CIRJE-F-890

Incentive Pay that Causes Inefficient Managerial Replacement

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June 2013; December 2013

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December 08, 2013

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First Version: December 24, 2012

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†This paper was presented at the Australian National University and Tokyo University. I appreciate the comments made by all participants. I am truly grateful to Kazuya Kamiya, Michihiro Kandori, Andrew Macrae, Hiroshi Osano, Dan Sasaki, and Akihiko Takahashi for their helpful comments and suggestions. This research is financially supported by the global Center of Excellence (COE) program, The Research and Training Center for New Development in Mathematics.

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Abstract

Using contract theory, this article considers the effect of stock-based compensation on managerial replacement. I show that while stock-based compensation solves the moral hazard problem, it creates a distortion in the principal's managerial replacement decisions. Specifically, I show the principal endogenously determines the agent's tenure in a way that maximizes her own expected payoff, where her rational choice to replace or retain the incumbent agent may depart from the total firm value maximization. I find that along the parametric range of control benefit, both long- and short-term vested stock options may exhibit over-replacement of the incumbent agent, but in some cases, long-term vested options may cause more inefficiency than short-term vested options. The article also indicates that only the short-term vested options may exhibit under-replacement.

Journal of Economic Literature Classifications: D86, G32, G34, M12, M52.

Keywords: Moral Hazard, Managerial Replacement, Stock-based Compensation, Long-term Vested Options, Short-term Vested Options.

1. Introduction

Although stock-based compensation is effective at solving the problem of inefficient agent effort, this paper argues that it may cause a distortion in the principal's decision to retain or replace the incumbent agent.[1,2] This is because stock-based compensation allows the principal's objective to depart from the maximization of firm value, which in turn causes excessive retention or replacement of the incumbent agent.[3] The conventional argument is that the generous stock options bias managers toward excessive continuation (see for example Inderst and Mueller 2010), and in the decades since 1990 there has been an increased use of executive stock-based compensation (Murphy 2012). However, theoretical research that links the effect of stock-based compensation and managerial tenure is still limited (Inderst and Mueller 2010, Laux 2012). In this article, following Laux (2012), I distinguish short-term vested options and long-term vested options and find that short-term vested options can lead to excessive retention of managers. I also find that both long-term and short-term vested options can cause excessive replacement of managers when a control benefit is given to the agent, and that this tendency is stronger with long-term vested options. In addition to these two main findings, I also study several scenarios in which the principal is more interested in replacing than retaining an incumbent agent who has shown he is likely to be inadequate.

The incumbent agent and the principal in this article can be: the CEO and board of directors of a listed company; the manager and founder and/or the family of a founder's company; the junior partner and senior partner in a partnership; or the manager and controlling shareholders such as parent companies.[4] The article mainly focuses on the context of CEO and board of directors.

I analyze the rational behavior of the principal that causes a misalignment of interest between the principal and the firm in a contractual framework. There is a principal and an incumbent agent as active players, and outside investors and a new agent as passive players. All players are risk neutral. The flow of the game is that: 1. the principal writes the contract; 2. the agent accepts the contract and exerts effort; 3. the principal observes a

[1] Stock-based compensation can be stock options or restricted stock.

[2] Theoretical studies that show stock options can mitigate the unobservable effort problems are provided by Hemmer, Kim, and Verrecchia 1999, Carpenter 2000 and Ross 2004, while Hanlon, Rajgopal and Shevlin 2003 provide an empirical analysis.

[3] Firm value is measured by the total equity value of the firm. Social welfare is measured by the sum of the total equity value of the firm, expected control benefit of the manager, and wages to the incumbent and new agents.

[4] Throughout this paper, 'she' is used for the principal and 'he' is used for the agent.

noisy but costless signal about the agent's performance; 4. the principal retains or replaces the incumbent agent given the signal; and 5. the firm's profit is realized and everyone receives their pay.

The incumbent agent owns no endowment and faces limited liability. The ability of the agent is unknown to all players as in Holmstrom (1999). The agent's ability is stochastically determined by his effort level, which is either high (optimal) or low, but unobservable. Hence, in addition to the base salary, the principal gives stock-based compensation to motivate the incumbent agent to exert optimal effort. An incumbent agent who stays in the same job receives a control benefit at the last stage; however, this is exogenous to the model. With rewards of fixed pay, stock options and a control benefit, the agent is induced to exert high effort to increase his chance of good performance. The job of the principal is to determine: the amount of fixed pay; the fraction of the stock given to the incumbent agent; and whether to dismiss the incumbent agent following a bad signal or to retain the incumbent agent irrespective of the signal. At the beginning of the game, the principal offers the agent a contract which includes each of these three things. The contract is renegotiation-proof (see Appendix A.1).

In order to study distortion in the replacement policy, I consider three models. These models all use the same environment, but differ in terms of the incentive pay the agent receives. The benchmark model is the constrained optimality model in which the agent's effort level is observable. In this model, the strategies undertaken by the principal are efficient because the agent is motivated to exert optimal effort without the need for incentives in the form of stock options. The other two models are both moral hazard models, where the incumbent agent's effort level is unobservable, so the principal gives him stock-based compensation to motivate him.[5] The first of these is the long-term vested stock options model, where the agent forfeits his options if he is dismissed. In the second moral hazard model, he is given short-term vested stock options, which he can exercise at an early stage, even upon dismissal—'thus after vesting, the CEO can keep the stock options even when he leaves the firm' (Laux 2012, p. 5).[6]

With all three models, I first conduct comparative statics with several parameters to study the environment in which the equilibrium contract is likely to be a replacement contract

[5] In this paper, there is a moral hazard problem but no adverse selection problem.

[6] In some Japanese companies such as Mitsubishi UFJ Financial Group, Mitsubishi Corporation and Shin-Etsu Chemical Co. Ltd., the managers can keep their right to exercise stock options after they are ousted. This practice can also be interpreted as a variant of short-term vested stock options.

rather than a retention contract. For example, under parameter B, which stands for control benefit, the principal is more likely to offer a replacement contract with a relatively small control benefit under the constrained optimality model, but with a relatively large control benefit under the two moral hazard models. The theoretical mechanism is as follows. Under the constrained optimality setting where the incumbent agent exerts the optimal effort level, he will receive a fixed salary with certainty and a control benefit if he is retained to the final stage. Then, if the contract ensures the incumbent agent a retention even following a bad signal, the incumbent agent is sure to receive a control benefit in the final stage, allowing the principal to save some salary. Therefore, the larger the control benefit, the more likely it is that the retention contract is the equilibrium contract. On the other hand, in the case of the moral hazard models, the principal gives stock-based compensation to the incumbent agent to motivate him to exert optimal effort. If the retention contract is offered in these models, the incumbent agent knows he will be retained even following a bad signal, which leaves him with little incentive to exert optimal effort. Thus, the principal needs to give a further more stock-based compensation to motivate him. However, if a replacement contract is offered, the incumbent agent is motivated to exert optimal effort in order to be retained and receive a control benefit, if the benefit is sufficiently large. Thus, the principal can reduce the agent's share when the control benefit is large. This means that the larger the control benefit, the more likely that the replacement contract is the equilibrium contract.

After conducting comparative statics with several parameters on all three models, I compare two moral hazard models and the constrained optimality model to study any inefficiencies that may arise when stock-based compensation is used to mitigate the agency problem.[7] That is, if the equilibrium contract taken under the moral hazard models is different from that taken under the constrained optimality model under a certain parametric range, it means the contract offered under the moral hazard model is inefficient: there is too much retention or too much replacement. I use the parametric range of the control benefit among several parameters for this comparison.

The main results of this article are summarized as follows. First, short-term vested options can yield under-replacement of the incumbent agent. Second, whether under the long-term vested options or short-term vested options, there is a possibility that he could be over

[7] To be more precise, as the grant of stock options is determined appropriately, and both the principal and the incumbent agent are risk neutral in this model, the agency problem regarding the unobservable effort level can be solved (rather than just 'mitigated') even if the agent faces the limited liability restriction.

replaced, but the former is more inefficient than the latter.[8]

The logic is as follows. Under the moral hazard setting in which the incentive compatibility constraint for the agent's effort is binding, the fear of being fired works as a useful threat to induce effort, given the fraction of the stock given to him. This is especially true when the agent is given long-term vested options, because he will lose his share options. If he knows he will be fired if there is a bad performance signal, he has an incentive to exert optimal effort to decrease his chances of losing both the control benefit and the stock option. Regardless of the size of the control benefit, he has an incentive to be retained in this scenario. Hence, the principal takes advantage of this and offers a replacement contract at the beginning (over-replacement under long-term vested options).[9]

However, if the agent is given short-term vested options, his fear of being fired is not as much of an incentive as it was under the long-term vested options. Although he will not obtain any control benefit, he can still receive short-term vested options. So in order to induce optimal effort, the principal must give him a larger number of stock options, compared with the case under long-term vested options. In addition, the control benefit must be sufficiently large if the principal wishes to motivate the agent with the threat of replacing him (over-replacement under short-term vested options). If the control benefit is small, (or even zero), the agent will not be so motivated to produce optimal effort. In order to motivate him, the principal must give him a lot more stock options, but because the principal will be worse off if too many stock options are given to the agent, she decides to promise to retain the agent and guarantees him a control benefit even if his performance is likely to be low (under-replacement under short-term vested options).

The theoretical and empirical implications of these results are explained as follows. First, although both the long- and short-term vested options can cause excessive replacement of managers, my results show that this is more likely to be caused by long-term vested options. Thus in certain cases, short-term vested options are a better form of incentive pay. This result contrasts with the conventional wisdom that short-term vested options are more likely to cause short-termism, thereby inducing managers to take moral hazard actions. This may

[8] This finding is similar to Laux (2010) and Laux (2012) in the sense that short-term vested options can mitigate inefficient CEO tenure.

[9] It is important to note that there will be no under-replacement when long-term vested option is given to the agent. The reason is that the principal need not trade off between her payment of stock options to the agent and the retention of poor-performing managers, because the retention of poor-performing managers merely reduces the agent's incentive to exert optimal effort.

explain why short-term vested stock options are used and why, for example, managers in some Japanese companies can keep their right of exercising stock options even after dismissal. Second, my results indicate that short-term vested options can cause excessive retention in some parametric ranges. Because short-term vested options can be interpreted as a form of severance payment, these options increase the cost of replacing managers. Third, these theoretical results also provide a new testable prediction that the CEO tenure for firms using long-term vested options is more likely to be shorter than that for firms using short-term vested options.

This article shows that there is a possibility that the rational decision the principal makes to maximize her own welfare can be inefficient from the perspective of firm value. That is, even with the existence of moral hazard problems, the principal could still achieve the same total firm value if she chooses the same replacement/retention strategy as in the constrained optimality setting. However, if she does this, her welfare becomes smaller in the moral hazard setting than in the constrained optimality setting. Alternatively, if she chooses a different replacement/retention strategy, total firm value decreases (compared with constrained optimality), but her welfare does not decrease as much as in the previous case. This means that despite the fact stock-based compensation is said to mitigate the agency problem, it can lead to situations where the principal's objective departs from that of the firm, causing inefficient retention/replacement of incumbent managers.

This study is closely related to that of Laux (2012), which distinguishes the length of the vesting period in stock options and explores its effect on the CEO's project choice. The CEO can keep the short-term vested options but must forfeit the long-term vested options upon dismissal. In Laux (2012), the board endogenously determines the remuneration package, which consists of stock options. The CEO endogenously determines his effort level and how much to invest between long- and short-term projects. A short-term project reveals the CEO's type at an early stage, hence the CEO who has revealed himself as likely to be substandard is fired. A long-term project is assumed to yield an efficient level of productivity but does not reveal the CEO's type. As a result, the CEO does not get fired. In this environment, Laux (2012) shows that if the CEO is given only the long-term vested options which he must forfeit upon his dismissal, he is more biased toward the long-term project so that his type stays hidden all the way through to the final stage. In order to avoid this managerial myopia, the board should grant the CEO some short-term vested options as well as the long-term vested options. In other words, short-term vested options can mitigate over-retention of the CEOs. Similar to Laux (2012), my result demonstrates that short-term vested options can be a more efficient measure than long-term vested options. The difference between our studies is that this article focuses on the replacement/retention decision of the firm, whereas his paper focuses on the project choice decision of the firm.

This article is somewhat related to the work of Inderst and Mueller (2010), who examine optimal managerial compensation and replacement policy. In their model, the manager privately observes an interim signal about the likely firm value under his continued leadership and quits when from his perspective this is incentive compatible. The manager's desire to continue to hold his position, together with his private information at the interim stage, creates managerial entrenchment. Inderst and Mueller (2010) find that a steep incentive scheme such as granting stock options used together with severance pay can mitigate managerial entrenchment and inefficient CEO retention. This article differs from Inderst and Mueller (2010) in developing a model that examines inefficient CEO retention from a perspective of long- and short- term vested options.

Finally, this article is also related to the literature on the role of the board of directors by interpreting the principal as the board and the agent as the CEO. The effectiveness of board monitoring and forced CEO replacement has been theoretically examined from the perspectives of the information environment and/or board composition (Hirshleifer and Thakor 1994; Raheja 2005; Adams and Ferreira 2007), the degree of board independence (Hermalin and Weisbach 1998; Almazan and Suarez 2003), board members securing their job (Warther 1998), and CEOs' specific human capital (Shleifer and Vishny 1989). This article offers a new approach to the research on board ineffectiveness in replacing inadequate CEOs. If the CEO receives stock-based compensation, there can be a misalignment of the interest between the board and the firm.[10]

The article is organized as follows. Section 2 presents the basic model framework common to both the constrained optimality and moral hazard problems. Section 3 studies the constrained optimality model and conducts comparative statics analyses. Section 4 develops two moral hazard models and conducts comparative statics analyses. Section 5 compares the constrained optimality and moral hazard settings and studies the inefficiencies exhibited in the moral hazard models. Section 6 concludes.

[10] Adachi-Sato (2013) examines a misalignment of the interest between the board and the firm using a Nash bargaining model without moral hazard problems.

2. The model

2.1 The basics A principal and an agent (who becomes the *incumbent* agent after accepting a job offer) are active players, while outside investors and a new agent are passive players. All players are risk neutral. The incumbent agent does not own an endowment, faces limited liability, and has zero reservation utility. The model consists of five stages.

In the first stage, the principal finds the agent. The principal offers the agent a take-it-or-leave-it employment contract specifying remuneration and termination policies. Specifically, the principal endogenously determines the fixed pay and the stock-based compensation the incumbent agent receives and whether to dismiss or retain him following a bad signal. The players commit to the contract agreements.[11]

In the second stage, the agent accepts the offer and makes an effort $e \in \{\overline{e}, \underline{e}\}$, $\overline{e} > \underline{e} > 0$ at cost $c(e)$ to implement the project. At the beginning, he must implement a project that is observable and acts as a signal of whether he is likely to succeed or fail in subsequent periods. He becomes either talented (H) or substandard (L), depending on the effort he has made during the implementation of the project. That is, the probability of his talent becoming H or L depends on his effort level e. His talent is interpreted as project quality τ, where $\tau \in \{H, L\}$. It is assumed that with probability $q(\overline{e})$, he achieves a high project quality $\tau = H$. Note that $q(\overline{e}) > q(\underline{e})$, while $c(\overline{e}) > c(\underline{e})$.

In the third stage, the project is implemented and the principal (as well as the incumbent agent) receives a noisy but observable and verifiable signal $\sigma\{\theta|\tau\}$ about the project quality τ. The signal is either bad $\theta = b$ or good $\theta = g$. Naturally, $\sigma\{g|H\} + \sigma\{b|H\} = \sigma\{g|L\} + \sigma\{b|L\} = 1$. I assume $\sigma\{g|H\} > \frac{1}{2}$ and $\sigma\{b|L\} > \frac{1}{2}$, meaning that if the incumbent agent's talent/project quality is high, he is more likely to produce a good signal and vice versa.[12]

In the fourth stage, following the replacement policy determined in the first stage, the principal retains or replaces the incumbent agent following a bad signal ($\theta = b$). If the incumbent agent is dismissed, a new agent whose reservation utility is zero will be hired and commences the job that has already been implemented.[13] To focus on the moral hazard

[11] If the contract includes retention of the agent, it is clear that it is a renegotiation-proof contract. If the contract includes the dismissal of the agent following a bad signal, the contract is a renegotiation-proof contract only if π_N is sufficiently large. See Appendix A.1.

[12] This implies $\frac{\sigma(g|H)q(\overline{e})}{\sigma(g|H)q(\overline{e})+\sigma(g|L)(1-q(\overline{e}))} > q(\overline{e}) > \frac{\sigma(b|H)q(\overline{e})}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$.

[13] It is assumed that the new agent cannot start a new project, which would require too much effort. The new agent, however, is assumed to be capable of carrying on the project once it has been started. Therefore, the replacement of the agent can occur after the fourth stage.

problem of the incumbent agent, I assume that the new agent always produces π_N, where $\pi_H > \pi_N > \pi_L = 0$.[14]

In the fifth stage, publicly observable firm profit is realized and both the active and passive players receive their pay based on the contract. The agent who is retained at this last stage also enjoys a control benefit, B, which is set exogenously.

In sum, the endogenously determined factors in the model are: the effort level of the agent e; the amount of fixed pay f always given to the incumbent agent; the fraction of the stock α that is granted to the incumbent agent; and whether to retain or dismiss the incumbent agent when a bad performance signal is observed. Let β denote the fraction of initial stock owned by the principal, which is exogenously determined before the principal endogenously determines α. As α is the fraction of the stock the incumbent agent receives, the principal will receive the fraction $\beta(1-\alpha)$ of the firm's stock, and outside investors will receive the fraction $(1-\alpha)(1-\beta)$.[15]

Below, I analyze the determinants of the principal's decision to retain or replace the incumbent agent even after the incumbent agent has produced a bad performance signal. To do so, I build three models. I first consider the *constrained optimality* model in which the incumbent agent's effort level is publicly observable and the optimal contract always maximizes the sum of the utilities of the principal and the incumbent agent.[16] Next, I discuss two *moral hazard* models in which the incumbent agent's effort level is unobservable by the principal and hence she motivates him by offering a fraction of stock-based compensation denoted by α. I consider two settings for the moral hazard model: one is that the incumbent agent forfeits his share α upon dismissal as is commonly observed in US firms; the other is that the incumbent agent does not forfeit α upon dismissal.[17] In practice, the former can be interpreted as long-term vested options which cannot be exercised by the incumbent agent until a certain time. The latter can be interpreted as short-term vested options which

[14] As the focus of this paper is on the effect of the incumbent agent's moral hazard action on the contract, for clarity I do not consider the moral hazard problem of the new agent. In other words, the new agent on average produces π_N, without being given α. For simplicity, I assume $\pi_N = \pi_{\widehat{N}} - \widehat{f}$, where $\widehat{f}$ is the base salary paid to the new agent.

[15] In the subsequent analysis, I focus on the managerial compensation contract observed in practice. That is, I restrict the analysis to the case in which the principal offers the incumbent agent a noncontingent base salary f and a fraction of the firm's stock α as a part of the return to the project. As is standard in the theoretical literature on executive compensation, I restrict the analysis to linear compensation contracts.

[16] The case of first-best is considered in Appendix A.2.

[17] As mentioned in Section 1, this can be observed in some Japanese firms, such as Mitsubishi UFJ Financial Group, Shin-etsu Chemical Co., Ltd. and Mitsubishi Corp.

the incumbent agent can exercise at any time, including the moment he is fired from the company.

In all three settings (constrained optimality and the two models of moral hazard), I first derive the principal's expected payoffs for the case in which the incumbent agent is dismissed following a bad signal and the case in which the incumbent agent is retained irrespective of the realized value of the signal. The difference between these expected profits determines the threshold above which the principal prefers to fire rather than retain the incumbent agent following a bad signal. I then conduct some comparative statics and show how some parameters (such as control benefit B and signal precision $\sigma(g|H)$) affect the principal's decision to dismiss or retain the incumbent agent. Finally, I cross-compare these three models to study any inefficiencies that may arise under moral hazard.

2.2. Assumptions Throughout this article, I make the following parametric assumptions.[18] For this purpose, I denote $\Delta c = c(\overline{e}) - c(\underline{e})$ and $\Delta q = q(\overline{e}) - q(\underline{e})$, where $\overline{e}$ represents a high effort level and $\underline{e}$ represents a low effort level.

Assumption A: $\frac{\Delta c}{\Delta q} \geqq [\sigma(g|H) - \sigma(g|L)]\, B$.

Assumption B: $\min\{\pi_H, \sigma(g|H)\pi_H + [\sigma(b|H) - \sigma(b|L)]\pi_N + [\sigma(g|H) - \sigma(g|L)]B\} \geq \frac{\Delta c}{\Delta q}$.

Assumption C: $\sigma(g|L)B + q(\overline{e})\frac{\Delta c}{\Delta q} \geqq c(\overline{e})$.

Assumption D: $c(\overline{e}) \geqq B$.

Assumption A implies that the difference between the cost of a high effort level $\overline{e}$ and a low effort level $\underline{e}$, denoted by Δc, is larger than or equal to $[\sigma(g|H) - \sigma(g|L)]B\Delta q$, which is the expected increment in the control benefit B by moving to a high effort level under the condition that the incumbent agent is fired following a bad signal.

Assumption B means that whether or not the incumbent agent is dismissed following a bad signal, total welfare is non-negative if the incumbent agent selects $\overline{e}$. That is, if the incumbent agent is dismissed following a bad signal, the sum of the additional expected profit and the expected control benefit generated by selecting $\overline{e}$ is larger than or equal to the additional cost incurred by selecting $\overline{e}$.[19] Alternatively, if the incumbent agent is retained following a bad

[18] See Appendix A.3. for the range of B that is specified by Assumptions A–D.

[19] The sum of the additional expected profit and the expected control benefit generated by selecting the higher effort level is expressed as $\{\sigma(g|H)\pi_H + [\sigma(g|H) - \sigma(g|L)]B\}\Delta q$ when α is to be returned, and $\{\sigma(g|H)\pi_H - [\sigma(b|L) - \sigma(b|H)]\pi_N + [\sigma(g|H) - \sigma(g|L)]B\}\Delta q$ when α is not to be returned to the firm.

signal, the additional expected profit generated by selecting the high effort level ($\pi_H \Delta q$) is larger than or equal to the additional cost incurred by selecting $\overline{e}$. Assumption B guarantees that the manager's equity holding ratio is nonnegative and does not exceed 1 in any of the cases studied below.

Assumption C guarantees that the incentive compatibility (IC) constraint binds under the optimal contract. To be more specific, suppose $c(\overline{e})$ is fixed, and $c(\underline{e})$ decreases, causing Δc to increase. Or suppose there is a high probability a less-talented agent can produce a good signal, causing $\sigma(g|L)$ to increase. In both cases, Assumption C is likely to be satisfied. Neither of these cases affects the individual rationality (IR) constraint, but encourages the IC constraint to bind in all analyses below. Figure 1 depicts the case that satisfies this situation.

Assumption D implies that the cost of effort level $\overline{e}$ is larger than the incumbent agent's control benefit B. This assumption ensures that the basic salary is nonnegative under constrained optimality.

3. Benchmark model – constrained optimality setting

In this section, I consider the constrained optimality model in which the principal can observe the incumbent agent's effort level and select $e = \overline{e}$. Under this constraint, the replacement policy chosen by the principal is *efficient* given the predetermined ownership between the principal and outsiders.[20] As the principal can set $e = \overline{e}$, there is no need to motivate the incumbent agent by giving him a fraction of stock, α. Hence, without loss of generality, I can set $\alpha = 0$. In addition, I can also exclude the case in which the incumbent agent is dismissed following a good signal of firm profit.

3.1. The incumbent agent is dismissed following a bad signal of firm profit I first analyze the case in which the incumbent agent is dismissed following a bad signal. The maximization problem for the principal is expressed as Ω_D, where D stands for *dismiss following a bad signal*:

$$\Omega_D(f) = \beta \left\{ \sigma(g|H) q(\overline{e}) \pi_H + \left[\sigma(b|H) q(\overline{e}) + \sigma(b|L)(1 - q(\overline{e})) \right] \pi_N \right\} - f. \tag{1}$$

[20] In this section, if there are no outside shareholders (formally expressed by $\beta = 1$), the first-best result is achieved.

The first term of the right-hand side is the share of the expected profit the principal obtains. The principal's expected payoff following a good signal (and hence retention of the incumbent agent) is given by $\sigma(g|H)q(\bar{e})\pi_H + \sigma(g|L)(1-q(\bar{e}))\pi_L$. As $\pi_L = 0$, this becomes $\sigma(g|H)q(\bar{e})\pi_H$. Next, the principal's expected payoff following a bad signal of firm profit (and hence replacement of the incumbent agent with a new agent) is given by $[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1-q(\bar{e}))]\,\pi_N$. Note that π_N is the net profit generated by a new agent. The second term of the right-hand side is the base salary f paid to the incumbent agent.

The IR constraint guarantees that the incumbent agent prefers to accept the contract. This is given by:

$$[\sigma(g|H)q(\bar{e}) + \sigma(g|L)(1-q(\bar{e}))]\,B + f - c(\bar{e}) \geqq 0. \tag{2}$$

The first term of the left-hand side shows that with probability $\sigma(g|H)q(\bar{e})+\sigma(g|L)(1-q(\bar{e}))$, the principal obtains a good signal of firm profit and retains the incumbent agent, and thus the incumbent agent receives the control benefit B. The second term indicates that the incumbent agent receives the base salary f. The third term, $c(\bar{e})$, is the cost of making the effort. The IR constraint implies that the total of these three terms must exceed the reservation utility of the incumbent agent, which is equal to zero.

Because an increase in f reduces the principal's objective function given by (1), it is better for the principal to keep f as small as possible. In order to do this, IR constraint must hold in equality:

$$f^*_{CO} = c(\bar{e}) - [\sigma(g|H)q(\bar{e}) + \sigma(g|L)(1-q(\bar{e}))]\,B \geqq 0, \tag{3}$$

where the last inequality follows from Assumption D. Substituting (3) into (1) leads to the principal's total payoff $\Omega_D(f^*_{CO})$ under the optimal contract:

$$\begin{aligned}\Omega_D(f^*_{CO}) = {} & \beta\sigma(g|H)q(\bar{e})\pi_H + \beta\,[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1-q(\bar{e}))]\,\pi_N \\ & - c(\bar{e}) + [\sigma(g|H)q(\bar{e}) + \sigma(g|L)(1-q(\bar{e}))]\,B.\end{aligned} \tag{4}$$

3.2. The incumbent agent is retained to the final stage irrespective of the signal

I next study the case in which the incumbent agent is not dismissed following a bad signal. This implies that the incumbent agent survives all five stages and hence no new agent is hired in the model. The maximization problem for the principal in this case is expressed by

Ω_S, where S stands for *survive all stages*:

$$\Omega_S(f) = \beta q(\overline{e})\pi_H - f. \tag{5}$$

The first term of the right-hand side is the principal's share from the expected firm profit. It is expressed as such because $\beta\left[\sigma(g|H) + \sigma(b|H)\right] q(\overline{e})\pi_H + \beta\left[\sigma(g|L) + \sigma(b|L)\right](1 - q(\overline{e}))\pi_L = \beta q(\overline{e})\pi_H$, because of $\sigma(g|H) + \sigma(b|H) = 1$ and $\pi_L = 0$. The second term of the right-hand side is the base salary f that the principal pays to the incumbent agent.

The IR constraint is given by:

$$B + f - c(\overline{e}) \geqq 0. \tag{6}$$

The first term of the left-hand side is the control benefit B. As the incumbent agent is not going to be dismissed regardless of the performance signal, he receives B as well as the base salary f. He incurs a cost $c(\overline{e})$ in making the selected level of effort. The total of these must exceed his reservation utility.

It is clear that (6) should hold in equality:

$$\overline{f}_{CO} = c(\overline{e}) - B \geqq 0, \tag{7}$$

where the last inequality follows from Assumption D. Substituting (7) into (5) leads to the principal's total payoff $\Omega_S(\overline{f}_{CO})$ under the optimal contract:

$$\Omega_S(\overline{f}_{CO}) = \beta q(\overline{e})\pi_H + B - c(\overline{e}). \tag{8}$$

3.3. Comparative statics and empirical implications Finally, I compare the principal's expected payoffs when the incumbent agent is dismissed and when he is retained following a bad signal, and then conduct comparative statics analyses. I denote $\kappa = \Omega_D(f^*_{CO}) - \Omega_S(\overline{f}_{CO})$ from (4) and (8). This is expressed by:

$$\begin{aligned}\kappa = &-\beta\sigma(b|H)q(\overline{e})\pi_H + \beta\left[\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1 - q(\overline{e}))\right]\pi_N \\ &+ \left[\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e})) - 1\right]B,\end{aligned} \tag{9}$$

where $\kappa = 0$ is the threshold above which the manager is dismissed.

Proposition 1:

(i) The larger the initial fraction of shares owned by the principal, β, the more likely it is that the incumbent agent is replaced.[21]

(ii) The smaller the control benefit, B, the more likely it is that the incumbent agent is replaced.

(iii) The larger the probability of the type-H agent producing a signal g, $\sigma(g|H)$, the more likely it is that the incumbent agent is replaced.

(iv) f^*_{CO} and $\overline{f}_{CO}$ are independent of β.

(v) The larger the control benefit B, the larger $(f^*_{CO} - \overline{f}_{CO})$.

(vi) The larger the probability of the type-H agent producing a signal g, $\sigma(g|H)$, the smaller $(f^*_{CO} - \overline{f}_{CO})$.

Proof:

Using (3), (7), (9), and $\sigma(g|H) + \sigma(b|H) = 1$:

$$\frac{\partial \kappa}{\partial \beta} = -\sigma(b|H)q(\overline{e})(\pi_H - \pi_N) + \sigma(b|L)(1 - q(\overline{e}))\pi_N > 0, \tag{10}$$

$$\frac{\partial \kappa}{\partial B} = \sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e})) - 1 < 0, \tag{11}$$

$$\frac{\partial \kappa}{\partial \sigma(g|H)} = \beta q(\overline{e})(\pi_H - \pi_N) + q(\overline{e})B > 0, \tag{12}$$

$$\frac{\partial (f^*_{CO} - \overline{f}_{CO})}{\partial B} = 1 - [\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e}))] > 0, \tag{13}$$

$$\frac{\partial f^*_{CO}}{\partial B} = -[\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e}))] < 0 \tag{14}$$

$$\frac{\partial \overline{f}_{CO}}{\partial B} = -1 < 0 \tag{15}$$

$$\frac{\partial (f^*_{CO} - \overline{f}_{CO})}{\partial \sigma(g|H)} = -q(\overline{e})B < 0 \tag{16}$$

are obtained.‖

The intuition behind Proposition 1 is as follows. First, Proposition 1 (i) shows that the

[21]Proposition 1 (i) holds when $\pi_H \doteqdot \pi_N$ as shown in Appendix A.1, that is, the sufficient condition for the renegotiation proof. This means, $\frac{\pi_N}{\pi_H} > \frac{\sigma(b|H)q(\overline{e})}{[\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1-q(\overline{e}))]}$ holds. When $\pi_H \doteqdot \pi_N$, the right-hand side is always smaller than 1, hence $\frac{\partial \kappa}{\partial \beta} > 0$.

larger the principal's share, β, the more likely it is that the equilibrium contract involves replacing the incumbent agent following a bad signal. This implies that the larger the value of β, the more the principal benefits from the increment in the expected firm profit (from π_L to π_N). As a result, the larger the principal's share in the expected firm profit, the more keen the principal is to offer the agent a contract that involves the replacement of the incumbent agent following a bad signal. However, β does not affect the IR constraint regardless of whether the agent is replaced or retained following a bad signal. Hence, as shown in Proposition 1(iv), f^*_{CO} and $\overline{f}_{CO}$ are independent of β.

Next, Proposition 1 (ii) and (v) indicate that the larger the control benefit, B, the more likely it is that the principal offers the agent a contract that retains him, irrespective of the realized signal. The control benefit is obtained by the incumbent agent if he is retained to the last stage. We can assume it is reputation that gives him bargaining power, access to a private jet and office or a privilege that comes from running the company. Therefore, if the exogenously given B is large enough, it may motivate the agent to participate in the contract even with a small base salary f. While B is not paid from the principal's pocket, the base salary f is paid by the principal herself. Therefore, the principal is better off if she can induce the agent to participate in the contract with a small f in exchange for ensuring the incumbent agent receives B. Inequalities (11), (13), (14) and (15) indicate that the principal is more likely to make f smaller for a sufficiently large B in both the case in which the incumbent agent is retained and the case in which the incumbent agent is dismissed following a bad signal, but the tendency is stronger in the former case. This is because if the contract involves the retention of the incumbent agent irrespective of his performance (signal), the incumbent agent is sure to receive B. As a result, the larger the value of B, the less likely it is that the principal offers the incumbent agent a contract that involves his replacement following a bad signal.

Lastly, with respect to Proposition 1 (iii) and (vi), the increase in $\sigma(g|H)$ means an increase in the accuracy of information regarding project quality. That is, the agent is more likely to produce a good signal (g) if the agent's true talent is high (H). This implies that when $\sigma(g|H)$ is high, the firm's profit is likely to be larger if the incumbent agent is replaced by a new agent following a bad signal. It also means that even if the parties contract to replace the agent following a bad signal (b), the risk of firing the talented incumbent agent is low, as the talented incumbent is more likely to produce a good signal (g). Therefore, when the precision of the signal is higher, the difference between f^*_{co} and $\overline{f}_{co}$ can be small, as shown in

(16). As a result, the higher the precision of the signal, the keener the principal is to offer the replacement contract as suggested by (12).

To conclude this section, these replacement policies chosen by the principal are efficient. In other words, this section has considered the 'constrained' Pareto optimal situation in which the optimal contract always maximizes the welfare of the principal and the agent. These results also hold even if $\beta = 1$, the first-best case.

4. Moral hazard models

If the principal cannot observe the incumbent agent's effort level, the incumbent agent may only exert the lowest effort level ($e = \underline{e}$). Therefore, the principal offers the fraction $\alpha > 0$ of the firm stock to induce a higher effort level ($e = \overline{e}$). α is determined in the contract and once determined does not change during the five stages. Below, I study the case in which the contract states that the incumbent agent is dismissed following a bad signal in Section 4.1, and the case in which the contract states that the incumbent agent is retained irrespective of the signal in Section 4.2.

4.1. The incumbent agent is dismissed following a bad signal of firm profit Under the moral hazard setting, two systems can be considered with respect to the stock option α. One is to have the incumbent agent return α to the firm upon dismissal (long-term vested options), considered in Section 4.1.1. The other is to have the incumbent agent keep α upon dismissal (short-term vested options), studied in Section 4.1.2. Distinguishing between keeping or forfeiting the stock-based compensation upon dismissal follows Laux (2012).

4.1.1. The incumbent agent returns α to the firm upon dismissal (long-term vested options) If the incumbent agent forfeits a portion of α to the firm after being dismissed, the maximization problem for the principal is expressed as:

$$\Phi_D(\alpha, f) = \beta(1-\alpha)\sigma(g|H)q(\overline{e})\pi_H + \beta\left[\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1-q(\overline{e}))\right]\pi_N - f. \tag{17}$$

The first term of the right-hand side is the expected payoff to the principal when the incumbent agent is retained following a good signal. As the agent receives the share α, the principal receives the fraction $\beta(1-\alpha)$ of the expected profit $\sigma(g|H)q(\overline{e})\pi_H$ (recall that $\pi_L = 0$). The second term of the right-hand side is the principal's expected payoff when the incumbent agent is dismissed following a bad signal and a new agent is hired. As the incumbent agent returns the share α upon dismissal, the principal obtains the fraction β of the expected firm profit $[\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1 - q(\overline{e}))]\, \pi_N$. Note that π_N is the firm profit generated by a new agent. The third term, f, is the base salary the principal pays to the incumbent agent.

The IR constraint is given by:

$$\alpha\sigma(g|H)q(\overline{e})\pi_H + [\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e}))]\, B + f - c(\overline{e}) \geqq 0. \tag{18}$$

The first term of the left-hand side is the expected payoff to the incumbent agent from the firm's profit if he is retained. The second term of the left-hand side shows that with probability $\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e}))$, the principal receives a good signal of firm profit and retains the incumbent agent. In this case, the incumbent agent receives the control benefit B. The third term of the left-hand side, f, is the base salary the incumbent agent receives regardless of the signal. The fourth term of the left-hand side, $c(e)$, is the cost of the selected effort level. The IR constraint ensures that the total of these four terms exceeds the reservation utility of the incumbent agent, which is equal to zero.

If the IC constraint is satisfied, the incumbent agent finds it in his own interest to exert effort. The IC constraint is given by:

$$\begin{aligned} &\alpha\sigma(g|H)q(\overline{e})\pi_H + [\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1 - q(\overline{e}))]\, B - c(\overline{e}) \\ &\geqq \alpha\sigma(g|H)q(\underline{e})\pi_H + [\sigma(g|H)q(\underline{e}) + \sigma(g|L)(1 - q(\underline{e}))]\, B - c(\underline{e}). \end{aligned} \tag{19}$$

The IC constraint consists of both explicit incentives derived from the contract and implicit incentives derived from being retained, and a disutility of exerting the effort. That is, the left-hand side of (19) is the expected payoff to the incumbent agent when he selects the high effort level $\overline{e}$, whereas the right-hand side shows the expected payoff to the incumbent agent when he selects the lower effort level $\underline{e}$.

Given Assumption C and Figure 1, it is straightforward that only the IC constraint is binding. Therefore, $f = 0$ and:

$$\alpha^* = \frac{[\sigma(g|L) - \sigma(g|H)]\, B\Delta q + \Delta c}{\sigma(g|H)\pi_H \Delta q}, \tag{20}$$

where $0 \leq \alpha^* \leq 1$, given Assumptions A and B.

Finally, substituting (20) and $f = 0$ into (17) yields the principal's total payoff under the optimal contract:

$$\Phi_D(\alpha^*, 0) = \beta\sigma(g|H)q(\bar{e})\pi_H + \beta\left[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))\right]\pi_N \\ + \beta[\sigma(g|H) - \sigma(g|L)]q(\bar{e})B - \beta\frac{\Delta c}{\Delta q}q(\bar{e}) > 0. \tag{21}$$

4.1.2. The incumbent agent keeps α upon dismissal (short-term vested options) If the incumbent agent does not forfeit α to the firm upon dismissal, the maximization problem for the principal is expressed as:

$$\Phi_{D'}(\alpha, f) = \beta(1 - \alpha)\left\{\sigma(g|H)q(\bar{e})\pi_H + \left[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))\right]\pi_N\right\} - f. \tag{22}$$

Expression (22) is different from (17) only when the incumbent agent is dismissed and a new agent is hired. In (17), the principal receives β fraction of $\left[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))\right]\pi_N$, because the incumbent agent returns the share α to the firm upon dismissal. In (22), however, the principal receives only $\beta(1 - \alpha)$ fraction of $\left[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))\right]\pi_N$ because the incumbent agent keeps the fraction α of it. The other terms are the same.

The IR constraint is given by:

$$\alpha\left\{\sigma(g|H)q(\bar{e})\pi_H + \left[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))\right]\pi_N\right\} \\ + \left[\sigma(g|H)q(\bar{e}) + \sigma(g|L)(1 - q(\bar{e}))\right]B + f - c(\bar{e}) \\ \geqq 0, \tag{23}$$

where (23) is different from (18) only when the incumbent agent is replaced. That is, with probability $\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))$, the incumbent agent will be fired but still receives the share α.

The IC constraint is given by:

$$\begin{aligned}&\alpha\left\{\sigma(g|H)q(\overline{e})\pi_H+\left[\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))\right]\pi_N\right\}\\&+\left[\sigma(g|H)q(\overline{e})+\sigma(g|L)(1-q(\overline{e}))\right]B-c(\overline{e})\\&\geqq\alpha\left\{\sigma(g|H)q(\underline{e})\pi_H+\left[\sigma(b|H)q(\underline{e})+\sigma(b|L)(1-q(\underline{e}))\right]\pi_N\right\}\\&+\left[\sigma(g|H)q(\underline{e})+\sigma(g|L)(1-q(\underline{e}))\right]B-c(\underline{e}),\end{aligned}\tag{24}$$

where (24) is different from (19) only with respect to the term $\alpha\left[\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))\right]\pi_N$ in the left-hand side and $\alpha\left[\sigma(b|H)q(\underline{e})+\sigma(b|L)(1-q(\underline{e}))\right]\pi_N$ in the right-hand side. This is because the incumbent agent keeps his share α under (24) after being fired.

Under Assumption C, only the IC constraint is binding. To see this, see Figure 1. As a result, $f=0$ and:

$$\alpha^{**}=\frac{\left[\sigma(g|L)-\sigma(g|H)\right]B\Delta q+\Delta c}{\Delta q\left\{\sigma(g|H)\pi_H+\left[\sigma(b|H)-\sigma(b|L)\right]\pi_N\right\}},\tag{25}$$

where $0\leq\alpha^{**}\leq 1$, given Assumptions A and B.

Finally, by substituting (25) and $f=0$ into (22), I obtain the principal's total expected payoff under the optimal contract:

$$\begin{aligned}&\Phi_{D'}(\alpha^{**},0)\\&=\beta\left\{1-\frac{\left[\sigma(g|L)-\sigma(g|H)\right]B\Delta q+\Delta c}{\Delta q\left\{\sigma(g|H)\pi_H+\left[\sigma(b|H)-\sigma(b|L)\right]\pi_N\right\}}\right\}\sigma(g|H)q(\overline{e})\pi_H\\&+\beta\left\{1-\frac{\left[\sigma(g|L)-\sigma(g|H)\right]B\Delta q+\Delta c}{\Delta q\left\{\sigma(g|H)\pi_H+\left[\sigma(b|H)-\sigma(b|L)\right]\pi_N\right\}}\right\}\left[\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))\right]\pi_N.\end{aligned}\tag{26}$$

To simplify the exposition, let $X=[\sigma(g|L)-\sigma(g|H)]B\Delta q+\Delta c$, $Y=\Delta q\{\sigma(g|H)\pi_H+[\sigma(b|H)-\sigma(b|L)]\pi_N\}$, and $Z=\sigma(g|H)q(\bar{e})\pi_H+[\sigma(b|H)q(\bar{e})+\sigma(b|L)(1-q(\bar{e}))]\pi_N$ where $X\geq 0$ from Assumption A and $Y>0$ and $Z>0$.[22] Then, (26) is rewritten as:

$$\Phi_{D'}(\alpha^{**},0)=\beta Z\left(1-\frac{X}{Y}\right)>0,\tag{26'}$$

where $\frac{X}{Y}=\alpha^{**}$.

[22] $Y=\Delta q\{\sigma(g|H)\pi_H+[\sigma(b|H)-\sigma(b|L)]\pi_N\}$ is positive because $\sigma(b|H)=1-\sigma(g|H)$. Substituting this into Y yields $\Delta q\{\sigma(g|H)(\pi_H-\pi_N)+[1-\sigma(b|L)]\pi_N\}>0$.

4.2. The incumbent agent is retained to the final stage, irrespective of the signal

This subsection analyzes the case in which the incumbent agent is retained, irrespective of the realized value of the signal. As the agent will be retained anyway, there is no need to consider the problem of whether the incumbent agent must or must not forfeit the share α to the firm. Hence, the principal's maximization problem is expressed as:

$$\Phi_S(\alpha, f) = \beta(1-\alpha)q(\overline{e})\pi_H - f, \tag{27}$$

where the first term of the right-hand side is derived from $\beta(1-\alpha)[\sigma(g|H)+\sigma(b|H)]q(\overline{e})\pi_H + \beta(1-\alpha)[\sigma(g|L)+\sigma(b|L)](1-q(\overline{e}))\pi_L = \beta(1-\alpha)q(\overline{e})\pi_H$, because of $\sigma(g|H)+\sigma(b|H) = 1$ and $\pi_L = 0$. The second term is the base salary of the incumbent agent.

The IR constraint is given by:

$$\alpha q(\overline{e})\pi_H + B + f - c(\overline{e}) \geqq 0, \tag{28}$$

where the first term of the left-hand side is the incumbent agent's expected payoff. The second term is the control benefit the incumbent agent obtains by being retained. The third term is the base salary and the fourth term is the cost of making the selected effort.

The IC constraint is given by:

$$\alpha q(\overline{e})\pi_H - c(\overline{e}) \geqq \alpha q(\underline{e})\pi_H - c(\underline{e}), \tag{29}$$

where the left-hand side represents the incumbent agent's payoff when he exerts a high effort level $\overline{e}$, and the right-hand side represents the incumbent agent's payoff when he exerts the lower effort level $\underline{e}$.

Similar to Section 4.1, from Assumption C and Figure 1, the IC constraint is binding. As a result, $f = 0$ and:

$$\overline{\alpha} = \frac{\Delta c}{\pi_H \Delta q}, \tag{30}$$

are chosen under the optimal contract, where $0 \leq \overline{\alpha} \leq 1$ given Assumption B.

Finally, substituting (30) and $f = 0$ into (27) yields the principal's total expected payoff under the optimal contract:

$$\Phi_S(\overline{\alpha}, 0) = \beta q(\overline{e})\pi_H - \beta\frac{\Delta c}{\Delta q}q(\overline{e}) > 0. \tag{31}$$

4.3. Comparative statics This subsection compares the principal's expected payoffs between the agent being retained and being fired following a bad signal, and then presents some comparative statics analyses. As examined in Sections 4.1.1 and 4.1.2, there are two settings to be considered if the agent is going to be dismissed following a bad signal: the incumbent agent forfeits α (long-term vested options) or keeps α (short-term vested options) upon dismissal. The principal's expected payoffs for these two settings must be compared with that of the case in which the incumbent agent is retained irrespective of the signal, as studied in Section 4.2.

Let ζ denote the difference in the principal's expected payoffs between the case in which the incumbent agent is dismissed and returns α, and the case in which the incumbent agent is retained following a bad signal. Formally, this is expressed as $\zeta = \Phi_D(\alpha^*, 0) - \Phi_S(\overline{\alpha}, 0)$. Similarly, let φ denote the difference in the principal's expected payoffs between the case in which the incumbent agent is dismissed but keeps α, and the case in which the incumbent agent is retained following a bad signal. Formally, this is expressed as $\varphi = \Phi_{D'}(\alpha^{**}, 0) - \Phi_S(\overline{\alpha}, 0)$.

4.3.1. The case of ζ From (21) and (31), the principal's net incremental payoff achieved by firing the incumbent agent (with a share returned) following a bad signal is given by:

$$\begin{aligned}\zeta = & -\beta\sigma(b|H)q(\overline{e})\pi_H + \beta\left[\sigma(b|H)q(\overline{e}) + \sigma(b|L)\left(1 - q(\overline{e})\right)\right]\pi_N \\ & + \beta[\sigma(g|H) - \sigma(g|L)]q(\overline{e})B,\end{aligned} \tag{32}$$

or it can be rewritten as:

$$\zeta = -\beta q(\overline{e})\pi_H + \beta Z + \beta[\sigma(g|H) - \sigma(g|L)]q(\overline{e})B. \tag{32'}$$

In other words, $\zeta = 0$ is the threshold above which the principal prefers to replace the agent and have the share returned rather than retaining him following a bad signal.

Proposition 2: *Suppose that the agent forfeits α upon dismissal. Then, the following can be said.*

(i) The larger the initial fraction of shares owned by the principal, β, the more likely it is

that the incumbent agent is replaced.[23]

(ii) The larger the control benefit, B, the more likely it is that the incumbent agent is replaced.

(iii) The higher the probability of the talented (H) incumbent agent producing a good signal (g), $\sigma(g|H)$, the more likely it is that the incumbent agent is replaced.

(iv) α^* and $\overline{\alpha}$ are independent of β. In other words, the principal must motivate the incumbent agent independently of the initial stock distribution.

(v) The larger the control benefit, B, the smaller $(\alpha^* - \overline{\alpha})$.

(vi) The higher the probability of the talented incumbent agent (H) producing a good signal (g), $\sigma(g|H)$, the smaller $(\alpha^* - \overline{\alpha})$.

Proof:

Differentiating (32) with respect to β yields:

$$\frac{\partial \zeta}{\partial \beta} = -\sigma(b|H)q(\overline{e})\pi_H + \{\sigma(b|H)q(\overline{e}) + \sigma(b|L)[1 - q(\overline{e})]\}\,\pi_N + [\sigma(g|H) - \sigma(g|L)]\,q(\overline{e})B > 0. \tag{33}$$

Similarly, differentiating (32) with respect to B leads to:

$$\frac{\partial \zeta}{\partial B} = \beta[\sigma(g|H) - \sigma(g|L)]q(\overline{e}) > 0. \tag{34}$$

Given $\sigma(g|H) + \sigma(b|H) = 1$, differentiating (32) with respect to $\sigma(g|H)$ yields:

$$\frac{\partial \zeta}{\partial \sigma(g|H)} = \beta q(\overline{e})(\pi_H - \pi_N) + \beta q(\overline{e})B > 0. \tag{35}$$

Moreover, it follows from (20) and (30) that:

$$\frac{\partial(\alpha^* - \overline{\alpha})}{\partial B} = \left[\frac{\sigma(g|L) - \sigma(g|H)}{\sigma(g|H) \cdot \pi_H}\right] < 0. \tag{36}$$

Lastly, from (20), (30) and $\sigma(g|H) + \sigma(b|H) = 1$, I have:

$$\frac{\partial(\alpha^* - \overline{\alpha})}{\partial \sigma(g|H)} = \frac{-\left[\sigma(g|L)B\Delta q + \Delta c\right]}{\pi_H \Delta q\left[\sigma(g|H)\right]^2} < 0. \tag{37}$$

[23] Proposition 2 (i) holds when π_N is nearly equal to π_H. That is, $\frac{\pi_N}{\pi_H} > \frac{\sigma(b|H)q(\overline{e})}{[\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1-q(\overline{e}))]}$. As shown in Appendix A.1, under the renegotiation-proof contract, π_N is sufficiently large.

$\|$

The logic behind Proposition 2 is as follows. First, similar to Proposition 1 (i) obtained under the constrained optimality setting, Proposition 2 (i) indicates that the larger the value of β, the more likely the equilibrium contract involves the replacement of the agent following a bad signal. Indeed, if the principal's share β is small, the principal does not benefit much from the increase in the net expected firm profit generated by replacing the incumbent agent.[24] Hence, even if the net expected firm profit increases, the principal must split this with outside shareholders. As a result, the principal is less likely to replace the incumbent agent following a bad signal if she has a smaller stake in the net expected firm profit.

Second, Proposition 2 (ii) suggests the larger the control benefit B, the more likely that the equilibrium contract involves replacement of the incumbent agent following a bad signal. This result is opposite to Proposition 1 (ii) obtained under the constrained optimality setting. Under Proposition 1 (ii), both f and B can be used to guarantee the agent's reservation utility, $c(\bar{e})$ (recall that there is no α in Proposition 1, as the effort level is observable). As B increases, the amount of f the principal pays to the incumbent agent decreases. This holds true whether the incumbent agent is retained or dismissed following a bad signal, but the tendency is stronger in the former case. Formally, $0 > \frac{\partial f^*_{CO}}{\partial B} > \frac{\partial \bar{f}_{CO}}{\partial B}$, and thus $|\frac{\partial \bar{f}_{CO}}{\partial B}| > |\frac{\partial f^*_{CO}}{\partial B}|$ holds. As the incumbent agent's base salary f is paid by the principal herself, she is better off the lower the value of f. As a result, the larger the value of B, the less keen the principal is to replace the incumbent agent.

In contrast, under Proposition 2 (ii) derived in the moral hazard setting, both α and B can be used to motivate the agent to exert his selected effort level instead of guaranteeing his reservation utility (recall that the principal sets $f = 0$). However, if the incumbent agent is to be retained even following a bad signal, he is sure to receive B (see (28)). In this case, regardless of whether he achieves the optimal effort level or not, an increase in B does not affect his effort level. Only α can give the agent an incentive to achieve $\bar{e}$, and hence the exogenous increase in B does not necessarily reduce α.[25] However, if the he will be dismissed following a bad signal, he can be motivated with the increase in B, allowing α to

[24] Under the moral hazard setting, the agent claims α of the expected firm profit. Therefore, the remainder of the expected firm profit shared between the principal and the outside shareholders is referred to as 'net' expected firm profit in Proposition 2.

[25] The IC constraint (29) shows that an increase in B has no effect on α.

be reduced.[26] As a result, the principal is more likely to dismiss the agent as the exogenously given control benefit B increases. This is also ensured by Proposition 2 (v), where $\alpha^* - \overline{\alpha}$ decreases.[27]

Finally, Proposition 2 (iii) shows that the higher the precision of the signal, the more likely that the equilibrium contract involves the replacement of the incumbent agent following a bad signal. Suppose the contract involved the retention of the agent irrespective of the signal, and the agent can survive without making any effort. Hence, the increase in $\sigma(g|H)$ has no effect on α. In this case, the precision of the signal is meaningless. However, if the contract involves the replacement of the incumbent agent following a bad signal, the precision of the signal becomes important. That is, if the incumbent agent achieves the selected effort level, the higher the probability of acquiring high talent (project quality) H, and the higher the probability of obtaining a good signal, g. This increases the probability that the agent is retained and that he obtains B. Therefore, the principal can motivate the agent to exert $e = \overline{e}$, instead of $e = \underline{e}$ with a smaller α if the contract stipulates the replacement of the agent following a bad signal. As a result, the principal will be more keen to dismiss the incumbent agent as the precision of the signal, $\sigma(g|H)$, increases. This is also ensured by Proposition 2(vi), where $\alpha^* - \overline{\alpha}$ decreases in response to an increase in $\sigma(g|H)$.

4.3.2. The case of φ From (26) and (31), the principal's net incremental payoff by discharging the incumbent agent (but having him keep his shares) instead of retaining him following a bad signal, is given by:

$$\varphi = \beta\left(1 - \frac{X}{Y}\right) Z - \beta q(\overline{e})\pi_H + \beta\frac{\Delta c}{\Delta q}q(\overline{e}). \tag{38}$$

In other words, φ is the threshold above which the principal prefers discharging the incumbent agent as opposed to retaining the incumbent agent following a bad signal and letting the agent keeps his shares α.

Proposition 3: *Suppose the agent keeps α upon dismissal.*

[26] This can be seen from (19); that is, an increase in B relaxes the IC constraint. In other words, it reduces α as indicated in (20).

[27] The logic is that the increase in B affects the IC constraint only when the incumbent agent will be dismissed following a bad signal. In other words, in order not to be dismissed, the incumbent agent makes more effort. As a result, the IC constraint is relaxed and the incumbent agent makes more effort even when he is offered a lower α.

(i) The effect of the initial fraction of shares owned by the principal, β, on the likelihood of the replacement of the incumbent agent is ambiguous.
(ii) The larger the control benefit, B, the more likely it is that the incumbent agent is replaced.
(iii) The higher the probability of a talented (H) incumbent agent producing a good signal (g), $\sigma(H|g)$, the more likely it is that the incumbent agent is replaced.
(iv) α^{**} and $\overline{\alpha}$ are independent of β. In other words, the principal must motivate the incumbent agent independently of the initial stock distribution.
(v) The larger the control benefit, B, the smaller is $(\alpha^{**} - \overline{\alpha})$.
(vi) The higher the probability of a talented (H) incumbent agent producing a good signal (g), $\sigma(H|g)$, the smaller is$(\alpha^{**} - \overline{\alpha})$.

Proof:

Differentiating (38) with respect to β yields:

$$\frac{\partial \varphi}{\partial \beta} = Z\left(1 - \frac{X}{Y}\right) + \left(\frac{\Delta c}{\Delta q} - \pi_H\right) q(\overline{e}), \tag{39}$$

where the effect of β is ambiguous. Similarly, differentiating (38) with respect to B leads to:

$$\frac{\partial \varphi}{\partial B} = -\beta \left[\sigma(g|L) - \sigma(g|H)\right] \Delta q \frac{Z}{Y} > 0, \tag{40}$$

as $\sigma(g|H) > \sigma(g|L)$. Differentiating (38) with respect to $\sigma(g|H)$, and using $\sigma(g|H) + \sigma(b|H) = 1$, I obtain:

$$\frac{\partial \varphi}{\partial \sigma(g|H)} = \beta q(\overline{e})(\pi_H - \pi_N)(1 - \frac{X}{Y}) + \frac{\beta Z \Delta q}{Y^2}\left[BY + (\pi_H - \pi_N)X\right] > 0, \tag{41}$$

where the last inequality follows from Assumption B.

Furthermore, from (25) and (30), it follows that:

$$\frac{\partial(\alpha^{**} - \overline{\alpha})}{\partial B} = \frac{\left[\sigma(g|L) - \sigma(g|H)\right] \Delta q}{Y} < 0. \tag{42}$$

Lastly, from (25), (30), and $\sigma(g|H) + \sigma(b|H) = 1$:

$$\frac{\partial(\alpha^{**} - \overline{\alpha})}{\partial\sigma(g|H)} = \frac{\Delta q\,[-BY - (\pi_H - \pi_N)X]}{Y^2} < 0, \tag{43}$$

is obtained.‖

The reason for the ambiguity of the effect of β on the likelihood of the replacement policy determined in the contract (Proposition 3 (i)) is because in order to motivate the incumbent agent to exert $e = \overline{e}$, he needs to receive a larger amount of α when he is allowed to keep α compared with when he must forfeit α upon dismissal. Hence, even for the larger β, there is a chance that the principal *may become* less likely to replace the incumbent agent. The intuition behind Proposition 3 (ii) (iii) (iv) (v) and (vi) are similar to that of Proposition 2.

4.3.3. Empirical implications Propositions 2 and 3 provide several empirical implications. The larger the values of β, B, and $\sigma(g|H)$, the more likely that the agent is replaced. First, a larger value of β implies an increase in the share of the parent company or an increase in the number of owners or founders of the company. Replacement of the incumbent manager is more likely in firms with a high stock ownership concentration than in firms with dispersed ownership. This finding is consistent with the existing empirical studies that examine venture-capital-backed companies (Hellmann and Puri 2002) and main banks or block ownership (Kang and Shivdasani 1995). Second, a large value of B implies that managers receive a large control benefit. I predict that such firms are more willing to commit to dismissing an agent who is granted stock-based compensation. Finally, a large value of $\sigma(g|H)$ implies that the precision of the signal is high. This holds for firms in which precise information about firm profit is easy to obtain, such as large firms or firms that belong to mature industries. Firms for which many security analysts provide a rating recommending an investment action to buy, sell or to hold can also be considered to have a high value of $\sigma(g|H)$. Firms whose accounting system is transparent are another example. Therefore, this article provides a new testable prediction that such firms are willing to replace a manager who produces a bad signal about firm performance.

5. Discussions

In Section 3, the principal's net gain from dismissing the agent rather than retaining him following a bad signal was derived as κ under the constrained optimality setting. Similarly, in Section 4, the principal's net gain from dismissing the incumbent agent following a bad signal was derived as ζ (the incumbent agent is given long-term vested options) or φ (the incumbent agent is given short-term vested options) under the moral hazard problem setting. In this section, I compare these net gains to study how the inefficiency in the managerial retention policy is created. I change the value of B in Propositions 4, 5 and 6 derived below, but there exists a range of B even if it is restricted by Assumptions A through D. See Appendix A.3 for this.

5.1. Efficiency arguments In this section, I compare both moral hazard models with the constrained optimality model. By comparing the strategies taken under the moral hazard models with the strategy taken under the constrained optimality model, this subsection investigates any inefficiencies brought about under the moral hazard models. If there exists a parametric range in which the strategies taken under the moral hazard models are different from that taken under the constrained optimality setting, then inefficiency occurs. Below, I conduct analyses with respect to the parameter B, which is the control benefit.

I first compare the model in which moral hazard is solved with long-term vested options and the constrained optimality model. Recall from Section 4 that ζ is an increasing function of B and κ is a decreasing function of B. If $\zeta > 0$ and $\kappa > 0$, or $\zeta < 0$ and $\kappa < 0$, the contract chosen under the moral hazard model (the case of ζ) is efficient because the strategy under the moral hazard model is the same as that under the constrained optimality model. The inefficiency arises when $\zeta > 0$ and $\kappa < 0$, because the strategy under the moral hazard model is different from that under the constrained optimality model.[28] See Figure 2.

Proposition 4:
Suppose there is a company that has an agency problem, and this company uses long-term vested options to motivate the agent. In other words, the agent has to forfeit his share α to the firm when dismissed. Then, the manager is over-replaced when $\zeta > 0$ and $\kappa < 0$ hold.

[28] As $\zeta - \kappa = \beta\left[\sigma(g|H) - \sigma(g|L)\right] q(\bar{e})B + [1 - \sigma(g|H)q(\bar{e}) - \sigma(g|L)(1 - q(\bar{e}))]B > 0$, there will not be a case where $\zeta < 0$ and $\kappa > 0$.

When ζ and κ are viewed as a function of B, *ζ and κ have the same intercept.*

i) Suppose, the intercept is positive. Then, $\zeta > 0$ *and* $\kappa < 0$ *(κ crosses the x-axis) holds when* $B \geq \frac{\beta\sigma(b|H)q(\overline{e})\pi_H - \beta[\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))]\pi_N}{[\sigma(g|H)q(\overline{e})+\sigma(g|L)(1-q(\overline{e}))-1]} \equiv \Xi_1$. *This is the parametric range in which the incumbent agent is over-replaced.*

ii) Suppose the intercept is negative. Then, $\zeta > 0$ *and* $\kappa < 0$ *(*ζ crosses the x-axis*) holds when* $B \geq \frac{\sigma(b|H)q(\overline{e})\pi_H - [\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))]\pi_N}{[\sigma(g|H)-\sigma(g|L)]q(\overline{e})} \equiv \Xi_2$. *This is the parametric range in which the incumbent agent is over-replaced.*

Next I compare the model in which the moral hazard problem is solved using short-term vested options and the constrained optimality model. Recall from Section 4 that φ is an increasing function of B and κ is a decreasing function of B. If $\varphi > 0$ and $\kappa > 0$, or $\varphi < 0$ and $\kappa < 0$, the contract chosen under the moral hazard model (the case of φ) is efficient. The inefficiency arises when $\varphi > 0$ and $\kappa < 0$ or $\varphi < 0$ and $\kappa > 0$. See Figure 3.

Proposition 5:

Suppose there is a company that has an agency problem, and this company uses short-term vested options to motivate the agent. In other words, the agent can keep his share α *when dismissed. Then, the agent is over-replaced when* $\varphi > 0$ *and* $\kappa < 0$ *hold, but is likely to be under-replaced when* $\varphi < 0$ *and* $\kappa > 0$ *hold. When* ζ and κ are viewed as a function of B, *ζ and κ have different intercepts.*

i) Suppose a) both φ *and* κ *have positive intercepts, or b)* φ *has a negative intercept but* κ *has a positive intercept and their intersection is positive. Then,* $\varphi > 0$ *and* $\kappa < 0$ *(κ crosses the x-axis) holds when* $B \geq \Xi_1$. *This is the parametric range in which the incumbent agent is over-replaced.*

ii) Suppose a) both φ *and* κ *have negative intercepts, or b)* φ *has a negative intercept but* κ *has a positive intercept and their intersection is negative. Then,* $\varphi > 0$ *and* $\kappa < 0$ *(φ crosses the x-axis) holds, when* $B \geq \frac{Y\left[Z - q(\overline{e})\pi_H + \frac{\Delta c}{\Delta q}q(\overline{e})\right] - \Delta c Z}{Z[\sigma(g|L)-\sigma(g|H)]\Delta q} \equiv \Xi_3$. *This is the parametric range in which the incumbent agent is over-replaced.*

iii) Suppose φ *has a negative intercept but* κ *has a positive intercept and their intersection is positive. Then,* $\varphi < 0$ *and* $\kappa > 0$ *hold(*φ crosses the x-axis) *when* $B \leq \Xi_3$. *This is the parametric range in which the incumbent agent is under-replaced.*

iv) Suppose φ *has a negative intercept but* κ *has a positive intercept and their intersection is negative. Then,* $\varphi < 0$ *and* $\kappa > 0$ *hold (*κ crosses the x-axis) *when* $B \leq \Xi_1$. *This is the* *parametric range in which the incumbent agent is under-replaced.*

Propositions 4 and 5 hold because both ζ and φ are increasing functions of B, but κ is a decreasing function of B. The principal chooses whether to offer the 'dismiss contract' or the 'retain contract' at the first stage, but if this contract choice is different between the moral hazard model (ζ and φ) and the constrained optimal model (κ), this implies that the choice under the moral hazard model is inefficient. The implication of Propositions 4 and 5 is that *inefficient* retention or replacement policies would be chosen when stock-based compensation is used to resolve the moral hazard problems related to the agent's effort. This happens because the principal can determine the agent's share in a way that benefits the principal herself (which may reduce the firm value).

In the case of over-replacement, when B is sufficiently large, inefficient replacement occurs because the principal can increase her gain by offering a dismiss contract under the moral hazard settings, whereas the retain contract is optimal under the constrained optimality setting. Recall from (21), (26) and (31) (or Propositions 2(ii) and 3(iii)), that as B increases, the principal's payoff is unaffected if she offers the contract that involves the retention of the agent, but her payoff increases if she offers the contract that involves replacing him following a bad signal. That is, when the stock-based compensation is used, by offering the replacement contract to the incumbent agent, the principal can reduce α under the moral hazard model (as long as the IC constraint is binding). The logic is that if the agent will be dismissed following a bad signal, he is more likely to have an incentive to produce a high effort level, $\overline{e}$, even for a small value of α. This is because the higher the effort level, the higher the probability of being retained and receiving the control benefit B.[29] Obviously, the larger the value of B, the more the agent is motivated to achieve $\overline{e}$ in order to be retained. This is true for both long-term and short-term vested options. That is, irrespective of whether or not the contract forces the agent to forfeit his share upon dismissal, the agent can survive and receive both α and B if he exerts the high effort level. As a result, there is a case in which the principal (firm) offers a replacement contract under the moral hazard settings even though a retention contract would be the efficient contract that should be offered. See Figures 2 and 3, where B is sufficiently large.

[29] The higher the effort level, the more likely it is that the agent obtains high-talent H, and in turn, is more likely to produce a good signal, g. As a result, he will be retained and obtains the control benefit, B. If B is sufficiently large, the agent has an incentive to be retained to the final stage.

However, if B is sufficiently small, there could be a case of under-replacement where short-term vested options are granted to the incumbent agent. This is because long-term vested options are returned to the principal upon the dismissal of the incumbent agent (hence the principal has an incentive to fire the incumbent agent) but short-term vested options stay with the incumbent agent even after he is dismissed (hence the principal is not keen on firing the incumbent agent in this case). Moreover, if B is sufficiently small, the incumbent agent is not so keen on receiving it. In other words, the threat of dismissal following a bad signal only gives the agent a small incentive to achieve $\bar{e}$, which is not attractive to the principal. Furthermore, in order to motivate the agent when B is very small, the principal must give the agent a large α, even after he is dismissed. Hence, the principal is better off if she offers the agent a retain contract, so that she can make α smaller. (The principal can reduce α because the agent will receive some portion of B if he is retained.) As a result, when B is sufficiently small, the principal tends to under-replace the incumbent agent when he can keep his share α upon dismissal. See Figure 3, where B is sufficiently small.

This finding adds a new dimension to the findings of Inderst and Mueller (2010), who examine optimal CEO compensation and timing of replacement. They find that steep incentive schemes, such as granting stock options, used together with severance pay can mitigate inefficient CEO retentions. Although there are several differences in my model structures, Propositions 4 and 5 suggest a different result from that of Inderst and Mueller (2010): granting stock-based compensation, either long-term or short-term vested, to the CEO can cause over-replacement, while there is also a possibility of under-replacement only in the case of short-term vested options which could be interpreted as severance pay. Note that the difference between the short-term vested options and severance pay is that the incumbent agent can receive the short-term vested options whether he is retained or fired.

This article also contributes to the theoretical literature on corporate boards that examines CEO replacement in a contracting framework (Almazan and Suarez 2003, Dow and Raposo 2005, Hermalin 2005, Inderst and Mueller 2010). Propositions 4 and 5 indicate that the board (principal) cannot completely solve the agency problem or achieve efficient CEO replacement through the use of stock-based compensation.

Remark 1 *The reason why $\zeta = \kappa$ holds when $B = 0$ is as follows. Both ζ and κ are the principal's incremental expected payoff when she chooses the action to dismiss the incumbent agent following a bad signal rather than the action to always retain the incumbent agent. In* *both cases, the effort level is $\overline{e}$. This means the difference between ζ and κ is attributed to the difference in the incremental expected payment to the incumbent agent the principal has to pay.*[30]

When $B = 0$, the incremental expected payment to the incumbent agent becomes the same (it becomes 0) between ζ and κ. More specifically:

In the case of ζ, the expected payment to the incumbent agent is $\pi_H \times \sigma(g|H) \times q(\overline{e}) \times \alpha^* = \frac{\Delta c q(\overline{e})}{\Delta q}$, when the principal chooses the action which involves the dismissal of the incumbent agent following a bad signal (recall that α is returned to the principal upon dismiss). The expected payment to the incumbent agent is also $\pi_H \times q(\overline{e}) \times \overline{\alpha} = \frac{\Delta c q(\overline{e})}{\Delta q}$, when the principal chooses the action which involves retention of the incumbent agent irrespective of the signal. Hence, the incremental expected payment to the incumbent agent is 0.

In the case of κ, the expected payment to the incumbent agent is $c(\overline{e})$ ($B = 0$ in (3)), when the principal chooses the action which involves the dismissal of the incumbent agent following a bad signal. The expected payment to the incumbent agent is also $c(\overline{e})$ ($B = 0$ in (7)), when the principal chooses to retain the incumbent agent irrespective of the signal. Hence the incremental expected payment to the incumbent agent is 0.

In sum, when $B = 0$ holds, the incremental expected payment to the incumbent agent by choosing the action that involves 'dismiss' rather than 'retain' is zero for both the long-term vested options model (the case of ζ) and the constrained optimality model (the case of κ), as represented by the same intercept in Figure 2.

5.2. Long-term vested stock options vs short-term vested stock options Below I compare both long- and short-term vested options together with the constrained optimality model. The difference between ζ and φ is expressed as:

$$\zeta - \varphi = \frac{\beta \sigma(b|L) \pi_N X}{Y} > 0, \tag{44}$$

where the inequality is derived from Assumption A.[31] Clearly, (44) always hold irrespective of the parameter values. From this inequality, there is not a case where φ has a positive intercept but κ and ζ have negative intercepts. Considering (44) together with Propositions

[30] I thank Michihiro Kandori for pointing this out.

[31] $\zeta : -\beta q(\overline{e})\pi_H + \beta Z + \beta\left[\sigma(g|H) - \sigma(g|L)\right] q(\overline{e}) B$. $\varphi : \beta\left(1 - \frac{X}{Y}\right) Z - \beta q(\overline{e})\pi_H + \beta \frac{\triangle c}{\triangle q} q(\overline{e})$. Hence, $\zeta - \varphi = \beta\left[\sigma(g|H) - \sigma(g|L)\right] q(\overline{e}) B + \beta \frac{X}{Y} Z - \beta \frac{\triangle c}{\triangle q} q(\overline{e})$. Rearranging this yields (44).

4 and 5, I obtain:

Proposition 6:
Suppose there is a company that has an agency problem, and this company uses stock-based compensation to motivate the incumbent agent.
i) Suppose a) the intercept of φ is negative but the intercept of κ is positive (the intersection can be either positive or negative), or b) both intercepts are positive. Then there exists a parametric range in which $\zeta > 0$, $\varphi > 0$ and $\kappa < 0$ hold. Here, when B is sufficiently large, both long- and short-term vested options yield over-replacement. However, for case (a), when B is sufficiently small, short-term vested options exhibit under-replacement but long-term vested options are always efficient (that is, $\zeta > 0, \varphi < 0$, and $\kappa > 0$).
ii) Suppose the intercepts of both φ and κ are negative (that is, κ, φ, and ζ all have negative intercepts). Then, there exists a parametric range in which $\zeta > 0$, $\varphi < 0$ and $\kappa < 0$ hold. Here, long-term vested options exhibit more inefficiency than the short-term vested options.

Proposition 6 can be easily obtained by including (44) in Figure 3. (Note that doing so would also cover the two cases considered in Figure 2 with (44).)

Proposition 6 indicates that when the control benefit B is large, there can be firms that offer replacement contracts when they should be offering retention contracts. This holds true whether the firm is using long- or short-term vested options to motivate the agent. Moreover, firms that offer long-term vested options and firms that offer short-term vested options exhibit the same degree of inefficiency most of the time, but under a certain condition (κ, φ, and ζ all have a negative intercept), firms that offer long-term vested options exhibit more inefficiency than firms that offer short-term vested options. This finding is similar to Laux (2012) in the sense that there is a possibility that short-term vested options can be better than long-term vested options, which is contrary to the pervasive understanding.

Nevertheless, as short-term vested options can function as a severance payment or golden parachute to the incumbent agent upon his dismissal, this finding is contrast to those of Lambert and Larcker (1985), Knoeber (1986), Harris (1990), and Almazan and Suarez (2003). That is, they show that severance pay can ease the departure of a CEO who does not really wish to leave. However, in this article, there exists a case in which short-term vested options can better mitigate the inefficiency than long-term vested options because short-term vested options can make it less likely to replace managers.

Finally, Proposition 6 also provides a new testable prediction that the CEO tenure for firms using long-term vested options is more likely to be shorter than that for firms using short-term vested options when the agent's control benefit is sufficiently small.

Remark 2 *If long- and short-term vested options are compared without considering the replacement policy, they are indifferent as an agency cost. That is, without the replacement policy, α^{**} from (25) will equal α^{*} from (20). Short-term vested options are kept by the incumbent agent even when he is fired following his ability signal θ. From the decision theory point of view, it may seem that short-term vested options allow the principal to design the contract using accurate information (in this model, it is τ). However, stock options, whether they are short or long-term vested options, are granted before information about the CEO's true ability is revealed. This implies that the amount of stock options in general are not determined contingent on the incumbent agent's true ability as it is modeled here. Only the profit the incumbent agent obtains from the stock options at the last stage is contingent on his true ability τ.*

In this article, moreover, the principal decides whether to replace or retain the incumbent agent as well as the number of stock options granted to the incumbent agent in the moral hazard context. In this scenario, short-term vested options deprive the incumbent agent of his incentive to make high effort when he knows he might be replaced, because he can keep his short-term vested options even after he is fired. To avoid this happening, the principal needs to motivate the incumbent agent by granting a much greater number of stock options, as shown by $\alpha^{**} > \alpha^{*}$. Indeed, when the control benefit is small, the principal has to give many more stock options to motivate the incumbent agent, but as this reduces her share, she is more likely to promise to retain the incumbent agent. But short-term vested options can be better than long-term vested options when the control benefit is sufficiently large. This is because long-term vested options could cause over-replacement of the incumbent agent. When considered from the point of view of decision theory, the model I present in this article, therefore, is not so simple as the contract designed with accurate information. [32]

Proposition 6 also indicates that when the control benefit B is sufficiently small, firms will inefficiently retain incumbent managers. This holds true only if firms are using short-term vested options. This finding is similar to the pervasive understanding that short-term vested options can be more inefficient than long-term vested options. Lastly, I have changed the

[32] I am grateful to Kazuya Kamiya, Michihiro Kandori, and Hiroshi Osano for pointing this out.

value of B in Propositions 4, 5, and 6, but a range of B exists even if it is restricted by Assumptions A through D. See Appendix A.3 for this.

6. Conclusion

It is well-known that stock-based compensation can induce effort by the agent and hence alleviate the moral hazard problem. This article, however, has shown that stock-based compensation used together with other remuneration schemes can give rise to another problem—the inefficient replacement/retention of incumbent agents. In particular, I have indicated that both long- and short-term vested options can cause over-replacement of the incumbent agent but in some cases, long-term vested options can cause greater inefficiency than short-term vested options. I have also shown that only short-term vested stock options can cause the under-replacement of the incumbent agent.

The article provides several fruitful avenues for future research. The first is to consider a model in which the choice to dismiss the manager and the choice to retain the manager are not discrete, but are somewhat continuous. For example, these two choices are determined by probability. The second is to introduce an exercise price into a simpler model and consider the repricing of the stock options. The third is to consider if stock-based compensation or incentive pay solves the agency problem with respect to unobservable effort but creates another problem, what type of compensation package would mitigate or solve this problem?[33] Finally, an empirical study could compare the CEO tenure for firms using long-term vested options and firms using short-term vested options. This could be done in relation to the volume of the control benefit, ownership concentration or information accuracy.

[33] Kamiya and Adachi-Sato (2013) attempt this analysis in a purely theoretical context.

Appendix

A.1. Renegotiation Proof: If the players were to renegotiate the contract, they would renegotiate after observing the bad signal. It is obvious that the contract is renegotiation proof under the constrained optimality model. Below, I show that both the long- and short-term vested options are renegotiation proof.

Assumption E: $\pi_N \doteqdot \pi_H$.

Assumption E is only used for the renegotiation proof, which is shown in Appendix A.1. It is the sufficient condition for the contract to be renegotiation proof. It implies that the difference between π_H and π_N is small.

Long-term vested options (α will be forfeited):

The utilities for the principal and the incumbent agent are $\beta\pi_N - f$ and f, respectively, when the incumbent agent is fired following a bad signal. The utilities for the principal and the incumbent agent are $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} - f$ and $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} + f + B$, respectively, when the incumbent agent is retained following a bad signal. As $f = 0$, the incumbent agent can pay a maximum of $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ to the principal in exchange for the retention. As a result, $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} + \beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} < \beta\pi_N$ is the sufficient condition for the players to not renegotiate. Substituting $\alpha^* = \frac{[\sigma(g|L)-\sigma(g|H)]B\Delta q+\Delta c}{\sigma(g|H)\pi_H\Delta q}$ into α, yields:

$$\left\{\frac{[\sigma(g|L)-\sigma(g|H)]\,B\Delta q+\Delta c}{\sigma(g|H)\pi_H\Delta q}(1-\beta)+\beta\right\}\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} < \beta\pi_N. \quad \text{(A1)}$$

As $\left[\frac{\Delta c}{\sigma(g|H)\pi_H\Delta q}(1-\beta)+\beta\right]\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$, $\alpha^* < \frac{\Delta c}{\sigma(g|H)\pi_H\Delta q}$. Therefore, the left-hand side of (A1) is smaller than $\frac{\frac{\Delta c}{\sigma(g|H)\pi_H\Delta q}(1-\beta)+\beta}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}\cdot\sigma(b|H)q(\overline{e})\pi$. If $\Delta c \doteqdot 0$ and $\pi_H = \pi_N$, $\frac{\beta\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} < \beta\pi_N$ holds. As a result, (A1) holds with strict inequality at $\Delta c \doteqdot 0$ and $\pi_H = \pi_N$. Then there exists $\overline{\pi_N}$ such that for all $\pi_H > \pi_N \geq \overline{\pi_N}$, the above inequality holds. This holds for both $\overline{e}$ and $\underline{e}$.$\|$

Short-term vested options (α will be kept):

The utilities for the principal and the incumbent agent are $\beta(1-\alpha)\pi_N - f$ and $\alpha\pi_N + f$, respectively, if the incumbent agent is dismissed following a bad signal. The utilities for the principal and the incumbent agent are $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} - f$ and $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} + B + f$, respectively, when the incumbent agent is retained following a bad signal. As $f=0$, the incumbent agent can pay a maximum of $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ to the principal in exchange for retention. The amount he can pay is ε, where $\varepsilon \leq \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ (recall that the incumbent agent faces limited liability and that B is nonpecuniary). At the same time, if $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\varepsilon$ is smaller than $\alpha\pi_N$, the incumbent agent is better off not paying ε. As a result, $\varepsilon \leq \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\alpha\pi_N$ holds.

Case 1: If $\alpha\pi_N \leq B$, $\varepsilon \leq \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$, and thus the incumbent agent pays a maximum of $\varepsilon = \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ to the principal. The sufficient condition for the players not to renegotiate is that the principal's utility $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ plus $\varepsilon = \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ must be smaller than $\beta(1-\alpha)\pi_N$. Formally, $\beta\left[\pi_N-\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}\right] \geq \alpha\left[(1-\beta)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+\beta\pi_N\right]$. Substituting $\alpha^{**}=\frac{[\sigma(g|L)-\sigma(g|H)]B\Delta q+\Delta c}{\Delta q\{\sigma(g|H)\pi_H+[\sigma(b|H)-\sigma(b|L)]\pi_N\}}$ into α yields:

$$\begin{aligned}&\beta\left[\pi_N-\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}\right] \\ &\geq \frac{[\sigma(g|L)-\sigma(g|H)]\,B\Delta q+\Delta c}{\Delta q\left\{\sigma(g|H)\pi_H+[\sigma(b|H)-\sigma(b|L)]\,\pi_N\right\}}\left[(1-\beta)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+\beta\pi_N\right],\end{aligned} \tag{A2}$$

which is the sufficient condition that there will not be any renegotiation. A2 holds with strict inequality at $\pi_H=\pi_N$. Then there exists $\overline{\pi_N}$ such that for all $\pi_N \geq \overline{\pi_N}$, the above inequality holds. This holds for both $\overline{e}$ and $\underline{e}$.

Case 2: If $\alpha\pi_N \geq B$, $\varepsilon \leq \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\alpha\pi_N$, the incumbent agent pays a maximum of $\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\alpha\pi_N$ to the principal. The sufficient condition for the players not to renegotiate is that the principal's utility $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}$ plus $\varepsilon = \alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\alpha\pi_N$ is smaller than $\beta(1-\alpha)\pi_N$. Formally, $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+B-\alpha\pi_N \leq \beta(1-\alpha)\pi_N$ is the sufficient condition for no renegotiation. As $\alpha\pi_N \geq B$, this can be reduced to $\beta(1-\alpha)\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))}+\alpha\frac{\sigma(b|H)q(\overline{e})\pi_H}{\sigma(b|H)q(\overline{e})+\sigma(b|L)(1-q(\overline{e}))} \leq \beta(1-\alpha)\pi_N$. Substituting $\alpha^{**}=\frac{[\sigma(g|L)-\sigma(g|H)]B\Delta q+\Delta c}{\Delta q\{\sigma(g|H)\pi_H+[\sigma(b|H)-\sigma(b|L)]\pi_N\}}$ into α yields (A2). As a result, the renegotiation proof is the same as for Case 1.‖

A.2. The First-best Setting: If the incumbent agent is going to be dismissed following a bad signal, the total utility of the principal, the incumbent agent, and outsiders is expressed by:

$$\sigma(g|H)q(\overline{e})\pi_H + [\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1-q(\overline{e}))]\,\pi_N$$
$$- [\sigma(b|H)q(\overline{e}) + \sigma(b|L)(1-q(\overline{e}))]\,c_f + [\sigma(g|H)q(\overline{e}) + \sigma(g|L)(1-q(\overline{e}))]\,B - c(\overline{e}). \tag{A3}$$

The total utility of the case in which the incumbent agent does not get dismissed is expressed by:

$$q(e)\pi_H + B - c(\overline{e}). \tag{A4}$$

‖

A.3. The Range of B: 1. From Assumptions A–D, the range of B is restricted by:

$$\min\left[c(\overline{e}), \frac{\frac{\Delta c}{\Delta q}}{\sigma(g \mid H) - \sigma(g \mid L)}\right]$$
$$\geq B \geq \max\left\{\frac{c(\overline{e}) - \frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g \mid L)}, \frac{\frac{\Delta c}{\Delta q} - \sigma(g \mid H)\pi_H - [\sigma(b \mid H) - \sigma(b \mid L)]\,\pi_N}{\sigma(g \mid H) - \sigma(g \mid L)}\right\}.$$

However, the possibility of $\frac{\frac{\Delta c}{\Delta q}}{\sigma(g|H)-\sigma(g|L)}$ being the minimum is excluded for the reason given below. Hence, the range of B is limited to:

$$c(\overline{e}) \geq B \geq \max\left\{\frac{c(\overline{e}) - \frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g \mid L)}, \frac{\frac{\Delta c}{\Delta q} - \sigma(g \mid H)\pi_H - [\sigma(b \mid H) - \sigma(b \mid L)]\,\pi_N}{\sigma(g \mid H) - \sigma(g \mid L)}\right\}.$$

Suppose, $c(\overline{e}) > \frac{\frac{\Delta c}{\Delta q}}{\sigma(g|H)-\sigma(g|L)}$. Then, $c(\overline{e})\,[\sigma(g \mid H) - \sigma(g \mid L)] > \frac{\Delta c}{\Delta q}$. In addition, under this condition:

$$\frac{c(\overline{e}) - \frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g \mid L)} - \frac{\frac{\Delta c}{\Delta q} - \sigma(g \mid H)\pi_H - [\sigma(b \mid H) - \sigma(b \mid L)]\,\pi_N}{\sigma(g \mid H) - \sigma(g \mid L)}$$
$$= \frac{\begin{array}{c} c(\overline{e})\,[\sigma(g \mid H) - \sigma(g \mid L)] - \{q(\overline{e})\sigma(g \mid H) + [1-q(\overline{e})]\,\sigma(g \mid L)\}\,\frac{\Delta c}{\Delta q} \\ +\sigma(g \mid L)\,\{\sigma(g \mid H)\pi_H + [\sigma(b \mid H) - \sigma(b \mid L)]\,\pi_N\} \end{array}}{\sigma(g \mid L)\,[\sigma(g \mid H) - \sigma(g \mid L)]}$$
$$> 0.$$

Thus, if $c(\overline{e}) > \frac{\frac{\Delta c}{\Delta q}}{\sigma(g|H)-\sigma(g|L)}$, the range of B restricted by Assumptions A–D is reduced to:

$$\frac{\frac{\Delta c}{\Delta q}}{\sigma(g \mid H) - \sigma(g \mid L)} \geq B \geq \frac{c(\overline{e}) - \frac{\Delta c}{\Delta q} q(\overline{e})}{\sigma(g \mid L)}.$$

To ensure that this range is nonempty, the following condition is required:

$$\begin{aligned} &\frac{\frac{\Delta c}{\Delta q}}{\sigma(g \mid H) - \sigma(g \mid L)} - \frac{c(\overline{e}) - \frac{\Delta c}{\Delta q} q(\overline{e})}{\sigma(g \mid L)} \\ &= \frac{\frac{\Delta c}{\Delta q}\sigma(g \mid L) - [\sigma(g \mid H) - \sigma(g \mid L)] \left[c(\overline{e}) - \frac{\Delta c}{\Delta q} q(\overline{e})\right]}{[\sigma(g \mid H) - \sigma(g \mid L)]\, \sigma(g \mid L)} \\ &> 0. \end{aligned}$$

This condition is reduced to:

$$\begin{aligned} &\frac{\Delta c}{\Delta q}\sigma(g \mid L) - [\sigma(g \mid H) - \sigma(g \mid L)] \left[c(\overline{e}) - \frac{\Delta c}{\Delta q} q(\overline{e})\right] \\ &\quad = \frac{\Delta c}{\Delta q} \left\{\sigma(g \mid H) q(\overline{e}) + \sigma(g \mid L)[1 - q(\overline{e})]\right\} - [\sigma(g \mid H) - \sigma(g \mid L)]\, c(\overline{e}) \\ &\quad > 0. \end{aligned}$$

However:

$$\sigma(g \mid H) q(\overline{e}) + \sigma(g \mid L)[1 - q(\overline{e})] = [\sigma(g \mid H) - \sigma(g \mid L)]\, q(\overline{e}) + \sigma(g \mid L) < 1.$$

Hence, if $c(\overline{e}) > \frac{\frac{\Delta c}{\Delta q}}{\sigma(g|H)-\sigma(g|L)}$, the range of B restricted by Assumptions A–D is empty.

Now, suppose that $c(\overline{e}) \leq \frac{\frac{\Delta c}{\Delta q}}{\sigma(g|H)-\sigma(g|L)}$. That is, $c(\overline{e})\, [\sigma(g \mid H) - \sigma(g \mid L)] \leq \frac{\Delta c}{\Delta q}$. If:

$$\begin{aligned} &c(\overline{e})\, [\sigma(g \mid H) - \sigma(g \mid L)] + \sigma(g \mid L) \left\{\sigma(g \mid H)\pi_H + [\sigma(b \mid H) - \sigma(b \mid L)]\, \pi_N\right\} \\ &- \left\{q(\overline{e})\sigma(g \mid H) + [1 - q(\overline{e})]\, \sigma(g \mid L)\right\} \frac{\Delta c}{\Delta q} \geq 0, \end{aligned}$$

the range of B restricted by Assumptions A–D is reduced to:

$$c(\overline{e}) \geq B \geq \frac{c(\overline{e}) - \frac{\Delta c}{\Delta q} q(\overline{e})}{\sigma(g \mid L)}. \tag{A5}$$

On the other hand, if:

$$
\begin{aligned}
& c(\overline{e})\left[\sigma(g \mid H)-\sigma(g \mid L)\right]+\sigma(g \mid L)\left\{\sigma(g \mid H)\pi_H+\left[\sigma(b \mid H)-\sigma(b \mid L)\right]\pi_N\right\} \\
-& \left\{q(\overline{e})\sigma(g \mid H)+\left[1-q(\overline{e})\right]\sigma(g \mid L)\right\}\frac{\Delta c}{\Delta q}<0,
\end{aligned}
$$

the range of B restricted by Assumptions A–D is reduced to:

$$
c(\overline{e}) \geq B \geq \frac{\frac{\Delta c}{\Delta q}-\sigma(g \mid H)\pi_H-\left[\sigma(b \mid H)-\sigma(b \mid L)\right]\pi_N}{\sigma(g \mid H)-\sigma(g \mid L)}. \tag{A6}
$$

Both (A5) and (A6) are nonempty under certain conditions. In short, there is a range of B that satisfies Assumptions A–D.

2. I next derive the conditions under which the following three thresholds are inside or outside the range defined by (A5) or (A6). Here, I check the range specified by (A5) to show that there exists a range of B:

$$
\begin{aligned}
B_1 &= \frac{Z\Delta c-Y[Z-q(\overline{e})\pi_H+\frac{\Delta c}{\Delta q}q(\overline{e})]}{\left[\sigma(g \mid H)-\sigma(g \mid L)\right]Z\Delta q}, \\
B_2 &= \frac{\beta\left\{[\sigma(b \mid H)q(\overline{e})+\sigma(b \mid L)(1-q(\overline{e}))]\pi_N-\sigma(b \mid H)q(\overline{e})\pi_H\right\}}{1-\sigma(g \mid H)q(\overline{e})-\sigma(g \mid L)(1-q(\overline{e}))}, \\
B_3 &= \frac{\sigma(b \mid H)q(\overline{e})\pi_H-\left[\sigma(b \mid H)q(\overline{e})+\sigma(b \mid L)(1-q(\overline{e}))\right]\pi_N}{\left[\sigma(g \mid H)-\sigma(g \mid L)\right]q(\overline{e})}.
\end{aligned}
$$

I first check the conditions under which B_1 is inside or outside the range defined by (A5). Then:

$$
\begin{aligned}
& Z\left\{\frac{\sigma(g \mid H)-\sigma(g \mid L)}{\sigma(g \mid L)}\left[q(\overline{e})\Delta c-\Delta q c(\overline{e})\right]+\Delta c\right\} \\
& \geq Y[Z-q(\overline{e})\pi_H+\frac{\Delta c}{\Delta q}q(\overline{e})] \geq Z\left\{\Delta c-\left[\sigma(g \mid H)-\sigma(g \mid L)\right]\Delta q c(\overline{e})\right\}.
\end{aligned} \tag{A7}
$$

Hence, I show the following:

(i) If (A7) is satisfied, B_1 is inside the range defined by (A5).

(ii) If only the second inequality of (A7) is satisfied, then $c(\overline{e}) \geq B_1$ but $\frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)} > B_1$ hold. Hence, B_1 is smaller than any other points inside the range defined by (A5).

(iii) If only the first inequality of (A7) is satisfied, then $B_1 \geq \frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)}$ but $B_1 > c(\overline{e})$ hold. Hence, B_1 is larger than any other points inside the range defined by (A5).

Second, I examine the conditions under which B_2 is inside or outside the range defined by (A5). Then:

$$
\begin{aligned}
& c(\overline{e})\left[1-\sigma(b \mid H) q(\overline{e})-\sigma(b \mid L)(1-q(\overline{e}))\right] \\
& >\beta\left\{[\sigma(b \mid H) q(\overline{e})+\sigma(b \mid L)(1-q(\overline{e}))] \pi_N-\sigma(b \mid H) q(\overline{e}) \pi_H\right\} \\
& >\frac{1-\sigma(b \mid H) q(\overline{e})-\sigma(b \mid L)(1-q(\overline{e}))}{\sigma(g \mid L)}\left[c(\overline{e})-\frac{\Delta c}{\Delta q} q(\overline{e})\right] .
\end{aligned} \tag{A8}
$$

Hence, I obtain the following:
(iv) If (A8) is satisfied, B_2 is inside the range defined by (A5).
(v) If only the first inequality of (A8) is satisfied, then $c(\overline{e}) \geq B_2$ but $\frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)} > B_2$ hold. Hence, B_2 is smaller than any other points inside the range defined by (A5).
(vi) If only the second inequality of (A8) is satisfied, then $B_2 \geq \frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)}$ but $B_2 > c(\overline{e})$ hold. Hence, B_2 is larger than any other points inside the range defined by (A5).

Finally, I investigate the conditions under which B_3 is inside or outside the range defined by (A5). Then:

$$
\begin{aligned}
& \sigma(g \mid L)\left\{\sigma(b \mid H) q(\overline{e}) \pi_H-[\sigma(b \mid H) q(\overline{e})+\sigma(b \mid L)(1-q(\overline{e}))] \pi_N\right\} \\
& \quad+[\sigma(g \mid H)-\sigma(g \mid L)][q(\overline{e})]^2 \frac{\Delta c}{\Delta q} \\
& \quad>[\sigma(g \mid H)-\sigma(g \mid L)] q(\overline{e}) c(\overline{e}) \\
& \quad>\sigma(b \mid H) q(\overline{e}) \pi_H-[\sigma(b \mid H) q(\overline{e})+\sigma(b \mid L)(1-q(\overline{e}))] \pi_N .
\end{aligned} \tag{A9}
$$

Hence, I obtain the following:
(vii) If (A9) is satisfied, B_3 is inside the range defined by (A5).
(viii) If only the second inequality of (A9) is satisfied, then $c(\overline{e}) \geq B_3$ but $\frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)} > B_3$ hold. Hence, B_3 is smaller than any other points inside the range defined by (A5).
(ix) If only the first inequality of (A9) is satisfied, then $B_3 \geq \frac{c(\overline{e})-\frac{\Delta c}{\Delta q}q(\overline{e})}{\sigma(g|L)}$ but $B_3 > c(\overline{e})$ hold. Hence, B_3 is larger than any other points inside the range defined by (A5).‖

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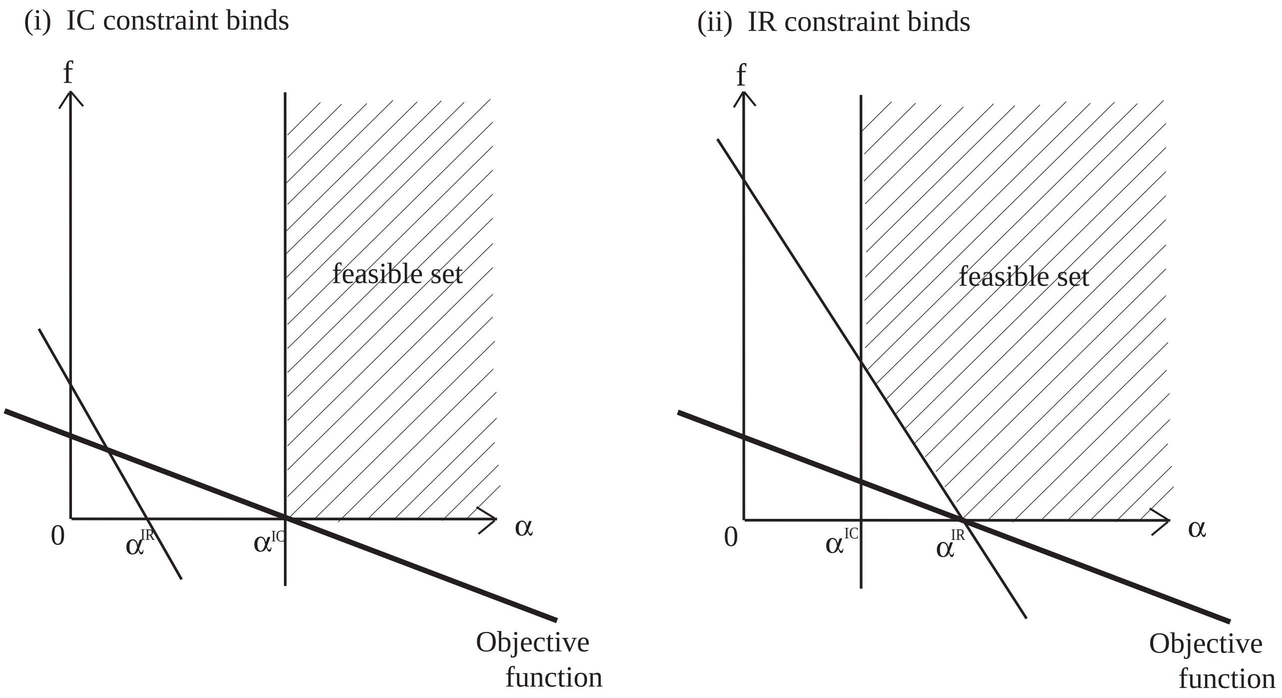
Figure One



Assumption C is the sufficient condition for $\alpha^{IC} > \alpha^{IR}$.
Therefore, under Assumpion C, the model only concerns the situation of Figure One, (i).

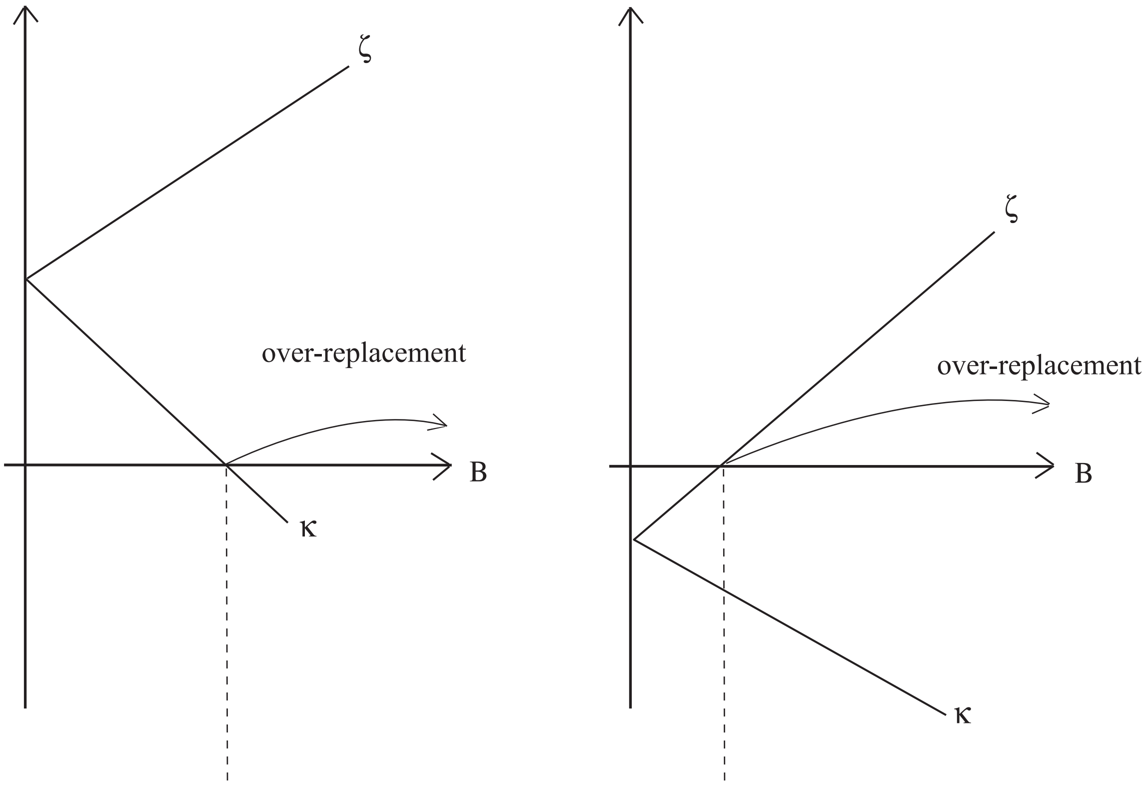
That is, if α^{IC} is $\begin{cases} \alpha^{*} \text{ in (20)}, \\ \alpha^{**} \text{ in (25)}, \\ \bar{\alpha} \text{ in (30)}, \end{cases}$ α^{IR} is $\begin{cases} \dfrac{c(\bar e) - [\sigma(g|H)\, q(\bar e) + \sigma(g|L)\,(1 - q(\bar e))]\, B}{\sigma(g|H)\, q(\bar e)\, \pi_H}, \\ \dfrac{c(\bar e) - [\sigma(g|H)\, q(\bar e) + \sigma(g|L)\,(1 - q(\bar e))]\, B}{\sigma(g|H) q(\bar e) \pi_H + [\sigma(b|H) q(\bar e) + \sigma(b|L)\,(1 - q(\bar e))]\, \pi_N}, \\ \dfrac{c(\bar e) - B}{q(\bar e)\, \pi_H}, \end{cases}$

where the slope of the objective function is

$$\begin{cases} -\beta\sigma(g|H)q(\bar e)\pi_H \\ -\beta\{\sigma(g|H)q(\bar e)\pi_H + [\sigma(b|H)q(\bar e) + \sigma(b|L)\,(1 - q(\bar e))]\,\pi_N\} \\ -\beta q(\bar e)\pi_H \end{cases}$$

respectively.

Figure 2

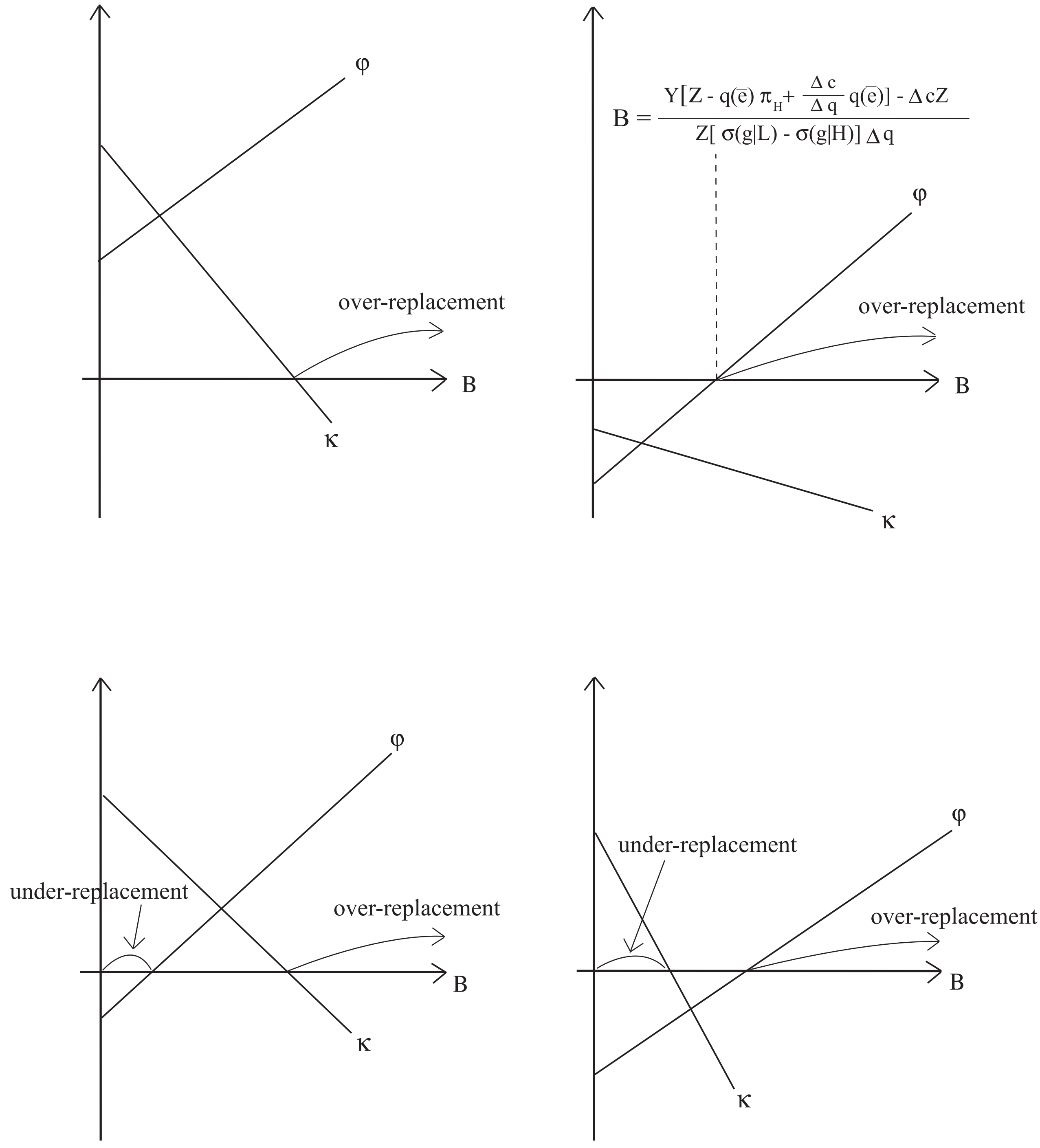


$$B = \frac{\beta\sigma(b|H)\, q(\bar{e})\pi_H - \beta[\,\sigma(b|H)\, q(\bar{e}) + \sigma(b|L)\,(1 - q(\bar{e}))]\,\pi_N}{[\sigma(g|H)\, q(\bar{e}) + \sigma(g|L)\,(1 - q(\bar{e})) - 1]}$$

$$B = \frac{\sigma(b|H)\, q(\bar{e})\pi_H - [\,\sigma(b|H)\, q(\bar{e}) + \sigma(b|L)\,(1 - q(\bar{e}))]\,\pi_N}{[\sigma(g|H) - \sigma(g|L)]q(\bar{e})}$$

note : $\zeta(o) = \kappa(o) = \beta\{[\sigma(b|H)q(\bar{e}) + \sigma(b|L)(1 - q(\bar{e}))]\,\pi_N - \sigma(b|H)\, q(\bar{e})\pi_H\}$

Figure 3



note : $\varphi(o) = \beta[Z - q(\bar{e})\,\pi_H + \frac{\Delta c}{\Delta q} q(\bar{e}) - \frac{\Delta c}{Y} Z\,]$