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Product Proliferation and First Mover Advantage in a Multiproduct Duopoly

Yi-Ling Cheng*　　Takatoshi Tabuchi†

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Abstract: This study aims to understand product proliferation and first mover advantage in the case of multiproduct firms that engage in Stackelberg competition on the number of varieties and prices. We show that when firms sequentially choose the number of varieties and then simultaneously decide prices, the leader produces more varieties and enjoys first mover advantage. By contrast, when the leader sets both the number of varieties and prices before the follower does, the follower tends to produce more varieties and enjoy second mover advantage in the case of a large demand and a small cost of expanding product lines. This result sharply contrasts with those of studies on the sequential entry of single-product firms. We also show that the market provides too few varieties relative to the social optimum.

Keywords: product proliferation, first/second mover advantage, sequential entry, multiproduct duopoly.

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1 Introduction

The literature on the sequential entry of multiproduct firms often focuses on market preemption by an incumbent applying product proliferation to deter the entry of other firms and maintain its monopoly power. For example, Schmalensee (1978) and Eaton and Lipsey (1979) indicate that an incumbent may deter entry by producing new goods and crowding the product spectrum to the extent that no niche remains for potential entrants.

Leading producers such as Quaker Oats and Campbell Soup proliferate brands, but they often fail to deter the entry of new firms. Some followers may even be more successful than leaders in the sense of earning higher profits, producing more products or gaining larger market share. A typical example is Amazon.com, an online bookstore that launched in 1995. Amazon's product lines were quickly expanded to include DVDs, CDs, software, video games, furniture, toys, and so on, and Amazon grew to become an S&P 100 company and America's largest online retailer. However, the very first online bookstore was Books.com (Book Stacks Unlimited), which was founded in 1991 and launched online in 1992. Other well-known companies have followed a similar path. For example, Boeing did not pioneer modern jet travel, nor did Google pioneer the Internet search engine. Nevertheless, both are now industry leaders.

Typically, in modern industries, both pioneers and followers produce multiple products. However, thus far, few studies have considered product scope with the sequential entry of firms. Judd (1985) shows that the incumbent firm withdraws a product if it is a close substitute for an entrant's product to avoid intense competition. Because a potential entrant anticipates this withdrawal, market preemption by the incumbent is not credible, and thus, the entry may not be deterred if the cost of withdrawing a product is low. Gilbert and Matutes (1993) and Murooka (2013) also show that brand proliferation is not a credible entry-deterring strategy if the degree of brand-specific differentiation is large or if price competition is intense. Furthermore, whereas Martinez-Giralt and Neven

(1988) and Ashiya (2000) show that firms do not proliferate brands if price competition is intense, Tabuchi (2012) shows that firms do proliferate brands under intense competition in the case of three or more firms. However, these studies consider rather limited numbers of products.

The literature on first mover advantage (FMA) often assumes single-product firms (Gal-Or, 1985; Dowrick,1986; Deneckere and Kovenock, 1992; Amir and Stepanova, 2006). It is well known that a single-product leader can move preemptively to gain FMA under Stackelberg quantity competition, and that a follower may copy the leader or undercut the leader's price to obtain second mover advantage (SMA) under Stackelberg price competition (Gal-Or, 1985; Baumol, 1982). However, in practice, firms often produce a significant number of varieties. The product range is an important strategy of multiproduct firms, and it affects the leader's and follower's advantages. Therefore, this study examines the product scopes of first and second movers and explores the conditions for FMA when firms can produce an arbitrary number of products. We also discuss the strategic and cannibalization effects of product proliferation by the leader and follower.

The literature includes numerous empirical findings on FMA and SMA. Whereas many studies demonstrate the existence of FMA in terms of market share or profits (Kekre and Srinivasan 1990; Robinson 1988; Robinson and Fornell 1985; Urban et al. 1986), a growing body of evidence suggests the existence of SMA. For example, Boulding and Christen (2003) empirically show that a leader is at a long-term profit disadvantage due to the costs of product proliferation. Golder and Tellis (1993) find that the first mover is the leader in terms of market share in only four of the fifty product categories studied. Lieberman and Montgomery (1998) provide a comprehensive survey of the theoretical and empirical literature and conclude that FMA is possible but is certainly not guaranteed.

The literature also includes empirical findings on product scope. Some evidence indicates that a broader product line is advantageous to a leader (Kalyanaram et al. 1995; Kekre and Srinivasan 1990; Robinson 1988; Robinson and Fornell 1985), but others find that product proliferation does not necessarily provide advantages to leaders (Boulding and Christen, 2003). Bayus and Putsis (1999) show that product proliferation has a negative net impact on a firm's profitability in personal computer industry because the higher cost associated with a broader product line dominates any potential increase in demand.

These empirical studies suggest the great importance of investigating whether FMA exists and whether a strategy of product proliferation is advantageous for a leader. For this purpose, we build a model of Stackelberg competition in a multiproduct duopoly and derive the rationale for FMA and SMA. Specifically, we first consider a case in which two multiproduct firms enter and choose the number of product varieties sequentially, and then set the prices of their products simultaneously. Second, we consider the case in which the firms engage in Stackelberg competition in both of varieties and prices. In this case, the leader chooses both of the number of varieties and the prices of its products before the follower does. Finally, we investigate optimal product diversification and compare it with the market one.

Our main findings are summarized as follows. In the three-stage game, in which firms sequentially choose the number of varieties and then simultaneously set prices, the leader produces more varieties and gains FMA. By contrast, in the two-stage game, in which the leader chooses both the number of varieties and prices before the follower does, the equilibrium outcome depends on product demand and the cost of expanding product lines. The leader is likely to produce more varieties and enjoy FMA when consumers' demand is low and the cost of expanding product lines is high. Otherwise, the follower tends to produce more varieties and enjoy SMA. These results sharply contrast with those of studies of the sequential entry of single-product firms.

The rest of this paper is organized as follows. Section 2 presents a duopolistic model of Stackelberg competition in the number of varieties. We characterize the subgame perfect Nash equilibrium (SPNE) and the product scopes and profits of the leader and follower. Section 3 examines Stackelberg competition in the number of varieties and price. Section

4 considers the socially optimal level of product diversification and investigates whether the market equilibrium number of products is too high or too low. Section 5 concludes.

2 Stackelberg Competition in Variety

The economy includes two firms $i = 1,\ 2$ that produce any number of varieties of a horizontally differentiated good. There is one unit mass of identical consumers, whose preferences are defined over a number of varieties of a horizontally differentiated good and a homogeneous good chosen as the numeraire. These preferences are given by:

$$U = \alpha \left[\sum_{k=1}^{n_1} q_1(k) + \sum_{k=1}^{n_2} q_2(k) \right] - \frac{\beta}{2} \left[\sum_{k=1}^{n_1} q_1(k)^2 + \sum_{k=1}^{n_2} q_2(k)^2 \right] - \frac{\gamma}{2} \left[\sum_{k=1}^{n_1} q_1(k) + \sum_{k=1}^{n_2} q_2(k) \right]^2 + q_0 \tag{1}$$

where q_0 denotes consumption of the homogeneous good, $q_i(k)$ is consumption of variety $k \in \{1, \ldots, n_i\}$ of the differentiated good produced by multiproduct firm $i = 1,\ 2$, and n_i is the number of varieties produced by firm i. The total number of varieties of the differentiated good in the economy is denoted by $N = n_1 + n_2$. The parameters α, β, and γ are positive. A higher α means a stronger preference towards the differentiated varieties relative to that for the numeraire, a higher β implies more bias toward love for variety, and a higher γ indicates closer substitutes between varieties.

The budget constraint of a consumer is given by:

$$\sum_{k=1}^{n_1} p_1(k) q_1(k) + \sum_{k=1}^{n_2} p_2(k) q_2(k) + q_0 = w \tag{2}$$

where w is the consumer's wage income and $p_i(k)$ is the price of variety k produced by firm i. Substituting the numeraire consumption in (2) into (1) and solving the first-order conditions with respect to $q_i(k)$, we obtain the linear demand for variety k of firm i as follows.

$$q_i(k) = \frac{\alpha}{\beta + \gamma N} - \frac{1}{\beta} p_i(k) + \frac{\gamma}{\beta\,(\beta + \gamma N)} \left[\sum_{k=1}^{n_1} p_1(k) + \sum_{k=1}^{n_2} p_2(k) \right] \tag{3}$$

where $k = 1, 2, ..., n_i$ and $i = 1, 2$.

The two firms have the same production technology. The fixed overhead cost of launching a variety is F, and the subsequent marginal cost of producing each variety is constant and normalized to zero. Given the demand (3), firm i maximizes its profit:[1]

$$\pi_i = \sum_{k=1}^{n_i} p_i(k) q_i(k) - F n_i \tag{4}$$

where $q_i(k)$ is given by (3).

We first examine the two-stage game, in which two firms simultaneously enter and choose the number of varieties (n_1, n_2), and then compete in price (p_1, p_2). We seek the SPNE by backward induction. Differentiating the profit (4) with respect to $p_i(k)$, we obtain the first-order conditions in the second stage:

$$\alpha\beta - 2\left(\beta + \gamma N\right) p_i(k) + \gamma \left[\sum_{k=1}^{n_1} p_1(k) + \sum_{k=1}^{n_2} p_2(k)\right] = 0$$

for $k = 1, 2, \ldots, n_i$ and $i = 1, 2$. Using the symmetry among the varieties produced by firm i, we compute the prices as

$$p_i(k) = p_i \equiv \frac{\alpha\beta}{\Psi}[2\beta + \gamma(2n_i + n_j)] \tag{5}$$

where $\Psi \equiv 4\beta^2 + 4\beta\gamma(n_i + n_j) + 3\gamma^2 n_i n_j$, $i \neq j$. Thus, we obtain

$$p_1 - p_2 = \frac{\alpha\beta\gamma}{\Psi}(n_1 - n_2)$$

which implies that a firm producing more varieties charges a higher price because it holds a dominant market position.

By substituting (5) into (4), and differentiating the resulting equation with respect to n_i, we obtain the two first-order conditions in the first stage as follows:

$$\begin{aligned} R_i(n_1, n_2) \quad &\equiv \quad \frac{\alpha^2\beta(\beta + \gamma n_j)}{\Psi}[2\beta + \gamma(2n_i + n_j)][8\beta^4 + 4\beta^3\gamma(4n_i + 5n_j) \\ + \quad & \quad 2\beta^2\gamma^2(4n_i^2{+}15n_i n_j{+}8n_j^2){+}\beta\gamma^3 n_j(10n_i^2{+}11n_i n_j{+}4n_j^2)\text{-}3\gamma^4 n_i n_j^3]\text{-}F{=}0 \end{aligned} \tag{6}$$

[1] We assume away entry costs to avoid a monopoly by the incumbent.

To ensure that two firms enter and provide a positive number of varieties, the relationship

$$\frac{d\pi_1}{dn_1}_{(n_1,n_2)=(0,0)} = \frac{\alpha^2}{4\beta} - F > 0$$

should hold. Therefore, we assume that $A \equiv \frac{\alpha}{\sqrt{F\beta}} > 2$ throughout the analysis. Based on this assumption, we can derive the following corollary.

Corollary 1 $(n_1, n_2)-(p_1, p_2)$: *When firms simultaneously choose the number of varieties and then select prices, there always exists an SPNE, which may be symmetric* (n^e, n^e) *or asymmetric* (n_1^e, n_2^e) *and* (n_2^e, n_1^e) *with* $n_1^e > n_2^e$.

Proof. See Appendixes 1 and 2. ■

Interestingly, the equilibrium outcome can be asymmetric despite the symmetric setting of the game. We can numerically show that the symmetric equilibrium (n^e, n^e) is a unique SPNE when the market is small, whereas the asymmetric equilibria (n_1^e, n_2^e) and (n_2^e, n_1^e) appear when the market is large.

The analysis thus far has assumed that the firms choose the number of varieties simultaneously. We next consider the case in which firms choose the number of varieties sequentially. This game consists of three stages. In the first stage, firm 1 enters the market and selects its number of varieties n_1. Then, in the second stage, firm 2 enters and chooses its number of varieties n_2. Finally, two firms compete in price (p_1, p_2) in the third stage.

Proposition 1 $(n_1) - (n_2) - (p_1, p_2)$: *When firms sequentially choose the number of varieties and then compete in price, the leader produces more varieties* $n_1 > n_2$ *and enjoys FMA* $\pi_1 > \pi_2$.

Proof. See Appendix 3. ■

This proposition vindicates the importance of the product proliferation strategy by the first mover, which is intuitive. In fact, the proposition is robust even when the third-stage price competition is absent. That is, in the two-stage game of $(n_1) - (n_2)$ with

$p_1 = p_2 = p$, we can also show that the leader always offers more varieties and enjoys FMA. This result is also robust if we replace price competition with quantity competition in the third stage. That is, we find product proliferation and FMA in the three-stage game of $(n_1) - (n_2) - (q_1, q_2)$.[2]

In sum, similar to their outputs, *firms' numbers of varieties are strategic substitutes*, implying that the leader proliferates product variety and enjoys FMA. This result may be understood by considering that product proliferation is not very different from an increase in output.

3 Stackelberg Competition in Price and Variety

In this section, we examine the case in which the leader's price is not flexible. More specifically, we consider the following two-stage game. In the first stage, firm 1 chooses the number n_1 and prices $p_1(k)$ of varieties for $k = 1, 2, ..., n_1$. In the second stage, firm 2 selects the number n_2 and prices $p_2(k)$ of varieties for $k = 1, 2, ..., n_2$.

By backward induction, we differentiate (4) with respect to n_2 and $p_2(k)$. Because of the symmetry among varieties $p_i(k) = p_i$ for all $k = 1, 2, \ldots, n_i$, we obtain the first-order conditions in the second stage, from which we can determine the product scope and price of firm 2 as follows:

$$n_2 = \frac{\alpha\beta + \gamma n_1 p_1 - 2\sqrt{F\beta}(\beta + \gamma n_1)}{2\sqrt{F\beta}\gamma} \tag{7}$$

$$p_2 = \frac{\alpha\beta + \gamma n_1 p_1}{2(\beta + \gamma n_1)} \tag{8}$$

By substituting (7) and (8) into (3), we can write the demand for firm 2's product as

$$q_2 = \sqrt{\frac{F}{\beta}}$$

which is independent of n_1 and p_1.

[2] The proofs of these two results are provided upon request to the authors.

Plugging (7) and (8) into equations for the profits (4) of the two firms, and differentiating the resulting equation with respect to n_1 and p_1, we obtain the first-order conditions:

$$\frac{\partial \pi_1}{\partial n_1} = \left[\frac{\beta^2(\alpha - p_1)}{(\beta + \gamma n_1)^2} - p_1\right]\frac{p_1}{2\beta} + \frac{\sqrt{F}}{\sqrt{\beta}}p_1 - F = 0 \tag{9}$$

$$\frac{\partial \pi_1}{\partial p_1} = \frac{n_1}{2}\left(\frac{2\sqrt{F}}{\sqrt{\beta}} + \frac{\alpha - 2p_1}{\beta + \gamma n_1} - \frac{2p_1}{\beta}\right) = 0 \tag{10}$$

From (10), we observe that $dp_1/dn_1 < 0$; when the leader increases the number n_1 of its varieties, its price p_1 decreases. This result is due to the direct effect of cannibalization as well as the subsequent reactions of the follower.

Solving (10) for p_1 and substituting the resulting equation into (9), we have

$$\frac{\partial \pi_1}{\partial n_1} = \frac{F}{8(B + n_1)^2(2B + n_1)^2} f(n_1) = 0 \tag{11}$$

where

$$f(n_1) \equiv 2(A-2)(A+6)B^4 + 16(A-4)B^3 n_1 - \left(A^2 - 8A + 60\right)B^2 n_1^2 - 24Bn_1^3 - 4n_1^4$$

and $B \equiv \beta/\gamma$. The equation $f(n_1) = 0$ is a fourth-order polynomial and its third and fourth derivatives are negative. Examining the signs of the first and second derivatives at $n_1 = 0$ and $n_1 \to \infty$ in the cases of $2 < A \leq 4$ and $A > 4$, we can readily show that a unique positive solution exists for all $A > 2$.

Proposition 2 $(n_1, p_1) - (n_2, p_2)$: *In the two-stage game in which the first firm chooses its number of varieties and price, and then the second firm selects the number of varieties and price, a unique SPNE exists with* $n_1^* > 0$ *and* $n_2^* > 0$.

Having established the existence of a unique SPNE, we investigate the following total derivatives to study how product proliferation affects the follower's price and product scope decisions.

$$\frac{dp_2}{dn_1} = \underset{(-)}{\frac{\partial p_2}{\partial n_1}} + \underset{(+)}{\frac{\partial p_2}{\partial p_1}} \underset{(-)}{\frac{dp_1}{dn_1}} < 0 \tag{12}$$

$$\frac{dn_2}{dn_1} = \frac{\partial n_2}{\partial n_1} + \underset{(+)}{\frac{\partial n_2}{\partial p_1}} \underset{(-)}{\frac{dp_1}{dn_1}} = \frac{1}{2}\left[\frac{(A-2)B^2}{(2B+n_1)^2} - 1\right] \tag{13}$$

The sign of each term in (12) is straightforward by the two reaction functions (7) and (8) as well as (10). The negative first term $\partial p_2/\partial n_1 < 0$ implies that when the leader produces more varieties for a given p_1, product competition intensifies, and the follower reacts by lowering its price p_2. The positive term $\partial p_2/\partial p_1 > 0$ indicates that when the leader lowers its price for a given n_1, the follower reacts by reducing its price because the two variables are strategic complements (Gal-Or, 1985). The negative term $dp_1/dn_1 < 0$ means that as the leader produces more varieties, it lowers its price p_1. Therefore, as shown in (12), the follower lowers its price when the leader produces more varieties.

By contrast, the sign of dn_2/dn_1 in (13) depends on the parameters. The first term $\partial n_2/\partial n_1$ in (13) is ambiguous, whereas the second term is negative because $dp_1/dn_1 < 0$ and $\partial n_2/\partial p_1 > 0$. The positive term $\partial n_2/\partial p_1 > 0$ indicates that when the leader lowers its price p_1 for a given n_1, the follower reacts by producing fewer varieties because the profit per variety falls. Thus, *the leader has an additional advantage of strategic pricing over the number of follower's varieties* in a multiproduct duopoly. Such an outcome never arises in a single-product duopoly.[3]

From (13), we can get $\frac{dn_2}{dn_1} \lesseqgtr 0$ for $A \lesseqgtr 18$.[4] Therefore, the leader's strategic pricing to reduce the number of follower's varieties is effective for small A (i.e., in the case of low demand and high fixed costs). In this case, the negative effect of the second term of

[3]In a single-product duopoly, when the leader lowers its price, the follower's only strategy is to lower its price, which necessarily leads to keen competition and results in an SMA (Gal-Or, 1985).

[4]Solving $\frac{dn_2}{dn_1} = 0$ in (13), we find that $n_1 = B\sqrt{A-2} - 2B$. Plugging it into $f(n_1)$, we get $f(B\sqrt{A-2} - 2B) \gtreqless 0$ for $A \gtreqless 18$. However, Proposition 2 implies that $f(n_1) \gtreqless 0$ for $n_1 \lesseqgtr n_1^*$ and (13) shows that $\frac{dn_2}{dn_1}$ decreases with n_1. Thus, we can establish that $\frac{dn_2}{dn_1} \lesseqgtr 0$ for $A \lesseqgtr 18$.

(13) dominates the positive effect of the first term, so that an increase in n_1 decreases n_2. In other words, the leader is less worried about the negative effects of cannibalization and variety competition. In contrast, n_2 increases with n_1 for large A. In this case, competition is fierce, and thus, the leader has more concerns about cannibalization and variety competition.

In sum, (12) and (13) show that when the leader produces more varieties, the follower reacts by lowering its price and, potentially, by producing fewer varieties because of keener competition. Although the former reaction puts the leader at a disadvantage, the latter gives the leader an advantage.

Next, using (8) and (10), we obtain

$$p_1 - p_2 = \frac{\sqrt{BF}\,[n_1 - (A-2)\,B/2]}{2(n_1 + B)}$$

Thus, $p_1 = p_2$ implies that $n_1 = (A-2)\,B/2$. Plugging it into $f(n_1)$, we find that $f((A-2)\,B/2) = -(A-2)^2\,A^2B^4/2 < 0$. However, we know from Proposition 2 that $f(n_1) \gtreqqless 0$ for $n_1 \lesseqqgtr n_1^*$. Hence, it follows that $p_1 - p_2 < 0$ when $n_1 = n_1^*$. That is, *the follower sets a higher price than the leader in a multiproduct oligopoly*, which is in sharp contrast to the result in a single-product oligopoly. Moreover, it can easily be shown that $q_1 - q_2 > 0$.

Regarding product proliferation and FMA/SMA, we obtain the following.

Proposition 3 $(n_1, p_1) - (n_2, p_2)$: *Three different outcomes may arise in equilibrium.*

(i) For $2 < A < 4.2$, *the leader produces more varieties* $n_1 > n_2$ *and enjoys FMA* $\pi_1 > \pi_2$.

(ii) For $4.2 < A < 4.54$, *the leader produces fewer varieties* $n_1 < n_2$ *and enjoys FMA* $\pi_1 > \pi_2$.

(iii) For $A > 4.54$, *the follower produces more varieties* $n_1 < n_2$ *and enjoys SMA* $\pi_2 > \pi_1$.

Proof. See Appendix 4. ■

In Proposition 3(iii), A is large, which means that the demand α for the product is large but the fixed costs F of a new variety are low. Then, the follower enters the market and provide many varieties so that SMA emerges in an SPNE. However, Proposition 3(i) shows the opposite result; if the demand α is low and the fixed costs F are high, the follower does not produce many varieties.

Proposition 3 may be explained by the strategic behaviors of Stackelberg competition in price and variety. As in our explanations of (12) and (13), when the leader produces more varieties, the follower reacts by lowering its price, which intensifies competition and puts the leader at a disadvantage. However, when the leader produces more varieties, its optimal price decreases, which reduces the number of follower's varieties and gives the leader an advantage. For small A, the leader's strategic pricing is more effective at reducing the number of follower's varieties, so that the leader produces more varieties and enjoys FMA. In contrast, for large A, the leader's strategic pricing is less effective, and thus, the follower produces more varieties and enjoys SMA.[5]

Proposition 3 may be intuitively understood as follows. Stackelberg competition in price leads to SMA because prices are strategic complements. In contrast, we know from Proposition 1 that Stackelberg competition in variety leads to FMA because the numbers of varieties are strategic substitutes in the same way as quantities are. Therefore, the two strategic variables have opposite effects; committing to the number of varieties first is advantageous to the first mover, but committing the price first is disadvantageous.

Proposition 3 is consistent with the empirical findings on FMA. Suarez and Lanzolla (2005) examine the conditions for FMA and argue that when both technological innovation and market demand develop rapidly, the leader is highly vulnerable. Netscape is often cited as an example of leading companies overturned by followers due to the rapid churning

[5] If three firms enter the market sequentially, we can verify that the first mover applies product proliferation to obtain FMA for small α, the second mover applies product proliferation to obtain SMA for intermediate A, and the last mover applies product proliferation to obtain last mover advantage for large A.

of technology and markets. Netscape's browser was invented in 1994 and ushered in the era of widespread Internet access, but, today, Netscape is used very infrequently. Further examples include Maxwell, the leading producer of freeze-dried coffee, which was overturned by Nestle, and Dreft, the leading producer of liquid laundry detergent, which eventually lost ground to Tide. Audi was a leading producer of hybrid cars in 1989, but it was overturned by Toyota and Honda in the late 1990s. Xerox invented the bitmap display and the mouse-centered interface in 1973, but Apple and Microsoft took over the market after several years.

By contrast, slower growth in both technology and market demand provides better conditions for a leader to create a dominant position. Suarez and Lanzolla (2005) refer to Hoover's vacuum cleaner as an example. In 1908, the first commercial vacuum cleaner was produced by Hoover, but, by 1930, fewer than 5% of households had purchased one. The technology changed as slowly as the market demand. In 1935, the Hoover designer encased the vacuum cleaner's components in a streamlined canister, which created a technological blueprint that persists to this day. Since Hoover had little trouble technologically keeping up-to-date and meeting market demand, the company maintained the dominant position in the industry. Other examples include Gillette in the safety razor industry and Sony in the personal stereo industry.[6]

We next discuss results for market share. We can show that the leader acquires a larger market share $n_1q_1 > n_2q_2$ for $2 < A < 4.62$, whereas the follower acquires a larger market share $n_1q_1 < n_2q_2$ for $A > 4.62$. Combining this result together with Proposition 3, we can say that when $4.54 < A < 4.62$, the leader obtains FMA in terms of market share but he does not enjoy FMA in terms of profits. This result implies that for a pioneer, achieving FMA in terms of profits is not as easy as achieving FMA in terms of market share.

[6] In empirical studies of FMA, another key research stream focuses on the impact of the order of market entry on the market share. Some empirical studies argue that market pioneers tend to maintain market share advantages over later entrants (Robinson and Fornell, 1985; Robinson, 1988).

Product proliferation is regarded as a strategy for pioneers, and it often yields FMA in terms of profit in the literature. However, Proposition 3(ii) shows that the leader produces fewer varieties but still enjoys FMA. Hence, a broader product line does not necessarily coincide with a higher profit and a larger market share.

Next, we investigate the effect of the substitutability γ between varieties, product demand α, fixed costs F, and consumers' love for variety β on the numbers of product varieties provided by the leader and follower. The comparative statics are summarized as follows and their proofs are given in Appendix 5.

(i) $\frac{\partial n_1}{\partial \gamma} < 0,\ \frac{\partial n_2}{\partial \gamma} < 0 \quad \text{for } A > 2$

(ii) $\begin{cases} \frac{\partial n_1}{\partial \alpha} > 0,\ \frac{\partial n_2}{\partial \alpha} > 0 & \text{for } 2 < A < 18 \\ \frac{\partial n_1}{\partial \alpha} \leq 0,\ \frac{\partial n_2}{\partial \alpha} > 0 & \text{for } 18 \leq A \end{cases}$

(iii) $\begin{cases} \frac{\partial n_1}{\partial F} < 0,\ \frac{\partial n_2}{\partial F} < 0 & \text{for } 2 < A < 18 \\ \frac{\partial n_1}{\partial F} \geq 0,\ \frac{\partial n_2}{\partial F} < 0 & \text{for } 18 \leq A \end{cases}$

(iv) $\begin{cases} \frac{\partial n_1}{\partial \beta} < 0,\ \frac{\partial n_2}{\partial \beta} < 0 & \text{for } 2 < A < 3.42 \\ \frac{\partial n_1}{\partial \beta} \geq 0,\ \frac{\partial n_2}{\partial \beta} \leq 0 & \text{for } 3.42 \leq A \leq 5.18 \\ \frac{\partial n_1}{\partial \beta} > 0,\ \frac{\partial n_2}{\partial \beta} > 0 & \text{for } A > 5.18 \end{cases}$

(i) When the varieties are closer substitutes (i.e., γ is larger), both the leader and follower provides fewer varieties, which is intuitive. (ii)-(iii) When the demand α is larger and/or the fixed cost F is lower, the follower provides more varieties, which is also intuitive. The leader does the same when $2 < A < 18$, but does the opposite when $A > 18$. The latter result stems from the fact that dn_2/dn_1 in (13) is positive for $A > 18$. Because the follower reacts the leader's increase in the number of varieties by providing more varieties, the leader wants to reduce the negative effects of cannibalization and variety competition by providing fewer varieties. (iv) When consumers' love for variety, β, becomes stronger, both the leader and follower offer fewer varieties for $2 < A < 3.42$; the leader provides more varieties and the follower offers fewer varieties for $3.42 < A < 5.18$; and both firms provide more varieties for $A > 5.18$. An increase in β has two opposite effects. Consumers obtain more utility if they consume more varieties, and thus, each firm has an incentive to increase the number of varieties. However, as the love for variety increases, the demand for each variety shrinks, which is confirmed by equation (3). For small demand α and large fixed costs F (i.e., small A), the latter effect dominates the former so that each firm has incentive to reduce its number of varieties.

4 Optimal Product Diversification

In this section, we explore the optimal product diversification in the market and investigate whether the equilibrium number of varieties is too high or too low. Because utility (1) is transferable, social welfare can be defined by the sum of consumer surplus and producer surplus as follows:

$$
\begin{aligned}
W &\equiv U + \pi_1 + \pi_2 \\
&= \alpha \left[\sum_{k=1}^{N} q(k) \right] - \frac{\beta}{2} \left[\sum_{k=1}^{N} q(k)^2 \right] - \frac{\gamma}{2} \left[\sum_{k=1}^{N} q(k)^2 \right]^2 - FN + w
\end{aligned}
$$

Differentiating the social welfare with respect to the product output and the number of varieties and solving the first-order conditions, we obtain the optimal number of varieties, output per product, and its price:

$$
N^o = \frac{\alpha\sqrt{\frac{2\beta}{F}} - 2\beta}{2\gamma}, \quad q^o = \sqrt{\frac{2F}{\beta}}, \quad p^o = 0 \tag{14}
$$

The optimal number of varieties is decreasing in the fixed cost F, whereas the output for each variety is increasing in F. This result occurs because when the fixed cost is high, the social planner reduces the total fixed cost by reducing the number of varieties and increasing the output per variety. The latter increase in output compensates for the former reduction in variety.

By comparing the optimal number of varieties with the equilibria obtained in the previous section, we obtain the following proposition.

Proposition 4 $(n_1, p_1) - (n_2, p_2)$: *If the first entrant chooses both of the number of varieties and prices before the second entrant does, the market provides too few varieties.*

Proof. See Appendix 6. ■

The market outcome of too few varieties is somewhat similar to that of too few quantities in the case of quantity competition with a homogeneous good in oligopoly. Oligopolists in a market of differentiated goods supply a limited number of varieties in order to reduce the fixed costs, avoid cannibalization, and relax competition, whereas oligopolists in the market of a homogeneous good supply a limited amount of output in order to relax competition. In both cases, high prices and profits are maintained. Furthermore, as the number of entrants increases sufficiently, the number of varieties or the amount of output increases to coincide with the optimum level.

5 Conclusion

We have examined the strategic behavior of multiproduct firms given Stackelberg competition in the number of varieties and price. We have focused on both product proliferation and FMA. In section 2, we showed that when firms sequentially choose the number of varieties and then simultaneously decide prices, the leader produces more varieties and gains FMA. This result is due to the advantage of committing to the number of varieties first. Therefore, numbers of varieties can be regarded as strategic substitutes in the same way that quantities are.

In contrast, we have shown in section 3 that when the leader selects both the number of varieties and the product prices before the follower does, either FMA or SMA emerges in SPNE. This is because if the leader produces many varieties, the follower reacts by reducing its prices, which disadvantages the leader, but the follower may also react by producing fewer varieties, which favors the leader. When the demand is high and/or the fixed cost of producing a new variety is low, the leader's strategic pricing is less effective in reducing the number of follower's varieties, which means the follower may produce more varieties and enjoy SMA. In contrast, when the demand is low and/or the fixed cost is high, the leader's strategic pricing is more effective in reducing the number of follower's varieties, which leads to product proliferation by the leader and FMA. When the demand and fixed cost are intermediate, the leader produces fewer varieties, but still enjoys FMA. This result implies that product proliferation does not necessarily ensure a higher profit (or a larger market share).

The results obtained in this analysis are in sharp contrast with those in the literature on product proliferation as well as those in the literature on Stackelberg competition in the case of a single-product duopoly.

Appendix 1: Proof of Lemma 1

Let $n_2 = g(n_1)$ be the implicit function of $R_1(n_1, n_2) = 0$.

Lemma 1 $(n_1, n_2) - (p_1, p_2) : n_2 = g(n_1)$ *is either (i) decreasing, or (ii) decreasing, then increasing, and then decreasing.*

Equation $R_1(n_1, n_2) = 0$ given by (6) is a fifth-order polynomial of n_1. It can be shown that its fifth and fourth derivatives with respect to n_1 are negative and that $\lim_{n_1 \to \infty} R_1(n_1, n_2) < 0$. If $R_1(0, n_2) < 0$, the third, second and first derivatives are shown to be negative, and thus, (6) has at most one solution of n_1. On the other hand, if $R_1(0, n_2) \geq 0$, (6) has at most three solutions of n_1. Hence, given n_2, (6) has at most three positive solutions of n_1.

Substituting $n_1 = 0$ into (6), we get $n_2 = \check{n}_2 \equiv \frac{2\beta(2-A)}{\gamma(A-4)}$. This means that if $2 < A < 4$, $n_2 = g(n_1)$ crosses the vertical axis at $(0, \check{n}_2)$ with $\check{n}_2 > 0$. If $A \geq 4$, $n_2 = g(n_1)$ does not cross the vertical axis, but approaches the vertical axis as $\lim_{n_1 \to 0} g(n_1) \to +\infty$. On the other hand, plugging $n_2 = 0$ into (6), we get $n_1 = \check{n}_1 \equiv \frac{\beta(A-2)}{2\gamma}$, implying that $n_2 = g(n_1)$ always crosses the horizontal axis at $(\check{n}_1, 0)$ with $\check{n}_1 > 0$.

In sum, $n_2 = g(n_1)$ crosses the vertical axis at $(0, \check{n}_2)$ where $\check{n}_2$ is a positive finite value or $+\infty$, and crosses the horizontal axis at $(\check{n}_1, 0)$. Hence, because (6) has at most one solution of n_1 given n_2, the curve $n_2 = g(n_1)$ is either (i) monotone decreasing or (ii) first decreasing, then increasing, and decreasing.

Appendix 2: Proof of Corollary 1

Because $\partial\pi_1/\partial n_1 = 0$ means $n_2 = g(n_1)$, we have

$$g'(n_1) = -\frac{\partial^2\pi_1/\partial n_1^2}{\partial^2\pi_1/\partial n_1 \partial n_2} \tag{15}$$

Since we can readily show that $\partial^2\pi_1/\partial n_1\partial n_2 < 0$, the slope $g'(n_1)$ is positive if $\partial^2\pi_1/\partial n_1^2 > 0$. This implies that any point on the increasing segment of $n_2 = g(n_1)$ is a local minimizer of π_1. On the other hand, the slope $g'(n_1)$ is negative if $\partial^2\pi_1/\partial n_1^2 < 0$. This means that any point on the decreasing segment of $n_2 = g(n_1)$ is a local maximizer of π_1.

Since implicit function $n_2 = g(n_1)$ in case (i) is decreasing, it is the locus of a unique maximizer of π_1, and hence, it is the reaction function of firm 1. In case (ii), however, because one increasing segment of $n_2 = g(n_1)$ is a local minimizer, firm 1's reaction function consists of two noncontiguous parts of decreasing segments of $n_2 = g(n_1)$. That is, firm 1's reaction function is discontinuous such that

$$n_2 = \begin{cases} g(n_1) & \text{for } 0 \leq n_1 < \widehat{n} \\ g(n_1) & \text{for } n_1 \geq \widetilde{n} \end{cases} \tag{16}$$

where $\pi_1(\widehat{n}, g(\widehat{n})) = \pi_1(\widetilde{n}, g(\widetilde{n}))$ and $g(\widehat{n}) = g(\widetilde{n})$ with $\widehat{n} < \widetilde{n}$. It can be rewritten as $n_1 = g^{-1}(n_2)$, which jumps at $n_2 = \overline{n}$. Firm 2's reaction function is a mirror image of (16) with respect to the 45 degree line and is given by

$$n_1 = \begin{cases} g(n_2) & \text{for } 0 \leq n_2 < \widehat{n} \\ g(n_2) & \text{for } n_2 \geq \widetilde{n} \end{cases} \tag{17}$$

which can be rewritten as $n_2 = g^{-1}(n_1)$, which jumps at $n_1 = \overline{n}$.

An example of these reaction functions is given in Figure 1. Firm 1's reaction function is bold curves KJ and AB, which correspond to the first and second lines in (16), respectively. Firm 2's reaction function is bold curves IH and CD, which correspond to the first and second lines in (17), respectively. Points B and I are given by $(\check{n}_1, 0)$ and $(0, \check{n}_1)$, respectively. Point F is the intersection of JA and HC and is on the 45 degree line. The discontinuity may occur differently such that both H and C are northwest of F or both H and C are southeast of F.

Let (n^e, n^e) be the intersection point of $n_2 = g_1(n_1)$ and the 45 degree line, which can be shown to be unique for all $A > 2$. For any reaction functions (16) and (17), two cases may arise in the simultaneous two-stage game $(n_1, n_2) - (p_1, p_2)$: the reaction functions are (a) continuous at (n^e, n^e); (b) discontinuous at (n^e, n^e). In case (a), (n^e, n^e) is an SPNE. In case (b), A is east of F and C is south of F as shown Figure 1. Furthermore, B is southwest of D from the proof of Lemma 1. Therefore, the two reaction functions intersect at E, which is an SPNE (n_1^e, n_2^e) with $n_1^e > n_2^e$. By symmetry, G (n_2^e, n_1^e) is also an SPNE.

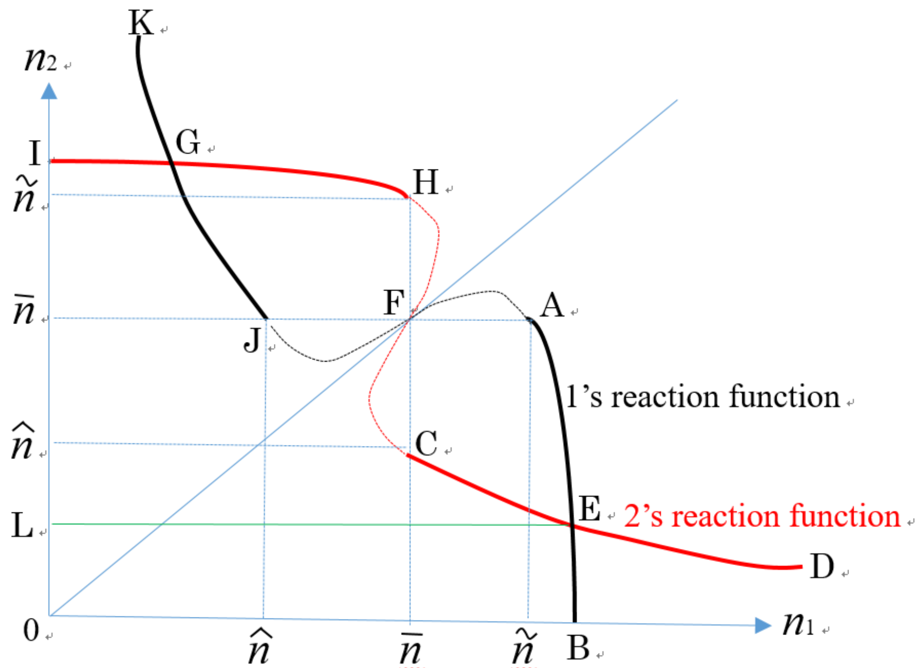

Figure 1: Reaction functions of simultaneous two-stage game $(n_1, n_2) - (p_1, p_2)$

Appendix 3: Proof of Proposition 1

(I) Suppose asymmetric SPNEs (n_1^e, n_2^e) and (n_2^e, n_1^e) exist. First, consider the domain $n_1 \geq n_2 > 0$. The first-order condition of the leader is given by

$$\frac{d\pi_1}{dn_1} = \frac{\partial \pi_1}{\partial n_1} + \frac{\partial \pi_1}{\partial n_2} g^{-1\prime}(n_1) \tag{18}$$

where $n_2 = g^{-1}(n_1)$ is the implicit function of $R_2(n_1, n_2) = 0$.

Because $\frac{\partial \pi_1}{\partial n_1} = 0$ holds on the firm 1's reaction function, it must be that $\frac{\partial \pi_1}{\partial n_1} > 0$ in the domain left of curve AB, and thus, $\frac{\partial \pi_1}{\partial n_1} > 0$ holds on curve CE. Sequential choice of the number of varieties implies that the Stackelberg equilibrium should be somewhere on

2's reaction function CD. However, on CE, we have

$$\left.\frac{d\pi_1}{dn_1}\right|_{\text{on CE}} = \underset{+}{\frac{\partial\pi_1}{\partial n_1}} + \underset{-}{\frac{\partial\pi_1}{\partial n_2}} \underset{-}{g^{-1\prime}(n_1)} > 0$$

because $\frac{\partial\pi_1}{\partial n_2} < 0$ is easily shown and $g^{-1}(n_1)$ is downward sloping. Hence, $\frac{d\pi_1}{dn_1} > 0$ for all (n_1, n_2) on curve CE. This implies that the Stackelberg equilibrium point must be somewhere on firm 2's reaction function $n_2 = g^{-1}(n_1)$ with $n_1 > n_1^e$, i.e., on curve ED.

Second, consider the whole domain. Since $\frac{\partial\pi_1}{\partial n_2} < 0$ always holds, $\pi_1(n_1, g^{-1}(n_1))$ on IHCE is lower than $\pi_1(n_1, n_2)$ on LE for any fixed n_1. However, $\frac{\partial\pi_1}{\partial n_1} > 0$ in the domain left of curve AB, $\pi_1(n_1, n_2)$ on LE is not higher than $\pi_1(n_1, n_2)$ at E. Hence, we have shown that Stackelberg equilibrium (n_1^s, n_2^s) with product proliferation exists somewhere on curve ED.

(II) Suppose (n_1^e, n_2^e) and (n_2^e, n_1^e) do not exist. Then, (n^e, n^e) is the unique SPNE in the simultaneous two-stage game $(n_1, n_2) - (p_1, p_2)$. We can apply the first part of case (I) in the above and show that Stackelberg equilibrium (n_1^s, n_2^s) is somewhere on firm 2's reaction function in the domain $n_1 > n_2 > 0$.

Hence, we have proven that $n_1^s > n_2^s$ and $\pi_1(n_1^s, n_2^s) > \pi_2(n_1^s, n_2^s)$.

Appendix 4: Proof of Proposition 3

Using (4), (7), (8), and (10), the profit differential can be written as

$$\begin{aligned}\pi_1 - \pi_2 \;=\; & \frac{F}{16(B+n_1)(2B+n_1)^2}(64B^4 - 64AB^4 + 16A^2B^4 + 208B^3n_1 - 144AB^3n_1 \\ & +20A^2B^3n_1 + 220B^2n_1^2 - 100AB^2n_1^2 + 7A^2B^2n_1^2 + 88Bn_1^3 - 20ABn_1^3 + 12n_1^4)\end{aligned}$$

Applying the Buchberger's algorithm to solve $f(n_1) = 0$ and $\pi_1 - \pi_2 = 0$ simultaneously, we compute the Gröbner bases, which leads to $725A^4 - 4700A^3 + 7268A^2 - 3104A - 3776 = 0$. Solving it, we can verify that $\pi_1 - \pi_2 \gtreqless 0$ for $A \lesseqgtr 4.54$.

Similarly, using (7) and (10), the variety differential is given by

$$n_1 - n_2 = \frac{8B^2 - 4A + 18Bn_1 - 3ABn_1 + 6n_1^2}{4(2B + \gamma n_1)}$$

Solving $f(n_1) = 0$ and $n_1 - n_2 = 0$ simultaneously, we obtain $9A^3 - 174A^2 + 416A + 656 = 0$. Thus, we verify that $n_1 - n_2 \gtreqless 0$ for $A \lesseqgtr 4.2$.

Appendix 5: Proof of comparative statics

Define $h \equiv F\gamma^4 f(n_1)$. Using the implicit function theorem, we have

$$\begin{aligned}
\frac{\partial n_1}{\partial \gamma} &= -\frac{\partial h/\partial \gamma}{\partial h/\partial n_1} = -\frac{n_1}{\gamma} < 0 \\
\frac{\partial n_1}{\partial \alpha} &= -\frac{\partial h/\partial \alpha}{\partial h/\partial n_1} = \frac{-B^{\frac{3}{2}}(4B^2 + 2AB^2 + 8Bn_1 + 4n_1^2 - An_1^2)}{\sqrt{F\gamma}(8AB^3 - 32B^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)} \\
\frac{\partial n_1}{\partial F} &= -\frac{\partial h/\partial F}{\partial h/\partial n_1} = \frac{2\left(B + n_1\right)^2\left(6B^2 - AB^2 + 4Bn_1 + n_1^2\right)}{F(8AB^3 - 32B^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)} \\
\frac{\partial n_1}{\partial \beta} &= -\frac{\partial h/\partial \beta}{\partial h/\partial n_1} \\
&= \frac{96B^3 - 28AB^3 - 6A^2B^3 + 192B^2n_1 - 40AB^2n_1 + 120Bn_1^2 - 12ABn_1^2 + A^2Bn_1^2 + 24n_1^3}{2\gamma(8AB^3 - 32B^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)}
\end{aligned}$$

It obvious that $\frac{\partial n_1}{\partial \gamma} < 0$. By applying the Buchberger's algorithm (Cox et al., 1997) to solve the equations (11) and $\frac{\partial n_1}{\partial \alpha} = 0$ simultaneously, we compute the Gröbner bases, one of which is the polynomial with one variable A. Then, we can solve for A and verify that $\frac{\partial n_1}{\partial \alpha} > 0$ for $A < 18$ and $\frac{\partial n_1}{\partial \alpha} \leq 0$ for $A \geq 18$. Similarly, we can verify that $\frac{\partial n_1}{\partial F} < 0$ for $A < 18$ and $\frac{\partial n_1}{\partial F} \geq 0$ for $A \geq 18$. Besides, $\frac{\partial n_1}{\partial \beta} < 0$ for $A < 3.42$ and $\frac{\partial n_1}{\partial \beta} \geq 0$ for $A \geq 3.42$.

The comparative statics of n_2 are also straightforward as follows:

$$
\begin{aligned}
\frac{\partial n_2}{\partial \gamma} &= \frac{\partial n_2}{\partial \gamma} + \frac{\partial n_2}{\partial n_1}\frac{\partial n_1}{\partial \gamma} = \frac{8B^2 - 4AB^2 + 10Bn_1 - 3ABn_1 + 2n_1^2}{4(2B+n_1)\gamma} \\
\frac{\partial n_2}{\partial \alpha} &= -\sqrt{B}(208B^5 - 80AB^5 + 4A^2B^5 + 672B^4n_1 - 144AB^4n_1 + 8A^2B^4n_1 + 864B^3n_1^2 \\
&\quad -88AB^3n_1^2 + 8A^2B^3n_1^2 + 556B^2n_1^3 - 16AB^2n_1^3 + 3A^2B^2n_1^3 + 180Bn_1^4 + 2ABn_1^4 + 24n_1^5) \\
&\quad / \left[4\sqrt{\gamma F}(2B+n_1)^2(-32B^3 + 8AB^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)\right] \\
\frac{\partial n_2}{\partial F} &= -(288B^6 - 352AB^6 + 72A^2B^6 + 960B^5n_1 - 1056AB^5n_1 + 160A^2B^5n_1 - 8A^3B^5n_1 \\
&\quad +1280B^4n_1^2 - 1224AB^4n_1^2 + 112A^2B^4n_1^2 - 10A^3B^4n_1^2 + 896B^2n_1^3 - 700AB^3n_1^3 \\
&\quad +24A^2B^3n_1^3 - 3A^3B^3n_1^3 + 360B^2n_1^4 - 204AB^2n_1^4 + 80Bn_1^5 - 24ABn_1^5 + 8n_1^6) \\
&\quad / \left[8(2B+n_1)^2\gamma(-32B^3 + 8AB^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial n_2}{\partial \beta} &= -(128B^5 - 16AB^5 - 80A^2B^5 + 12A^3B^5 + 128B^4n_1 + 96AB^4n_1 - 112A^2B^4n_1 \\
&\quad +8A^3B^4n_1 - 288B^3n_1^2 + 336AB^3n_1^2 - 80A^2B^3n_1^2 + 4A^3B^3n_1^2 - 496B^2n_1^3 \\
&\quad +332AB^2n_1^3 - 28A^2B^2n_1^3 + 3A^3B^2n_1^3 - 256Bn_1^4 + 132ABn_1^4 + 2A^2Bn_1^4 - 48n_1^5 + 24An_1^5) \\
&\quad / \left[8(2B+n_1)^2\gamma(-32B^3 + 8AB^3 - 60B^2n_1 + 8AB^2n_1 - A^2B^2n_1 - 36Bn_1^2 - 8n_1^3)\right]
\end{aligned}
$$

Solving $\frac{\partial n_2}{\partial \gamma} = 0$, we obtain a unique solution of $n_2 > 0$. By substituting it into (11), we can verify that $\frac{\partial n_2}{\partial \gamma} < 0$. Besides, applying the Buchberger's algorithm to solve the equations (11) and $\frac{\partial n_1}{\partial \alpha} = 0$ simultaneously, we compute the Gröbner bases, one of which is the polynomial with one variable A. Then, we can solve for A and verify that $\frac{\partial n_2}{\partial \alpha} > 0$ for all $A > 2$. Similarly, we can also verify that $\frac{\partial n_1}{\partial F} < 0$ for $A > 2$, and $\frac{\partial n_1}{\partial \beta} < 0$ for $A < 5.18$ and $\frac{\partial n_1}{\partial \beta} \geq 0$ for $A \geq 5.18$.

Appendix 6: Proof of Proposition 4

Using (7), (8) and (14), the difference between the equilibrium number of varieties and the optimal one can be written as

$$N - N^o = \frac{2n_1^2 + \left[(3-2\sqrt{2})A + 2\right] Bn_1 - 4(\sqrt{2}-1)AB^2}{4\,(2B + n_1)}$$

Accordingly, for $N - N^o \gtreqless 0$, we should have

$$n_1 \gtreqless \tilde{n}_1 = \frac{B}{4}\left[(2\sqrt{2}-3)A - 2 + \psi\right]$$

where $\psi \equiv \sqrt{4 + A(17A - 12\sqrt{2}A - 20 + 24\sqrt{2})}$. Substituting $n_1 = \tilde{n}_1$ into (11), we have

$$\begin{aligned} \frac{\partial \pi_1}{\partial n_1} &= \frac{1}{32(9-4\sqrt{2})}\Big\{(152\sqrt{2}-215)A^2 + (412-296\sqrt{2})A + 144\sqrt{2} - 316 \\ &\quad + \psi[(26\sqrt{2}-37)A + 50 - 24\sqrt{2}]\Big\} < 0 \end{aligned}$$

for all $A > 2$. This implies $n_1 < \tilde{n}_1$, and hence, we have $N < N^o$.

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