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Abstract

This study investigates a behavioral aspect of bank runs that occur as a consequence of depositors' panic. We assume that each depositor is motivated not only by his/her financial interest but also by reciprocity. Our results show that bank-run equilibria can be eliminated by introducing minimum reciprocity, which affects depositors to the extent that it does not infringe on their financial interests. This permissive result holds even in the absence of suspension of convertibility and restrictions on depositors' right of withdrawal. To prove this result, we design a priority-based deposit contract and investigate a scenario in which depositors can always observe the bank's remaining payment capacity. We allow the bank to include a clause in the deposit contract that each depositor has an option to reserve his/her withdrawal. Even in this scenario, we can eliminate any bank-run equilibrium by considering minimum reciprocity.

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1. Introduction

This study investigates bank runs from a behavioral point of view. We assume that each depositor is motivated not only by financial interest, but also by social preferences (non-financial interests, moral sentiment) such as *reciprocity*. This study shows that we can eliminate any bank-run equilibrium by considering depositors' *minimum* reciprocity. This permissive result holds even in the absence of a third party's restrictions on depositors' right of premature withdrawal.

A bank run is likely to occur as a consequence of depositors' panic (Diamond and Dybvig, 1983) when depositors are only motivated by their financial interests. Suppose each depositor expects other depositors to prematurely withdraw money from their deposit accounts. Then, all depositors have the incentive to withdraw money, as the bank is expected to go bankrupt due to the massive withdrawals that may occur before the bank's long-term illiquid investments mature. As a result, all depositors request full withdrawals even in the absence of exceptional needs that justify such early withdrawals. This scenario represents a self-fulfilling bank run.

Fundraising becomes easier once the bank permits depositors to prematurely withdraw. However, banks are more likely to profitably invest in long-term illiquid assets. Hence, the role of financial intermediation is to eliminate or reduce the mismatch between depositors' liquidity demands and banks' investment illiquidity. Bank runs seem, in principle, inevitable if depositors are only motivated by their financial interests. In fact, the likelihood of a bank run does not depend on whether the bank is solvent in the sense that its long-term investment is profitable enough and its promise of payments to depositors is reliable.

Previous research has emphasized that restrictions of depositors' rights are essential to avoid bank runs. Diamond and Dybvig (1983) proposed a model in which each depositor cannot observe other depositors' requests of withdrawal when his/her turn comes and emphasized that the suspension of convertibility is essential to eliminate the risk of a bank run. Hence, the government should force deposits to be frozen once the total withdrawals exceed some predetermined threshold. Despite its importance, this policy has several drawbacks and requires strong commitment to overcome the time inconsistency between ex-ante and ex-post efficiency (Ennis and Keiser, 2009a). The effectiveness of this policy depends on the absence of public uncertainty regarding the aggregate shocks across depositors' liquidity demands. In the absence of this assumption, the suspension policy eliminates even bank runs that should occur for fundamental causes (Bryant, 1980). In this scenario, a bank never goes bankrupt even if it operates improperly.

In contrast, this study does not consider any suspension of convertibility. In the proposed scenario, a bank fully responds to depositors' requests based on its payment capacity. We assume that depositors are not only driven by their financial interests, but also by minimum reciprocity.

A depositor may lose confidence in a bank when its promise is not executable. In this case, by negative reciprocity, the depositor will withdraw as much money as possible from his/her deposit account until the withdrawal does not result in a financial loss. On the other hand, the depositor increases his/her confidence in the bank when its promise is executable. In this case, by positive reciprocity, the depositor will withdraw as little as possible until this withdrawal does not result in a financial loss.

In other words, we assume lexicographical preferences in that depositors are affected by reciprocity to the extent that it does not infringe on their financial interests. In the presence of multiple possible best responses, a depositor selects the minimum (maximum) withdrawal among his/her best responses whenever the bank's promise is executable (not executable).

We can show that the introduction of minimum reciprocity, along with a priority-based deposit contract design, successfully deters the occurrence of a bank run. When all depositors are motivated by minimum reciprocity, any patient depositor, who has no special use that demands liquidity, can increase the bank's long-term investment, which successfully matures, by unilaterally decreasing his/her withdrawal. We specify a deposit contract such that the bank responds to depositors' requests in a fixed order of *priority*. With this specification, any patient depositor with high priority can fully receive the increased return of the long-term investment at the maturity date by unilaterally decreasing his/her withdrawal.

The proposed model does not contemplate the suspension of convertibility or other outside policies such as deposit insurance (Cooper and Ross, 2002), interbank liquidity markets, credit lines, and lenders' last resort (Rochet and Vives, 2004).[3]

The theoretical logic behind the this result depends on the assumption that each depositor is unable to observe other depositors' withdrawal requests. Depositors sequentially request their withdrawals, and the bank sequentially responds to their requests. In a process of *sequential services*, however, any depositor may obtain information about the bank's residual capacity when his/her turn comes. Therefore, even if a depositor decreases his/her withdrawal to avoid the occurrence of a bank run, this does not necessarily allow the bank's long-term investment to successfully mature. By observing the resultant increase in the bank's remaining capacity, any impatient depositor whose priority order is lower than that of this depositor is willing to increase his/her withdrawal. The capacity-contingent withdrawal by the low-priority depositor makes bank-run equilibria more difficult to eliminate.

To overcome this difficulty, the second part of this study considers a situation in which the bank permits any depositor to have an option to *reserve* a fraction of his/her deposit for a while, rather than withdraw it immediately. By reserving his/her deposit, the depositor obtains the right to withdraw any amount up to the reserve right after all impatient depositors terminate their immediate withdrawals.

By reserving instead of decreasing their immediate withdrawal, any patient depositor can increase the share of the bank's long-term investment that successfully matures without increasing low-priority withdrawals. Importantly, because of minimum reciprocity, any patient depositor dislikes making an unnecessary reserve that results in unfairly preventing impatient depositors' withdrawals. Hence, by adding a reserve as an option in addition to immediate withdrawal, minimum reciprocity can effectively deter the occurrence of a bank run even if depositors can observe the bank's remaining capacity in the process of sequential services.

Green and Lin (2003) emphasized that banks need to control depositors' withdrawal allowances at any time according to the bank's remaining capacity.[4] In contrast, this

[3] See Tirole (2006) and Brunnermeir et al. (2013) for surveys on bank runs and financial crises.
[4] See also De Nicolo (1996), Peck and Shell (2003), Andolfatto, Nosal, and Wallace (2007), Ennis and Keister (2009a, 2009b), and Andolfatto, Nosal, and Sultanum (2017).

study does not allow banks to use any such discretionary control: instead of this discrepancy, banks permit each depositor to reserve his/her deposit as he/she likes, which effectively restricts withdrawals of low-priority depositors depending on the bank’s remaining capacity.

This study contributes to behavioral finance in the following sense. Several works in behavioral finance assume that players (depositors, traders) are irrational and make mistakes according to regular patterns of psychological biases such as loss aversion, momentum, and ambiguity aversion; see, for instance, Kahneman and Tversky (1979), Shleifer and Vishny (1997), and Caballero and Krishnamurthy (2008). Sophisticated portfolio managers strategically utilize noise traders' psychological biases to enhance their financial gains (Abreu and Brunnermeier, 2003; Matsushima, 2013a). The government utilizes libertarian-paternalism to induce the public to follow its intention (Thaler and Sunstein, 2003); see also Köszegi (2014) for recent trends in behavioral contract design.

In contrast with these works, this study focuses on the possibility that real players are motivated not only by financial interest but also by moral sentiment (social preferences). This study shows the importance of designing financial systems that motivate people to behave according to moral sentiment. In this sense, this study is also related to behavioral implementations such as Matsushima (2008a, 2008b, 2013b) and Dutta and Sen (2012), which uniquely implement a social choice function by incorporating moral sentiment such as preference for honesty into the mechanism design.[5]

There have been various attempts to apply global game techniques to the bank-run problem, which show the uniqueness of equilibria under incomplete information; see, for example, Allen and Morris (2001) and Morris and Shin (2001). By assuming the presence of a possible state in which the bank is strongly solvent even if all patient and impatient depositors request withdrawals at once, these works showed that a threshold level of fundamentals exists below which the occurrence of a bank run is the unique equilibrium outcome, while above which no bank run is the unique equilibrium outcome. In contrast, this study shows the presence of a threshold that virtually matches the boundary between the bank's executability and un-executability, without assuming such strong solvency. As

[5] See also Kartik and Tercieux (2012), Kartik et al. (2014), and Ortner (2015).

a result, this study can eliminate depositors' self-fulfilling panics and reduce the bank-run problem to whether the bank is solvent if only impatient depositors request to withdraw.

The remainder of this paper is organized as follows. Section 2 investigates a three-period model in line with Diamond and Dybvig (1983), in which each depositor cannot observe the bank's remaining capacity, and the bank responds to depositors' requests in a fixed order of priority. We introduce the *Nash equilibrium with minimum reciprocity* and show its uniqueness by demonstrating that a bank run never occurs if the bank's promise is executable. We further show that a bank run inevitably occurs if the bank's promise is far from executable (beyond a limit). Section 3 investigates a modification of the model introduced in Section 2, in which we assume that each depositor can observe the bank's remaining capacity in the process of sequential services. We permit each depositor to either reserve or immediately withdraw. By replacing the Nash equilibrium with a subgame perfect equilibrium, we extend the permissive result in Section 2 to the case of sequential services with observability. Section 4 presents our concluding remarks.

2. Simultaneous Services

This section investigates a situation in which a financial intermediary simultaneously responds to all depositors' withdrawal requests We assume that each depositor cannot observe the other depositors' withdrawal requests.

2.1. The Model

We consider a three-period model in which the bank (financial intermediary) enters into a *deposit contract* with multiple agents ($n \geq 2$ depositors) and invests their deposits in a short-term liquid asset and a long-term illiquid asset. In period 0, each agent $i \in N = \{1,...,n\}$ deposits an amount equal to 1 in the bank. The bank makes a *promise* to each agent, denoted by $x > 1$, to respond to his/her withdrawal request in period 1 by paying an amount equal to 1 per unit of deposit and to respond to his/her repayment request in period 2 by paying the monetary amount x per unit.

Along with x, the bank writes a *deposit contract* with these agents, which is defined as $c = (c^1, c^2)$, where $c^t = (c_i^t)_{i \in N}$ denotes a *payment scheme* in period $t \in \{1,2\}$, $c_i^t : [0,1]^n \to [0,\infty)$, $c_i^1(m)$ is non-decreasing in m_i, and $c_i^2(m)$ is non-increasing in m_i. A deposit contract c describes how the bank provides partial responses to agents' requests when it cannot fully respond as promised.[6]

The bank invests a fraction of its deposits rn into a short-term (liquid) asset and the remaining fraction $(1-r)n$ into a long-term (illiquid) asset, where $r \in [0,1]$. The bank's *capacity* in period 1 is given by:

(1) $$Z^1 \equiv \{r + l(1-r)\}n,$$

where $0 < l < 1$. By liquidating the entire investment, the bank can respond to agents' withdrawal requests up to Z^1. Since $l < 1$, liquidating its long-term investment is costly for the bank, while liquidating its short-term investment is costless. The bank can collect an amount equal to 1 per unit by liquidating its short-term investment, while it can collect only $l < 1$ per unit by liquidating its long-term investment. Since $l < 1$, the bank cannot fully respond when all agents request full withdrawals.

By assuming the set of depositors is finite rather than a continuum, this study implicitly excludes the presence of small depositors who are fully protected by deposit insurance. To make the problem non-trivial, we assume that there exists *no large depositor*:

(2) $$n - 1 > Z^1,$$

which implies that, even if a depositor requests no withdrawal, the bank run can occur in principle.

The agents' deposits mature in period 2. At maturation, the short-term investment yields 1, and the long-term investment yields $R > 0$ per unit.[7] The bank knows R in period 0, while depositors are informed about R in period 1, but R is not contractible.

The long-term investment is *ex-ante profitable* if its yield is greater than that of the short-term investment ($R > 1$). On the contrary, the long-term investment is *ex-post profitable* if its yield is greater than its liquidation value ($R > l$).

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[6] The details of the design of the deposit contract are explained in the next subsection.
[7] To eliminate irrelevant complexity, we assume $R \neq lx$ and $R \neq l$.

In period 1, each agent i simultaneously selects his/her *action* $m_i \in M_i \equiv [0,1]$. Agent i requests to withdraw m_i in period 1 and to receive $(1-m_i)x$ in period 2. Let $m = (m_i)_{i\in N} \in M \equiv \times_{i\in N} M_i$ denote an action profile. According to c and m, each agent i receives from the bank $c_i^1(m) \in [0,\infty)$ in period 1 and $c_i^2(m) \in [0,\infty)$ in period 2. His/her payoff is given by:

$$u_i(m) = c_i^1(m) + \alpha_i c_i^2(m),$$

where $\alpha_i \in \{0,1\}$. An action profile $m \in M$ is said to be a *Nash equilibrium* of the game in period 1 if for every $i \in N$:

$$u_i(m) \geq u_i(m_i', m_{-i}) \text{ for all } m_i' \in [0,1].$$

An agent i is said to be *patient* (*impatient*) if $\alpha_i = 1$ ($\alpha_i = 0$, respectively).[8] Let:

$$\overline{N} \equiv \{i \in N \mid \alpha_i = 0\}, \quad \overline{N}^c \equiv \{i \in N \mid \alpha_i = 1\},$$

$$\overline{n} \equiv \left|\overline{N}\right|, \text{ and } \overline{n}^c \equiv \left|\overline{N}^c\right| = 1 - \overline{n}.$$

To eliminate irrelevant complexity, we assume:

(3) $$\overline{n} = rn.$$

From (3), by *only* liquidating its short-term investment, the bank can respond to all impatient agents' requests of withdrawal. From (1) and (3), we obtain:

$$Z^1 = \overline{n}^c l + \overline{n}.$$

2.2. Priority

We specify c^1 as a *priority* scheme in period 1, according to which the bank responds to agent's withdrawal requests in a certain order of priority, that is, from agent 1 to agent n. To be more precise, if the bank cannot fully respond to all agents' withdrawal requests, that is,

[8] The mechanism implied by a deposit contract is not a direct mechanism in this study because each agent's type is either "patient" or "impatient," but the action space is a continuum. We need an indirect mechanism design to derive the uniqueness of the equilibrium.

$$Z^1 < \sum_{i \in N} m_i \,,$$

then, there exist $h \in N$ and $\hat{m}_h \equiv Z^1 - \sum_{i<h} m_i \in (0, m_h]$, such that:

$$c_i^1(m) = m_i \qquad \text{for all } i < h,$$

$$c_i^1(m) = 0 \qquad \text{for all } i > h,$$

$$c_h^1(m) = \hat{m}_h .$$

If the bank can fully respond to all agents' withdrawal requests, that is,

$$Z^1 \geq \sum_{i \in N} m_i \,,$$

then:

$$c_i^1(m) = m_i \quad \text{for all } i \in N .$$

This priority-based contract design implies *no suspension of convertibility* in period 1 as the bank responds to agents' requests as much as possible:

$$\sum_{i \in N} c_i^1(m) = \min[\sum_{i \in N} m_i, Z^1].$$

The bank never pays each agent more than his/her request:

$$c_i^1(m) \leq m_i \quad \text{for all } i \in N .$$

We denote by $Z^2(m)$ the bank's *capacity* in period 2 when agents select $m \in M$:

$$[\sum_{i \in N} m_i \geq Z^1] \Leftrightarrow [Z^2(m) = 0],$$

$$[\sum_{i \in N} m_i \leq rn] \Rightarrow [Z^2(m) = rn - \sum_{i \in N} m_i + R(1-r)n],$$

$$[rn < \sum_{i \in N} m_i < Z^1] \Rightarrow [Z^2(m) = \frac{Z^1 - \sum_{i \in N} m_i}{l} R].$$

We specify c^2 as a *priority* scheme in period 2; if:

$$Z^2(m) < x \sum_{i \in N} (1 - m_i),$$

then, there exist $h \in N$ and $\hat{m}_h \in [m_h, 1]$, such that:

$$c_i^2(m) = x(1 - m_i) \qquad \text{for all } i < h,$$

$$c_i^2(m) = 0 \qquad \text{for all } i > h,$$

$$c_h^2(m) = x(1-\hat{m}_h),$$

where $\hat{m}_h$ is specified by:

$$\{\sum_{i<h}(1-m_i)+1-\hat{m}_h\}x = Z^2(m).$$

If

$$Z^2(m) \geq x\sum_{i\in N}(1-m_i),$$

then,

$$c_i^2(m) = x(1-m_i) \text{ for all } i \in N.$$

This priority-based contract design implies *no suspension of convertibility* in period 2:

$$\sum_{i\in N} c_i^2(m) = \min[x\sum_{i\in N}(1-m_i), Z^2(m)].$$

The bank never pays each agent more than his/her request in period 2:

$$c_i^2(m) \leq x(1-m_i) \text{ for all } i \in N.$$[9]

2.3. Bank run

An action profile $m \in M$ is said to i*nduce a bank run* if the bank is insolvent in period 1, that is, the sum of all agent's withdrawal requests is not less than the bank's capacity in period 1:

$$Z^2(m) = 0.$$

Let us denote by $\bar{m} \equiv (1,...,1)$ the action profile according to which all agents request full withdrawals, and the bank run occurs.

Proposition 1: *The action profile* $\bar{m}$ *is a Nash equilibrium.*

[9] We assume that the order of priority is equivalent between period 1 and period 2. This equivalence assumption is essential in our study. See also De Nicolo (1996), which considers deposit contracts in which the order of priority is reversed.

Proof: According to $\bar{m}$, any agent may receive nothing in period 2 irrespective of his/her action selection because he/she is not a large depositor (i.e., inequality 2), and the other agents request full withdrawals. This implies that $\bar{m}$ is a Nash equilibrium.

Q.E.D.

We define an action profile $m^* \in M$ as:

$$m_i^* = 1 \qquad \text{if } \alpha_i = 0,$$

$$m_i^* = 0 \qquad \text{if } \alpha_i = 1.$$

According to m^*, only impatient agents request withdrawals in period 1, and the bank run never occurs.

A promise x is said to be *executable* if $R \geq x$. In this case, the bank can fully respond to all agents' requests in both periods 1 and 2 if their requests are in line with m^*. This implies that m^* achieves the Pareto-dominant outcome: any impatient agent receives 1 in period 1, and any patient depositor receives x in period 2. This automatically implies a Nash equilibrium.

Proposition 2: *If* $R \geq x$, $m^* \in M$ *is a Nash equilibrium.*

2.4. Quasi-Executability

Fix an arbitrary $q \in N$, and define an action profile m^q as follows:

$$m_i^q = 1 \qquad \text{for all } i \in \bar{N},$$

$$m_i^q = 0 \qquad \text{if } i \leq q \text{ and } i \in \bar{N}^c,$$

$$m_i^q = 1 \qquad \text{if } i > q \text{ and } i \in \bar{N}^c.$$

According to m^q, any impatient agent requests full withdrawal, any patient agent whose priority order is not lower than q requests no withdrawal, and any patient agent whose priority order is lower than q requests full withdrawal. Note that $m^* = m^n$.

A promise x is said to be *quasi-executable* if there exists $q \in \bar{N}^c$ such that m^q is a Nash equilibrium. It is clear from the Nash equilibrium properties that if x is quasi-executable, that is, if there exists $q \in \bar{N}^c$ such that m^q is a Nash equilibrium, then such q is unique, and $q = n$ must hold. Hence, x is quasi-executable if and only if m^* is a Nash equilibrium.

Note that if x is executable, then it is automatically quasi-executable. However, x is not necessarily executable even if it is quasi-executable. If x is quasi-executable but $R < x$, then, the patient agent whose priority order is the lowest among all patient agents fails to receive x in period 2.

Proposition 3: *If x is quasi-executable, m^* is the only Nash equilibrium that does not induce the bank run. If x is not quasi-executable, any Nash equilibrium induces the bank run.*

Proof: Consider an arbitrary Nash equilibrium $m \in M$ that does not induce the bank run. Note that any impatient agent prefers to withdraw as much as possible from his/her deposit. Since there is no bank run, he/she can withdraw more money by increasing the withdrawal associated with his/her action. Hence,

$$m_i = 1 \qquad \text{if } \alpha_i = 0.$$

Any patient agent whose priority order is sufficiently high has the incentive to decrease his/her action because he/she prefers receiving $x > 1$ in period 2 rather than 1 in period 1. In contrast, any patient agent whose priority order is sufficiently low has the incentive to increase his/her action because he/she may receive nothing in period 2, and no bank run occurs. From these observations, we deduce that there exists a patient agent $\tilde{q} \in \{1,...,n\}$ such that, for every $i \in \bar{N}^c$:

$$m_i = 0 \qquad \text{if } i < \tilde{q},$$

$$m_i = 1 \qquad \text{if } i > \tilde{q}.$$

Note that agent $\tilde{q}$ gains Δ in period 1 and loses $\frac{R}{l}\Delta$ in period 2 by increasing the withdrawal associated with his/her action by Δ. If $R < l$, then, $\Delta > \frac{R}{l}\Delta$: the depositor has the incentive to increase the withdrawal associated with his/her action, which implies $m_{\tilde{q}} = 1$. If $R > l$, then, $\Delta < \frac{R}{l}\Delta$: the depositor has the incentive to decrease the withdrawal associated with his/her action, which implies $m_{\tilde{q}} = 0$. Hence, any Nash equilibrium m that does not induce the bank run must have a patient agent $\bar{q}$ ($\leq \tilde{q}$) such that $m = m^{\bar{q}}$. This implies that x must be quasi-executable, and $m = m^*$.

Q.E.D.

The bank run inevitably occurs whenever the bank's promise is not quasi-executable. If it is quasi-executable, m^* is a Nash equilibrium that deters the bank run and achieves the desired outcome. The action profile m^* is the only Nash equilibrium that deters the bank run, but, as Proposition 1 indicates, bank-run equilibria exist even if the promise is quasi-executable.

2.5. Minimum Reciprocity

To eliminate bank-run equilibria in the case of quasi-executability and guarantee the uniqueness of the equilibrium, we introduce a refinement of equilibrium from a behavioral point of view. We assume that an agent has a bad feeling regarding the bank when he/she is informed that the bank's promise is not executable: the depositor attempts to liquidate the bank by withdrawing the *maximal* amount among all his/her best responses as a sign of *negative reciprocity*. On the other hand, the agent has a good feeling about the bank when he/she is informed that the bank's promise is executable: the depositor attempts to support the bank by withdrawing the *minimal* amount among all his/her best responses as a sign of *positive reciprocity*. An action profile $m \in M$ is said to be a *Nash equilibrium with minimum reciprocity* if it is a Nash equilibrium, and for every $i \in N$ and $m_i' \in M_i$:

$$[R \geq x \text{ and } u_i(m) = u_i(m_i', m_{-i})] \Rightarrow [m_i \leq m_i'],$$

$$[R < x \text{ and } u_i(m) = u_i(m_i', m_{-i})] \Rightarrow [m_i \geq m_i'].$$

The following theorem shows its *unique* implementation in a Nash equilibrium with minimum reciprocity, which strongly supports the statement that *the bank run never occurs if the bank's promise is executable, while the bank run inevitably occurs if the bank's promise is not quasi-executable.*

Theorem 4: *If x is executable, m^* is the unique Nash equilibrium with minimum reciprocity. If x is not quasi-executable, $\bar{m}$ is the unique Nash equilibrium with minimum reciprocity.*

Proof: Suppose that x is executable. Provided that other agents play m^*, any patient agent i, who selects $m_i^* = 0$, decreases his/her payoff by increasing the withdrawal associated with his/her action because of $x > 1$. Any impatient agent i, who selects $m_i^* = 1$, decreases his/her payoff by decreasing the withdrawal associated with his/her action since the bank can fully respond to his/her withdrawal request. Hence, m^* is a Nash equilibrium with minimum reciprocity.

Consider an arbitrary Nash equilibrium $m \neq m^*$. From Proposition 3, this equilibrium induces the bank run, that is, $Z^2(m) = 0$. Since x is executable, if it satisfies (positive) reciprocity, $c_i^1(m) = m_i$ must hold for all $i \in N$, that is, $\sum_{i \in N} m_i = Z^1$. Since $R > x > l$, the patient agent whose priority order is the highest among all patient agents has the incentive to decrease the withdrawal associated with his/her action by a positive amount $\Delta > 0$: he/she loses Δ in period 1 but gains $\min[y, \frac{R}{l}]\Delta > \Delta$ in period 2, which contradicts the Nash equilibrium property. Hence, any Nash equilibrium with minimum reciprocity does not induce the bank run. From Proposition 3, if x is executable, then, m^* is the unique Nash equilibrium with minimum reciprocity.

Suppose that x is not quasi-executable. From Proposition 3, any Nash equilibrium with minimum reciprocity induces the bank run. From Propositions 1 and the above definitions, $\bar{m}$ is a Nash equilibrium with minimum reciprocity because it is a Nash equilibrium and all agents select their highest withdrawals. Suppose that an action profile m induces the bank run but $m \neq \bar{m}$: there exists an agent $i \in N$ such that $m_i < 1$. Note that his/her receipts never decrease when he/she changes his/her action from m_i to $\bar{m}_i = 1$. This contradicts negative reciprocity. Hence, $\bar{m}$ is the unique Nash equilibrium with minimum reciprocity.

Q.E.D.

2.6. Remarks

Remark 1: If x is quasi-executable but not executable, both $\bar{m}$ and m^* are Nash equilibria and satisfy negative reciprocity. Note that, in this case, m^* also satisfies positive reciprocity, while $\bar{m}$ does not satisfy positive reciprocity. Hence, if we modify the definition of a Nash equilibrium with minimum reciprocity by replacing "executable" with "quasi-executable," we can prove in the same manner as in Theorem 4 that m^* *is the unique Nash equilibrium with minimum reciprocity if* x *is quasi-executable, while* $\bar{m}$ *is the unique Nash equilibrium with minimum reciprocity if* x *is not quasi-executable.*

Remark 2: The action profile $\bar{m}$ is a Nash equilibrium, but it is not a Nash equilibrium with minimum reciprocity when the bank's promise is executable. According to $\bar{m}$, any patient agent i hesitates to decrease his/her request for withdrawal below his/her receipt $c_i^1(\bar{m})$ because this reduction does not raise the bank's capacity in period 2 but increases the other agents' receipts in period 1. Theorem 4 indicates that the requirement of positive reciprocity successfully eliminates such an obstacle.

Remark 3: Our priority-based contract design implies heterogeneity across depositors because the bank raises funds by issuing different subordinated bonds for each depositor. Theorem 4 depends on this heterogeneity. To clarify this point, let us replace priority with *neutrality*: for every $m \in M$ and $i \in N$:

$$c_i^1(m) = \min[k^1(m), m_i],$$

$$c_i^2(m) = x \min[k^2(m), 1 - m_i],$$

where $k^1(m)$ and $k^2(m)$ are specified as follows:

$$\sum_{i \in N} \min[k^1(m), m_i] = Z^1 \qquad \text{if } \sum_{i \in N} m_i \geq Z^1,$$

$$k^1(m) = \max_{i \in N} m_i \qquad \text{if } \sum_{i \in N} m_i < Z^1,$$

$$x \sum_{i \in N} \min[k^2(m), 1 - m_i] = Z^2(m) \qquad \text{if } x \sum_{i \in N} (1 - m_i) \geq Z^2(m),$$

$$k^1(m) = \max_{i \in N} (1 - m_i) \qquad \text{if } x \sum_{i \in N} (1 - m_i) < Z^2(m).$$

Assume $n > R$. With this replacement, $\tilde{m} \equiv (\frac{Z^1}{n}, \ldots, \frac{Z^1}{n})$ becomes a Nash equilibrium with minimum reciprocity even if $R > x$. Agent i has no incentive to decrease the withdrawal amount associated with his/her action: by decreasing $\tilde{m}_i$ to $\tilde{m}_i - \Delta$, he/she loses Δ in period 1 and receives only $R\Delta/n < \Delta$ in period 2: the other agents receive the remaining amount $\frac{(n-1)R\Delta}{n}$.

Remark 4: Our priority-based contract design makes each depositor's receipt insensitive to his/her request when the bank run is about to occur. This contrasts with a deposit contract design based on *proportionality*, which always makes each depositor's receipt sensitive to his/her request. However, Theorem 4 depends on this insensitivity. To clarify this point, let us replace priority with this proportionality; for every $m \in M$ and $i \in N$:

$$c_i^1(m) = \min[\frac{m_i}{\sum_{j \in N} m_j} Z^1, m_i],$$

$$c_i^2(m) = \min[\frac{1 - m_i}{\sum_{j \in N} (1 - m_j)} Z^2(m), (1 - m_i)x].$$

With this replacement, $\bar{m}$ becomes a Nash equilibrium with minimum reciprocity even if $R > x$. Since $c_i^1(m)$ is strictly increasing in m_i, the requirement of reciprocity becomes irrelevant to agent i's incentive in this case: the Nash equilibrium automatically implies Nash equilibrium with minimum reciprocity.

Remark 5: Several experimental works support the relevance for strategic behavior of social preferences such as reciprocity. An experimental subject sometimes retaliates against others at the cost of his/her financial interest, depending on the context; see, for instance, Fehr and Gächter (2000), Camerer (2003), and Sobel (2005).[10]

In contrast, this study does not assume such strong reciprocal motives. We allow reciprocity to affect an agent's motive only when there exist multiple best responses and he/she selects one among them.

This point is related to Matsushima (2008a, 2008b, 2013) and Dutta and Sen (2012), who investigate the unique implementation of a social choice function by introducing a small (or lexicographically low-ordered) cost of dishonest announcements. This tiny non-financial motive plays a decisive role in eliminating unwanted equilibria. Subsequently, Ohashi (2016) incorporates such behavioral aspects into the problem of eliminating bank runs. Ohashi designed a deposit contract as an abstract indirect mechanism that allows the bank to restrict the depositors' right of withdrawal contingent on their messages.

These works commonly considered preferences for honesty in that a player (depositor) prefers announcing honestly as along as this honesty does not significantly impair his/her financial gain. Instead of such preferences, this study considers reciprocal motives regarding the soundness of the bank's management. In this case, whether a depositor is motivated by positive reciprocity or negative reciprocity depends on whether the bank operates properly or improperly, in other words, whether its promise is executable or not. Hence, it is crucial in this study that depositors are well informed about the bank's executability in period 1. If depositors are not informed about it and are always motivated by negative reciprocity, the bank-run equilibrium $\bar{m}$ is always the unique equilibrium. If depositors are always motivated by positive reciprocity, there exists no equilibrium when the bank's promise is not quasi-executable.

[10] See also the important experimental works by Cohn et al. (2014) and Dufwenberg and Rietzke (2016), showing that experimental subjects who play roles in financial intermediation are significantly motivated by non-financial interests such as reciprocity and (dis)honesty, contingent on business cultures.

3. Sequential Services

This section considers a situation in which agents sequentially request their withdrawals, and the bank sequentially responds to their requests from agent 1 to agent n in order of priority. We assume that any agent can observe how much the agents with higher priority withdrew, thus acknowledging the bank's remaining capacity when his/her turn comes. Hence, each agent can make his/her request contingent on this observation.

Because of this contingency, even if a patient agent decreases his/her request for withdrawal, the bank's capacity in period 2 does not necessarily increase: an impatient agent whose priority order is lower than that of this patient agent may be willing to increase his/her request for withdrawal, making the bank run more likely to occur.

Consider a situation in which $n = 4$, agents 1 and 2 are patient, agents 3 and 4 are impatient, $r = 1/2$, $l = 1/2$, and x is executable ($R > x$). Consider $m = (1,1,1,0)$. Note that this action profile induces the bank run because $Z^1 = \sum_{i \in N} m_i$ ($= 3$). Importantly, this bank run is attained as a subgame perfect equilibrium: whenever a patient agent decreases his/her withdrawal request, impatient agent 4, who observes this reduction, is willing to increase his/her request for withdrawal. This bank run is consistent with positive reciprocity: any agent's reduction in his/her request of withdrawal decreases his/her utility.

3.1. Immediate Withdrawal and Reserve

We propose an additional device to avoid the bank run as follows. Consider the above-mentioned example. We assume that each agent i, who selects $m_i \in [0,1]$, is required to determine whether to *immediately* withdraw m_i or *reserve* m_i up to the end of period 1. By reserving, he/she can withdraw any amount d_i up to m_i at the end of period 1.

An impatient agent prefers immediate withdrawal to reserve, while a patient agent weakly prefers reserve to immediate withdrawal. By reserving his/her entire deposit but selecting no withdrawal at the end of period 1, the agent can receive the promised payment x in period 2 without increasing agent 4's withdrawal. In fact, this is a subgame perfect equilibrium outcome such that agents select $m = (1,1,1,0)$ in period 1, patient agents 1 and 2 receive x in period 2, impatient agent 3 immediately withdraws his/her entire deposit, and impatient agent 4 withdraws nothing. This outcome surely avoids the bank run, but it is not satisfactory with respect to welfare: since $R > x$, impatient agent 4 should have withdrawn his/her entire deposit without harming the other agents' welfare.

To overcome this inefficiency, we extend the interpretation of positive reciprocity in the following manner. Suppose that patient agent 1 selects $m_1 = 0$ instead of $m_1 = 1$. In response to agent 1's selection change, impatient agent 4 increases his/her request from no withdrawal (0) to full withdrawal (1). For patient agent 2, who reserves his/her entire deposit and withdraws nothing at the end of period 1, the entire long-term investment $(1-r)n = 2$ successfully matures even if agent 1 changes his/her selection in this manner. This, along with $R > x$, implies that both patient agents can receive the promised payment x in period 2. In fact, the action profile $m = (0,0,1,1)$ is the only profile that is consistent with the subgame perfect equilibrium property. This action profile induces no bank run and is even Pareto-dominant because both patient agents 1 and 2 can receive x in period 2, and both impatient agents 3 and 4 can receive 1 in period 1.

3.2. The Model

This section investigates the following modification of the three-period model addressed in Section 2. We decompose period 1 into $n+1$ stages (i.e., stage 1, stage 2, …, stage n, and stage $n+1$). Each agent $i \in N$ selects his/her action $m_i \in [0,1]$ at stage i. Agent i can observe $m^{i-1} \equiv (m_1,...,m_{i-1})$ when his/her turn comes. Agent i also decides whether to withdraw m_i immediately, or reserve it up to stage $n+1$, at stage i.

An impatient agent prefers immediate withdrawal to reserve, while a patient agent is (almost) indifferent between immediately withdrawing or reserving. To simplify the analysis, we assume that an impatient agent decides to immediately withdraw, while a patient agent decides to reserve. Since impatient agents obtain no benefit from the payment in period 2 (also at stage $n+1$), we further assume that any impatient agent i closes his/her deposit account at stage i right after his/her immediate withdrawal.[11]

Since the bank cannot respond beyond its capacity, impatient agent $i's$ immediate withdrawal m_i is restricted to be not greater than the bank's remaining capacity, that is:

$$\max[0, Z^1 - \sum_{j=1}^{i-1} m_j].$$

Since an impatient agent prefers immediately withdrawing as much money as possible, we assume that:

$$m_i = \max[0, Z^1 - \sum_{j=1}^{i-1} m_j] \text{ for all } i \in \overline{N}.$$

The bank responds to impatient agents' withdrawal requests by:

$$c_i^1(m,d) = m_i \text{ for all } i \in \overline{N}.$$

In contrast with impatient agents, we assume that a patient agent can select any level of reserve:

$$m_i \in [0,1] \text{ for all } i \in \overline{N}^c.$$

Let $M^i \subset \times_{j=1}^{i} M_j$ denote the set of possible action profiles up to stage i. Note that:

$$[m^i \in M^i] \Leftrightarrow [m_h = \max[0, Z^1 - \sum_{j=1}^{h-1} m_j] \text{ if } h \in \overline{N} \text{ and } h \le i].$$

At stage $n+1$, a patient agent $i \in \overline{N}^c$ is required to select his/her withdrawal request $d_i \in [0, m_i]$. Let $d \equiv (d_i)_{i \in \overline{N}^c}$. The bank responds to patient agents' withdrawal requests in order of priority; if the bank cannot fully respond to all agents' withdrawal requests:

[11] We can make the same argument even if we permit impatient depositors to have a slight preference for payment at stage $n+1$.

$$Z^1 - \sum_{i \in \bar{N}} m_i < \sum_{i \in \bar{N}^c} d_i \ .$$

Then, there exist $h \in \bar{N}^c$ and $\hat{d}_h \equiv Z^1 - \sum_{i \in \bar{N}} m_i - \sum_{i \in \bar{N}^c, i<h} d_i \in [0, d_h]$ such that for every $i \in \bar{N}^c$:

$$c_i^1(m,d) = d_i \quad \text{if} \quad i < h,$$

$$c_i^1(m,d) = 0 \quad \text{if} \quad i > h,$$

$$c_h^1(m,d) = \hat{d}_h .$$

If the bank can fully respond to all withdrawal requests,

$$Z^1 - \sum_{i \in \bar{N}} m_i \geq \sum_{i \in \bar{N}^c} d_i \ .$$

Then,

$$c_i^1(m,d) = d_i \quad \text{for all} \quad i \in \bar{N}^c .$$

Since $\sum_{i \in N} c_i^1(m,d) \geq \bar{n}$, the entire short-term investment is liquidated before stage $n+1$. Hence, the remaining capacity of the bank in period 2 is given by:

$$Z^2(m,d) = \{Z^1 - \sum_{i \in N} c_i^1(m,d)\} \frac{R}{l} .$$

The bank responds to patient agents' requests in period 2 in order of priority; if $Z^2(m,d) = 0$, then

$$c_i^2(m,d) = 0 \quad \text{for all} \quad i \in N .$$

If

$$0 < Z^2(m,d) < x \sum_{i \in \bar{N}^c} (1 - d_i),$$

then there exist $h \in \bar{N}^c$ and $\tilde{d}_h \in [d_h, 1]$ such that for every $i \in \bar{N}^c$:

$$c_i^2(m,d) = x(1-d_i) \qquad \text{for all} \quad i < h,$$

$$c_i^2(m,d) = 0 \qquad \text{for all} \quad i > h,$$

$$c_h^2(m,d) = x(1-\tilde{d}_h),$$

where $\tilde{d}_h$ is specified by:

$$x\{\sum_{i\in\bar{N}^c, i<h}(1-m_i)+1-\tilde{d}_h\}=Z^2(m,d).$$

If

$$Z^2(m,d)\geq x\sum_{i\in\bar{N}^c}(1-d_i),$$

then,

$$c_i^2(m,d)=x(1-d_i) \quad \text{for all} \quad i\in\bar{N}^c.$$

The resultant utility for each patient agent $i\in\bar{N}^c$ is given by

$$u_i(m,d)=c_i^1(m,d)+c_i^2(m,d).$$

3.3. Subgame Perfect Equilibrium with Minimum Reciprocity

We define the strategic aspect of the bank's sequential services with depositors' observability as a dynamic game that only patient agents play in period 1. Patient agent $i's$ strategy is defined as $s_i=(\mu_i,\eta_i)$, where $\mu_i: M^{i-1}\to M_i$, $\eta_i: M^n\to M_i$, $(m^{i-1},\mu_i(m^{i-1}))\in M^i$, and $\eta_i(m)\in[0,m_i]$. Let S_i denote the set of all strategies for patient agent $i\in\bar{N}^c$. Let $S\equiv\times_{i\in N}S_i$. We denote by $u_i(s)$ the utility for patient agent i induced by a strategy profile $s\in S$. We denote by $u_i(s\,|\,m^{i-1})$ the utility for patient agent i induced by a strategy profile $s\in S$ contingent on m^{i-1}. We denote by $u_i(s\,|\,m)$ the utility for patient agent i induced by a strategy profile $s\in S$ contingent on m. A strategy profile $s\in S$ is said to be a *subgame perfect equilibrium* in the dynamic game if for every $i\in\bar{N}^c$, $m\in M^n$, and $s_i'\in S_i$:

$$u_i(s\,|\,m^{i-1})\geq u_i(s_i',s_{-i}\,|\,m^{i-1}),$$

$$u_i(s\,|\,m)\geq u_i(s_i',s_{-i}\,|\,m).$$

A strategy profile s is said to be a *subgame perfect equilibrium with minimum reciprocity* if it is a subgame perfect equilibrium, satisfies positive reciprocity in the case of executability, and satisfies negative reciprocity in the case of un-executability. In other words, for every $i\in\bar{N}^c$, $m\in M^n$, and $s_i'\in S_i$:

$$[R > x \text{ and } u_i(s \mid m^{i-1}) = u_i(s_i', s_{-i} \mid m^{i-1})] \Rightarrow [\mu_i(m^{i-1}) \le \mu_i'(m^{i-1})],$$

$$[R > x \text{ and } u_i(s \mid m) = u_i(s_i', s_{-i} \mid m)] \Rightarrow [\eta_i(m) \le \eta_i'(m)],$$

$$[R < x \text{ and } u_i(s \mid m^{i-1}) = u_i(s_i', s_{-i} \mid m^{i-1})] \Rightarrow [\mu_i(m^{i-1}) \ge \mu_i'(m^{i-1})],$$

$$[R < x \text{ and } u_i(s \mid m) = u_i(s_i', s_{-i} \mid m)] \Rightarrow [\eta_i(m) \ge \eta_i'(m)].$$

3.4. No Bank Run

This subsection considers the case in which the promise x is executable ($R > x$). We specify a strategy profile $s^* = (s_i^*)_{i \in \bar{N}^c}$ as follows: for every $i \in \bar{N}^c$ and $m \in M^n$:

$$\mu_i^*(m_{-i}) = 0 \text{ and } \eta_i^*(m) = 0,$$

where $s_i^* = (\mu_i^*, \eta_i^*)$. According to s^*, any patient agent requests no reserve, the bank run never occurs, and the Pareto-dominant outcome is achieved.

Theorem 5: *If $R > x$, then s^* is the unique subgame perfect equilibrium with minimum reciprocity.*

Proof: Clearly, s^* is a subgame perfect equilibrium with minimum reciprocity: any patient agent always receives x in period 2, which implies the Pareto-dominant outcome. This along with the satisfaction of positive reciprocity automatically implies a subgame perfect equilibrium with minimum reciprocity.

Consider an arbitrary subgame perfect equilibrium with minimum reciprocity, $s \in S$. At stage $n+1$, the patient agent whose priority order is the highest among all patient agents has the incentive to select no withdrawal because he/she can receive x in period 2. Recursively, given that all patient agents whose priority orders are higher select no withdrawal, any patient agent has the incentive to select no withdrawal because he/she can receive x in period 2. Hence, according to s, any patient agent selects no withdrawal at stage $n+1$ irrespective of $m \in M^n$. Because of positive reciprocity, no patient agent $i \in \bar{N}^c$ selects positive reserve irrespective of $m^{i-1} \in M^{i-1}$. Hence, $s = s^*$ must hold.

Q.E.D.

3.5. Bank Run

This subsection considers the case in which the promise x is not executable ($R < x$). Based on negative reciprocity, this subsection assumes that any patient agent selects full reserve:

$$m_i = 1 \quad \text{for all} \quad i \in \bar{N}^c .$$

Instead of a subgame perfect equilibrium with minimum reciprocity, this subsection considers a Nash equilibrium with minimum reciprocity in the static game at stage $n+1$, which only patient agents play if they reserve their entire deposits. We specify $m^+ \in M^n$ as follows:

$$m_i^+ = 1 \qquad \text{for all} \quad i \in \bar{N}^c ,$$

$$m_i^+ = \max[0, \min[1, Z^1 - \sum_{j<i} m_j^+]] \quad \text{for all} \quad i \in \bar{N} .$$

This subsection assumes that all agents select m^+ before stage $n+1$. A profile of patient agents' withdrawals $d \in [0,1]^{\bar{n}^c}$ is said to be a *Nash equilibrium* in the game at stage $n+1$ if for every $i \in \bar{N}^c$ and $d_i' \in [0,1]$:

$$u_i(m^+, d) \geq u_i(m^+, d_i', d_{-i}) .$$

A profile d is said to be a *Nash equilibrium with negative reciprocity* in this game if it is a Nash equilibrium and satisfies negative reciprocity. In other words, for every $i \in \bar{N}^c$ and $d_i' \in [0,1]$:

$$[\, u_i(m^+, d) = u_i(m^+, d_i', d_{-i}) \,] \Rightarrow [\, d_i \geq d_i' \,].$$

This subsection also assumes

(4) $$\bar{n}^c - 1 > Z^1 - \sum_{i \in \bar{N}} m_i^+ ,$$

which is more restrictive than the absence of large depositors assumed by inequality (2). Assumptions (2) and (4) are equivalent if and only if $\sum_{i \in \bar{N}} m_i^+ = \bar{n}$. Note that this equivalence holds if the first $\bar{n}$ agents are impatient, that is, if $\bar{N} = \{1, \ldots, \bar{n}\}$. On the other hand, this equivalence does not hold if the last $\bar{n}$ agents are impatient, that is, if $\bar{N}^c = \{1,...,\bar{n}^c\}$.

We specify $d = \bar{d}$ by

$$\bar{d}_i = 1 \quad \text{for all} \quad i \in \bar{N}^c .$$

According to $\bar{d}$, all patient agents request full withdrawals at stage $n+1$, and the bank run occurs. We can prove the following proposition in the same manner as Proposition 1, implying that $\bar{d}$ is a bank-run Nash equilibrium.

Proposition 6: *The specified profile* $\bar{d}$ *is a Nash equilibrium with negative reciprocity in the game at stage* $n+1$.

Fix an arbitrary patient agent $q \in \bar{N}^c$, and define d^q as follows: for every $i \in \bar{N}^c$:

$$d_i^q = 0 \qquad \text{if } i \le q ,$$

$$d_i^q = 1 \qquad \text{if } i > q .$$

According to d^q, any patient agent whose priority order is not lower than q does not withdraw anything, while any patient agent whose priority is lower than q withdraws his/her entire deposit. Denote

$$d^* \equiv d^n = (0,...0) ,$$

which implies that all patient depositors do not withdraw anything at stage $n+1$.

A promise x is said to be *quasi-executable associated with reserve* if there exists $q \in \bar{N}^c$ such that d^q is a Nash equilibrium. Note that

$$(c_i^1(m^+, d^q), c_i^2(m^+, d^q)) = (d_i^q, (1-d_i^q)x) \quad \text{for all} \quad i \in \bar{N}^c ,$$

and d^q does not induce the bank run. Therefore, if x is quasi-executable associated with reserve, that is, if there exists $q \in \bar{N}^c$ such that d^q is a Nash equilibrium with negative reciprocity, then such q is unique, and $q = n$ must hold. Hence, x is quasi-executable associated with reserve if and only if d^* is a Nash equilibrium with negative reciprocity.

Note that if x is executable, then it is automatically quasi-executable associated with reserve. However, x is not necessarily executable even if it is quasi-executable associated with reserve. If x is quasi-executable associated with reserve but $R < x$, then, the patient agent whose priority is the lowest among all patient agents fails to receive x in period 2.

We can prove the following proposition in the same manner as Theorem 4, implying that the bank run inevitably occurs whenever the promise is not quasi-executable associated with reserve.

Theorem 7: *If the promise* x *is not quasi-executable associated with reserve,* $\bar{d}$ *is the only Nash equilibrium with negative reciprocity in the game at stage* $n+1$.

From Theorems 5 and 7, we can conclude that, *even in the case of sequential services with depositors' observability, by considering the option to reserve, we can avoid the occurrence of a bank run whenever the promise is executable, while the bank run inevitably occurs whenever the promise is far from executable beyond the limit.*

Remark 6: This subsection assumed that any patient agent selects full reserve. However, even if the promise is not executable, and negative reciprocity is required, there may exist a subgame perfect equilibrium that does not induce the bank run once we permit patient agents to select no reserve.

Consider an example with three patient agents and no impatient agents, where $l = 1/3$, $r = 0$, and $2x+1 < 3R < 3x$. Assume $m = (1,0,1)$. In this case, $d = (0,0,0)$ is a Nash equilibrium in the game at stage $n+1$ associated with $m = (1,0,1)$. In period 2, agents 1 and 2 can receive the promised payment x, and agent 3 can receive $3R - 2x > 1$. Even if negative reciprocity is required, agent 2 hesitates to increase his/her reserve. Suppose that agent 2 selects $m_2' > 0$ instead of $m_2 = 0$, and the subsequent equilibrium is switched from $d = (0,0,0)$ to $d' \equiv (1, m_2', 1)$. This equilibrium switch induces the bank run as a subgame perfect equilibrium: at stage $n+1 = 4$, agent 1 receives 1, but agents 2 and 3 receive nothing, and the bank run inevitably occurs irrespective of any agent's unilateral change in withdrawal at stage $n+1$. Hence, agents have no incentive to deviate from d'. This implies a subgame perfect equilibrium.

4. Concluding Remarks

In this study, we investigated the occurrence and deterrence of a bank run from a behavioral point of view. In the proposed setting, each depositor was motivated not only by his/her financial interest, but also by minimum reciprocity, which affects depositors to the extent it does not infringe on their financial interests. We showed that it is possible to eliminate any bank-run equilibrium if the bank's promise is executable. According to depositors' positive reciprocity, the bank run never occurs as a Nash equilibrium with minimum reciprocity. In this case, the bank can fully respond to all impatient depositors' withdrawal requests and make maturity payments to all patient depositors as promised.

To prove our permissive results, we designed a priority-based deposit contract. We did not permit outside parties to restrict depositors' right of withdrawal. We also did not allow suspension of convertibility. Moreover, if each depositor can observe the bank's remaining payment capacity in the process of sequential services, we permitted the bank to include a clause in the deposit contract that each depositor has an option to reserve his/her withdrawal.

This study also showed that if the bank's promise is not executable, according to depositors' negative reciprocity, the bank run inevitably occurs as the unique Nash equilibrium outcome. Hence, thanks to depositors' minimum reciprocity, we can eliminate the possibility of depositors' panic and focus only on the possibility of a bank run that occurs from fundamental causes.

A bank run inherently has the power to induce a bank's bankruptcy whenever it is insolvent. Our uniqueness results implied that, due to minimum reciprocity, the depositors' behavioral pattern effectively motivates the bank to look for profitable investment opportunities and only make executable promises. As Calomiris and Kahn (1991) indicated, this behavioral pattern also motivates depositors to collect information regarding whether the bank's promise is executable. This inhibits the trade-off between the inefficiency induced by liquidation and the benefit of disciplining the bank's morality.

To make minimum reciprocity function, this study assumed that depositors are fully informed about the bank's executability in period 1. Without this assumption, we need to introduce a stronger reciprocal motive that is not completely compatible with financial interest. If depositors are ambiguity-averse, the range in which they are motivated by negative reciprocity expands and the bank run will be more likely to occur.

This study assumed that apart from minimum reciprocity, depositors have no non-financial motive. Another non-financial motivation may push minimum reciprocity and promote the occurrence of bank runs, although we do not clearly know what motivation should be considered in this context

Moreover, this study assumed that whether depositors are motivated by positive reciprocity or negative reciprocity depends on whether the bank's promise is executable. However, if the bank's bankruptcy has significant spillover effects on financial systems and depositors recognize this effect, they may not be motivated by negative reciprocity even if the bank's promise is far from executable beyond the limit. This suggests that depositors' manner of moral sentiment may change depending on the system context.

Weakening these assumptions creates room for another essential discussion and should be left for future research. Although this problem remains, we believe that this study is sufficient to point out that conventional research has unduly underestimated the possibility of stabilizing financial systems by associating moral sentiment with decision making. This will promote the stability of financial systems by complementing the reestablishment of a more moralistic business culture in financial industries (Cohn et al., 2014).

This study did not consider any outside policy devices to diminish the occurrence of bank runs, such as suspension of convertibility, deposit insurance, interbank liquidity markets, credit lines, and lenders' last resort. Future research should carefully analyze the relationship between these devices and social preferences. For example, it was implicit in this study to assume that small depositors were fully protected by deposit insurance. Hence, we could focus on a limited finite number of depositors who are not small enough to be fully protected, without considering the non-financial motives of small depositors. This observation suggests that unlike general wisdom, deposit insurance could rather contribute to preventing bank runs without promoting banks' moral hazard.

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