right)y_m(\lambda)\left[\left(1-\lambda\right)\mu\right] - f'\left(N_m\right)y_c(\lambda)\left(\lambda\mu + 1 - \mu\right)}{\left[y_m(\lambda)\right]^2}.$$

Since $f'\left(N_c\right) < f'\left(N_m\right)$[1], $y_m(\lambda) < y_c(\lambda)$, $\forall\lambda \in [0,1]$, and $\left(1-\lambda\right)\mu < \lambda\mu + 1 - \mu$, then $\frac{\partial\left(y_c(\lambda)/y_m(\lambda)\right)}{\partial \varepsilon} < 0$. ▮

Proof of Lemma 2: $\Gamma(\lambda;\alpha)$ is defined as:

$$\begin{aligned}\Gamma(\lambda;\alpha) &\equiv (\alpha+\sigma)\ln\frac{y_c(\lambda)}{y_m(\lambda)} - \delta\ln\frac{D_c(\lambda)}{D_m(\lambda)} \\ &= (\alpha+\sigma)\ln\frac{f(N_c)}{f(N_m)} - \delta\ln d\left(P\right),\end{aligned}$$

where

$$\begin{aligned}N_c &\equiv \lambda\mu + 1 - \mu + \varepsilon(1-\lambda)\mu, \\ N_m &\equiv (1-\lambda)\mu + \varepsilon(\lambda\mu + 1 - \mu), \\ d\left(P\right) &\equiv \frac{D_c(\lambda)}{D_m(\lambda)} \text{ where } P \equiv \frac{1-\mu}{\lambda\mu + 1 - \mu}.\end{aligned}$$

Let us differentiate $\Gamma(\lambda;\alpha)$ with respect to λ. We obtain:

$$\frac{\partial\Gamma(\lambda;\alpha)}{\partial\lambda} = (\alpha+\sigma)(1-\varepsilon)\mu\left(\frac{f'(N_c)}{f(N_c)} + \frac{f'(N_m)}{f(N_m)}\right) + \frac{\delta\lambda d'(P)}{d(P)\left(\lambda\mu + 1 - \mu\right)^2} > 0,$$

[1] Indeed, since $N_c > N_m$ and $f''\left(.\right) < 0$, then $f'\left(N_c\right) < f'\left(N_m\right)$.

and

$$\lim_{\mu\to 0}\frac{\partial\Gamma(\lambda;\alpha)}{\partial\lambda}=\frac{\delta\lambda d'(1)}{d(1)}=\frac{\delta\lambda}{\overline{d}}d'(1)=0.$$

This completes the proof. ▮

Proof of Proposition 3: To prove this proposition, we state the following lemma:

Lemma A3

(*i*) *The higher is μ, the fraction of minority individuals in the population, the lower is $\Gamma(0;\alpha)$ and the higher is $\Gamma(1;\alpha)$, i.e. $\partial\Gamma(0;\alpha)/\partial\mu<0$ and $\partial\Gamma(1;\alpha)/\partial\mu>0$.*

(*ii*) *The higher is ε, the productivity spillover effect, the lower are $\Gamma(0;\alpha)$ and $\Gamma(1;\alpha)$, i.e. $\partial\Gamma(0;\alpha)/\partial\varepsilon<0$ and $\partial\Gamma(1;\alpha)/\partial\varepsilon<0$. Moreover, $\lim_{\varepsilon\to 1}\Gamma(0;\alpha)<0$ and $\lim_{\varepsilon\to 1}\Gamma(1;\alpha)<0$.*

Proof of Lemma A3: We know that

$$\begin{aligned}\Gamma(0;\alpha) &= (\alpha+\sigma)\ln\frac{f(1-\mu+\varepsilon\mu)}{f(\mu+\varepsilon(1-\mu))}-\delta\ln\overline{d},\\ \Gamma(1;\alpha) &= (\alpha+\sigma)\ln\frac{f(1)}{f(\varepsilon)}-\delta\ln d(1-\mu),\end{aligned}$$

(*i*): By differentiating these functions, we obtain:

$$\begin{aligned}\frac{\partial\Gamma(0;\alpha)}{\partial\mu} &= -(\alpha+\sigma)(1-\varepsilon)\left[\frac{f'(1-\mu+\varepsilon\mu)}{f(1-\mu+\varepsilon\mu)}+\frac{f'(\mu+\varepsilon(1-\mu))}{f(\mu+\varepsilon(1-\mu))}\right]<0,\\ \frac{\partial\Gamma(1;\alpha)}{\partial\mu} &= \frac{\delta d'(1-\mu)}{d(1-\mu)}>0.\end{aligned}$$

(*ii*): By differentiating these functions, we obtain:

$$\frac{\partial\Gamma(0;\alpha)}{\partial\varepsilon}=(\alpha+\sigma)\left[\frac{\mu f'\left(1-\mu+\varepsilon\mu\right)}{f\left(1-\mu+\varepsilon\mu\right)}-\frac{\left(1-\mu\right)f'\left(\mu+\varepsilon\left(1-\mu\right)\right)}{f(\mu+\varepsilon(1-\mu))}\right]<0,$$

since $\mu<1-\mu$, $\mu+\varepsilon(1-\mu)<1-\mu+\varepsilon\mu$, $f(\mu+\varepsilon(1-\mu))<f\left(1-\mu+\varepsilon\mu\right)$ and $f'\left(1-\mu+\varepsilon\mu\right)<f'(\mu+\varepsilon(1-\mu))$, and

$$\frac{\partial\Gamma(1;\alpha)}{\partial\varepsilon}=-(\alpha+\sigma)\frac{f'(\varepsilon)}{f(\varepsilon)}<0.$$

Moreover, taking limits, we obtain:

$$\begin{aligned}\lim_{\varepsilon\to 1}\Gamma(0;\alpha) &= -\delta\ln\overline{d} < 0,\\ \lim_{\varepsilon\to 1}\Gamma(1;\alpha) &= -\delta\ln d(1-\mu) < 0.\end{aligned}$$

Using Lemmas 2 and A3, it is then straightforward to prove Proposition 3. ∎

Proof of Proposition 4: As is well known in urban economics, an agent having a steeper bid rent at an intersection of bid rent curves of heterogeneous agents lives closer to the CBD. From (7), we know that at any intersection of Φ_{mc} and Φ_{cc} (i.e., $\check{x}$ that satisfies $\Phi_{mc}(\check{x},\lambda) = \Phi_{cc}(\check{x},\lambda) \equiv \Phi(\check{x})$), the slopes of the two bid rent curves are the same:

$$\frac{\partial\Phi_{mc}(\check{x},\lambda)}{\partial x} = -\frac{t\Phi(\check{x})}{(1-a)\left(y_c(\lambda)-t\check{x}\right)} = \frac{\partial\Phi_{cc}(\check{x},\lambda)}{\partial x}.$$

Hence, the assimilated ethnic minorities and the majority individuals reside in the same area. Moreover, at any intersection of Φ_{mm} and Φ_{cc} (i.e., at any $\tilde{x}$ that satisfies $\Phi_{mm}(\tilde{x},\lambda) = \Phi_{cc}(\tilde{x},\lambda) = \Phi(\tilde{x})$), we can see that

$$\frac{\partial\Phi_{mm}(\tilde{x},\lambda)}{\partial x} = -\frac{t\Phi(\tilde{x})}{(1-a)\left(y_m(\lambda)-t\tilde{x}\right)} < 0, \qquad \frac{\partial\Phi_{cc}(\tilde{x},\lambda)}{\partial x} = -\frac{t\Phi(\tilde{x})}{(1-a)\left(y_c(\lambda)-t\tilde{x}\right)} < 0,$$

which, combined with the fact that $y_c(\lambda) > y_m(\lambda)$ under $0 < \varepsilon < 1$ implies that

$$\left|\frac{\partial\Phi_{mm}(\tilde{x},\lambda)}{\partial x}\right| > \left|\frac{\partial\Phi_{cc}(\tilde{x},\lambda)}{\partial x}\right|.$$

Hence, the minority individuals who do not assimilate live closer to the CBD than the majority individuals and the minority individuals who assimilate. This segregation pattern under income heterogeneity is very standard in the urban economics literature (see Fujita, 1989). ∎

Proof of Proposition 6

To prove this proposition, we need to know: *(i)* $V_{mc}(1)$, the utility level of all ethnic minorities when all of them assimilate to the majority group, *(ii)* $V_{mm}(1)$, the utility level of the ethnic minorities who reject the majority's norm, *(iii)* $V_{mm}(0)$, the utility level of all ethnic minorities when all of them decide to reject the norm of the majority group, (*iv*) $V_{mc}(0)$, the utility level of a some ethnic minorities who decide to assimilate to the majority group.

From (B.2), we obtain:

$$\begin{aligned} \Gamma_{mon}(1) &= V_{mc}(1) - V_{mm}(1) \\ &= (1+\sigma)\ln\left(\frac{y_c(1)}{y_m(1)}\right) - \delta \ln d(1-\mu). \end{aligned} \tag{A.1}$$

where $d(.)$ is the perceived distance defined in Section 2.3.1.

Similarly, we can solve the model when ethnic minorities reject the majority norm to obtain

$$\begin{aligned} \Gamma_{mon}(0) &= V_{mc}(0) - V_{mm}(0) \\ &= \ln\left(\frac{y_c(0) - t\tilde{x}}{y_m(0) - t\tilde{x}}\right) + \sigma \ln\left(\frac{y_c(0)}{y_m(0)}\right) - \delta \ln \overline{d}, \\ t\tilde{x}_m &= \left\{1 - \left[\frac{H + (1-\mu)t}{H+t}\right]^{1-a}\right\} y_m(0). \end{aligned} \tag{A.2}$$

By differentiating $\Gamma_{mon}(1)$ and $\Gamma_{mon}(0)$ with respect to t and H, we obtain:

$$\begin{aligned} \frac{\partial \Gamma_{mon}(1)}{\partial t} &= \frac{\partial \Gamma_{mon}(1)}{\partial H} = 0, \\ \frac{\partial \Gamma_{mon}(0)}{\partial t} &= \frac{y_c(0) - y_m(0)}{(y_m(0) - t\tilde{x})(y_c(0) - t\tilde{x})} \frac{\partial (t\tilde{x})}{\partial t} > 0, \\ \frac{\partial \Gamma_{mon}(0)}{\partial H} &= \frac{y_c(0) - y_m(0)}{(y_m(0) - t\tilde{x})(y_c(0) - t\tilde{x})} \frac{\partial (t\tilde{x})}{\partial H} < 0. \end{aligned}$$

This proves the result. ▮

Proof of Proposition 7

By proceeding as for the proof of Proposition 6, we can differentiate (A.1) and (A.2) to obtain:

$$\begin{aligned} \frac{\partial \Gamma_{mon}(1)}{\partial \varepsilon} &< 0, \quad \frac{\partial \Gamma_{mon}(1)}{\partial \mu} > 0 \quad, \frac{\partial \Gamma_{mon}(1)}{\partial \delta} < 0 \quad, \frac{\partial \Gamma_{mon}(1)}{\partial \sigma} > 0 \quad, \frac{\partial \Gamma_{mon}(1)}{\partial a} = 0, \\ \frac{\partial \Gamma_{mon}(0)}{\partial \varepsilon} &< 0, \quad \frac{\partial \Gamma_{mon}(0)}{\partial \mu} < 0 \quad, \frac{\partial \Gamma_{mon}(0)}{\partial \delta} < 0 \quad, \frac{\partial \Gamma_{mon}(0)}{\partial \sigma} > 0 \quad, \frac{\partial \Gamma_{mon}(0)}{\partial a} = 0, \end{aligned}$$

This proves the result. ▮

Proof of Proposition 8

For the ASIUE, equations (12) and (B.1) yield:

$$\begin{aligned} t\bar{x}_A &= \left[1-\left(\frac{H}{H+t}\right)^{1-a}\right] y_c(1), \\ R_A(x) &= \begin{cases} \Phi_{cc}(x,1)\ (=\Phi_{mc}(x,1)) & \text{if } x\in[0,\bar{x}_A] \\ 1 & \text{if } x\in(\bar{x}_A,\infty) \end{cases}, \\ \Phi_{cc}(x,1) &= \left(\frac{y_c(1)-tx}{y_c(1)-t\bar{x}_A}\right)^{1/(1-a)}, \end{aligned}$$

For the OSIUE, we have:

$$\begin{aligned} t\tilde{x}_O &= \left\{1-\left[\frac{H+(1-\mu)t}{H+t}\right]^{1-a}\right\} y_m(0), \\ t\bar{x}_O &= y_c(0)-(y_c(0)-y_m(0))\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} - y_m(0)\left(\frac{H}{H+t}\right)^{1-a}, \\ R_O(x) &= \begin{cases} \Phi_{mm}(x,0) & \text{if } x\in[0,\tilde{x}_O] \\ \Phi_{cc}(x,0)\ (=\Phi_{mc}(x,0)) & \text{if } x\in(\tilde{x}_O,\bar{x}_O] \\ 1 & \text{if } x\in(\bar{x}_O,\infty) \end{cases}, \\ \Phi_{cc}(x,0) &= \left(\frac{y_c(0)-tx}{y_c(0)-t\bar{x}_O}\right)^{1/(1-a)}, \\ \Phi_{mm}(x,0) &= \left(\frac{y_m(0)-tx}{y_m(0)-t\tilde{x}_O}\right)^{1/(1-a)} \left(\frac{y_c(0)-t\tilde{x}_O}{y_c(0)-t\bar{x}_O}\right)^{1/(1-a)}, \end{aligned}$$

Because

$$\frac{H}{H+(1-\mu)t} > \frac{H}{H+t},$$

we know that

$$\begin{aligned} t\bar{x}_O &= \left\{1-\left[\frac{H}{H+(1-\mu)t}\right]^{1-a}\right\} y_c(0) + \left\{\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} - \left(\frac{H}{H+t}\right)^{1-a}\right\} y_m(0) \\ &< \left\{1-\left[\frac{H}{H+(1-\mu)t}\right]^{1-a}\right\} y_c(0) + \left\{\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} - \left(\frac{H}{H+t}\right)^{1-a}\right\} y_c(0) \\ &= \left[1-\left(\frac{H}{H+t}\right)^{1-a}\right] y_c(0) \\ &< \left[1-\left(\frac{H}{H+t}\right)^{1-a}\right] y_c(1) = t\bar{x}_A. \end{aligned}$$

Moreover, the total land rents (TLR) are given by

$$\begin{aligned}
TLR_A &= H\int_0^{t\bar{x}_A} \Phi_{cc}(x,1)dx \\
&= \frac{(H+t)(1-a)}{t(2-a)}\left[1-\left(\frac{H}{H+t}\right)^{2-a}\right] y_c(1), \\
TLR_O &= H\int_0^{t\tilde{x}_O} \Phi_{mm}(x,0)dx + H\int_{t\tilde{x}_O}^{t\bar{x}_O} \Phi_{cc}(x,0)dx \\
&= \frac{H(1-a)}{t(2-a)}\left[\left(y_c(0)+\frac{\mu t}{H+(1-\mu)t}y_m(0)\right)\left[\frac{H+(1-\mu)t}{H}\right]\right. \\
&\quad \left. -y_c(0)\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} + y_m(0)\left\{\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} - \left(\frac{H}{H+t}\right)^{1-a}\right\}\right].
\end{aligned}$$

From this, we can show that

$$\begin{aligned}
TLR_O &< \frac{H(1-a)}{t(2-a)}\left[\left(y_c(0)+\frac{\mu t}{H+(1-\mu)t}y_c(0)\right)\left[\frac{H+(1-\mu)t}{H}\right]\right. \\
&\quad \left. -y_c(0)\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} + y_c(0)\left\{\left[\frac{H}{H+(1-\mu)t}\right]^{1-a} - \left(\frac{H}{H+t}\right)^{1-a}\right\}\right] \\
&= \frac{(H+t)(1-a)}{t(2-a)}\left[1-\left(\frac{H}{H+t}\right)^{2-a}\right] y_c(0) \\
&< \frac{(H+t)(1-a)}{t(2-a)}\left[1-\left(\frac{H}{H+t}\right)^{2-a}\right] y_c(1) = TLR_A.
\end{aligned}$$

B Equilibrium values of all variables in any urban equilibrium

Letting $\bar{x}$ denote the edge of the city, the market land rent is given by

$$R(x,\lambda) = \begin{cases} \Phi_{mm}(x,\lambda) & \text{if } x \in [0,\tilde{x}] \\ \Phi_{cc}(x,\lambda)\ (= \Phi_{mc}(x,\lambda)) & \text{if } x \in (\tilde{x},\bar{x}] \\ 1 & \text{if } x \in (\bar{x},\infty) \end{cases}.$$

At the city edge $\bar{x}$, the utility of the majority individuals is equal to:

$$V_{cc}(\lambda) = \ln(y_c(\lambda) - t\bar{x}) + \sigma \ln\left[\frac{y_c(\lambda)}{\overline{y}(\lambda)}\right].$$

Plugging this into (12) with $i = J = c$ yields the bid rent of the majority individuals:[2]

$$\Phi_{cc}(x,\lambda) = \left(\frac{y_c(\lambda) - tx}{y_c(\lambda) - t\bar{x}}\right)^{1/(1-a)}.$$

At the border $\tilde{x}$ between the residential area of the minority individuals who do not assimilate and that of other individuals, we have $\Phi_{mm}(\tilde{x},\lambda) = \Phi_{cc}(\tilde{x},\lambda)$, implying that we can write the indirect utility of the minority individuals who do not assimilate as

$$V_{mm}(\lambda) = \ln\left(y_m(\lambda) - t\tilde{x}\right) + \sigma \ln\frac{y_m(\lambda)}{\overline{y}(\lambda)} - (1-a)\ln\Phi_{cc}(\tilde{x},\lambda).$$

Plugging this into (12) with $i = J = m$, we obtain the bid rent of the minority individuals who do not assimilate:

$$\Phi_{mm}(x,\lambda) = \left(\frac{y_m(\lambda) - tx}{y_m(\lambda) - t\tilde{x}}\right)^{1/(1-a)} \left(\frac{y_c(\lambda) - t\tilde{x}}{y_c(\lambda) - t\bar{x}}\right)^{1/(1-a)}.$$

[2] We can obtain the same bid rent function by deriving the utility of the group m individuals who assimilate, and plugging it into (12) with $i = m$ and $J = c$.

Hence, using (10), the housing demands are given by:

$$\begin{aligned} h_{mm}(x,\lambda) &= (1-a)\frac{(y_m(\lambda)-t\tilde{x})^{1/(1-a)}}{(y_m(\lambda)-tx)^{a/(1-a)}}\left(\frac{y_c(\lambda)-t\bar{x}}{y_c(\lambda)-t\tilde{x}}\right)^{1/(1-a)}, \\ h_{mc}(x,\lambda) &= h_{cc}(x,\lambda) = (1-a)\frac{(y_c(\lambda)-t\bar{x})^{1/(1-a)}}{(y_c(\lambda)-tx)^{a/(1-a)}}. \end{aligned}$$

The population constraints (14) are then equal to:

$$(1-\lambda)\mu = \int_0^{\tilde{x}} \frac{H}{h_{mm}(x,\lambda)}dx, \qquad \lambda\mu + 1 - \mu = \int_{\tilde{x}}^{\bar{x}} \frac{H}{h_{cc}(x,\lambda)}dx.$$

We can solve them with respect to $\tilde{x}$ and $\bar{x}$, respectively, and obtain

$$\begin{aligned} t\tilde{x} &= \left\{1-\left[\frac{H+(\lambda\mu+1-\mu)t}{H+t}\right]^{1-a}\right\} y_m(\lambda), \qquad \text{(B.1)} \\ t\bar{x} &= y_c(\lambda) - (y_c(\lambda)-y_m(\lambda))\left[\frac{H}{H+(\lambda\mu+1-\mu)t}\right]^{1-a} - y_m(\lambda)\left(\frac{H}{H+t}\right)^{1-a}. \end{aligned}$$

Plugging the above equations into (11), we obtain the indirect utility as

$$\begin{aligned} V_{mm}(\lambda) &= \ln[(y_m(\lambda)-t\tilde{x}) - \ln\frac{y_c(\lambda)-t\tilde{x}}{y_c(\lambda)-t\bar{x}} + \sigma\ln\frac{y_m(\lambda)}{\overline{y}(\lambda)}, \qquad \text{(B.2)} \\ V_{mc}(\lambda) &= \ln(y_c(\lambda)-t\bar{x}) - \delta\ln D_{mc}(\lambda) + \sigma\ln\frac{y_c(\lambda)}{\overline{y}(\lambda)}, \\ V_{cc}(\lambda) &= \ln(y_c(\lambda)-t\bar{x}) - \delta\ln D_{cc}(\lambda) + \sigma\ln\frac{y_c(\lambda)}{\overline{y}(\lambda)}. \end{aligned}$$

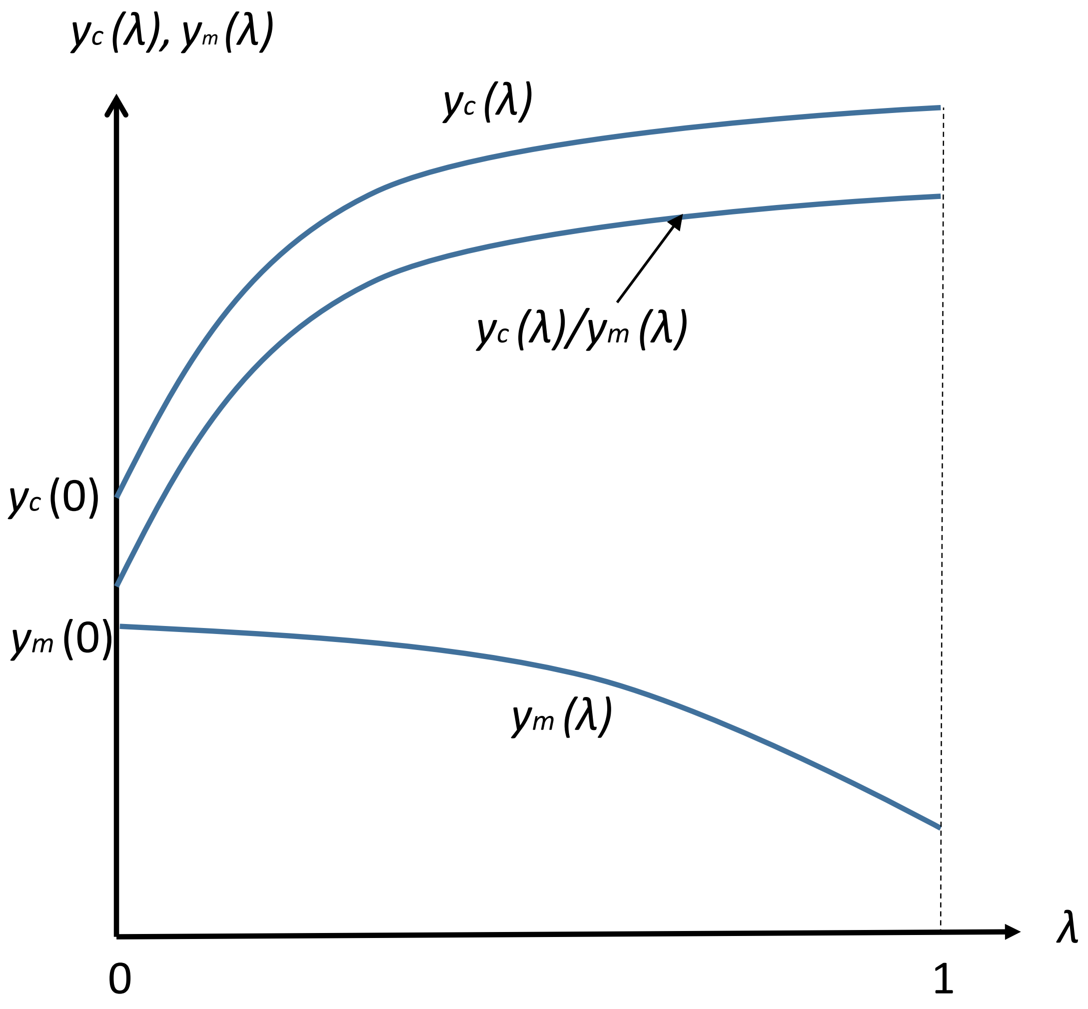


Figure 1(a): Effect of λ on incomes

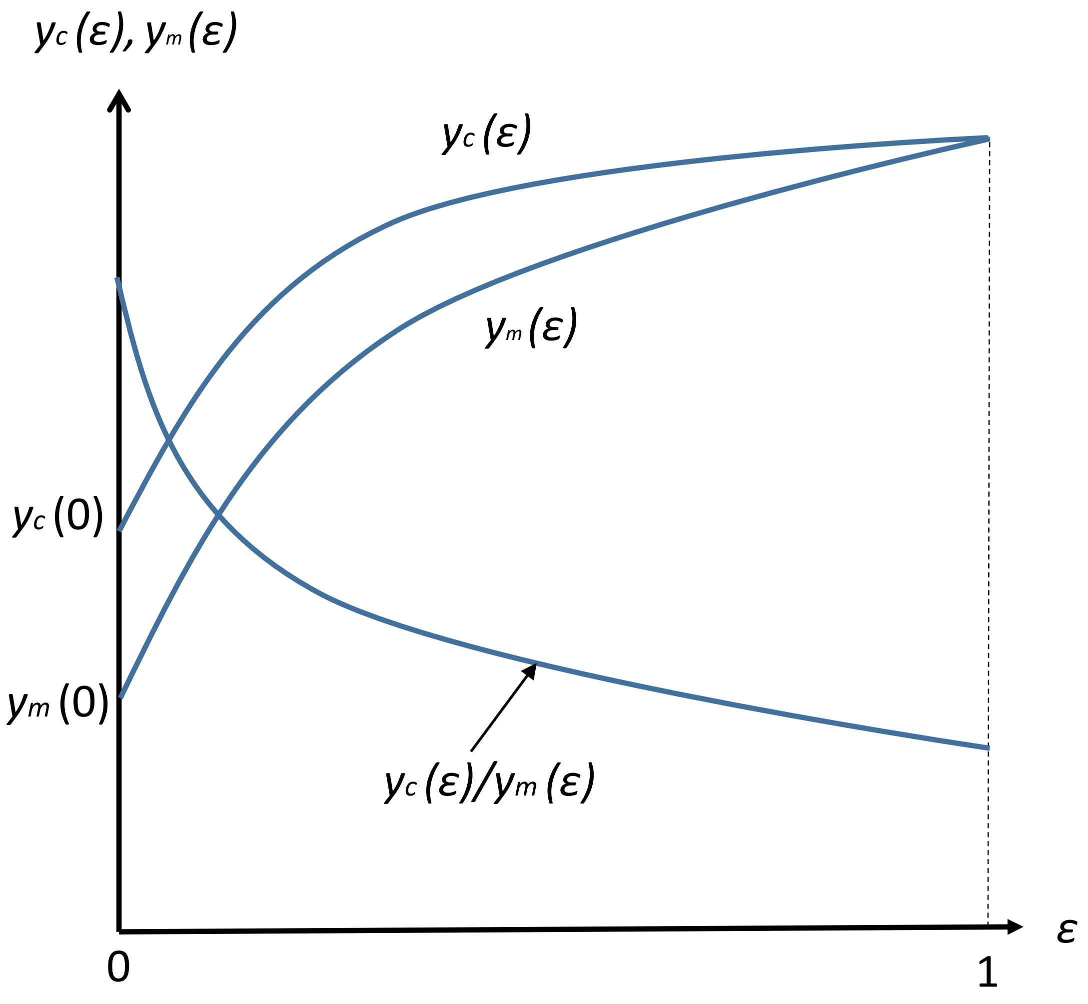


Figure 1(b): Effect of ε on incomes

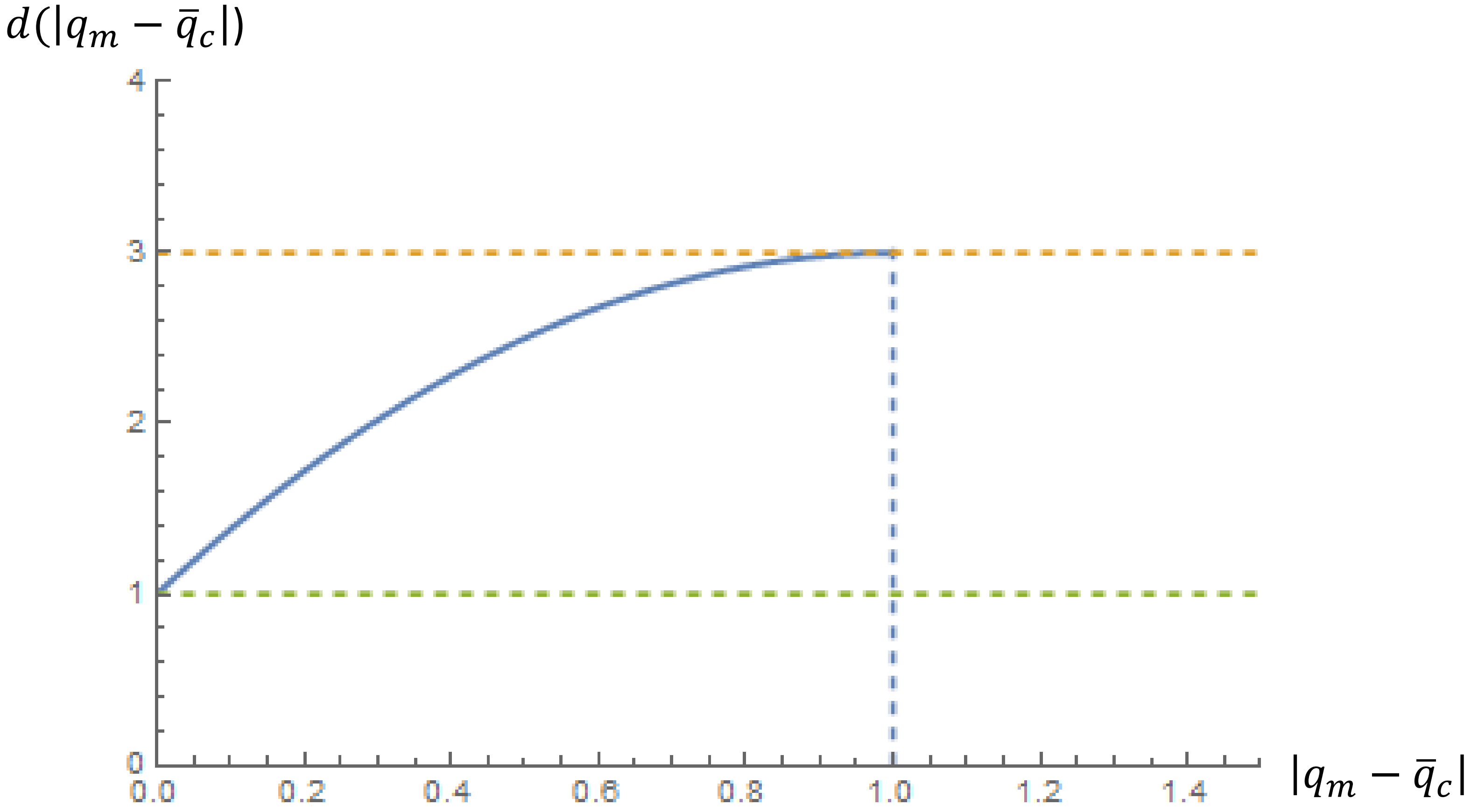


Figure 2: Perceived distance function when $d(x) = 3 - 2(x-1)^2$

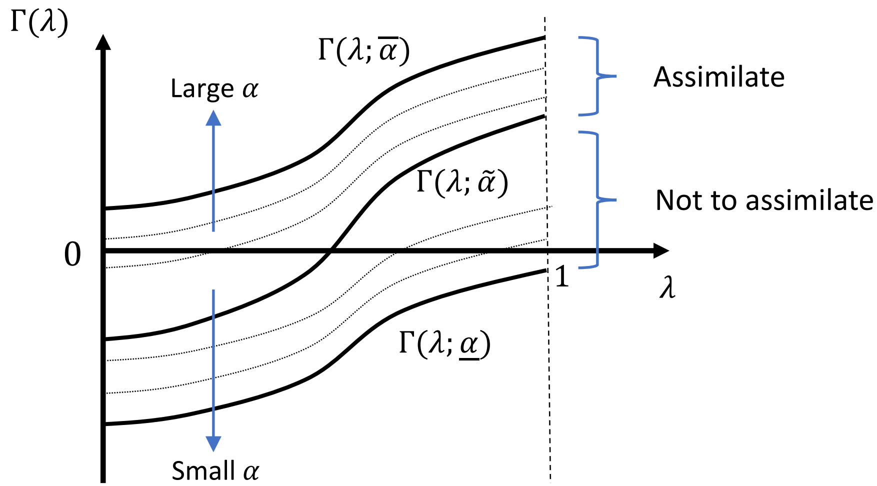
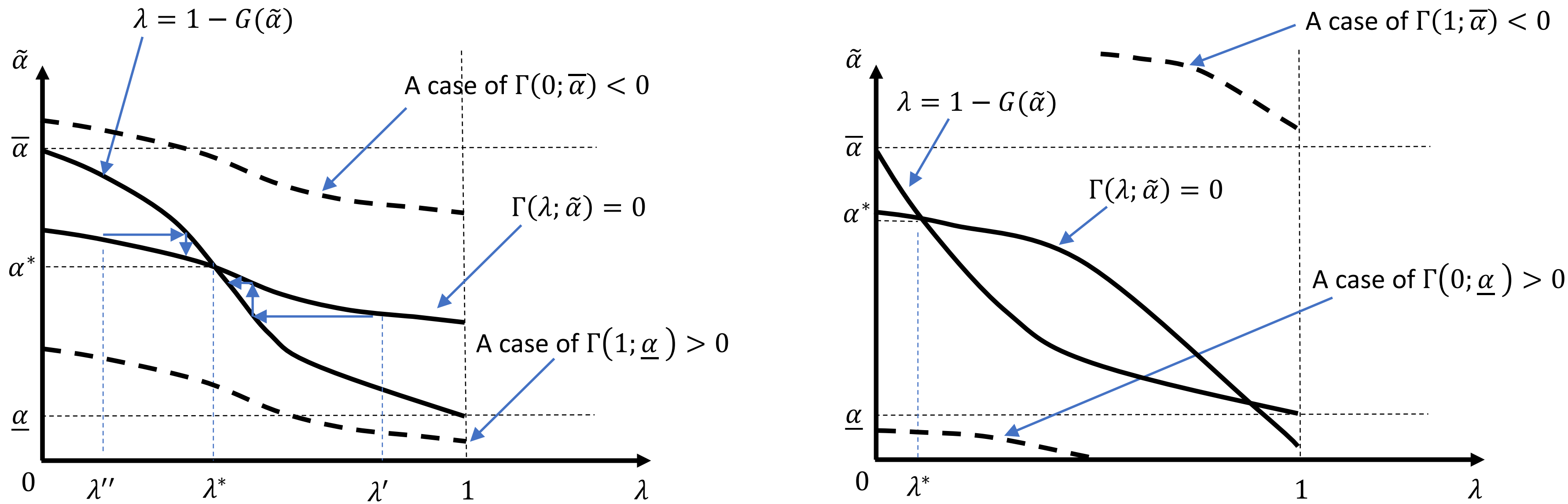


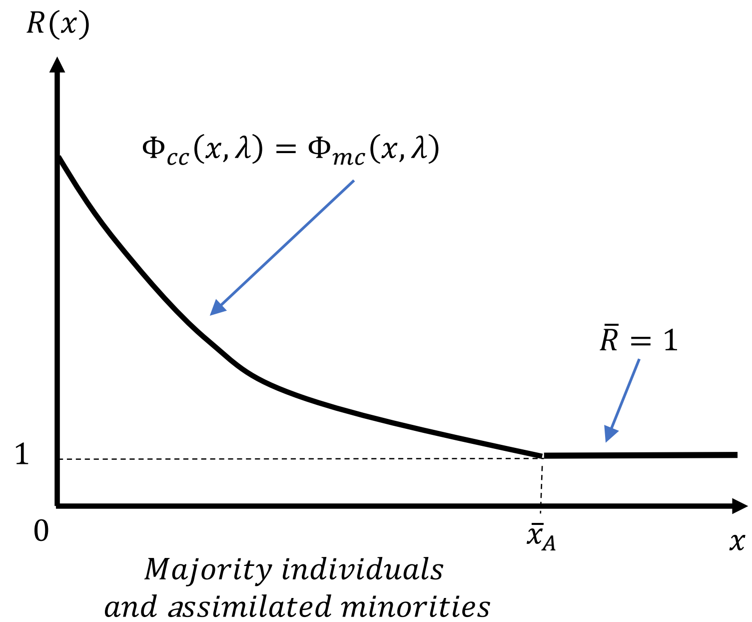
Figure 3: Assimilation decision



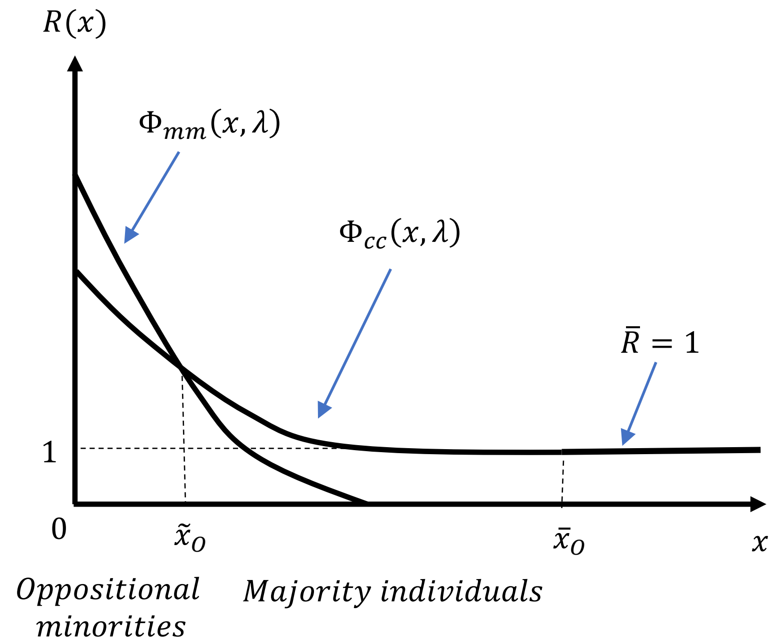
(a) The condition (6) holds true.

(b) The condition (6) does not hold true.

Figure 4: Different possible equilibria

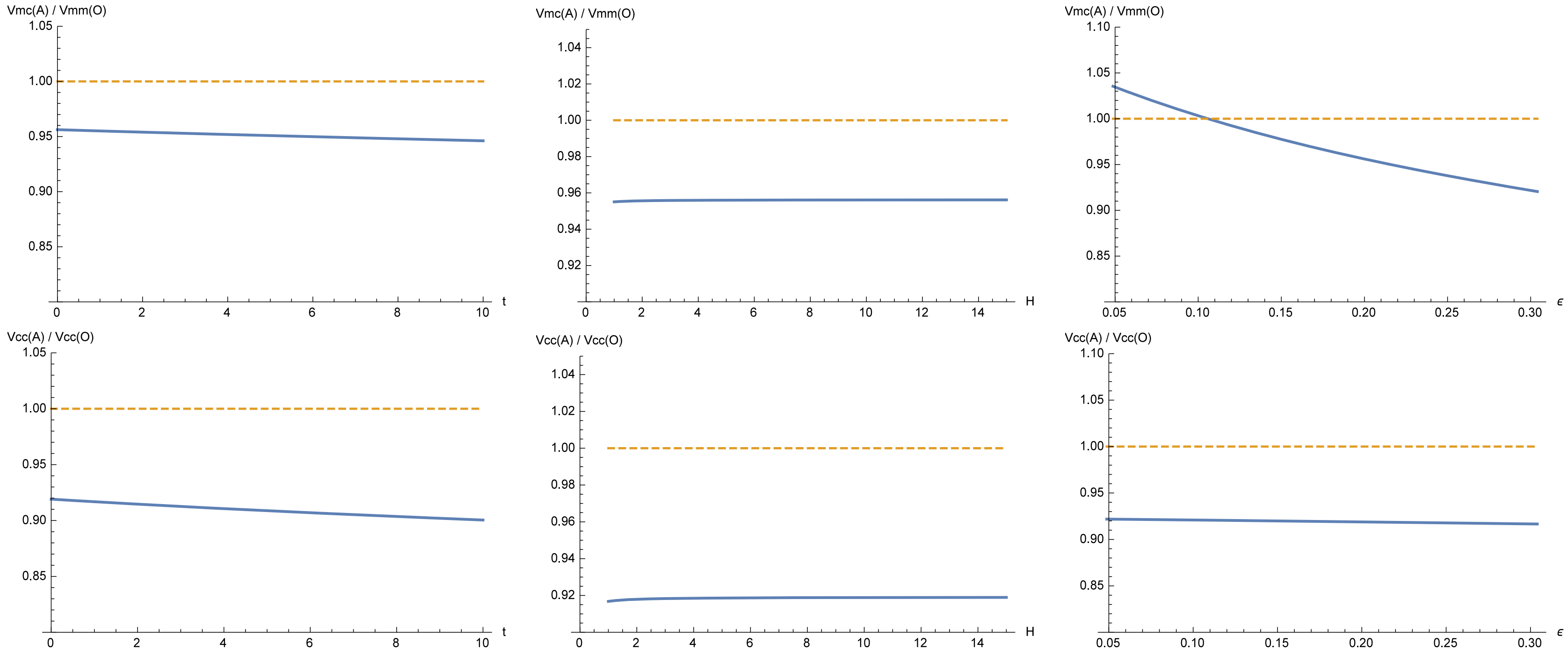


(a) Assimilation Social Identity Urban Equilibrium (ASIUE)



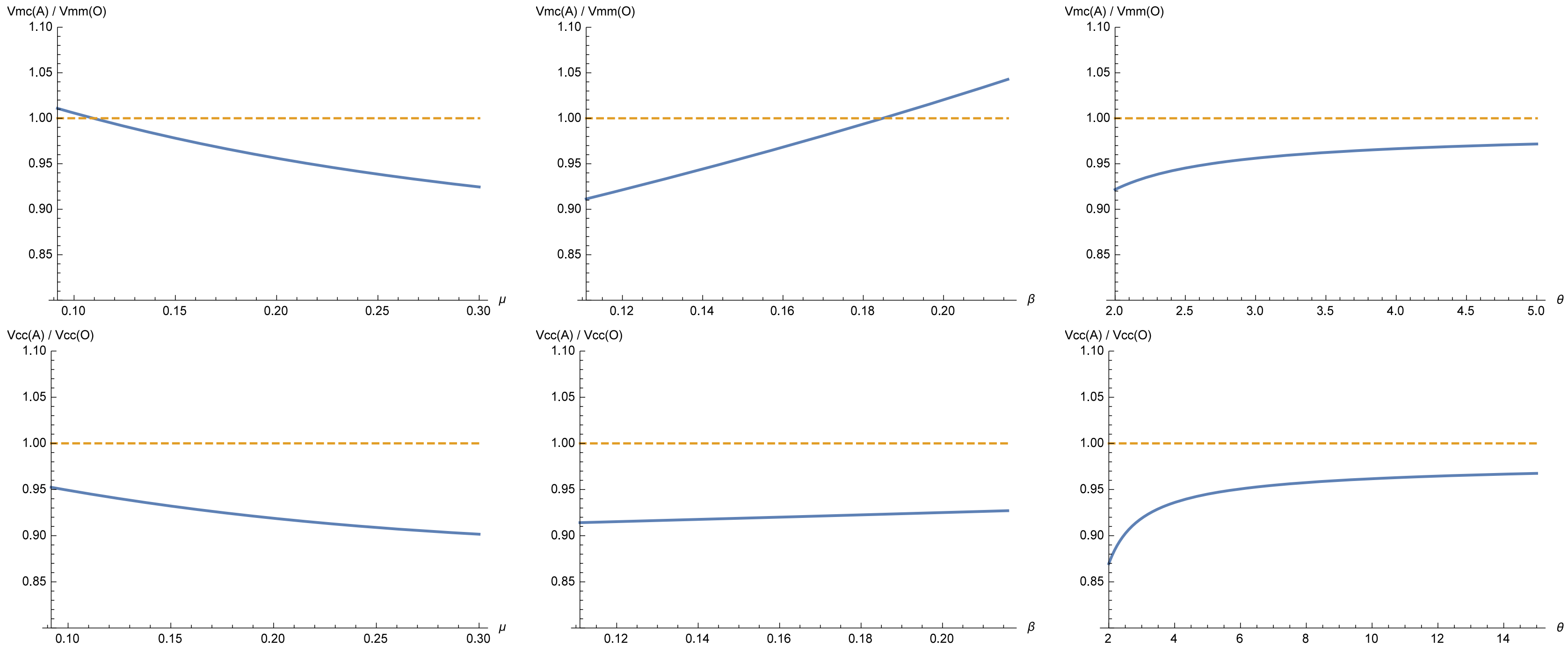
(b) Oppositional Social Identity Urban Equilibrium (OSIUE)

Figure 5: Different urban equilibrium configurations



(1) Effects of changes in t (2) Effects of changes in H (3) Effects of changes in ε

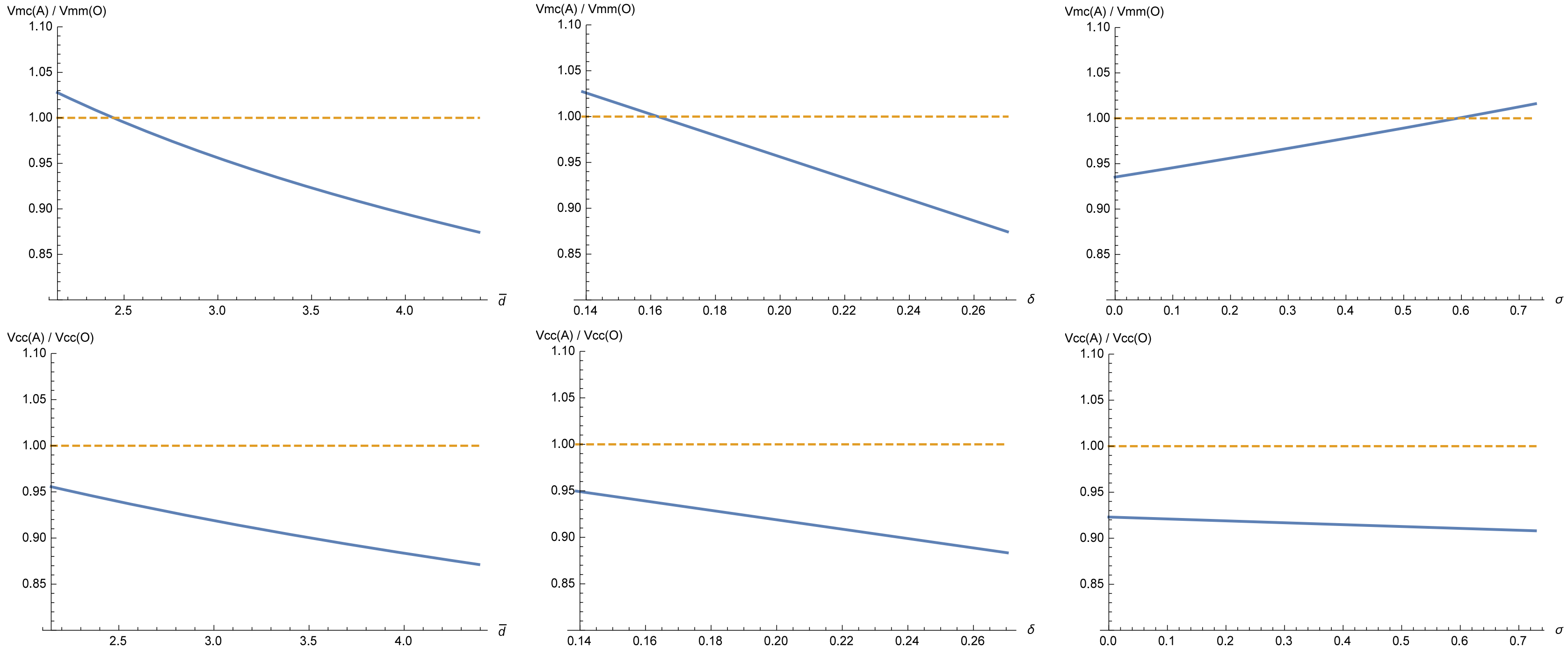
Figure 6: Utility differences between the ASIUE and the OSIUE
Notes: In the baseline case, we set $a = 0.75$, $\beta = 0.15$, $\overline{d} = 3$, $\delta = 0.2$, $\varepsilon = 0.2$, $H = 10$, $\mu = 0.2$, $\sigma = 0.2$, $t = 0.1$, and $\theta = 3$.



(4) Effects of changes in μ (5) Effects of changes in β (6) Effects of changes in θ

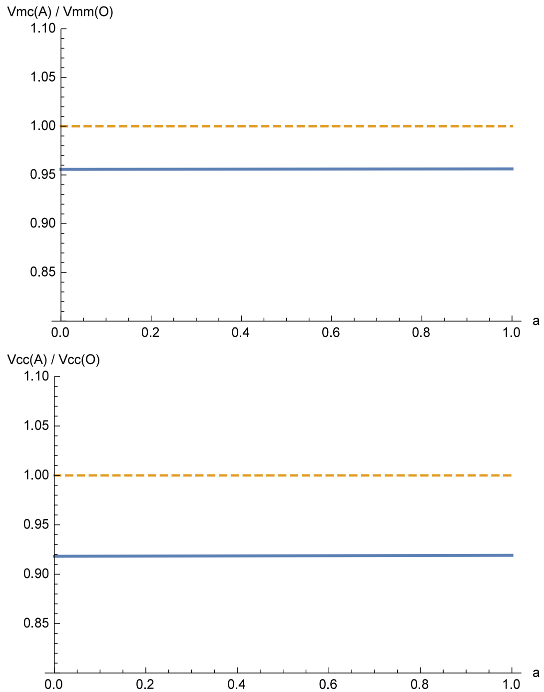
Figure 6 (cont.): Utility differences between the ASIUE and the OSIUE

Notes: In the baseline case, we set $a = 0.75$, $\beta = 0.15$, $\overline{d} = 3$, $\delta = 0.2$, $\varepsilon = 0.2$, $H = 10$, $\mu = 0.2$, $\sigma = 0.2$, $t = 0.1$, and $\theta = 3$.



(7) Effects of changes in $\overline{d}$ (8) Effects of changes in δ (9) Effects of changes in σ

Figure 6 (cont.): Utility differences between the ASIUE and the OSIUE
Notes: In the baseline case, we set $a = 0.75$, $\beta = 0.15$, $\overline{d} = 3$, $\delta = 0.2$, $\varepsilon = 0.2$, $H = 10$, $\mu = 0.2$, $\sigma = 0.2$, $t = 0.1$, and $\theta = 3$.



(10) Effects of changes in a

Figure 6 (cont.): Utility differences between the ASIUE and the OSIUE

Notes: In the baseline case, we set $a = 0.75$, $\beta = 0.15$, $\overline{d} = 3$, $\delta = 0.2$, $\varepsilon = 0.2$, $H = 10$, $\mu = 0.2$, $\sigma = 0.2$, $t = 0.1$, and $\theta = 3$.