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Abstract

Why do some employees receive only fixed or incentive pay, while others receive a mix of fixed and incentive pay? Moreover, why do the lengths of wage contracts differ across these types of pay? This paper attempts to respond to this economic puzzle by developing a theoretical model of multiperiod contracts that incorporates short-, medium-, and long-term contracts with different wage profiles. We obtain different combinations of these contracts as equilibria when the efficiency of investment in human capital changes endogenously over time.

Keywords: Differing Length Contracts; Unverifiable Outputs; Unverifiable Investments; Unverifiable Ability; Holdup Problems

JEL Codes: D86; J41; J31

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1 Introduction

Why do we observe varying wage profiles in practice? CEOs and other executives of firms often receive most of their pay in the form of annual bonus plans and stock-based compensation, including executive stock options and restricted stock (Murphy 1999). Other employees, such as fund managers and salespersons, are also paid by commission, which fluctuates a great deal depending on outputs (see Coughlan and Narasimhan 1992, Elton, Gruber, and Blake 2003, Misra, Coughlan, and Narasimhan 2005, Ma, Tang, and Gomez 2012). Conversely, public servants and many businesspersons typically receive a fixed wage throughout their tenure (Lazear 1986, Itoh 1995). That is, their wage remains fixed until it is renewed to another fixed amount at the time of a promotion or during wage negotiation. Of course, these are two extreme examples where the agent mainly receives either only a commission or a fixed wage during the contracting period. For the most part, these differences in compensation structure exist in practice because employee outputs are verifiable in the former and mostly unverifiable in the latter (Kamiya and Sato 2011). However, when we consider the real world, there are many employees who begin with a fixed wage, but later receive commission until they retire or are dismissed from their job. Alternatively, some other employees start their careers with commissions but later receive a fixed wage until they retire from or quit their job.

In this paper, we first confine our analyses to simple wage contracts and explore why these various combinations of wage profiles exist and explain the optimal length of each wage. In short, we examine how a multiperiod optimal contract is designed as a combination of contracts of different lengths and different wage profiles. Suppose there are n contractible periods. We define a wage contract in which the initial wage agreement remains unchanged for all n periods as a long-term contract, whereas the agreement is only valid for a single period as a short-term contract, and for anything between one and n periods as a medium-term contract.[1] We then investigate a general mechanism, which

[1] This is because the purpose of this analysis is to prove theoretically that an optimal wage contract can be a combination of contracts of different lengths. As in Fudenberg, Holmstrom, and Milgrom (1990), the agent is not dismissed (or fired) on the equilibrium path, even in the case of an agent repeating a number of short-term contracts.

includes menu and option contracts, and show that any mechanism cannot perform better than simple wage contracts.

The overview of our model is as follows. There is a principal and an agent, and both are risk neutral. The agent undertakes two types of investment (efforts) to accumulate the human capital necessary to produce the two types of output, x and y. The first type of output x is observable and verifiable (contractible) whereas the second type of output y is observable but unverifiable (noncontractible). Examples of x are the annual profits of the firm or the amount of sales a salesperson makes. Examples of y are the extent to which an employee has contributed to the work of a team, or the leadership of a high-ranking employee. These investments are denoted I_c and I_n, and while both are observable, they are unverifiable. When the agent makes an investment I_c, he obtains the skill to produce x. The principal can then write a wage that depends on x. When the agent makes an investment I_n, he obtains the skills needed to produce y. Because y is unverifiable, the wage cannot reflect y. We assume that the investment in the current period becomes effective in the next period. Both investments are made in each period, after the wage contract is agreed and before the outcome of each period is realized.[2]

In this environment, suppose there are three periods, where the possible combinations of contracts the principal can offer are as follows: a) a long-term contract in which the wages for all three periods are determined by the principal and offered to the agent at the beginning of the first period; b) short-term contracts for all three periods in which the principal and the agent determine the wage for the second and third periods at the beginning of each period; c) a short-term contract for the wage in the first period and a medium-term contract for the wages in the second and third periods; and d) a medium-term contract in which the wages for the first and second periods are offered at the beginning of the first period and a short-term contract in which the wage for the third period is agreed to at the beginning of the third period.[3]

[2] As the wage for the first period is determined prior to the investment in human capital, and as the investment in the first period becomes effective in the second period, the first-period wage does not affect the agent's behavior (investment decision). In other words, the first-period wage is irrelevant for the choice of the agent's investment and hence we focus on the wages in the second period onward.

[3] We define a medium-term contract as anything between a one-period contract and the entire-period contract.

In a), the principal offers the agent wages for all three periods at the beginning of the first period. Within this contract, the agent has no incentive to increase his bargaining power. The principal designs each period's wage to depend on the verifiable output x produced in each period, but she cannot design wages to reflect unverifiable output y in each period. In short, a long-term contract deprives the agent of an incentive to invest in I_n (an effort to improve his skill to produce unverifiable output y), but motivates the agent to undertake a great deal more I_c (more than he could under short-term contracts).[4] As a result, the principal offers a commission payment to the agent under a long-term contract. Typically, this is the CEO receiving his salary as stock-based compensation and holding his position for a long time.

In b), the bargaining position of the agent at the beginning of the second period depends on his skill in producing both unverifiable y and verifiable output x. In other words, the amount of I_c and I_n made during the first period determines the agent's bargaining position. This provides the agent with an incentive to make I_n as well as I_c during the first period, but triggers a *holdup* for both I_c and I_n. As the wage is determined in the negotiations between the two parties in each period, the principal has no incentive to offer incentive pay to the agent under risk neutrality. As a result, the principal and the agent agree on the fixed wage for every short-term contract. This is similar to a bureaucrat or businessperson being promoted and their fixed wage increasing each time they are promoted.

In c), the parties Nash bargain over the wages for the second and third periods at the beginning of the second period (a medium-term contract). This gives the agent an incentive to make I_n as well as I_c during the first period, as the bargaining position at the beginning of the second period depends on both I_c and I_n. The third-period wage, however, is determined at the beginning of the second period, meaning that it depends on the verifiable output x in the third period. This implies that the agent has no incentive to invest in I_n during the second period, but does have an incentive to invest (a great deal) in I_c during the second period. The fixed wage part in the second- and third-period wages

[4]In Appendix B, we show that long-term contracts can achieve the first-best level in verifiable outputs.

is the result of the investment (I_c and I_n) level in the first period, whereas the commission parts in the second- and the third-period wages are meant to induce the agent to invest in I_c in both periods.

In d), the principal posts a take-it-or-leave-it offer to the agent for both the first- and second-period wages at the beginning of the first period (a medium-term contract). Here, the design of the second-period wage is to reflect only the verifiable output x that the agent is going to produce in the second period. Without any bargaining at the beginning of the second period, the agent would normally have no incentive to increase his skill in producing unverifiable output y during the first period. In d), however, as the agent and the principal will negotiate the third-period wage at the beginning of the third period (a short-term contract), the agent has some incentive to invest in the skills needed to produce y as well as x during the first period in order to increase his skill in the third period, where the amount of investment depends on the convexity of the agent's cost function. Moreover, because the wage of the third period is determined by Nash bargaining at the beginning of the third period, the agent still undertakes investment in both I_c and I_n during the second period.

We show that the choice between a), b), c), and d) is made based on the endogenously determined efficiency of investment and the relative value of the verifiable and unverifiable outputs. In other words, the choice is made depending on the accumulation level of human capital the agent makes. That is, if the human capital useful in producing (un)verifiable outputs does not accumulate much, even though the agent had made substantial effort, the investment in human capital can be considered as inefficient. If the human capital accumulates with a small amount of effort, the investment in human capital is then said to be efficient. We show that if investment efficiency remains fairly similar in all periods[5], the principal chooses either a) or b) (see Kamiya and Sato (2011) for details).

If investment efficiency changes between periods, the principal chooses c) or d).[6] We show that c) is chosen when the investment I_n becomes inefficient after the second period,

[5] For example, if it is (in)efficient in the first period, it is (in)efficient in the remaining periods.

[6] These two cases can be considered in relation to the career concerns model. We discuss this at the beginning of Section 2.

in turn making the investment I_c relatively efficient. In reality, if a worker on probation has invested time and effort in learning about the match between himself and his position in the company, he does not need to spend as much time and effort in learning this same information when he becomes a full-time employee. Even though such human capital is essential in conducting the position, the investment in I_n actually becomes inefficient if he keeps doing that forever. On the other hand, d) is chosen when the investment I_c becomes inefficient after the second period, in turn making the investment I_n relatively efficient. This can explain the case of salespersons becoming middle managers. Obviously, he will need some field experience in becoming a successful middle manager, but this field experience for him is mostly verifiable output: sales. That is, I_c was important in the past. However, after becoming a middle manager and receive mainly fixed salary, he obviously needs some knowledge in leading his team, and thinking about strategic plans for sales in the long run, as denoted by I_n.

In an n-period model, we can obtain a more complicated combination of contracts as an equilibrium, such as repeating medium-term contracts. We show that such a combination contract is offered when the agent's human capital depreciates. Suppose that a skill needed to produce y depreciates at some given depreciation rate. In this case, the principal wishes the agent to make occasional efforts I_n to maintain the skills needed to produce y at some certain level. To do so, the principal repeatedly offers medium-term contracts. If the principal and the agent bargain over wages every two periods, the agent is given an incentive to invest in I_n which compensates for any depreciation in human capital.

The structure of the remainder of this paper is as follows. Section 2 reviews the relevant literature. Section 3 analyzes a three-period model with simple wage contracts in which we do not impose limited liability constraints. We devote Section 4 to the five-period case. In Section 5, we discuss limited liability constraints as an extension, and show that we obtain nearly the same results as in the preceding section. Section 6 analyzes a general mechanism that includes menu contracts and option contracts, and show that any mechanism cannot perform better than simple wage contracts. Section 7 presents our conclusion.

2 Literature

Labor contracts tend to be depicted as either short- or long-term contracts (for example, Fudenberg, Holmstrom and Milgrom 1990, Ray and Salanié 1990, Dutta and Reichelstein 2003, and Kamiya and Sato 2011)[7]. Moreover, although many outputs in practice are observable but unverifiable, most models tend to incorporate only verifiable outputs (for example, Mirrlees 1976, Harris and Raviv 1979, Holmstrom 1979, and Grossman and Hart 1983). The main contribution of our model is that it is the first to combine contracts of different lengths in an incomplete contracting environment where both verifiable and unverifiable outputs are incorporated.

This paper is related to earlier work by Fudenberg, Holmstrom, and Milgrom (1990), Ray and Salanié (1990), and Salanié (2005, Chapter 6) in which they discuss the environment where an efficient long-term contract can be implemented as a sequence of one-period short-term contracts. However, they do not investigate the situation when a multiperiod optimal contract is implemented as a combination of contracts of different lengths. Therefore, as far as we are aware, this paper is the first to show that the optimal contract in the multiperiod principal–agent relationship comprises several contracts of different lengths.

Our model is related to Hellmann and Thiele (2011), in that they consider a compensation scheme where the agent is confronted with a multitasking choice between a standard task (verifiable) and the development of innovation (unverifiable). Hellmann and Thiele (2011) show that a low-powered incentive wage can motivate the effort for innovation by sacrificing effort for the standard task, as an externality exists between the tasks. This result is similar to our result in that a fixed wage can be derived as an optimal wage schedule, even when an outcome is not completely unverifiable. Our analysis, however, is quite different from theirs in at least three respects. First, our main objective is in exploring the efficient timing of a contract when the agent is expected to produce both verifiable and unverifiable outputs, which requires a multiperiod analysis. On the other

[7] In Dutta and Reichelstein (2003), the principal sometimes chooses a short-term contract even when outputs are verifiable. This is because in their model, the optimal short-term contract requires dismissing (or firing) the incumbent agent and hiring a new agent in the second period. It is clear that this scenario is ruled out under long-term contracting with the same incumbent agent.

hand, their model is essentially a one-period model in which the agent makes each effort just once. Second, we consider the situation where there are no externalities between tasks, while in their model externalities are essential for the results. Third, we consider the situation in which the agent's compensation for the verifiable and unverifiable outputs is not separable. In practice, there are many types of work in which the wage contract cannot be separable for each task the agent is expected to undertake. That is, a baseball player receives pay which is supposed to reflect both hits (verifiable) and teamwork skills (unverifiable), a tenured professor receives a wage that reflects not only the quantity of papers published (verifiable), but also their quality (observable but unverifiable), along with administrative/collegial work (observable but unverifiable). In Hellmann and Thiele (2011), their starting point is that the wage for the verifiable (standard) task and the unverifiable task (innovation) can be separable. That is, even if wage negotiation over the innovation output collapses, the agent can continue to receive pay for the verifiable output.

Bernheim and Whinston (1998) demonstrate that if there are some unverifiable actions and if agents' actions are sequential, there are cases in which an efficient outcome is obtained only by the incomplete contracting of verifiable actions. More precisely, restricting the second mover's (verifiable) action space in the contract, the shape of the second mover's best-response function can be modified such that the first mover chooses an (unverifiable) action that leads to an efficient outcome. Although our paper incorporates both verifiable and unverifiable outputs, and wages for verifiable outputs are sometimes unspecified in the contract, the underlying logic is quite different from that in Bernheim and Whinston (1998). For example, suppose the wages for future verifiable outputs are not specified as in the short-term contract in our model. This gives the agent an incentive to make I_n as well as I_c during the current period, as the bargaining position at the beginning of the next period depends on both I_c and I_n. In short, our core logic is quite different from that in Bernheim and Whinston (1998).

Edlin and Reichelstein (1996), Maskin and Tirole (1999a, 1999b), Moore and Repullo (1988), Kahn and Huberman (1988), and Noldeke and Schmidt (1995) also examine relatively complicated contracts. In Section 6, we investigate a general mechanism, and show that any mechanism cannot perform better than simple wage contracts. Our mechanism is very general and therefore includes most complicated contracts, including menu and option contracts.

Finally, seminal work by Fama (1980) suggests that there is no need to resolve incentive problems using explicit output-contingent contracts, because the agent is concerned about his reputation in the labor market. However, Holmstrom (1998) provides a formal model in which Fama's conclusion is only correct under some narrow assumptions: namely, career concerns induce the efficient action of the agent. That is, in most cases, explicit contracts play an important role. In our model, investment and human capital are to some extent firm specific. One way to relate our model to Holmstrom (1998) is to interpret firm specificity as an observable signal of the agent's investment. Firm specificity allows the firm to observe the investment perfectly, but the market can only receive a noisy signal about the agent's investment. The agent knows that the market can learn the agent's investment through this noisy signal over time; hence, he might make some efforts to influence this learning process of the market. However, the learning process of the market is imperfect and slow, so an explicit contract is the only way to induce a large amount of agent investment in the environment concerned within this model.

3 Three-period Model

3.1 Model

In this section, we assume that both the principal and the agent live for three periods, and show that several interesting combinations of contracts are chosen depending on the parameters. For example, the principal chooses to contract the wage for the first period in a short-term contract, and contract the wages for both the second and third periods in a medium-term contract at the beginning of the second period. Another example is that the principal may contract the wages for both the first and second periods in a medium-term contract at the beginning of the first period, and contract the third-period wage in a short-term contract at the beginning of the third period. For simplicity, we assume that the principal and the agent are both risk neutral. Although we do not impose limited liability constraints in Section 3, we discuss limited liability constraints in Section 4 and suggest that almost the same results can be obtained using the additional constraints.

There is a principal and an agent. We assume both are risk neutral. There are two types of outputs: an observable and contractible output x and an observable but noncontractible output y. The two contractible output levels are x^H and x^L, where $x^H > x^L > 0$. The probabilities of x^H and x^L are denoted by $P^H \in [0,1]$ and $P^L = 1 - P^H$. The two noncontractible output levels are θy^H and θy^L, where $y^H > y^L > 0$. Note that $\theta \geq 0$ is a parameter introduced for later use. The probabilities of y^H and y^L are denoted by $Q^H \in [0,1]$ and $Q^L = 1 - Q^H$. As will be formally stated below, we assume that the random variables x and y are stochastically independent.

To investigate the three-period model, we introduce human capital (the skills needed to produce outputs), α_c and α_n, and investments, I_c and I_n. We assume that investment (effort) accumulates the human capital. Let $P^H(\alpha_c) \in [0,1]$ be the probability that x^H occurs when a skill corresponding to a contractible output is $\alpha_c \in [0,\infty)$. Let $P^L(\alpha_c) = 1 - P^H(\alpha_c)$. Let $Q^H(\alpha_n) \in [0,1]$ be the probability that y^H occurs when the skill corresponding to a noncontractible output is $\alpha_n \in [0,\infty)$. Let $Q^L(\alpha_n) = 1 - Q^H(\alpha_n)$. Let $f_c : R_+^2 \to R_+$ and $f_n : R_+^2 \to R_+$ be the transition function of human capital. That is, for $i = c, n$, $\alpha_i' = f_i(I_i, \alpha_i)$ means that when the skill in the current period is α_i and the investment is I_i, the skill in the next period, denoted by α_i', is $f_i(I_i, \alpha_i)$. The investments and human capital in period t are denoted $I_t = (I_{ct}, I_{nt})$ and $\alpha_t = (\alpha_{ct}, \alpha_{nt})$. For a given parameter $\theta \geq 0$, $g(\alpha_n, \theta) = \sum_{i=H,L} Q^i(\alpha_n)\theta y^i$ denotes the expected value of noncontractible output. We assume that the two types of human capital, α_c and α_n, and the investments, I_c and I_n, are observable.

We assume the agent incurs disutility in undertaking investment, denoted by $D_c(I_c)$ and $D_n(I_n)$. Let $\delta \in (0,1)$ be the discount factor. The payment of wages for each period is at the end of each period following the realization of output. The wage depends on the realization of x only, as x is the only verifiable output. The wage w^i, $i = H, L$, in period t is denoted by $w_t^i, t = 1, 2, 3$. Note that, because of the assumed risk neutrality, w_2^i and

w_3^i need not depend on the realization of an output in previous periods.[8]

Throughout this section, we make the following three assumptions. The assumptions on D_c, D_n, P^H, and Q^H are standard.

Assumption 1 1. $\frac{dD_i}{dI_i} > 0$, $\frac{d^2D_i}{dI_i^2} > 0$, $D_i(0) = 0$, and $\frac{d^2D_i(0)}{dI_i^2} = 0$, $i = c, n$.

2. $\frac{dP^H}{d\alpha_c} > 0$ and $\frac{d^2P^H}{d\alpha_c^2} < 0$.
3. $\frac{dQ^H}{d\alpha_n} > 0$ and $\frac{d^2Q^H}{d\alpha_n^2} < 0$.
4. The random variables x and y are stochastically independent.

For simplicity, we make the following assumption.

Assumption 2 In the first period, the two types of human capital, α_c and α_n, are zero, and $P^H(0) = Q^H(0) = 0$.

By this assumption, the principal only has to offer w_1^L in the first period. Note that even if we allow for $P^H(0) > 0$ or $Q^H(0) > 0$, the following analyses do not change much because the investments in the first period do not affect P^H and Q^H in the first period but affect P^H and Q^H in the second period onward. That is, even if x^H can be realized with a positive probability and the principal offers w_1^H, it does not affect the investment levels.

For the bargaining process, we suppose the market for workers without firm-specific skills is competitive. We also assume that the agent obtains some firm-specific skills in the first period without any particular investment (that is, working in the firm without much effort can provide some experience, and the agent acquires some level of firm-specific skills through this experience) and hence he obtains bargaining power to negotiate the wage at the beginning of the second and third periods.[9] Therefore, when the principal hires

[8] Suppose the agent has a strictly concave (risk-averse) utility function. The risk-averse agent prefers intertemporal consumption smoothing, whereas the risk-neutral principal needs not smooth the consumption stream. In this case, the principal can be better off offering wages with a small variance, which depend on the realization of outputs in previous periods. However, the risk-neutral agent does not require consumption smoothing, and hence, the principal does not have to offer wages that depend on the realization of an output in previous periods.

[9] Alternatively, we assume that the agent who undertook investment in the first or second period obtains bargaining power. We could assume that bargaining power is only given to the agent with $\alpha_c > 0$ or $\alpha_n > 0$. However, we can obtain the same result even when we assume that the agent obtains bargaining power through experience and without undertaking any particular investment.

an agent without firm-specific skills, she posts a take-it-or-leave-it wage offer. Note that we obtain similar results even if we assume the agent has certain bargaining power at the beginning of the first period, and hence Nash bargaining is used for the negotiation process (instead of a take-it-or-leave-it offer). What we are trying to emphasize here is that the change in bargaining power among different periods is not critical in obtaining our results.

After the agent has obtained skills, the principal and the agent might negotiate the wage at the beginning of the second and third periods. For simplicity, we use Nash bargaining with the threat point set at $(0, 0)$. That is, we assume that the principal and the agent have the same bargaining power and that they cannot find a new partner if they lose the current partner; i.e., they can access the labor market only once and their reservation utilities are zero. The case of nonzero reservation utilities is also important. Indeed, if the agent can reenter the job market or the principal can hire a new agent, the threat point is nonzero. It is worthwhile noting that we obtain similar results even if they have different bargaining power or their reservation utilities are nonzero in the second and third periods (see Subsection 3.9). We also note that in the discussion of renegotiation-proofness in the following theorems, we consider Nash bargaining games in which the status quo is the wage contract signed in the previous periods.

Assumption 3 The principal posts a take-it-or-leave-it offer for the length and wages of a contract with the agent's reservation utility $u \geq 0$ for a contract signed at the beginning of the first period (hereafter, period 1). The principal and the agent Nash bargain over the wages with the threat point $(0, 0)$ for a contract signed at the beginning of the second or third period (hereafter, periods 2 and 3, respectively).

3.2 Timing

At the beginning of period 1, the parties sign a contract. The contract could be either a short-, medium-, or long-term contract. At the middle of period 1, the agent makes investments, I_c and I_n. At the end of period 1, the outputs are realized and the wage agreed in the contract is paid. At the end of period 1, the parties can renegotiate the contract if they agree on medium or long-term contracts.

At the beginning of period 2, the parties sign either a short- or medium-term contract if the contract in period 1 was a short-term contract. Then, the same sequence of events as in period 1 occurs.

At the beginning of period 3, the parties sign a contract (only a short-term contract could be the case) when the contract in period 1 was either a medium-term contract or if the parties have been repeating short-term contracts. At the end of period 3, the outputs are realized and the wage agreed in the contract is paid. Note that there is no incentive to invest in period 3 as it is the final period and investment in the current period only becomes effective in the next period.

As described in Assumption 3, at the beginning of period 1, the principal posts a take-it-or-leave-it offer for the length and wages of a contract maximizing her discounted sum of expected utility subject to the agent's individual rationality and incentive compatibility constraints for investment. If the contract in period 1 is short term, then at the beginning of period 2, the parties bargain over the length and wages of the contract, maximizing the Nash product of their discounted sum of utilities in periods 2 and 3, subject to the incentive compatibility constraint on investment. If the contract in period 2 is short term, or that in period 1 is medium term, the parties bargain over the wages of the contract at the beginning of period 3, maximizing the Nash product of their utilities in period 3.

At the end of periods 1 and 2, the parties can renegotiate contracts and choose a new contract if both become better off by so doing. For example, suppose the parties sign a long-term contract at the beginning of period 1, which is before the first investment decision is made. In this case, it might be better to change the contract at the end of period 1, as the agent has already chosen investments I_c and I_n in period 1, and a new contract for the second- and third-period wages could lead to a Pareto improvement.

3.3 Equilibria

We adopt the dynamic programming approach. Let the values of the principal and the agent (that is, the discounted sums of expected utilities when a contract is optimally chosen) at the beginning of period $t = 1, 2, 3$ be $V_t^p(\alpha)$ and $V_t^a(\alpha)$, where $\alpha = (\alpha_c, \alpha_n)$ is the human capitals at the beginning of period t. That is, $V_t^p(\alpha)$ and $V_t^a(\alpha)$ are the principal's and the agent's values of α when the wages from period t onward are not yet determined at the beginning of period t. $V_t^p(\alpha)$ and $V_t^a(\alpha)$ satisfy the following.

At the beginning of the first period, the principal posts a take-it-or-leave-it wage offer. She has three choices: contract only the first-period wage (a short-term contract), contract the first- and second-period wages (a medium-term contract), or contract the wages for all three periods (a long-term contract). Hence,

$$V_1^p(0,0) = \max\{V_1^{p3}(0,0), V_1^{p2}(0,0), V_1^{p1}(0,0)\} \tag{1}$$

holds, where $V_1^{p3}(0,0), V_1^{p2}(0,0)$, and $V_1^{p1}(0,0)$ are the values in the cases of the long- (three-period), medium- (two-period), and short-term (one-period) contract, respectively. Namely, $V_1^{pk}(0,0)$ is the discounted sum of expected utilities when a k-period contract is chosen. Note that $\alpha = (0,0)$ at the beginning of period 1. $V_1^{pk}(0,0)$, the principal's value of k-period contract, $k = 1, 2, 3$, satisfies the following:

$$\begin{aligned}
V_1^{pk}(0,0) \quad &= \quad \max_{w_1,\dots,w_k,I_1,\dots,I_k} \sum_{j=1}^{k} \delta^{j-1} \left[\sum_{i=H,L} P(\alpha_{cj})(x^i - w_j^i) + \sum_{i=H,L} Q(\alpha_{nj})\theta y^i \right] + \delta V_{k+1}^p(\alpha_{k+1}) \\
\text{s.t.} \quad & \sum_{j=1}^{k} \delta^{j-1} \left[\sum_{i=H,L} P(\alpha_{cj}) w_j^i - \sum_{i=c,n} D_i(I_{ij}) \right] + \delta V_{k+1}^a(\alpha_{k+1}) \geq u, \qquad (2) \\
& \sum_{j=1}^{k} \delta^{j-1} \left[\sum_{i=H,L} P(\alpha_{cj}) w_j^i - \sum_{i=c,n} D_i(I_{ij}) \right] + \delta V_{k+1}^a(\alpha_{k+1}) \qquad (3) \\
\geq \quad & \sum_{j=1}^{k} \delta^{j-1} \left[\sum_{i=H,L} P(\alpha'_{cj}) w_j^i - \sum_{i=c,n} D_i(I'_{ij}) \right] + \delta V_{k+1}^a(\alpha'_{k+1}), \ {}^{\forall} I'_1, \dots, I'_k,
\end{aligned}$$

where u is the reservation utility, $w_j = (w_j^H, w_j^L)$, and $I_j = (I_{cj}, I_{nj})$. Note that w_j^i does not depend on the realization of x in the previous periods as the agent is risk neutral and there is no need for consumption smoothing (see footnote 8). In the objective function, the first term on the right-hand side (RHS) is the discounted sum of the principal's expected utility in the k-period contract and the second term is the principal's value of $\alpha_{k+1} = (\alpha_{c,k+1}, \alpha_{n,k+1})$, the discounted sum of expected utilities when a contract is optimally chosen in period $k+1$. Note that α_t (α'_t) is derived from f_c, f_n and $I_1, \ldots, I_k$ ($I'_1, \ldots, I'_k$) and that $\alpha_1 = \alpha'_1 = (0, 0)$. For example, $\alpha_{c2} = f_c(I_{c1}, 0)$. Expression (2) is the individual rationality constraint and (3) is the incentive-compatibility constraint. In (2), the first term is the discounted sum of the agent's expected utility in the k-period contract and the second term is the agent's value of α_{k+1}. Note that $V_4^p(\alpha_4) = V_4^a(\alpha_4) = 0$.

Suppose that the levels of human capital at the beginning of the second period are $\alpha = (\alpha_c, \alpha_n)$. If the wages for the second and third periods are not yet determined at the beginning of the second period, the principal and the agent have two choices: contract only the second-period wage—i.e., a one-period (short-term) contract—or contract the second- and third-period wages at the same time—i.e., a two-period (medium-term) contract. Note that w_3 does not depend on the realization of outputs in period 2 as the agent is risk neutral and there is no need for consumption smoothing (see footnote 8). The Nash bargaining problem for the k-period contract, $k = 1, 2$, is expressed as follows:

$$\max_{w_2,\ldots,w_{k+1}, I_2,\ldots,I_{k+1}} \left(\sum_{j=2}^{k+1} \left[\delta^{j-2} \sum_{i=H,L} P^i(\alpha_{cj})(x^i - w_j^i) + g(\alpha_{nj}, \theta) \right] + \delta V_{k+2}^p(\alpha_{k+2}) \right)$$
$$\times \left(\sum_{j=2}^{k+1} \delta^{j-2} \left[\sum_{i=H,L} P^i(\alpha_{cj}) w_j^i - \sum_{i=c,n} D_i(I_{ij}) \right] + \delta V_{k+2}^a(\alpha_{k+2}) \right).$$

$$\text{s.t.} \quad \sum_{j=2}^{k+1} \delta^{j-2} \left[\sum_{i=H,L} P^i(\alpha_{cj}) w_j^i - \sum_{i=c,n} D_i(I_{ij}) \right] + \delta V_{k+2}^a(\alpha_{k+2})$$
$$\geq \sum_{j=2}^{k+1} \delta^{j-2} \left[\sum_{i=H,L} P^i(\alpha'_{cj}) w_j^i - \sum_{i=c,n} D_i(I'_{ij}) \right] + \delta V_{k+2}^a(\alpha'_{k+2}) \text{ for all } I'_2, \ldots, I'_{k+1},$$

where $(\alpha_{c2}, \alpha_{n2}) = (\alpha_c, \alpha_n)$ and α_3 (α'_3) is derived from f_c, f_n, and I_2 (I'_2). In the objective function, the term in the first (second) parentheses is the discounted sum of the principal's (agent's) expected utility, and the constraint is the incentive compatibility condition.

Note that the individual rationality constraint is included in the Nash bargaining with the reservation utilities (threat point) $(0,0)$. Note that $V_4^p(\alpha_4) = V_4^a(\alpha_4) = 0$. Clearly, $I_{n3} = 0$ is chosen. The values of the principal and the agent (the utilities obtained from the bargaining) are expressed as $V_2^{pk}(\alpha)$ and $V_2^{ak}(\alpha)$. We obtain $V_2^{pk}(\alpha) = V_2^{ak}(\alpha)$. It is clear that

$$V_2^p(\alpha) = \max\{V_2^{p2}(\alpha), V_2^{p1}(\alpha)\}.$$

As stated in the above, $V_2^{p1}(\alpha) = V_2^{a1}(\alpha)$ and $V_2^{p2}(\alpha) = V_2^{a2}(\alpha)$ hold. Thus, if $V_2^p(\alpha) = V_2^{p1}(\alpha)$, then $V_2^a(\alpha) = V_2^{a1}(\alpha)$, and otherwise $V_2^a(\alpha) = V_2^{a2}(\alpha)$.

Suppose that the levels of human capital at the beginning of the third period are $\alpha = (\alpha_c, \alpha_n)$. If the wage for the third period is not yet determined at the beginning of the third period, the contracting problem (Nash bargaining) is expressed as follows:

$$\max_{w_3} \left(\sum_{i=H,L} P^i(\alpha_c)(x^i - w_3^i) + g(\alpha_n, \theta) \right) \left(\sum_{i=H,L} P^i(\alpha_c) w_3^i \right), \tag{4}$$

where the terms in the first and second parentheses are the utilities of the principal and the agent in the third period, respectively. Note that there is no incentive-compatibility constraint, given that the agent has no incentive to invest in the third period. With risk neutrality, each party obtains half of the total utility available. The values (the utilities obtained from the bargaining) of the principal and the agent are expressed as $V_3^p(\alpha)$ and $V_3^a(\alpha)$, respectively.

We adopt a pure strategy subgame-perfect equilibrium as a solution concept. By the standard argument (backward induction), there exists a subgame-perfect equilibrium: given α in period 3, $V_3^p(\alpha)$ and $V_3^a(\alpha)$ are obtained as in the above Nash bargaining problem in period 3, then given α in period 2, $V_2^p(\alpha)$ and $V_2^a(\alpha)$ are obtained as in the above Nash bargaining problem in period 2, and finally $V_1^p(0,0)$ and $V_1^a(0,0)$ are obtained as in the above principal–agent problem in period 1. There are four types of equilibria.

Definition 1 1. A long-term equilibrium contract, i.e., $V_1^p(0,0) = V_1^{p3}(0,0)$: the wages for all periods are determined at the beginning of the first period on the equilibrium path.

2. A short–short–short-term equilibrium contract, i.e., $V_1^p(0,0) = V_1^{p1}(0,0)$ and $V_2^p(\alpha) = V_2^{p1}(\alpha)$: the wages for each period are determined at the beginning of each period in an equilibrium contract on the equilibrium path.

3. A short–medium-term equilibrium contract, i.e., $V_1^p(0,0) = V_1^{p1}(0,0)$ and $V_2^p(\alpha) = V_2^{p2}(\alpha)$: if the wage for the first period is determined at the beginning of the first period, and the remaining wages are determined in the second period on the equilibrium path.

4. A medium–short-term equilibrium contract, i.e., $V_1^p(0,0) = V_1^{p2}(0,0)$: the wages for the first and second periods are determined at the beginning of the first period, and the remaining wages are determined at the beginning of the third period, on the equilibrium path.

We next discuss the incentives for investing I_c and I_n. The choice of contract depends on the relative importance of the verifiable and unverifiable outputs and the relative efficiency of the investments in human capital made for each output. As the relative efficiency endogenously varies over time according to human capital accumulation, various types of combinations of contracts of different lengths are obtained as equilibria. If a multiperiod contract (i.e., a long- or medium-term contract) is chosen, the agent is sometimes deprived of an incentive to increase α_n after signing the contract, as the wages do not depend on the realization of y. For example, if a medium-term contract is chosen at the beginning of the first period, the agent has no incentive to increase α_n in the second period. However, the agent has an incentive to increase α_n in the third period, as he can obtain half of the gain from the investment through the Nash bargaining process at the beginning of the third period. The benefit of the long-term (or multiperiod) contract is that the principal can motivate the agent to undertake a greater amount of I_c than she could under short-term contracts, as the contract can induce the first-best level of I_c in the contracting periods (see Appendix A). Conversely, under the short-term contract, the agent can obtain half of the total utility in the following period through Nash bargaining. Therefore, the agent has an incentive to make I_n, which is also beneficial for the principal.

Moreover, the relative efficiency of investment endogenously varies over time according to human capital accumulation. For example, suppose at the beginning of period 1, I_n is relatively more efficient than I_c. In this case, the principal chooses a short-term contract to induce I_{n1}. Suppose at the beginning of period 2, $\alpha_{n2} = I_{n1}$ is sufficiently large and I_c becomes relatively more efficient than I_n. Then, the parties choose a medium-term contract to induce I_{c2}. On the other hand, suppose at the beginning of period 1, I_c is relatively more efficient than I_n and that the difference in efficiency is not very large. Then, the principal chooses a medium-term contract to induce I_{c1}, predicting that in period 2 $\alpha_{c2} = I_{c1}$ will be sufficiently large and I_n will become relatively more efficient than I_c, and so at the beginning of period 3 the parties choose a short-term contract to induce I_{n2}.

To sum up, to induce I_c, a multiperiod contract should be chosen, and to induce I_n, a short-term contract should be chosen, and as the relative investment efficiency varies over time, then various combinations of contracts of different lengths are obtained as equilibria. In the following subsections, we present specifications of θ, f_c, f_n, D_c, D_n, P^H, and Q^H, where we obtain various types of contracts as equilibria.

3.4 Renegotiation

This subsection discusses renegotiation. We consider Nash bargaining games in which the status quo is the wage contract signed in the previous periods. The parties can renegotiate the contract at the end of period 1 and/or period 2. If the wage in period 3 has been already signed by the end of period 2, the Nash bargaining problem (renegotiation) at the end of period 2 is expressed as follows:

$$\max_{w_3^{H\prime}, w_3^{L\prime}} \left(\sum_{i=H,L} P^i(\alpha_c)(x^i - w_3^{i\prime}) + \sum_{i=H,L} P^i(\alpha_c)(x^i - w_3^i) \right) \left(\sum_{i=H,L} P^i(\alpha_c) w_3^{i\prime} - \sum_{i=H,L} P^i(\alpha_c) w_3^i \right), \tag{5}$$

where (w_3^H, w_3^L) is the status quo wages. Note that renegotiation at the end of period 2 does not improve the status quo utilities.

If the wage in period 2 has been already signed by the end of period 1, the parties can renegotiate the contract at the end of period 1. The renegotiation is similar to the contract in period 2 in Subsection 3.3. That is, they choose a short-term contract or a medium-term contract at the end of period 1: in each contract they maximize the Nash product with the threat point $(V_2^p(\alpha), V_2^a(\alpha))$ and they choose the better one. If both the principal and the agent can be made better off, then they renegotiate the contract.

The definition of renegotiation-proofness is as follows:

Definition 2 An equilibrium contract is said to be renegotiation-proof if the parties do not renegotiate the contract even when they can renegotiate.

3.5 Long-Term Contract

If θ is sufficiently small, it is better to induce an incentive for I_c and thus a long-term contract is chosen. Note that this holds for any f_c, f_n, D_c, D_n, P^H, and Q^H satisfying the above assumptions.

Theorem 1 *There exists a $\bar{\theta}$ such that $^\forall\theta \in [0, \bar{\theta})$, the principal chooses a long-term contract. In the contract, the second- and third-period wages depend on x to induce I_c. The contract is renegotiation-proof.*

Proof: See Appendix A. ∎

3.6 Short–Short–Short-Term Contract

If θ is sufficiently large, it is always better to induce an incentive for I_n and offer a short–short–short-term contract under some additional condition. The additional condition is that the investment I_n is sufficiently costly and the cost function is sufficiently convex. Then, α_n is not saturated in all periods and the principal always wishes to induce an incentive for I_n. For simplicity, in this subsection, we suppose $Q^H(\alpha_n) = \alpha_n$, $f_n(I_n, \alpha_n) = \min\{I_n + \alpha_n, 1\}$, and $D_n(I_n) = bI_n^2$, where $b > 0$ is a parameter. Note that f_c, D_c, and P^H can be any function satisfying the above assumptions.

Theorem 2 *Suppose $b > \frac{1}{4}\theta(2\delta+\delta^2)(y^H - y^L)$. Then, there exists a $\bar{\theta} > 0$ such that $^\forall\theta \geq \bar{\theta}$, the equilibrium contract is of short–short–short term (note that it is clearly renegotiation-proof). In the contract, the wages for the second and third periods can be fixed.*

Proof: See Appendix B. ∎

As the investment I_c has already been made during the first period, the principal does not have to offer incentive pay for the second period depending on the realization of x. The same argument applies to the third period.

3.7 Short–Medium-Term Contract

Suppose that I_n is relatively efficient and that the skill (α_n) is easily saturated. In this case, the principal chooses a short-term contract for the first-period wage to induce I_n. After the agent makes an investment I_n in the first period, they agree on a medium-term contract for the second- and third-period wages at the beginning of the second period. That is, following the saturation of α_n, the principal wishes to induce I_c. To illustrate this point, in this subsection, we suppose $f_n(I_n, \alpha_n) = I_n + \alpha_n$, $D_n(I_n) = aI_n$, where $a > 0$ is a parameter, and

$$Q^H(\alpha_n) = \begin{cases} b\alpha_n & \text{if } 0 \leq \alpha_n \leq \bar{\alpha}_n \\ 1 & \text{if } \bar{\alpha}_n \leq \alpha_n, \end{cases}$$

where $b > 0$ and $\bar{\alpha}_n = \frac{1}{b}$. Note that f_c, D_c, and P^H can be any functions satisfying the above assumptions.

Theorem 3 *Suppose $a < \frac{1}{2}\delta\theta b(y^H - y^L)$. Then, there exists a $\bar{\theta} > 0$ such that, for all $\theta \geq \bar{\theta}$, the equilibrium contract is short–medium term. It is also renegotiation-proof. In the contract, the third-period wage is incentive pay depending on x, while the second-period wage can be a fixed wage.*

Proof: See Appendix C. ∎

Low-powered wage incentives are very often observed in the real world, and this is theoretically proven by the above theorem. Indeed, in the above environment, the fixed wage parts of w_2 and w_3, i.e., w_2^L and w_3^L, include the payment for I_{c1}. Thus the incentive wage part of w_3, i.e., $w_2^H - w_2^L$ and $w_3^H - w_3^L$, which induces I_{c2}, is relatively small.

3.8 Medium–Short-Term Contract

Suppose that θ is sufficiently large and the agent should accumulate the first type of skill (α_c) to obtain the second type of skill (α_n). For example, the agent needs experience in sales to attain a position of leadership in the sales department. Hence, the principal writes the wages for the first and second periods in a medium-term contract to induce I_c during the first period, and she writes the third-period wage in a short-term contract to induce I_n during the second period. In this subsection, we suppose $P^H(\alpha_c) = \alpha_c, f_c(I_c, \alpha_c) = \min\{I_c + \alpha_c, 1\}$, $D_c(I_c) = I_c^2$, $Q^H(\alpha_n) = \alpha_n, f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, and $D_n(I_n) = I_n^2$. Note that the transition function f_n depends not only on I_n and α_n, but also on α_c. More precisely, if α_c is small, then the investment I_n is not efficient, as in $f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, I_n is multiplied by α_c.

Theorem 4 [10] *There exists a $\bar{\theta}$ such that $^\forall\theta \geq \bar{\theta}$, the equilibrium contract is of medium–short term. In the contract, the second-period wage depends on x, whereas the third-period wage can be a fixed wage. Moreover, it is renegotiation-proof.*

Proof: See Appendix D. ▮

3.9 An Extension

The arguments in the previous subsections can be extended to the case with different bargaining power and a nonzero threat point. In particular, it is worthwhile noting that if the agent can reenter the job market or the principal can hire a new agent, then the threat point is nonzero. In this case, the bargaining problem in period 2 is as follows:

[10] We can also show that a medium–short-term contract is an equilibrium if I_c is relatively efficient and easily saturated. That is, the principal chooses a medium-term contract for the first- and second-period wages to induce I_c, and after the investments she chooses a short-term contract on the third-period wages to induce I_n.

$$\max_{w_2,\ldots,w_{k+1},I_2,\ldots,I_{k+1}} \quad \left(\sum_{j=2}^{k+1}\left[\delta^{j-2}\sum_{i=H,L}P^i(\alpha_{cj})(x^i-w_j^i)+g(\alpha_{nj},\theta)\right]+\delta V_{k+2}^p(\alpha_{k+2})-T_2^p\right)^{\beta}$$
$$\times\left(\sum_{j=2}^{k+1}\delta^{j-2}\left[\sum_{i=H,L}P^i(\alpha_{cj})w_j^i-\sum_{i=c,n}D_i(I_{ij})\right]+\delta V_{k+2}^a(\alpha_{k+2})-T_2^a\right)^{1-\beta}$$
$$\text{s.t.}\quad \sum_{j=2}^{k+1}\delta^{j-2}\left[\sum_{i=H,L}P^i(\alpha_{cj})w_j^i-\sum_{i=c,n}D_i(I_{ij})\right]+\delta V_{k+2}^a(\alpha_{k+2})$$
$$\geq \sum_{j=2}^{k+1}\delta^{j-2}\left[\sum_{i=H,L}P^i(\alpha'_{cj})w_j^i-\sum_{i=c,n}D_i(I'_{ij})\right]+\delta V_{k+2}^a(\alpha'_{k+2})\text{ for all } I'_2,\ldots,I'_{k+1},$$

where (T_2^p, T_2^a) is the threat point in period 2; i.e., the utilities when the principal hires a new agent and the agent reenters the job market, and $(\beta, 1-\beta)$ is the vector of bargaining powers. The bargaining problem in period 3 is as follows:

$$\max_{w_3}\left(\sum_{i=H,L}P^i(\alpha_c)(x^i-w_3^i)+g(\alpha_n,\theta)-T_3^p\right)^{\beta}\left(\sum_{i=H,L}P^i(\alpha_c)w_3^i-T_3^p\right)^{1-\beta}, \tag{6}$$

where (T_3^p, T_3^a) is the threat point in period 3.

Even in this case, the same arguments as in the proofs of Theorems 1–4 can be applied. In the case of Theorem 1, the proof is based on the comparison of the gains from verifiable output in each contract; more precisely, the first-best I_c cannot be obtained in short–short–short-, short–medium- and medium–short-term contracts but only in a long-term contract, and thus a long-term contract is chosen when θ is close to zero. In the case of the other theorems, all proofs are based on the comparison of the gains from the verifiable and unverifiable outputs in the contracts; more precisely, comparison of the limit of differences as $\theta \to \infty$. Even in the case with different bargaining power and a nonzero threat point, the first-best I_c can be obtained only in a long-term contract. In addition, the comparisons of the limit of differences as $\theta \to \infty$ are essentially the same as in the case of $\beta = \frac{1}{2}$ and the threat point $(0,0)$, though the incentive to invest is different. In the bargaining in period t, the agent can obtain a $1-\beta$ fraction of gain from the investments; more precisely, $(1-\beta)((\text{total gain})-(T_t^p+T_t^a))+T_t^a$, and the investments decrease as $1-\beta$ becomes small.

Thus the gain from unverifiable output depends on $1-\beta$. However, as $1-\beta>0$ and the agent has an incentive to invest, the same arguments as in the proof of Theorems 1–4 can be applied, although the threshold $\bar{\theta}$ in the theorems becomes large as $1-\beta$ becomes small.

4 Five-Period Model

In this section, we assume that the principal and the agent live for five periods. All other things being equal, the principal has a greater variety of combinations of contracts to offer the agent. For example, if α_n depreciates, the principal wishes to occasionally induce an incentive for I_n to compensate for the depreciation. In this case, if the principal and the agent bargain over wages every two periods, then for every two periods the agent has an incentive to invest in α_n, which compensates for any depreciation.

Under the five-period model, contracting over two, three, or four periods are all considered medium-term contracts. Therefore, to avoid confusion, instead of referring to them as "medium-term contracts" we refer to them by the length of the periods included, for example, a *two–one–two* contract.

We assume that α_n is a function of the investments in the previous two periods: namely, $\alpha_{nt}=I_{n,t-2}+I_{n,t-1}$, i.e., the investments before period $t-2$ have entirely depreciated. Moreover, we suppose $D_n(I_n)=I_n$, and

$$Q^H(\alpha_n)=\begin{cases}\alpha_n & \text{if } 0\leq\alpha_n\leq 1\\ 1 & \text{if } 1\leq\alpha_n.\end{cases}$$

We adopt the same environment and assumptions as in the three-period model, other than the length of life and the arguments for f_n. Note that f_c, D_c, and P^H can be any functions satisfying the above assumptions.

We can define equilibrium contracts as in Section 3. That is, $V_t^p(\alpha)$ and $V_t^a(\alpha)$, $t=1,\ldots,5$, can be defined as in Section 3. Even though many types of equilibrium contracts could exist in this model, we focus on the following contract.

A one–two–two-term contract (which is a short–medium–medium-term contract): on the equilibrium path, the wage for the first period is determined at the beginning of the first period, the wages for the second and third periods are determined at the beginning of the second period, and the remaining wages are determined at the beginning of the fourth period.

Note that the other combinations, such as the two–two–one-term (medium–medium–short-term) contract, are similarly defined.

Theorem 5 *Suppose $x^H - x^L \leq 2$, $y^H - y^L \geq 2$, and $\theta \geq 5$. Then, there exists a $\bar{\delta} \in (0,1)$ such that the equilibrium contract is a one–two–two-term contract for $\delta \in (\bar{\delta}, 1]$. Moreover, it is renegotiation-proof.*

Proof: See Appendix E. ∎

5 Limited Liability Constraints

In this section, we discuss limited liability constraints, and show that nearly the same results can be obtained. For simplicity, we only investigate the three-period model. It is easy to see that almost the same arguments can be applied to the five-period model.

A standard limited liability constraint is (i) $w_1^L \geq 0$, $w_1^L + \delta w_2^i \geq 0, i = H, L$, and $w_1^L + \delta w_2^i + \delta^2 w_3^j \geq 0, i, j = H, L$: that is, the case in which the agent can save money. In contrast, the most conservative limited liability constraint is (ii) $w_t^i \geq 0, i = H, L, t = 1, 2, 3$. Below, we show that under a wide class of limited liability constraints, we can obtain the same results as in the previous section with only slight modification. More precisely, if there exists a real number $A \geq 0$ such that $w_t^i > A$ implies the limited liability constraint is not binding. In the case of (i), $w_1^L \geq 0$, $w_2^i \geq -\frac{1}{\delta} w_1^L$, and $w_3^j \geq -\frac{1}{\delta^2}(w_1^L + \delta w_2^i), i, j = H, L$. Thus, the minimum wages are $0, -\frac{1}{\delta} w_1^L$, and $-\frac{1}{\delta^2}(w_1^L + \delta w_2^i)$, and they are at most zero. Let $A = 0$. Then, for example, in the case of a medium-term contract in period 2, if $w_t^i > A$ for all $t = 2, 3, i = H, L$, the limited liability constraint in the contract is not binding.

Theorem 6 *Suppose there exists a real number $A \geq 0$ such that $w_t^i > A$ for all t, i implies the limited liability constraint is not binding. Then, the results in Theorems 1–4 hold. However, the thresholds are different from those in Theorems 1–4.*

Proof: See Appendix F. ∎

Below, we briefly explain the proof. Suppose $\theta = 0$. This is synonymous with saying that there are no unverifiable outputs. Given that even under some limited liability constraint any wage contract (such as a medium–short contract) can be replicated by a long-term contract (see Appendix F), I_{c1} and I_{c2} in the contract can be induced by the long-term contract. We can also show that the principal can make her utility strictly larger than under any other contract. Thus, the principal chooses a long-term contract. It is then obvious that the principal chooses a long-term contract even for a small θ. Therefore, the same result as in Theorem 1 holds. However, the threshold is different from that in the theorem.

As for the other theorems in Section 3, all proofs are based on the comparison of the gains from verifiable and unverifiable outputs in contracts; more precisely, the comparison of the limit of differences as $\theta \to \infty$. As the gains from the verifiable output do not depend on θ, the agent's gain from I_n, which is a part of wage, goes to ∞ and exceeds A as $\theta \to \infty$ and thus the relevant limited liability constraint is not binding. Thus the utility differences go to ∞ as $\theta \to \infty$ no matter what the limited liability constraint is. Thus, the results in the theorems hold.

6 A General Mechanism

In the previous sections, confining our attention to simple wage contracts, we found that combinations of contracts of different lengths arise as equilibrium contracts. One may consider some sophisticated contracts, if they are available, to be more efficient. Below, we investigate a general mechanism and show that any mechanism cannot perform better than simple two-period wage contracts. Of course, we assume risk neutrality of parties and renegotiation-proofness of equilibria. Note that our mechanism is very general and includes menu contracts, changes in ownership, such as a 'selling option to the agent', and some types of penalties.

6.1 Two-Period Case

For simplicity, we first discuss the two-period model, and show that any mechanism cannot perform better than simple two-period wage contracts.

6.1.1 Mechanisms without a Change of Ownership

We suppose that at the beginning of period 1 the principal offers a contract that involves the first period wage w_1^L and a mechanism specified below. The agent accepts it if and only if the discounted sum of expected utilities is not less than the reservation utility u. We first focus on the case in which the principal always has ownership. We then discuss the case in which the mechanism can change the ownership. In other words, the case in which the ownership can be moved from the principal to the agent.

The parties play a game (a mechanism) at the beginning of the second period, which is after observing (I_{c1}, I_{n1}) and before the realization of x and y. A mechanism is a pair of a function f and a message space $M = M^p \times M^a$, where f is a function from the message space to the space of outcomes Ω. Note that M^p, the principal's message space, and M^a, the agent's message space, can be any sets. Each element of the message spaces is observable and verifiable. The space of outcomes Ω is $R \times R^2$, where the first R is the set of transfers from the agent to the principal, denoted by q, paid before the realization of x and y, and R^2 is the set of the payments to the agent in the second period, denoted by (v^H, v^L), which depend on the realization of x. That is, the agent obtains v^i when x^i is realized. It is straightforward that the simple two-period wage contract is the contract that the message space M is a singleton, i.e., the payoff only depends on the realization of x.

In principle, the mechanism includes all possible outcomes when the principal always has ownership. First, the mechanism can force the agent to pay a 'penalty' q depending on the message. However, given renegotiation-proofness, a penalty cannot be paid to a third party (see Maskin and Tirole 1999a). Thus, the principal must obtain the penalty. In general, any monetary transfer between parties is included in the mechanism. Note that if at least one of the parties were strictly risk averse, a penalty using stochastic payment is useful. Suppose that only the agent is risk averse and that the mechanism forces the agent to pay q with probability $\frac{1}{2}$ to the principal and to obtain q (pay $-q$) from the principal with probability $\frac{1}{2}$. This mechanism is clearly renegotiation-proof because q is not paid to the third party, and the agent's utility is less than the case without the payment due to the risk averseness. Maskin and Tirole (1999) investigate mechanisms with such penalties. However, there is no point to use it in our case because both parties are risk neutral.

Second, the mechanism includes the case that the principal (the agent) chooses a contract from a menu of several simple wage contracts: i.e., the case that f assigns a simple wage contract depending on the message.

Remark 1 Moore and Repullo (1988) use a sequential-type mechanism and investigate implementation by subgame-perfect equilibria. Considering our message space as the set of bundles of messages in all nodes of the game tree, sequential-type mechanisms are covered by our model. In other words, we show below that any Nash equilibria, including subgame-perfect equilibria, cannot be better than the simple wage contract equilibria.

Remark 2 Even if the parties are risk averse, Maskin and Tirole's mechanism does not work in our environment. That is, the welfare neutrality that is the necessary condition for their theorem is violated in standard principal–agent models as in this paper (see Sections 2 and 8 in Maskin and Tirole 1999a).

After observing $I_1 = (I_{c1}, I_{n1})$ the principal (the agent) chooses $m^p \in M^p$ ($m^a \in M^a$), and thus a strategy of the principal is a function of I_1, denoted by $s^p(I_1) \in M^p$. The agent chooses I_1 in the first period and a message $s^a(I_1) \in M^a$ in the second period.

Let $m = (m^p, m^a)$ and $f(m) = (q(m), v^H(m), v^L(m))$. Then we define the agent's (discounted sum of) utilities in periods 1 and 2, denoted by u_1^a and u_2^a, and the principal's utility in period 2, denoted by u_2^p, as follows:

$$\begin{aligned}
u_1^a(w_1^L, I_1, f(m)) &= w_1^L - D_c(I_c) - D_n(I_n) - \delta q(m) + \delta \sum_{i=H,L} P^i(I_{c1}) v^i(m), \\
u_2^a(I_1, f(m)) &= -q(m) + \sum_{i=H,L} P^i(I_{c1}) v^i(m), \\
u_2^p(I_1, f(m)) &= q(m) + \sum_{i=H,L} P^i(I_{c1})(x^i - v^i(m)) + \delta \sum_{i=H,L} Q^i(I_{n1}) y^i.
\end{aligned}$$

For simplicity, $D_i(\alpha_{i1})$, $P^i(\alpha_{c1})$, and $Q^i(\alpha_{n1})$ are denoted by $D_i(I_{i1})$, $P^i(I_{c1})$, and $Q^i(I_{n1})$.

Definition 3 *For a given (w_1^L, f, M), $(s^p, (\hat{I}_1, s^a))$ is said to be an equilibrium strategy if*

$$u_1^a(w_1^L, \hat{I}_1, f(s^p(\hat{I}_1), s^a(\hat{I}_1))) \geq u_1^a(w_1^L, I_1, f(s^p(I_1), s^a(I_1))) \text{ for all } I_1,$$

and, for any given $I_1 = (I_{c1}, I_{n1})$,

$$u_2^p(I_1, f(s^p(I_1), s^a(I_1)) \geq u_2^p(I_1, f(m^p, s^a(I_1)) \text{ for all } m^p \in M^p,$$

$$u_2^a(I_1, f(s^p(I_1), s^a(I_1)) \geq u_2^a(I_1, f(s^a(I_1), m^a)) \text{ for all } m^a \in M^a.$$

The first inequality is the condition that the agent's choice in the first period is optimal, and the second and third inequalities imply that the principal's and the agent's choices of messages are optimal. Note that, given a (w_1^L, f, M), the principal does not choose anything in the first period and thus the principal's optimization in period 1 is not included in the definition.

We first show the revelation principle.

Theorem 7 *Given (w_1^L, f, M), suppose $(s^p, (\hat{I}_1, s^a))$ is an equilibrium strategy. Then there exists a direct revelation mechanism $(\tilde{f}, R_+^2 \times R_+^2)$ where the first (second) R_+^2 is the set of principal's (agent's) messages of investments, denoted by I_1^p (I_1^a), which can be different from the true investments, such that*

1. *$\tilde{s}^i(I_1) = I_1$, the truth telling, is a second-period equilibrium strategy of the game $(w_1^L, \tilde{f}, R_+^2 \times R_+^2)$ for $i = p, a$, where $\tilde{s}^i$ is a party i's strategy,*

2. *$\tilde{f}(I_1, I_1) = f(s^p(I_1), s^a(I_1))$.*

Proof: See Appendix G. ∎

Theorem 8 *Any mechanism cannot improve welfare in the simple two-period wage contract. (That is, the principal's utility cannot be better off. Note that the agent's utility is always equal to the reservation level u.)*

Proof: See Appendix H. ∎

6.1.2 Mechanisms with a Change of Ownership

In the above arguments, the principal is the residual claimer because the payment to the agent in the second period can only depend on the realization of x and the principal has ownership. The other possible outcome is to sell ownership to the agent and the agent becomes the residual claimer. That is, the mechanism assigns utilities (g^H, g^L) to the principal depending on the realization of x and the agent is the residual claimer. Therefore, the space of outcome is now extended to $\Omega' = R \times R^2 \times \{O^p, O^a\}$. Here, the first R is the same as the case of Ω, and $(v^H, v^L, O^p) \in R^2 \times \{O^p, O^a\}$ implies (v^H, v^L) is the payment to the agent and the principal has ownership and is thus the residual claimer. Similarly, $(g^H, g^L, O^a) \in R^2 \times \{O^p, O^a\}$ implies (g^H, g^L) is the payment to the principal and the agent has ownership and thus is the residual claimer. Note that the principal's selling option to the agent is included; that is, $(q, (0, 0), O^a) \in R \times R^2 \times \{O^p, O^a\}$ implies the principal sells the project to the agent with a price q. In addition, a future payment is allowed; that is, $(0, (g^H, g^L), O^a) \in R \times R^2 \times \{O^p, O^a\}$ implies the agents pays g^H (g^L) in the case of x^H (x^L) after the realization of x and y.

Clearly, as in the same argument above, the revelation principle holds. Below, we show that under limited liability constraints and some additional condition, it is sufficient to investigate simple two-period wage contracts. Of course, we keep assuming the risk neutrality of the parties and renegotiation-proofness of the equilibria.

Assumption 4 *1. Limited Liability Constraint 1: $w_1^L \geq 0$.*

2. Limited Liability Constraint 2: $\delta^{-1} w_1^L - q \geq 0$.

3. (a) *A future payment is not allowed, i.e., when the agent has ownership, f always assigns $(g^H, g^L) = (0, 0)$, or*

   (b) *$U^{p*} > \delta(x^H + y^L)$ holds, where U^{p*} is the principal's utility in the simple two-period wage contract equilibrium.*

As in the case without a change of ownership, any direct revelation mechanism cannot improve the welfare in the simple two-period wage contract when the mechanism assigns an outcome in $R \times R^2 \times \{O^p\}$. When the mechanism assigns an outcome in $R \times R^2 \times \{O^a\}$, the principal must obtain utility from negative w_1^L, negative $\delta^{-1}w_1^L - q$, or (g^H, g^L). However, from the above assumption, the first two options cannot be used. If Assumption 4-3-(a) holds, then the principal cannot obtain utility from (g^H, g^L). Suppose Assumption 4-3-(b) holds. The utility of the principal cannot be at most U^{p*}, because the payment to the principal cannot depend on the realization of y. Indeed, when x^i is realized, the mechanism can assign at most $g^i = x^i + y^L$, because g cannot depend on y. As a result, the principal's expected utility is $\delta \sum_{i=H,L} P(I_{c1}) g^i \leq \delta(x^H + y^L)$, which is less than U^{p*}. This leads to the following theorem.

Theorem 9 *Under Assumption 4, any mechanism cannot improve the welfare in the simple two-period wage contract equilibria.*

6.2 Three- and Five-Period Cases

In the case of the three-period model, there are two cases: (i) a contract is chosen at the beginning of the first period, and (ii) a contract is chosen at the beginning of the second period. In case (ii), although a contract is chosen using Nash bargaining, it is clear that the same argument as in the two-period model can be applied and it cannot improve the welfare in the case of the simple wage contract. In case (i), by the same arguments as in the proof in the two-period case, $\tilde{f}$ does not depend on I_{n1} and I_{n2}, and thus the agent chooses $I_{n1} = I_{n2} = 0$ when $\tilde{f}$ assigns O^p. Clearly, $\tilde{f}$ cannot assign O^a under similar assumptions as for Assumption 4.

Remark 3 Finally, note that Edlin and Reichelstein (1996) show that in a certain environment, where the threat point of a renegotiation is a function of investment, an appropriately chosen initial contract can provide the correct incentive for investment and lead to the first-best output. As shown in the proof of the renegotiation-proofness of the long-term contract in the three-period model (Theorem 1), the investments do not affect the threat point of renegotiation in the second period. Although the investments affect the threat point of renegotiation in the third period, the risk-neutral parties do not renegotiate because they share only the outputs in the third period. Thus, the initial contracting has no value, as verified in Che and Hausch (1999).

7 Conclusion

In this paper, we investigated multiperiod contracts of different lengths in an incomplete contracting framework. Confining our attention to simple wage contracts, we found that combinations of contracts of different lengths arise as equilibrium contracts when the principal's output is determined by both verifiable and unverifiable outputs and the investment efficiency endogenously changes over time. We also showed that any sophisticated contract (mechanism) could not do better than the simple wage contracts.

Appendices

A The Proof of Theorem 1

We first show that the first-best level investments of I_{c1} and I_{c2} are obtained in a long-term contract. Given the risk neutrality of the principal and the agent, the first-best investments are the maximizer of the following problem:

$$\max x^L - D_c(I_{c1}) + \delta \left[\sum_{i=H,L} P^i(\alpha_{c2})x^i - D_c(I_{c2}) \right] + \delta^2 \left[\sum_{i=H,L} P^i(\alpha_{c3})x^i \right],$$

where $\alpha_{c2} = f_c(I_{c1}, 0)$ and $\alpha_{c3} = f_c(I_{c2}, \alpha_{c2})$. Then, setting $w_2^j = x^j - r_2, j = H, L$ and $w_3^j = x^j - r_3, j = H, L$, where r_2 and r_3 are the principal's utilities in periods 2 and 3, the incentive-compatibility constraint in the long-term contract indeed yields the maximizer of the above problem. That is, the verifiable part of the constraint is reduced to the above problem. On the other hand, in the cases of the short–short–short-term contract, the medium–short-term contract, and the short–medium-term contract, the total utilities obtained from the contractible output, denoted by S_c, MS_c, and SM_c, are smaller than that of the long-term contract, denoted by L_c. Indeed, in the cases of the short–short–short-term and short–medium-term contracts, the agent chooses I_{c1} to maximize half of the second- and third-period utilities obtained from the contractible output, and in the case of the medium–short-term contract, the agent chooses I_{c2} to maximize half of the third- period utility obtained from the contractible output. Thus, in these cases, I_{c1} and/or I_{c2} are different from their first-best levels. Thus,

$$S_c < L_c, SM_c < L_c, MS_c < L_c$$

holds. Accordingly, if $\theta = 0$, the long-term contract shown above is chosen. Given that the first-best value of the gain from the unverifiable output is a continuous function of θ, there exists a $\bar{\theta}$ such that the principal chooses the long-term contract for $\theta \in [0, \bar{\theta})$.

Next, we show that the above long-term equilibrium contract is renegotiation-proof. There is no need to discuss renegotiation at the beginning of the third period, as the parties do not invest, and any renegotiation on the wages does not induce a Pareto improvement because both the principal and the agent are risk neutral. Below, we investigate a renegotiation at the beginning of the second period, where the threat point is the discounted sums of the utilities in periods 2 and 3 obtained from the long-term contract. Suppose their utilities resulting from this renegotiation are Pareto superior to those of the threat point at the beginning of the second period. Given this, further suppose the agent chooses $\hat{I}_{c1}$ and $\hat{I}_{n1}$ by maximizing the discounted sum of his expected utility. (Note that the investments do not affect the threat point of the renegotiation in period 2.) Then, their utilities are even Pareto superior to those of the long-term contract, even at the beginning of the first period, as the agent can choose I_{c1} and I_{n1} in the long-term contract, and the outputs in the first period are assumed to be always x^L and y^L. If a medium-term contract is chosen in the renegotiation, then $\hat{I}_{c1}$ and $\hat{I}_{n1}$ can be considered as the investments in the case of the short–medium-term contract, and if a short-term contract is chosen in the renegotiation, then $\hat{I}_{c1}$ and $\hat{I}_{n1}$ can be considered as the investments in the case of the short–short–short-term contract. However, it has been shown that the total utility is larger under the long-term contract than under the short–medium-term contract $^\forall\theta \leq \underline{\theta}$. This is a contradiction. Thus, the long-term contract is renegotiation-proof if $\theta \leq \underline{\theta}$.

B The Proof of Theorem 2

In (i)–(vi) below, we focus on each contract and obtain its total equilibrium utility from the noncontractible output. We use backward induction (if necessary). Then, in (v), we show that if θ is sufficiently large, the principal chooses a short–short–short-term contract.

(i) We first focus our attention on a long-term contract. That is, we derive the maximum total utility from the noncontractible output produced under the long-term contract. The agent chooses $I_{n1} = I_{n2} = 0$, and thus $g(\alpha_n, \theta) = \theta y^L$ holds throughout all periods. The total utility obtained from the noncontractible output, denoted by L_n, is $(1 + \delta + \delta^2)\theta y^L$.

(ii) Next, we focus on a short–short–short-term contract. The wages for the second and third periods are determined by Nash bargaining at the beginning of each period. Below, we consider only the utilities obtained from the noncontractible outputs. In the third period, the agent obtains half of the total utility, i.e., $\frac{1}{2}\sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i$. Thus, in the second period, the agent chooses I_{n2}, satisfying the incentive compatibility constraint:

$$\max_{I_{n2}} \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI_{n2}^2,$$

where $\alpha_{n3} = \min\{\alpha_{n2} + I_{n2}, 1\}$. Below, suppose that the optimal α_{n2} and α_{n3} are less than one, i.e., the optimal I_{n1} and I_{n2} are determined by the first-order condition. (Later, we show that the optimal α_{n2} and α_{n3} are indeed less than one.) Then,

$$I_{n2}^* = \tfrac{1}{4b}\delta\theta(y^H - y^L).$$

Note that I_{n2}^* does not depend on α_{n2}. In the second period, the agent obtains half of the total utility:

$$\frac{1}{2}\left(\sum_{i=H,L} Q^i(\alpha_{n2})\theta y^i - b(I_{n2}^*)^2\right) + \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n2} + I_{n2}^*)\theta y^i,$$

where $\alpha_{n2} = I_{n1}$. Thus, in the first period, the agent chooses I_{n1}, satisfying the incentive-compatibility constraint:

$$\max_{I_{n1}} \frac{1}{2}\left(\sum_{i=H,L} Q^i(I_{n1})\theta y^i - b(I_{n2}^*)^2\right) + \frac{1}{2}\delta \sum_{i=H,L} Q^i(I_{n1} + I_{n2}^*)\theta y^i - bI_{n1}^2.$$

Then, the optimal I_{n1} is obtained as follows:

$$I_{n1}^* = \tfrac{1}{4b}\theta(\delta + \delta^2)(y^H - y^L).$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds; thus, $\alpha_{n2}^* = I_{n1}^*$ and $\alpha_{n3}^* = I_{n1}^* + I_{n2}^*$ are indeed less than one because $I_{n1}^* + I_{n2}^* = \frac{1}{4b}\theta(2\delta + \delta^2)(y^H - y^L)$.

Then, the total utility obtained from the noncontractible output, denoted by $S_n(\theta)$, is obtained as follows:

$$\begin{aligned} S_n(\theta) &= \theta y^L - b(I_{n1}^*)^2 + \delta\left(I_{n1}^*\theta y^H + (1 - I_{n1}^*)\theta y^L - b(I_{n2}^*)^2\right) \\ &\quad + \delta^2\left((I_{n1}^* + I_{n2}^*)\theta y^H + (1 - I_{n1}^* - I_{n2}^*)\theta y^L\right) \\ &= (1 + \delta + \delta^2)\theta y^L + \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta^2 + 3\delta + 1). \end{aligned}$$

(iii) Next, we focus on a medium–short-term contract. The agent's utility in the third period is $\frac{1}{2}\sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i$. The wage for the second period is determined by a take-it-or-leave-it offer at the beginning of the first period. Thus, the agent is interested only in α_{n3}, as the wage for the second period does not depend on α_{n2}. That is, the agent solves the following problem with respect to I_{n1} and I_{n2} in the first period:

$$\max_{I_{n1}, I_{n2}} \frac{1}{2}\delta^2 \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI_{n1}^2 - \delta bI_{n2}^2,$$

where $\alpha_{n1} = \min\{I_{n1}, 1\}$ and $\alpha_{n2} = \min\{\alpha_{n1} + I_{n2}, 1\}$. Suppose the optimal $\alpha_{n3} = I^*_{n1} + I^*_{n2}$ is less than one. Then, I^*_{n1} and I^*_{n2} are obtained as follows:

$$\begin{aligned} I^*_{n1} &= \tfrac{1}{4b}\delta^2\theta(y^H - y^L), \\ I^*_{n2} &= \tfrac{1}{4b}\delta\theta(y^H - y^L). \end{aligned}$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds; thus, $I^*_{n1} + I^*_{n2}$ is indeed less than one. Then, $MS_n(\theta)$, the total utility obtained from the noncontractible output, is obtained as follows:

$$MS_n(\theta) = (1 + \delta + \delta^2)\theta y^L + \frac{1}{16b}(y^H - y^L)^2\delta^3\theta^2(3\delta + 7).$$

(iv) Finally, we consider a short–medium-term contract. By definition, the wage for the third period is determined at the beginning of the second period. Thus, the agent chooses $I_{n2} = 0$ in the second period. Therefore, the agent solves the following problem with respect to I_{n1} in the first period:

$$\max_{I_{n1}} \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n2})\theta y^i + \frac{1}{2}\delta^2 \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI^2_{n1},$$

where $\alpha_{n2} = \min\{I_{n1}, 1\}$ and $\alpha_{n3} = \alpha_{n2}$. Suppose the optimal $\alpha_{n3} = I^*_{n1}$ is less than one. Then, it is obtained as follows:

$$I^*_{n1} = \tfrac{1}{4b}\theta(\delta + \delta^2)(y^H - y^L).$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds; thus, I^*_{n1} is indeed less than one. Then, the total utility obtained from the noncontractible output, denoted by $SM_n(\theta)$, is obtained as follows:

$$SM_n(\theta) = (1 + \delta + \delta^2)\theta y^L + \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta + 1)^2.$$

Below in (v), we compare the total utilities obtained from both the contractible and noncontractible outputs.

(v) As shown in the proof of Theorem 1, in the case of a long-term contract, the total utilities obtained from the contractible output, denoted by L_c, are first-best. In the cases of the short–short–short-term contract, the medium–short-term contract, and the short–medium-term contract, the total utilities obtained from the contractible output, denoted by S_c, MS_c, and SM_c, are smaller than L_c. That is,

$$S_c < L_c, SM_c < L_c, MS_c < L_c.$$

On the other hand,

$$\begin{aligned} S_n(\theta) - L_n &= \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta^2 + 3\delta + 1) > 0, \\ S_n(\theta) - MS_n(\theta) &= \frac{1}{16b}(y^H - y^L)^2\delta^2\theta^2(2\delta + 3) > 0, \\ S_n(\theta) - SM_n(\theta) &= \frac{3}{16b}(y^H - y^L)^2\delta^3\theta^2 > 0 \end{aligned}$$

hold. As $S_n(\theta) - L_n$, $S_n(\theta) - MS_n(\theta)$, and $S_n(\theta) - SM_n(\theta)$ are strictly increasing functions of θ and go to $+\infty$ as θ goes to $+\infty$, there exists a $\bar{\theta} > 0$ such that ${}^\forall\theta \geq \bar{\theta}$,

$$L_c + L_n < S_c + S_n(\theta), SM_c + SM_n(\theta) < S_c + S_n(\theta), MS_c + MS_n(\theta) < S_c + S_n(\theta).$$

That is, $S_c + S_n(\theta)$ is the largest. Thus, the short–short–short-term contract is chosen for ${}^\forall\theta \geq \bar{\theta}$. Suppose the contrary. Then, the equilibrium contract derived from backward induction is one of long, medium–short, or short–medium term, and the equilibrium total utility is larger than $S_c + S_n(\theta)$. This contradicts the above inequalities.

Clearly, there is no need to discuss the renegotiation-proofness of the short–short–short-term contract. ∎

C The Proof of Theorem 3

As in the proof of Theorem 2, L_c, $L_n, S_c, S_n(\theta), MS_c, MS_n(\theta), SM_c$, and $SM_n(\theta)$ are obtained.

Suppose a short–medium-term contract is chosen. By $a < \frac{1}{2}\delta\theta b(y^H - y^L)$, the marginal cost of I_n is strictly smaller than the marginal utility, so that the agent chooses $I_{n1} = \bar{\alpha}_n$ in the first period. Thus,

$$SM_n(\theta) = \theta y^L + \delta\theta y^H + \delta^2\theta y^H - \frac{a}{b}$$

holds. Note that $S_n(\theta) = SM_n(\theta)$ holds.

Then, suppose a medium-term contract on the first- and second-period wages is signed at the beginning of the first period. Given that the marginal cost of I_n is strictly smaller than the marginal utility, then the agent chooses $I_{n1} = 0$ and $I_{n2} = \bar{\alpha}_n$ because of the discount factor. Thus,

$$MS_n(\theta) = \theta y^L + \delta\theta y^L + \delta^2\theta y^H - \frac{\delta a}{b}.$$

For a sufficiently large θ,

$$SM_n(\theta) - MS_n(\theta) = \delta\theta(y^H - y^L) - \frac{a}{b} + \frac{\delta a}{b} > 0$$

holds. Given $SM_n(\theta) - MS_n(\theta)$ and $SM_n(\theta)$ are strictly increasing, and $SM_n(\theta) - MS_n(\theta) \to +\infty$ and $SM_n(\theta) \to +\infty$ as $\theta \to +\infty$,

$$L_c + L_n < SM_c + SM_n(\theta), MS_c + MS_n(\theta) < SM_c + SM_n(\theta)$$

holds for a sufficiently large θ. Moreover, as shown in the previous section, $S_c < SM_c$ holds and thus

$$S_c + S_n(\theta) < SM_c + SM_n(\theta).$$

Using the same argument as in the proof of Theorem 2, the short–medium-term contract is an equilibrium contract.

Finally, as shown in the proof of Theorem 1, the above equilibrium is renegotiation-proof, as we should consider only the renegotiation in the third period. ∎

D The Proof of Theorem 4

By $f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, α_n in the second period is zero even if $I_n > 0$ in the first period, because $\alpha_c = 0$ holds at the beginning of the first period. Thus, the principal does not choose a short-term contract at the beginning of the first period; she instead chooses either a long- or a medium–short-term contract. If a medium–short-term contract is chosen, Q^H in the third period is $\alpha_{n3} = \min\{\alpha_{c2} I_{n2}, 1\}$. Note that α_{c2} is positive and an increasing function of θ in these contracts. Accordingly, the agent maximizes

$$\delta(\alpha_{n3}\theta y^H + (1 - \alpha_{n3})\theta y^L) - I_{n2}^2$$

with respect to I_{n2}. Suppose the optimal α_{n3} is less than one, then the optimal I_{n2} is equal to $\frac{1}{2}\delta\theta\alpha_{c2}(y^H - y^L)$. Therefore, the following total utility from noncontractible output is obtained:

$$(1 + \delta + \delta^2)\theta y^L + \delta^2\theta^2\alpha_{c2}(y^H - y^L)^2(\frac{1}{2} - \frac{1}{4}\alpha_{c2}). \quad (7)$$

Suppose the optimal α_{n3} is equal to one, then the agent chooses $I_{n2}^* = \frac{1}{\alpha_{c2}}$. Therefore, the following total utility from noncontractible output is obtained:

$$(1 + \delta + \delta^2)\theta y^L + \delta^2\theta y^H - \frac{1}{\alpha_{c2}^2}. \quad (8)$$

If the principal chooses a long-term contract, $I_{n1}^* = I_{n2}^* = 0$ holds and the total utility from the noncontractible output becomes $(1 + \delta + \delta^2)\theta y^L$. Given that α_{c2} is an increasing function of θ, (7) and (8) go to $+\infty$ as $\theta \to +\infty$. Using the same arguments as in the proof of Theorem 2, there exists a $\bar{\theta}$ such that $^\forall\theta \geq \bar{\theta}$, the medium–short-term contract is an equilibrium contract.

Finally, as shown in the proof of Theorem 1, the above equilibria are renegotiation-proof. ∎

E The Proof of Theorem 5

Suppose $\delta = 1$. Under the one–two–two-term contract (short–medium–medium-term contract), the agent chooses $I_{n1} = I_{n3} = 1$ and $I_{n2} = I_{n4} = 0$; as in periods 2 and 4, the principal and the agent bargain over the wages; and in periods 1 and 3, the marginal cost of I_n is one, and the marginal utility is $\frac{1}{2}(\delta+\delta^2)\theta(y^H-y^L)=\theta(y^H-y^L)\geq 10$ for $I_n<1$. Thus, $\alpha_n=1$ holds in periods 2, 3, 4, and 5, and the total utility obtained from I_n is

$$(\delta+\delta^2+\delta^3+\delta^4)\theta y^H-I_{n1}-\delta^2 I_{n3}=4\theta y^H-I_{n1}-I_{n3}\geq 38.$$

Other than a one–two–two-term contract, logically there are three other combinations of contracts under a five-period principal–agent relationship: (i) combinations that involve a contract that covers at least three periods, e.g., a one–three–one-term contract (short–medium–short-term contract); (ii) combinations that involve at most one contract that covers two periods, e.g., a one–one–two–one-term contract (short–short–medium–short-term contract); and (iii) combinations that include two contracts that cover two periods, e.g., a two–two–one-term contract (medium–medium–short-term contract). In (i), as there exists a period in which $\alpha_n=0$, they lose at least $\theta(y^H-y^L)-1\geq 9$ in the total noncontractible utility and obtain at most $4(x^H-x^L)\leq 8$ in the total contractible utility, where the number of periods in which α_c could be increased is four. In (ii), although the total utility obtained from I_n is the maximum amount, the total utility obtained from I_c is smaller than the case of the one–two–two-term contract. This is because combinations that fall in the category of (ii) involve more bargaining periods than the one–two–two-term contract. In the case of (iii), a two–two–one-term contract (medium–medium–short-term contract) and a two–one–two-term contract (medium–short–medium-term contract) are the only possibilities. In both cases, given $I_{n1}=0$ in the first period, they lose at least $\theta(y^H-y^L)-1\geq 9$ in the total noncontractible utility and obtain at most $4(x^H-x^L)\leq 8$ in the total contractible utility, where the number of periods in which α_c could be increased is four. Therefore, using the same arguments as in the proof of Theorem 2, the one–two–two-term contract is an equilibrium contract. The above arguments apply to any $\delta>0$ close to one. The maximal length of the contract is two, and the parties might renegotiate on wages in the last period of each contract. However, given risk neutrality, a Pareto improvement is impossible. Thus, the equilibrium is renegotiation-proof. ∎

F The Proof of Theorem 6

Suppose $\theta = 0$. In other words, this is the case in which there are only verifiable outputs (no unverifiable outputs). Below, we show that any contract can be replicated by a long-term contract. Consider a short–short–short-term contract as an example. Then, setting $w_t^H = \frac{1}{2}x^H > 0$ and $w_t^L = \frac{1}{2}x^L > 0, t = 2, 3,$ I_{c1} and I_{c2} in the short–short–short-term contract can be induced by the long-term contract. On the other hand, in the long-term contract,

$$\begin{aligned} w_1^L &= u - \delta \sum_{i=H,L} P^i(\alpha_{c2}) w_2^i - \delta^2 \sum_{i=H,L} P^i(\alpha_{c3}) w_3^i + D_c(I_{c1}) + \delta D_c(I_{c2}) \\ &= u - \frac{1}{2}\delta \sum_{i=H,L} P^i(\alpha_{c2}) x^i - \frac{1}{2}\delta^2 \sum_{i=H,L} P^i(\alpha_{c3}) x^i + D_c(I_{c1}) + \delta D_c(I_{c2}). \end{aligned}$$

The last line is equal to the first-period wage in the short–short–short-term contract. Note that the discounted sum of wages is the same in the contracts. Moreover, the principal can choose wage differences $(w_2^H - w_2^L)$ larger than $\frac{1}{2}(x^H - x^L)$ and can also keep the expected wages constant. Therefore, she can obtain a larger gain. Thus, the principal strictly prefers a long-term contract. The same argument applies to medium–short and short–medium contracts. Clearly, she chooses a long-term contract even for a small θ. Thus, the same results as in Theorem 1 hold. However, the threshold is different from that of Theorem 1.

Next, we prove the results in Theorems 2–4. If θ is small, the limited liability constraint might be binding and the agent might not invest the same I_n as in the case without the constraint. However, if θ is sufficiently large, half of the gain from I_n, which is a part of wages, is larger than A, and thus it is better to invest the same I_n as in the case without the constraint. In the proofs of these theorems, the gains from the unverifiable output are compared, and we take the limit of differences as $\theta \to \infty$. For example, in Theorem 2, $S_n(\theta) - L_n$, $S_n(\theta) - MS_n(\theta)$, and $S_n(\theta) - SM_n(\theta)$ are strictly increasing functions of θ and go to $+\infty$ as θ goes to ∞. Given that the least upper bound of gains from the verifiable output does not depend on θ, the utility differences of the principal go to ∞ as $\theta \to \infty$, no matter what the limited liability constraint. Thus, the results in the theorems hold. However, the thresholds differ from those in the theorems.

G The Proof of Theorem 7

Setting $\tilde{f}(I_1, I_1') = f(s^p(I_1), s^a(I_1'))$, the truth telling is an equilibrium strategy. Indeed, from the definition of equilibria,

$$u_1^a(w_1^L, \hat{I}_1, \tilde{f}(\hat{I}_1, \hat{I}_1)) \geq u_1^a(w_1^L, I_1, \tilde{f}(I_1, I_1)) \text{ for all } I_1,$$

and for any given $I_1 = (I_{c1}, I_{n1})$,

$$u_2^a(I_1, \tilde{f}(I_1, I_1)) \geq u_2^a(I_1, \tilde{f}(I_1, I_1^{a\prime})) \text{ for all } I_1^{a\prime},$$

$$u_2^p(I_1, \tilde{f}(I_1, I_1)) \geq u_2^p(I_1, \tilde{f}(I_1^{p\prime}, I_1)) \text{ for all } I_1^{p\prime}.$$

It is clear that $\tilde{f}(I_1, I_1) = f(s^p(I_1), s^a(I_1))$. ∎

H The Proof of Theorem 8

(i) We first show that, for any given I_{c1}, the agent's expected utility in the second period does not depend on the first period choice I_{n1} in equilibria. Suppose the contrary. Then there exist $I_1 = (I_{c1}, I_{n1})$ and $I_1' = (I_{c1}, I_{n1}')$, where $I_{n1} \neq I_{n1}'$, such that

$$u_2^a(I_1, \tilde{f}((I_{c1}, I_{n1}), (I_{c1}, I_{n1}))) > u_2^a(I_1', \tilde{f}((I_{c1}, I_{n1}'), (I_{c1}, I_{n1}')) \tag{9}$$

holds in an equilibrium. Of course, at least one of I_1 and I_1' must be an off-equilibrium choice. From the definition of an equilibrium,

$$u_2^a(I_1', \tilde{f}((I_{c1}, I_{n1}'), (I_{c1}, I_{n1}')) \geq u_2^a(I_1', \tilde{f}((I_{c1}, I_{n1}'), (I_{c1}, I_{n1})) \tag{10}$$

holds. That is, even if the agent announces I_{n1} instead of the true investment I_{n1}', he cannot be better off. Moreover, since the first arguments in I_1 and I_1' are the same and thus from the definition of u_2^a

$$u_2^a(I_1', (q, v^H, v^L)) = u_2^a(I_1, (q, v^H, v^L)) \text{ for all } (v^H, v^L). \tag{11}$$

Thus, from (9), (10), and (11),

$$u_2^a(I_1, \tilde{f}((I_{c1}, I_{n1}), (I_{c1}, I_{n1}))) > u_2^a(I_1, \tilde{f}((I_{c1}, I'_{n1}), (I_{c1}, I_{n1}))$$

holds. As

$$u_2^p(I_1, \tilde{f}((I_{c1}, I_{n1}), (I_{c1}, I_{n1}))) = \sum_{i=H,L} P^i(I_{c1})x^i + \sum_{i=H,L} Q^i(I_{n1})y^i - u_2^a(I_1, \tilde{f}((I_{c1}, I_{n1}), (I_{c1}, I_{n1})))$$

and

$$u_2^p(I_1, \tilde{f}((I_{c1}, I_{n1}), (I'_{c1}, I_{n1}))) = \sum_{i=H,L} P^i(I_{c1})x^i + \sum_{i=H,L} Q^i(I_{n1})y^i - u_2^a(I_1, \tilde{f}((I_{c1}, I'_{n1}), (I_{c1}, I_{n1})))$$

hold, then

$$u_2^p(I_1, \tilde{f}((I_{c1}, I'_{n1}), (I_{c1}, I_{n1}))) > u_2^p(I_1, \tilde{f}((I_{c1}, I_{n1}), (I_{c1}, I_{n1})).$$

That is, the principal chooses I'_{n1} instead of the true investment I_{n1}. This contradicts the definition of equilibrium.

(ii) Next, we show that the agent's investments in an equilibrium $(\hat{I}_{c1}, \hat{I}_{n1})$ satisfies $\hat{I}_{n1} = 0$. Suppose the contrary. Then $\hat{I}_{n1} > 0$. If the agent chooses $(I_{c1}, I_{n1}) = (\hat{I}_{c1}, 0)$, then from (i),

$$u_2^a((\hat{I}_{c1}, \hat{I}_{n1}), \tilde{f}((\hat{I}_{c1}, \hat{I}_{n1}), (\hat{I}_{c1}, \hat{I}_{n1}))) = u_2^a((\hat{I}_{c1}, 0), \tilde{f}((\hat{I}_{c1}, 0), (\hat{I}_{c1}, 0)))$$

holds. However, by $D_c(\hat{I}_{n1}) > D_c(0)$, the agent prefers $I_{n1} = 0$ in period one. Thus, $\hat{I}_{n1} = 0$ holds in equilibria. That is, the mechanism cannot induce I_{n1} at all, and it can induce at most the first-best I_{c1}. Recall that a simple two-period wage contract can induce the first-best I_{c1}, but cannot induce I_{n1}. ∎

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