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Comparative Advantage and Skill Premium of Regions

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Abstract

This paper provides one explanation of why there is observed a positive correlation between skill premium and income of regions. In doing so, this paper provides a model of self-organized sorting and skill premium with a continuum of heterogeneous individuals and that of industries or tasks within a production process. It is found that the positive correlation merges through the interaction between the location-occupation choice by individuals and regional comparative advantage. The spatial equilibrium, sorting, and product differentiation play a key role in determining the way how such interaction works.

Keywords: comparative advantage, skill premium, sorting

JEL classification: F12, J24, R12

1 Introduction

In the United States, there is observed a tendency that the skill premium in a region is positively correlated with the size of the region. If the skill premium is defined as the ratio of the average annual wage rate of high-skilled workers to that of low-skilled workers and if cities and their size are defined as the Metropolitan Statistical Areas (MSAs) and GDP, respectively, the positive correlation

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is depicted as in Figure 1.[1] It plots the relationship for each major occupation group in the *2000 Standard Occupational Classification System* which seems to be categorized into high-skilled workers, where the skill premium here is the ratio of the annual nominal wage rate of each of these occupations to that of production workers.[2]

This paper provides one explanation of why such relationship emerges by providing a comparative advantage model with a continuum of mobile heterogeneous individuals and that of final goods sectors different in their skill intensities of intermediate goods. All the individuals choose their occupations depending on their productivities, and any occupation can freely migrate across regions unlike footloose entrepreneur models such as Forslid and Ottaviano (2003). This location-occupation choice then interacts with regional comparative advantage in final goods sectors which depends on regional offer prices of two different types of intermediate goods, one of which features monopolistic competition à la Dixit and Stiglitz (1977). Although regions are ex-ante identical, interactions between individuals' location-occupation choices and regional comparative advantage results in a self-organized positive correlation between skill premium and income of regions. If necessary, the theory can also accommodate the interpretation that regional difference in skill premium is caused by specialization in task trade within firms not industries.

The basic mechanism is simple and described as follows: Since regions are ex-ante identical in their environment including the endowment, land, some initial shock or history, which reallocates the economy's expenditure across regions unevenly, results in a cross-region variation in land rents. Free-migration of workers is then associated with a compensating differential, i.e., wage rates in regions with higher land rents must be associated with higher wage rates in order for workers to reside in such regions. Due to cross-variation in factor prices, regions with higher prices have no comparative advantage in producing non-differentiated intermediate goods. However, higher land rents, by making the average productivities of high-skilled workers higher through sorting, make such

[1] As reported by Davis and Dingel (2012), the positive correlation also applies to the case where the skill premium and the size of a region are measured by the college premium and population, respectively. Beside skill premium, within-region inequalities in general is also positively correlated with the size of regions (Behrens and Robert-Nicoud, 2008; Glaeser *et al.*, 2009; Baum-Snow and Pavan, 2010).

[2] The data sources for the skill premium and GDP are the *Occupational Employment Statics* by the Bureau of Labor Statistics and the *Regional Economic Accounts* by the U.S. Bureau of Economic Analysis, respectively. The year of the data is 2009, and the nominal GDP is used in order to preserve consistency with the model. These two variables for each MSA are matched using MSA or CBSA (Core Based Statistical Area) code.

regions to have comparative advantage in producing skill-intensive intermediate goods. Reflecting this regional comparative advantage, final goods sectors relocate across regions, and such relocation of industries makes the initial reallocation of expenditures sustainable. Thus, there is observed a positive correlation between skill premium and the size of regions.

This paper is related to at least two lines of researches. The first is trade models with Ricardian comparative advantage. The current model is an application of Matsuyama (2010) to the regional context. His model itself is basically an extension of Dornbusch *et al.* (1977), where comparative advantage of countries is determined endogenously through firms' entry in a monopolistic competition environment, and the number of countries is increased arbitrarily. Although one of his motivation is to build a theory of income distribution across a large number of countries, I focus on two-region case. Unlike the international context, the regional economy is more complicated in that individuals are mobile across regions, and this makes intractable to derive the distribution of regional income explicitly. In addition to individuals' mobility, the current model differs from his in that individuals choose their occupations, either workers or entrepreneurs; that there are two types of intermediate goods sectors, one of which is characterized by monopolistic competition; and that land, which is one of usual elements in the urban economics literature, is introduced.

The second is models of the spatial sorting of individuals. Amongst those, Davis and Dingel (2012) is the most related in the sense that both papers share the same motivation and also the assumption of identical cities or regions and zero trade costs of some goods. Although both researches feature self-organized positive correlation between the skill premium and the size of regions, the key mechanism is quite different. In their paper, knowledge exchange works as a agglomeration force while regional specialization in different industries or tasks in mine. Models in the New Economic Geography (hereafter NEG) literature such as Mori and Turrini (2005) and Behrens *et al.* (2010) are also related. However, this paper differs from those in the sense that the environment here is that there are multiple industries not single, and there is no trade costs of final goods. Instead of focusing on endogenous comparative advantage through agglomeration within a single industry, this paper focus on the comparative advantage which is still endogenous but not related to a single industry. In addition, this paper differs from the former in that all the individuals are mobile and that there is limited supply of land in light of Helpman (1998) and Pflüger and Tabuchi (2010) among others. The latter also differs from this paper in that there is distinction between individuals' talent and productivity, which allows them to reproduce imperfect spatial sorting. In this sense, this paper is close to Nocke (2006) which discusses sorting in a partial equilibrium setting.

Before proceeding, one qualification should be made clear by mentioning what the model cannot explain or what kind of concentration is most suitable to be explained by the model. Specifically, the current model is basically the one which is suitable for the discussion on the concentration of industries, tasks, or income in the sense that any clear-cut mechanism guaranteeing the concentration of population or a particular type of occupation is not embedded. Stated differently, this paper provides an explanation of the positive correlation between skill premium and the size of regions which does not resort to the concentration of the skilled.

The remaining structure of this paper is organized as follows: I first introduce the model in Section 2. Then I discuss the definition of an equilibrium of interest and its properties in Section 3, which is followed by another interpretation of the model and a few discussions about the validity of the modeling strategy described in Section 4. Finally, in Section 5, I conclude this paper.

2 The Model

I consider a closed economy which consists of two ex-ante identical regions: Region 1 and Region 2. These regions are ex-ante identical in the sense that each region ex-ante has the same environment including the amount of endowment, land. Individuals, the mass of which is normalized to unity, are ex-ante heterogeneous in their entrepreneurial productivity, and they choose their occupation, worker or entrepreneur, depending on their productivity as well as their residential choice. As in Dornbusch *et al.* (1977), there is a $[0, 1]$-continuum of final goods sectors. Further, like Matsuyama (2010), each sector is different in its share parameters of two types of intermediate goods: labor- and skill- intensive intermediate goods, of which the former and latter are characterized by the monopolistic competition à la Dixit and Stiglitz (1977) and perfect competition, respectively. Including land services in both production and preference and location-occupation choice by individuals is the main difference from an international model of Matsuyama (2010). Each subsection below discusses the maximization problem of each agent.

2.1 Final Goods Sectors

Competitive final goods sectors exist on $[0,1]$ interval. Each sector $s \in [0,1]$ is characterized by its skill intensity $\gamma(s) \in [0,1]$ and, without loss of generality, ordered in a way that $\gamma(s)$ is monotonically increasing. Specifically, assuming a Cobb-Douglas production technology with constant returns to scale, $\gamma(s)$ is the expenditure share parameter of local differentiated skill-intensive intermediate goods. The remaining input is local homogeneous labor-intensive intermediate goods, the share parameter of which is equal to $1-\gamma(s)$.

As in Dornbusch *et al.* (1977), the location of each sector s is determined by consumers' demand. Thus, the price $P(s)$ of sector-s final good must be equal to the lowest unit cost of production given perfect competition and constant returns to scale technology, i.e., letting $\chi_j(s)$ and $\mathbb{S}_j \subseteq [0,1]$ denote the unit cost of production of sector s active in Region j and the set of sectors active in Region j, respectively, it holds that $P(s) = \chi_j(s)$ if $s \in \mathbb{S}_j$.

Formally, a typical firm in sector s residing in Region j, i.e., $s \in \mathbb{S}_j$, solves

$$\begin{aligned}
&\max_{\{M_{i,j}(s)\}_{i\in\{E,L\}}\{m_{i,j}(\varphi,s)\}_{\varphi}} P(s)M_{E,j}(s)^{\gamma(s)}M_{L,j}(s)^{1-\gamma(s)} - \int_0^\infty p_{E,j}(\varphi)m_{E,j}(\varphi,s)N_{E,j}g_j^*(\varphi)d\varphi - P_{L,j}M_{L,j}(s)\\
&s.t.\\
&M_{E,j}(s) = \left[\int_0^\infty m_{E,j}(\varphi,s)^{\frac{\sigma-1}{\sigma}}N_{E,j}g_j^*(\varphi)d\varphi\right]^{\frac{\sigma}{\sigma-1}}.
\end{aligned}$$

$m_{E,j}(\varphi,s)$ is the sector-s demand for a variety of skill-intensive intermediate goods produced locally by a firm with productivity φ (hereafter variety-φ skill-intensive intermediate good). As specified later, each variety is produced by one entrepreneur who is characterized by her entrepreneurial productivity φ. Here, this specification is explicitly taken into account, and thus the integral is taken over productivity φ. $N_{E,j}$ is the mass of skill-intensive intermediate goods or entrepreneurs in Region j. $g_j^*(\varphi)$ denotes the density function of productivity conditional on location. These differentiated goods are aggregated to $M_{E,j}(s)$ by technology with constant elasticity $\sigma > 1$ of substitution. $M_{E,j}(s)$ is the sector-s demand for homogeneous labor-intensive intermediate goods produced locally. Prices are denoted by $p_{E,j}(\varphi)$ and $P_{L,j}$ for variety-φ skill-intensive intermediate good and homogeneous labor-intensive intermediate goods, respectively.

The profit maximization implies the following demand for variety-φ skill-intensive good:

$$m_{E,j}(\varphi, s) = \left[\frac{p_{E,j}(\varphi)}{P_{E,j}}\right]^{-\sigma} M_{E,j}(s), \tag{1}$$

where $P_{E,j}$ is the price index of Region-j skill-intensive intermediate goods defined by

$$P_{E,j} \equiv N_{E,j}^{-\theta} \left[\int_0^\infty p_{E,j}(\varphi)^{-\frac{1}{\theta}} g_j^*(\varphi) d\varphi\right]^{-\theta}, \tag{2}$$

where $\theta \equiv 1/(\sigma - 1) > 0$. Further, Cobb-Douglas technology implies that $P(s) = \zeta(s) P_{E,j}^{\gamma(s)} P_{L,j}^{1-\gamma(s)}$, where $\zeta(s) \equiv \gamma(s)^{-\gamma(s)}(1-\gamma(s))^{-(1-\gamma(s))}$; and that the expenditure share of skill-intensive (labor-intensive) goods is $\gamma(s)$ $(1-\gamma(s))$.

2.2 Labor-intensive Intermediate Goods Sectors

The local labor-intensive intermediate goods sector in each region is competitive. Firms can access to a Cobb-Douglas production technology with constant returns to scale, inputs of which consist of workers' labor services $L_{L,j}$ and land $T_{L,j}$. These intermediate goods can be interpreted as labor services, provided by production workers in particular places, which the final goods firms receive when outsourcing their production of goods.

Specifically, a typical firm in this sector solves

$$\max_{L_{L,j}, T_{L,j}} P_{L,j} B L_{L,j}^{\beta_L} T_{L,j}^{1-\beta_L} - W_{L,j} L_{L,j} - R_j T_{L,j}, \quad B \equiv \beta_L^{-\beta_L}(1-\beta_L)^{-(1-\beta_L)},$$

where $\beta_L \in (0,1)$ and R_j are the share parameter of labor and the land price in Region j, respectively. The Cobb-Douglas technology implies that $P_{L,j} = W_{L,j}^{\beta_L} R_j^{1-\beta_L}$; and that the share of labor (land) cost is β_L $(1-\beta_L)$.

2.3 Skill-intensive Intermediate Goods Sectors

Contrary to the other sectors in the economy, the local skill-intensive intermediate goods sector is characterized by monopolistic competition à la Dixit and Stiglitz (1977), where each entrepreneur produces one variety of goods using workers' labor services and land as production inputs. Specifically, each entrepreneur must rent f units of land for her office and then use workers' labor services and land as variable inputs. Each variety of skill-intensive intermediate goods can be interpreted as skill-intensive labor services, which are difficult to replicate and thus differentiated, provided by innovative people, i.e., entrepreneurs, together with supports by office workers in particular places which the final goods firms receive when outsourcing their skill-intensive production processes.

Therefore, the income $\pi_j(\varphi)$ of an entrepreneur residing in Region j with productivity φ is given by her sales net of input costs:

$$\begin{aligned}\pi_j(\varphi) \quad &= \quad \max_{p_{E,j}(\varphi),q_{E,j}(\varphi)} \left[p_{E,j}(\varphi) - W_j^{\beta_E} R_j^{1-\beta_E} \varphi^{-1}\right] q_{E,j}(\varphi) - R_j f \\ & \quad\quad s.t. \\ & \quad\quad q_{E,j}(\varphi) = \int_{\mathbb{S}_j} m_{E,j}(\varphi, s) ds = \int_{\mathbb{S}_j} \left[\frac{p_{E,j}(\varphi)}{P_{E,j}}\right]^{-\sigma} M_{E,j}(s) ds,\end{aligned}$$

where $q_{E,j}(\varphi)$ and m are the output of variety-φ skill-intensive intermediate good produced in Region j and the shift parameter of the unit cost of production, respectively. Here, it is assumed that the unit cost of production is some amount of the Cobb-Douglas composite of workers' labor services and land, in which β_E governs the labor cost share. The constraint, i.e., the demand for variety-φ good comes from aggregating the demands (1) from all the final goods sectors locating in Region j, i.e., $\mathbb{S}_j$.

The associated optimal pricing rule is then $p_{E,j}(\varphi) = (1 + \theta) W_j^{\beta_E} R_j^{1-\beta_E} \varphi^{-1}$. Substituting this into (2) results in

$$P_{E,j} = (1 + \theta) W_j^{\beta_E} R_j^{1-\beta_E} \left(\tilde{\varphi}_j N_{E,j}^{\theta}\right)^{-1}, \tag{3}$$

where $\tilde{\varphi}_j$ is the average productivity in Region j defined by

$$\tilde{\varphi}_j = \left[\int_0^\infty \varphi^{\frac{1}{\theta}} g_j^*(\varphi) d\varphi \right]^\theta . \tag{4}$$

In the following, I assume that the entrepreneurial productivity φ follows a Pareto distribution with coefficient δ and a lower bound $\underline{\varphi}$. Under the assumption that $\delta > 1/\theta$, the individual variable profit $\pi_j^V(\varphi)$ and output $q_j(\varphi)$ are expressed as functions of the productivity ratio $\varphi/\tilde{\varphi}_j$ and the average variables as in Melitz (2003):

$$\begin{aligned} \pi_j^V(\varphi) &= \left(\frac{\varphi}{\tilde{\varphi}_j}\right)^{\frac{1}{\theta}} \pi_j^V(\tilde{\varphi}_j), \\ \pi_j^V(\tilde{\varphi}_j) &= \theta W_j^{\beta_E} R_j^{1-\beta_E} \tilde{\varphi}_j^{-1} N_{E,j}^{-(1+\theta)} \int_{\mathbb{S}_j} M_{E,j}(s) ds, \\ q_j(\varphi) &= \left(\frac{\varphi}{\tilde{\varphi}_j}\right)^{\sigma} q_j(\tilde{\varphi}_j), \\ q_j(\tilde{\varphi}_j) &= N_{E,j}^{-(1+\theta)} \int_{\mathbb{S}_j} M_{E,j}(s) ds. \end{aligned} \tag{5}$$

2.4 Individuals

Individuals are ex-ante heterogeneous in their entrepreneurial productivity φ. Depending on this productivity, each individual chooses her occupation and location freely in order to maximize her utility. Let $U_j(\varphi)$ and $e_j(\varphi)$ denote the utility and income of an individual having the productivity of φ and residing in Region j, respectively.

2.4.1 Occupational Choice

Suppose that an individual chose to reside in Region j. Then she chooses her occupation which maximizes her income. If she chooses to become an entrepreneur, she earns the entrepreneur's profit $\pi_j(\varphi)$ which is a function of both location j and productivity φ as discussed in the previous section. Instead if she chooses to work as a worker, she earns W_j which is independent of productivity φ. Thus, without adjustment costs, her income $e_j(\varphi)$ is given as the maximal of these two income sources, i.e., $e_j(\varphi) = \max\{\pi_j(\varphi), W_j\}$.

Figure 2 clarify this step. It shows the wage rate W_j and entrepreneur's profit $\pi_j(\varphi)$ as a function of the entrepreneurial productivity φ. The line for W_j is flat since it is independent of φ while that for $\pi_j(\varphi)$ is increases monotonically. Here, it is assumed that there is a cut-off level φ_j^* of productivity given by

$$W_j = \sigma^{-1}\tilde{A}_j\varphi_j^{*\frac{1}{\theta}} - R_j f. \tag{6}$$

That is, individuals with productivity φ_j^* are indifferent between two occupations given that they reside in Region j. Individuals having productivity φ greater than the cut-off level φ_j^* chooses to become entrepreneurs for the fixed location j. Contrastingly, those with φ lower than φ_j^* become workers.

For the given income $e_j(\varphi)$ as well as the given location j, she then consumes final goods and housing services:

$$\begin{aligned} U_j(\varphi) \quad = \quad & \max_{\{c_j(s,\varphi)\}_{s\in[0,1]},h_j(\varphi)} \exp\left[\alpha\int_0^1 \ln(c_j(s,\varphi))ds\right] h_j(\varphi)^{1-\alpha}, \quad \alpha\in(0,1), \\ & s.t. \\ & \int_0^1 P(s)c_j(s,\varphi)ds + R_j h_j(\varphi) = e_j(\varphi), \end{aligned}$$

where the preference is specified by two-tier Cobb-Douglass utility. α is the expenditure share of the consumption goods. $c_j(s,\varphi)$ and $h_j(\varphi)$ denote the quantities of goods and housing services consumed by an individual with φ residing in Region j. Equal weights in the lower-tier aggregation of the final goods imply that the expenditure share of each final good s is equal. Together with the upper-tier aggregation, it holds that $P(s)c_j(s,\varphi) = \alpha e_j(\varphi)$ for all s. It also holds that $R_j h_j(\varphi) = (1-\alpha)e_j(\varphi)$.

2.4.2 Residential Choice

Having specified utilities, I can now discuss individuals' residential choice. Each individual chooses her location in order to maximize her utility. Assuming that there is no adjustment costs, an individual with φ computes the utility ratio $u(\varphi) \equiv U_2(\varphi)/U_1(\varphi)$ and decides to live in Region 2

(Region 1) if $u(\varphi) \geq 1$ ($u(\varphi) < 1$), where using the above result, $u(\varphi)$ is given by

$$u(\varphi) = \left[\frac{e_2(\varphi)}{e_1(\varphi)}\right] \left(\frac{R_2}{R_1}\right)^{-(1-\alpha)}.$$

The following result then states that if both regions host positive measure of production activities, or stated more weakly, if there exists a threshold $\bar{\varphi}$ such that entrepreneurs with productivity of $\bar{\varphi}$ are indifferent between two location choices, the sorting of entrepreneurs is always associated:

Proposition 1. *Suppose that there exists $\bar{\varphi}$ such that $\bar{\varphi} > \max_j\{\varphi_j^*\}$; and that $u(\varphi) = [\pi_2(\bar{\varphi})/\pi_1(\bar{\varphi})]/(R_2/R_1)^{1-\alpha} =$ 1. Then, if $R_1 < R_2$, it holds that*

$$\left(\frac{R_2}{R_1}\right)^{1-\alpha} < \frac{\tilde{A}_2}{\tilde{A}_1} < \frac{R_2}{R_1}; \textit{and}$$

that $\pi_2(\varphi)/\pi_1(\varphi)$ is monotonically increasing in a well-defined region. Or, if $R_1 = R_2$, it must hold that $\tilde{A}_1 = \tilde{A}_2$ and thus $u(\varphi) = 1$ for all $\varphi \geq \underline{\varphi}$.

In sum, there are two types of thresholds. The one is a cut-off level productivity φ_j^* which determines the occupation of each individual for a given location j. The other is a cut-off level productivity $\bar{\varphi}$ which determines the location of individuals with sufficiently high φ.

3 Equilibrium Analysis

In the following, I focus on the case where regions are ex-post heterogeneous. The symmetric case is not interesting since it does not provide any information about the relationship between the size of regions and skill premium. In addition, the symmetric equilibrium, though always exists, is unstable in the sense that some shock to the economy breaks the symmetric configuration.

Since regions are ex-ante identical, an asymmetric equilibrium, if existed, is always associated with an equilibrium which is the mirror image of the equilibrium. Thus, without loss of generality, I focus on equilibria in which the land rent in Region 2 is greater than that in Region 1, i.e., $R_1 < R_2$. Therefore, given Proposition 1, an interior equilibrium is associated with a unique threshold $\bar{\varphi}$ such that $\bar{\varphi} > \max_j\{\varphi_j^*\}$ and there is a spatial sorting of entrepreneurs. More specifically, such an equilibrium is characterized by the ranking $\varphi_1^* < \varphi^I$. In this case, individuals with φ higher than or equal to φ^I reside in Region 2 and work as entrepreneurs. Those with φ less than φ^I but higher than or equal to φ_1^* reside in Region 1 and work still as entrepreneurs. Workers consist of individuals with φ less than φ_1^*. Since workers' income is independent of φ, the following free-migration condition or compensated differential for workers must be satisfied:

$$\frac{W_1}{P^\alpha R_1^{1-\alpha}} = \frac{W_2}{P^\alpha R_2^{1-\alpha}}, \quad \text{or} \quad \frac{W_2}{W_1} = \left(\frac{R_2}{R_1}\right)^{1-\alpha}, \tag{7}$$

which states that utility levels are equalized across regions. Otherwise, workers concentrate on one of the two regions, and production of goods in the other region becomes impossible.

3.1 Definition and Properties of an Equilibrium

The following definition of an equilibrium takes account of the above discussion:

Definition. *An interior sorting equilibrium is wage rates $\{W_j\}_{j=1}^2$, land rents $\{R_j\}_{j=1}^2$, worker-entrepreneur cut-off level φ_1^* of productivity, entrepreneurs' location threshold $\bar{\varphi}$ of productivity, and a spatial distribution $\{\mathbb{S}_j\}_{j=1}^2$ of final goods sectors such that (i) individuals optimally choose their location and occupation as well as quantities; (ii) firms including entrepreneurs maximize their profits; (iii) markets clear; and (iv) $\{\mathbb{S}_j\}_{j=1}^2$ is consistent with comparative advantage, i.e., $\chi_j(s) = \min\{\chi_1(s), \chi_2(s)\}$ for all $s \in \mathbb{S}_j$ and all $j \in 1, 2$.*

Before proceeding to the computation of the equilibrium, the following result is very convenient in the sense that the spatial distribution of final goods sectors characterized by *sets* $\{\mathbb{S}_j\}_{j=1}^2$ is actually summarized by a single variable S_1, and all s's greater than or equal to S_1 reside in Region 2 while the rest in Region 1:

Proposition 2. *Suppose an asymmetric interior equilibrium exists. Then, the spatial distribution of final goods sectors is summarized by a threshold $S_1 \in (0, 1)$ such that $\mathbb{S}_1 = [0, S_1)$ and $\mathbb{S}_2 = [S_1, 1]$.*

The implication of this result for the computation of an equilibrium is that the system of an equilibrium can be now interpreted as a fixed-point problem of S_1. The discussion, which is described in C, proceeds in two steps. (i) Given the spatial distribution S_1, the system of an equilibrium is consolidated into two simultaneous equations with two unknowns: the ratio of φ^I to φ_1^* and the ratio of φ_1^* to $\underline{\varphi}$. All the other variables except for S_1 are given as functions of these two and S_1. (ii) The rest of the computation is to search for S_1 that is consistent with the comparative advantage of regions. Stated differently, S_1 must be a solution to the following nonlinear equation

$$\frac{\chi_2(S_1)}{\chi_1(S_1)} = \left(\frac{P_{L,2}}{P_{L,1}}\right)^{1-\gamma(S_1)} \left(\frac{P_{E,2}}{P_{E,1}}\right)^{\gamma(S_1)} = 1, \tag{8}$$

and the Region 2-1 ratio $\chi_2(s)/\chi_1(s)$ of offer prices must be decreasing in s. If the latter condition does not hold, all s's greater than or equal to S_1 reside in Region 1 not Region 2, clearly contradicting the assumption.

The intuitive mechanism which can work in the model is summarized as follows: Suppose that some shock hits the economy consisting of two ex-ante identical regions in a way that expenditures concentrate on one of the regions (here Region 2), i.e., $|\mathbb{S}_1| < |\mathbb{S}_2|$. Since both regions have the same amount of land, it then holds that the land rent in Region 2 becomes higher than in Region 1, i.e., $R_1 < R_2$.[3] Due to the free-migration of workers or the compensating differential, i.e., (7), the wage rate in Region 2 also becomes higher than that in Region 1. Thus, unit costs or prices of non-differentiated goods are higher in Region 2 than in Region 1, i.e., $P_{L,1} < P_{L,2}$.[4] Instead, thanks to the sorting of entrepreneurs, i.e., (3), Region 2 becomes to have comparative advantage in producing skill-intensive intermediate goods, i.e., $P_{E,1} > P_{E,2}$. Reflecting these comparative advantage of regions, the spatial distribution of final goods sectors settles down in such a way that the reallocation of expenditures caused by the initial shock is actually preserved as an equilibrium outcome.

3.2 Numerical Exercise

In order to verify the mechanism in the previous subsection, I resort to a numerical exercise since an equilibrium cannot be computed analytically. The result shows that an equilibrium with such

[3] Strictly speaking, this relationship between the two rankings holds only if expenditures are the most important determinant of land rents as suggested by the land market clearing condition (18) derived in C.

[4] Note that the price of homogeneous labor-intensive intermediate goods is a weighted geometric mean of the wage rate and land rent.

mechanism actually exists. It is also verified that the equilibrium is unique in the sense that there is only one interior sorting equilibrium with the assumed regional rankings of variables.

In this exercise, parameters are set as follows: The elasticity of substitution σ between skill-intensive intermediate goods is set to 3 which is a frequently used value in the literature. The expenditure share α of final goods is set to 0.7, which is based on the calculation using the *Consumer Expenditure Survey* by the U.S. Bureau of Labor Statistics. The lower bound $\underline{\varphi}$ of the Pareto distribution of entrepreneurial productivities is set to 1 since the absolute value itself does not matter. The coefficient δ of the Pareto distribution is set to 4.2. The labor share parameter of the labor-intensive intermediate goods sector is set to 0.6 which is close to that of the manufacturing sector reported by Valentinyi and Herrendorf (2008). I use the same number for the labor share in variable costs of the skill-intensive intermediate goods sector. The fixed requirement f of land is set to 1. This value is chosen in a way that the demand of entrepreneurs for land does not affect land prices so much, and these prices are mainly determined by housing expenditures and housing demands associated with variable inputs. As for the specification of $\gamma(s)$, I simply assume that $\gamma(s) = s$ for $s \in [0, 1]$.

The equilibrium is summarized by Figure 3 and 4. The lower panel of Figure 3 depicts the relationship between the entrepreneurial productivity φ and the wage and entrepreneurs' profit schedules for each region. As already mentioned, the wage schedule is flat since workers' income is independent of their entrepreneurial productivity φ. Meanwhile, entrepreneurs' income $\pi_j(\varphi)$ is monotonically increasing in φ. That $W_1 < W_2$ is implied by the free-migration condition for workers together with the ranking $R_1 < R_2$. As for $\pi_j(\varphi)$, it is not always the case that $\pi_1(\varphi) < \pi_2(\varphi)$ for all φ. What is important here is that $u(\varphi)$ is monotonically increasing in φ (Proposition 1); and that $u(\varphi) = 1$ at φ^I, which are shown in the upper panel of Figure 3.

Figure 4 shows that the Region 2-1 ratio $\chi_2(s)/\chi_1(s)$ of offer prices is monotonically decreasing in s, and there is actually exists a threshold S_1 which summarizes the spatial distribution of final goods sectors. Since $|\mathbb{S}_j|$ is proportional to the regional GDP, the result that $|\mathbb{S}_1| < |\mathbb{S}_2|$ implies that the size of Region 2 is greater than that of Region 1 in terms of income. Importantly, the monotonicity of $\chi_2(s)/\chi_1(s)$ is the consequence of two results: $P_{L,2}/P_{L,1} > 1$; and $P_{E,2}/P_{E,1} < 1$.

The former result is simply due to the fact that $R_1 < R_2$ and $W_1 < W_2$ as discussed before. The latter result suggests that there actually exists a case where the cost-reducing effect of product differentiation and sorting on the aggregate price level dominates the cost push due to higher land rents and thus higher wage rates.[5]

4 Discussion

In order to make the derivation of the intuition of the theory simple, some simplifying assumptions are made. Because of these, the model produces several important deviations from facts presented in the literatures. In this section, I mention three of those and discuss possible extensions for further research.

4.1 Perfect vs. Imperfect Sorting

When focusing on *active* entrepreneurs, the model features perfect sorting in terms of their productivity φ. However, as mentioned by Behrens *et al.* (2010) among others, the contrasting property is observed in the data. In general, the data shows that there is no cut-off of the density function, and instead, larger regions have a distribution skewed to the right.

However, this perfect sorting does not constitute the main building blocks of the current theory. As suggested by previous discussions, what is important for Region 2 to have comparative advantage in producing skill-intensive intermediate goods is not the perfect sorting of entrepreneurs but a higher average productivity augmented by the number of varieties of goods, i.e., $\tilde{\varphi}_j^{-1} N_{E,j}^{-\theta}$.

If taking account of not only active entrepreneurs but also *inactive* entrepreneurs, i.e., workers, there is no perfect sorting. Unlike the typical assumption on the mobility of unskilled workers used in NEG, the current model allows all the individuals to move across regions freely. Thus, the spatial distribution of the unskilled, corresponding to workers in the model, is endogenously determined, and some fraction of them optimally choose to reside in a region with higher living costs thanks to

[5] It should be noted that the numerical exercise also shows that in equilibrium Region 1 has a greater number of entrepreneurs than Region 2 has, i.e., $N_{E,1} > N_{E,2}$. This suggests that for the theory considered in this paper, what is important for Region 2 to have comparative advantage in producing skill-intensive goods is not the number of entrepreneurs but also their productivity. However, this result clearly contradicts the fact that larger regions are likely to host a larger number of skilled workers. Possible extensions are discussed in Section 4.

the compensating differential guaranteed by the free migration.

4.2 Industries/Sectors vs. Production processes

As in Matsuyama (2010), each s is interpreted as a final good sector or industry. The most important implication of this interpretation is that the difference between regions in skill composition is mainly due to the regional difference in sectors or industries in which regions specialize. Stated differently, the cross-region variation in skill is determined by the cross-region variation in sectors or industries. This clearly contradicts the fact, reported by Hendricks (2011), that the variation in skill within sectors accounts for 80% of cross-city variation in skill.

However, there is another way to interpret each s such that the cross-region variation in skill is due to some difference within sectors. Specifically, if s is interpreted as a task or a production process within an industry or a sector and if it is assumed that there is a single competitive final goods sector producing a homogeneous good with the technology specified by the Cobb-Douglas function as in individuals' utility, production of each one unit of the final good requires firms to process a continuum of tasks $s \in [0, 1]$, each different in its skill intensity $\gamma(s)$.[6] Under this interpretation, regions sort themselves into those hosting different tasks each other. Region 2 hosts tasks requiring skill while Region 1 those requiring less skill. Location choice of tasks is now determined as an organizational choice of a firm rather than independent decisions of firms in different sectors.

4.3 Extreme assumptions on trade costs

Finally, I mention extreme assumptions on trade costs of both final and intermediate goods. The model features zero trade costs for the final goods and infinite trade costs for both types of intermediate goods. The first assumption has a role to remove the so-called "forward linkage" in the NEG literature in order to clarify the basic mechanism working in the model. Without trade costs of final goods, individuals do not need to consider the effect of their residential choice on the accessibility to goods, and thus the cumulative mechanisms between migration and the local market size of the final goods disappears. As suggested by the numerical exercise, this mechanism is not necessary in

[6] The specification of a continuum of tasks is used in the trade literature, e.g., Grossman and Rossi-Hansberg (2008).

order for Region 2 to become a larger region than Region 1.

As for the second assumption, even if an intermediate level of trade costs is considered, only expressions of $\tilde{A}_j$ and $P_{E,j}$ change, and the intuition of the theory, if an equilibrium existed, does not change. One of important implications of such intermediate level of trade costs, e.g., trade costs of skill-intensive intermediate goods, is that the spatial distribution of skill can be discussed in a more meaningful way than in the current model. As already mentioned in the numerical exercise, it can happen that a larger region in terms of GDP hosts a smaller number of entrepreneurs. However, if the trade costs of skill-intensive intermediate goods are reduced from infinity to some intermediate level, concentration of entrepreneurs might emerge, filling the gap between the model and the fact.[7]

5 Conclusion

There is observed a positive correlation between skill premium and the size of regions, of which the former and latter are measured by the income ratio of high-skilled and low-skilled workers and the regional income, respectively. The paper theoretically investigates one of possible explanations of this fact by providing a model with heterogeneous individuals and final and intermediate goods sectors, in which ex-ante identical regions specialize in different sectors, and interactions between individuals' location-occupation choices and regional comparative advantage self-organize the positive correlation between skill premium and income of regions. If necessary, the theory can also accommodate the interpretation that regional difference in skill premium is caused by specialization in task trade not industries. Although perfect sorting featuring the equilibrium itself is not a crucial element of the theory, filling the gap between the model and reality could be one of important future directions.

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[7] Of course, this is not the only way to make the model to close to the reality. For example, technological externalities among entrepreneurs, e.g., knowledge exchange through face-to-face communications, can work as a agglomeration force as argued by Davis and Dingel (2012).

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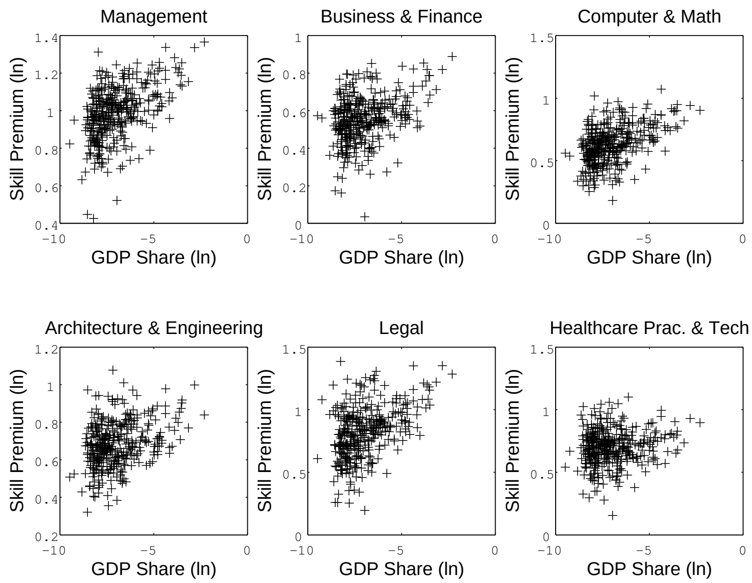

Figure 1: The Size and Skill Premium of Metropolitan Statistical Areas

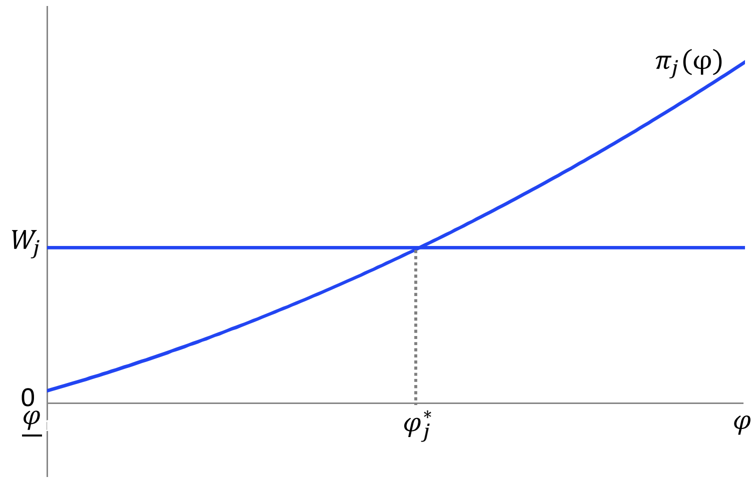

Figure 2: Occupational Choice by Individuals Residing in Region j

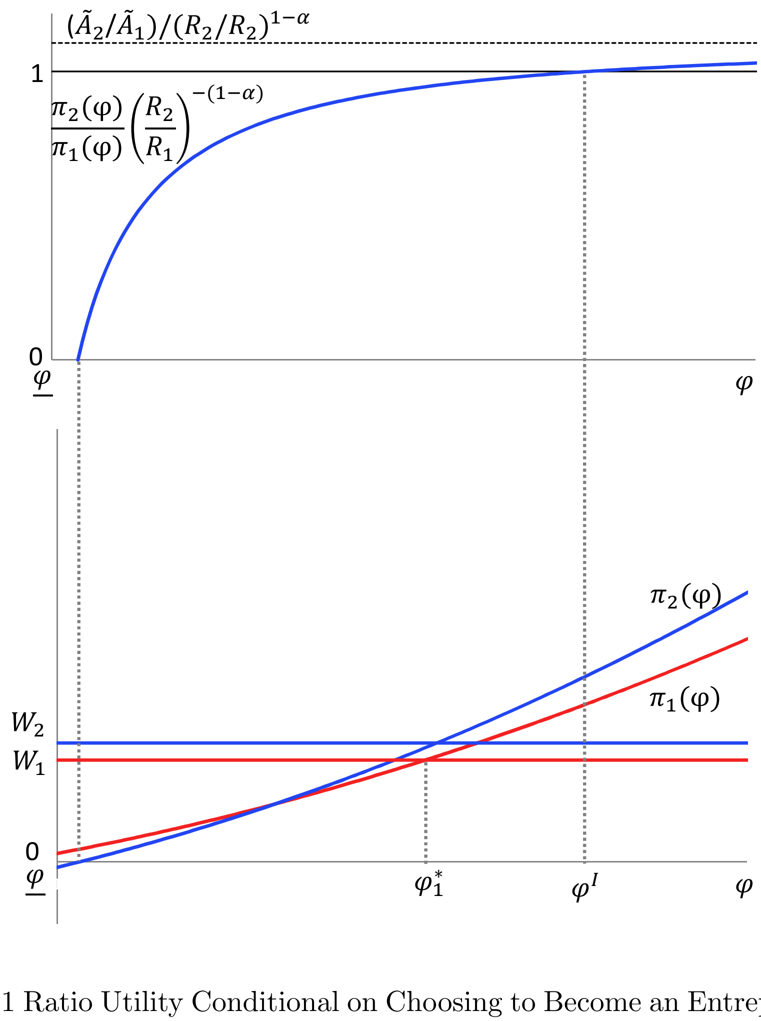

Figure 3: Region 2-1 Ratio Utility Conditional on Choosing to Become an Entrepreneur (The Upper Panel) and Entrepreneurs' and Workers' Incomes (The Lower Panel)

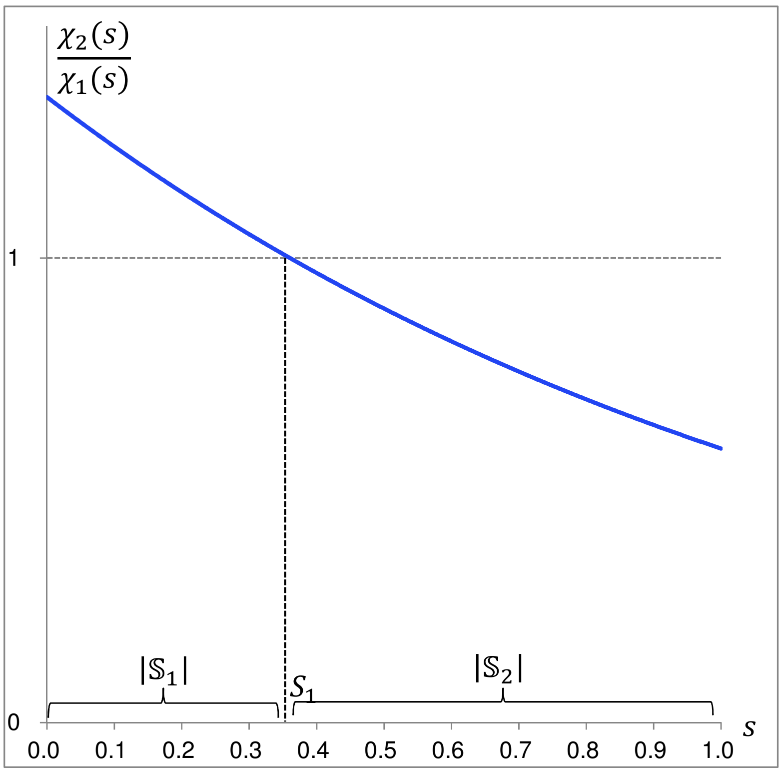

Figure 4: Region 2-1 Ratio of Offer Prices of Final Goods

A Proof of Proposition 1

Proof. The proof here focuses on the case where $R_1 < R_2$ since the result for the symmetric case is immediate. For notational convenience, let $\tilde{a}$ and r denote $\tilde{A}_2/\tilde{A}_1$ and R_2/R_1, respectively. Thus, by assumption, $r > 1$. With this notation, the Region 2-1 ratio of entrepreneurs at $\varphi = \bar{\varphi}$ is expressed by

$$\frac{\pi_2(\bar{\varphi})}{\pi_1(\bar{\varphi})} = \frac{\sigma^{-1}\tilde{a}\tilde{A}_1\bar{\varphi}^{\frac{1}{\theta}} - R_1 r f}{\sigma^{-1}\tilde{A}_1\bar{\varphi}^{\frac{1}{\theta}} - R_1 f}.$$

Now assume by contradiction that $\tilde{a} \geq r$. Then, it holds that $\pi_2(\bar{\varphi})/\pi_1(\bar{\varphi}) \geq r > r^{1-\alpha}$, where the last inequality holds by the assumption that $r > 1$. This contradicts the definition of $\bar{\varphi}$. Therefore it must hold that $\tilde{a} < r$. That $r^{1-\alpha} < \tilde{a}$ is also shown in a similar manner.

The first result is then used in order to show that $\pi_2(\varphi)/\pi_1(\varphi)$ is monotonically increasing in φ. Specifically, it is used in order to determine the sign of the derivative of the ratio given by

$$\frac{\partial[\pi_2(\varphi)/\pi_2(\varphi)]}{\partial\varphi} = \frac{(\sigma\theta)^{-1}\varphi^{\frac{1}{\theta}-1}}{\left(\sigma^{-1}\tilde{A}_1\varphi^{\frac{1}{\theta}} - R_1 f\right)^2} f R_1 \tilde{A}_1 (r - \tilde{a}),$$

which is positive given that $r > \tilde{a}$.

□

B Proof of Proposition 2

Proof. For an interior equilibrium on which I focus, the system of an equilibrium suggests that prices of composite intermediate goods, i.e., $P_{E,j}$ and $P_{L,j}$, are positive and bounded. Therefore, it holds that $P_{i,1}/P_{i,2}$ is strictly greater than, equal to, or strictly less than unity for each $i \in \{E, L\}$. In the following, I examine nine possible cases in order. For convenience, the Region 1-2 ratio of the unit cost of production of sector-s final good is reproduced:

$$\frac{\chi_1(s)}{\chi_2(s)} = \left(\frac{P_{E,1}}{P_{E,2}}\right)^{\gamma(s)} \left(\frac{P_{L,1}}{P_{L,2}}\right)^{1-\gamma(s)}, \quad \forall s \in [0,1].$$

Case 1: $P_{E,1}/P_{E,2} > 1$ and $P_{L,1}/P_{L,2} > 1$

In this case, it holds that $\chi_1(s)/\chi_2(s) > 1$ for all $s \in [0, 1]$, which implies that $\mathbb{S}_1 = \emptyset$ and $\mathbb{S}_2 = [0, 1]$. Thus, this case cannot be observed when $|\mathbb{S}_j| > 0$ for all j.

Case 2: $P_{E,1}/P_{E,2} > 1$ and $P_{L,1}/P_{L,2} = 1$

Again, it holds that $\chi_1(s)/\chi_2(s) > 1$ for all $s \in [0, 1]$, and this case cannot be also observed when $|\mathbb{S}_j| > 0$ for all j.

Case 3: $P_{E,1}/P_{E,2} > 1$ and $P_{L,1}/P_{L,2} < 1$

In this case, $\chi_1(s)/\chi_2(s)$ is increasing in s. Thus, the case is consistent with the hypothesis that $|\mathbb{S}_j| > 0$ for all j only if there exists a threshold $\hat{S} \in (0, 1)$. The associated distribution of final goods sectors is a partition, where $\mathbb{S}_1 = [0, \hat{S}]$ and $\mathbb{S}_2 = (\hat{S}, 1]$. Note that the location of sector $\hat{S}$ does not matter since it is of measure zero.

Case 4: $P_{E,1}/P_{E,2} = 1$ and $P_{L,1}/P_{L,2} > 1$

Since $\chi_1(s)/\chi_2(s) > 1$ for all $s \in [0, 1]$, this cannot be consistent with the hypothesis that $|\mathbb{S}_j| > 0$ for all j.

Case 5: $P_{E,1}/P_{E,2} = 1$ and $P_{L,1}/P_{L,2} = 1$

In this case, $\chi_1(s) = \chi_2(s)$ for all $s \in [0, 1]$. The implied distribution of final goods sectors is $\mathbb{S}_j = [0, 1]$ for all j.

Case 6: $P_{E,1}/P_{E,2} = 1$ and $P_{L,1}/P_{L,2} < 1$

Since $\chi_1(s)/\chi_2(s) < 1$ for all $s \in [0, 1]$, the implied distribution of final goods sectors is $\mathbb{S}_1 = [0, 1]$ and $\mathbb{S}_2 = \emptyset$. Thus, this cannot be observed when $|\mathbb{S}_j| > 0$ for all j.

Case 7: $P_{E,1}/P_{E,2} < 1$ and $P_{L,1}/P_{L,2} > 1$

This is a case where $\chi_1(s)/\chi_2(s)$ is decreasing in s. Thus, this case is consistent with the hypothesis that $|\mathbb{S}_j| > 0$ for all j only if there exists a threshold $\hat{S} \in (0, 1)$. The associated distribution of final goods sectors is a partition, where the order of skill intensities of regions is contrasting compared with Case 3: $\mathbb{S}_1 = (\hat{S}, 1]$ and $\mathbb{S}_2 = [0, \hat{S}]$.

Case 8: $P_{E,1}/P_{E,2} < 1$ and $P_{L,1}/P_{L,2} = 1$

Since $\chi_1(s)/\chi_2(s) < 1$ for all $s \in [0, 1]$, this cannot be consistent with the hypothesis that $|\mathbb{S}_j| > 0$ for all j.

Case 9: $P_{E,1}/P_{E,2} < 1$ and $P_{L,1}/P_{L,2} < 1$

This case also implies that $\chi_1(s)/\chi_2(s) < 1$ for all $s \in [0, 1]$ and thus cannot be consistent with the assumption that $|\mathbb{S}_j| > 0$ for all j.

In summary, there are only three cases (Case 3, 5, 7) which can be consistent with the hypothesis that $|\mathbb{S}_j| > 0$ for all j. In addition, if regions are ordered in a way that higher index j implies higher skill intensities, Case 3 and Case 7 are equivalent.

□

C Equilibrium System as a Fixed-Point Problem of S_1

In this section, I show that the equilibrium system of an equilibrium of interest is summarized as a fixed-point problem of S_1.

C.1 Number of Entrepreneurs, Conditional Densities, and Average Productivities

First of all, given the Pareto distribution $g(\varphi)$ of entrepreneurial productivity φ and the ranking of thresholds, i.e., $\varphi_1^* < \bar{\varphi}$, the number $\{N_{E,j}\}_{j=1}^2$ of entrepreneurs in each region, densities $\{g_j^*\}$ of productivity conditional on sorting, and average productivities $\{\varphi_j^*\}_{j=1}^2$ of regions are given as functions of thresholds $(\varphi_1^*, \bar{\varphi})$:

$$
\begin{aligned}
N_{E,1} &= G(\bar{\varphi}) - G(\varphi_1^*) = \left(\varphi_1^{*-\delta} - \bar{\varphi}^{-\delta}\right) \underline{\varphi}^{\delta}, && (9)\\
N_{E,2} &= 1 - G(\bar{\varphi}) = \left(\frac{\underline{\varphi}}{\bar{\varphi}}\right)^{\delta}, && (10)\\
g_1^*(\varphi) &= \frac{1}{N_{E,1}} \mathbf{1}\{\varphi_1^* \leq \varphi < \bar{\varphi}\} g(\varphi) = \left(\varphi_1^{*-\delta} - \bar{\varphi}^{-\delta}\right)^{-1} \mathbf{1}\{\varphi_1^* \leq \varphi < \bar{\varphi}\} \delta \varphi^{-(\delta+1)}, && (11)\\
g_2^*(\varphi) &= \frac{1}{N_{E,2}} \mathbf{1}\{\bar{\varphi} \leq \varphi\} g(\varphi) = \bar{\varphi}^{\delta} \mathbf{1}\{\bar{\varphi} \leq \varphi\} \delta \varphi^{-(\delta+1)}, && (12)\\
\tilde{\varphi}_1 &= \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta} \left[\frac{(\varphi_1^*/\bar{\varphi})^{\frac{1}{\theta}-\delta} - 1}{(\varphi_1^*/\bar{\varphi})^{-\delta} - 1}\right]^{\theta} \bar{\varphi}, \quad \text{or} \quad \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta} \left[\frac{1 - (\bar{\varphi}/\varphi_1^*)^{\frac{1}{\theta}-\delta}}{1 - (\bar{\varphi}/\varphi_1^*)^{-\delta}}\right]^{\theta} \varphi_1^*, && (13)\\
\tilde{\varphi}_2 &= \left(\frac{\delta}{\delta - 1/\theta}\right)^{\theta} \bar{\varphi}, && (14)
\end{aligned}
$$

where $\mathbf{1}\{\cdot\}$ is the indicator function which is equal to one if the statement in the braces is true and zero otherwise.

C.2 Factor Prices as Functions of Three Thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$

Next, $\tilde{A}_j$ is computed as a function of thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$ with the help of market clearing conditions: Cobb-Douglas preference suggests that the economy-wide expenditure for final goods is given by αE, where E denotes the economy-wide income excluding land rents. If $\mathbb{S}_j$ set of industries locate in Region j, equal weights of industries in preference and the production technology of the final goods sectors then imply that two market clearing conditions, one for the final goods and the other for the skill-intensive intermediate goods, are consolidated into

$$
\int_{\mathbb{S}_j} P_{E,j} M_{E,j}(s) ds = \alpha \Gamma_j |\mathbb{S}_j| E, \quad \text{or} \quad \int_{\mathbb{S}_j} M_{E,j}(s) ds = P_{E,j}^{-1} \alpha \Gamma_j |\mathbb{S}_j| E,
$$

where $\Gamma_j \equiv |\mathbb{S}_j|^{-1} \int_{\mathbb{S}_j} \gamma(s) ds$, implying that $\alpha \Gamma_j |\mathbb{S}_j| E = \int_{\mathbb{S}_j} \alpha E \gamma(s) ds$, the sum of expenditure for the skill-intensive intermediate goods in Region j, which in turn is related to consumers' demand. Substituting (3) into this equation, I get

$$
\int_{\mathbb{S}_j} M_{E,j}(s) ds = (1+\theta)^{-1} \left(W_j^{\beta_H} R_j^{1-\beta_H}\right)^{-1} \tilde{\varphi}_j N_{E,j}^{\theta} \alpha \Gamma_j |\mathbb{S}_j| E.
$$

Finally, substituting this equation into (5) results in

$$\pi_j^V(\tilde{\varphi}) = \sigma^{-1}\frac{\alpha\Gamma_j|\mathbb{S}_j|E}{N_{E,j}},$$

which gives

$$\tilde{A}_j = \frac{\alpha\Gamma_j|\mathbb{S}_j|E}{\tilde{\varphi}_j^{\frac{1}{\theta}} N_{E,j}}. \tag{15}$$

That is, $\tilde{A}_j$ is the normalized average market size of skill-intensive intermediate goods in Region j. Note that, given (9)-(14) and Proposition 2, $\tilde{A}_j$ is a function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$.

This derivation of $\tilde{A}_j$ is useful for the computation of factor prices $\{(W_j, R_j)\}_{j=1}^2$ in relating labor and land market clearing conditions with thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$, which I turn next.

Since the sales of an entrepreneur with productivity φ is $\tilde{A}_j\varphi^{\frac{1}{\theta}}$ and since the variable profit $\pi_j^V(\varphi) = \sigma^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$, the variable cost is equal to $(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$. Thus the Cobb-Douglas technology implies that the associated variable labor and land costs are given by $\beta_h(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$ and $(1-\beta_H)(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}}$, respectively. Factor market clearing conditions, which aggregate these firm-level costs, then pin down factor prices and the spatial distribution of workers.

The labor market clearing condition for each region is given as follows:

$$N_{E,j}\int_{\underline{\varphi}}^{\infty} \beta_H(1+\theta)^{-1}\tilde{A}_j\varphi^{\frac{1}{\theta}} g_j^*(\varphi)d\varphi + \beta_L\alpha(1-\Gamma_j)|\mathbb{S}_j|E = W_j\lambda_j N_W,$$

where $\lambda_j \in (0,1)$ denotes the share of Region j in workers, and N_W the total number of workers, i.e., $N_W = G(\varphi_1^*) = 1 - (\underline{\varphi}/\varphi_1^*)^{\delta}$. Together with (4) and (15), the first term becomes $\beta_H(1+\theta)^{-1}\alpha\Gamma_j|\mathbb{S}_j|E$, i.e., the clearing condition simplifies to

$$\beta_H(1+\theta)^{-1}\alpha\Gamma_j|\mathbb{S}_j|E + \beta_L\alpha(1-\Gamma_j)|\mathbb{S}_j|E = W_j\lambda_j N_W \quad \text{for all } j = 1, 2. \tag{16}$$

The second term on the left hand side is the demand from the labor-intensive sector, where the total sales $\alpha(1-\Gamma_j)|\mathbb{S}_j|E$ is derived in a similar way as in the case of the skill-intensive intermediate goods sector, and the Cobb-Douglas technology then implies β_L fraction of these must be distributed to workers.

Thus, noting that N_W is a function of φ_1^*; and that both Γ_j and $|\mathbb{S}_j|$ are functions of S_1, the labor market clearing condition together with the free-migration condition for workers, i.e., (7), gives the wage rate and the spatial distribution of workers as functions of two thresholds (φ_1^*, S_1) and the land rents ratio R_2/R_1:

$$W_1 = \frac{\tilde{\beta}_1 A_1}{(1-\lambda_2)N_W}, \quad W_2 = W_1 \left(\frac{R_2}{R_1}\right)^{1-\alpha}, \quad \lambda_2 = \frac{\frac{\tilde{\beta}_2 A_2}{\tilde{\beta}_1 A_1}}{\frac{\tilde{\beta}_2 A_2}{\tilde{\beta}_1 A_1} + \left(\frac{R_2}{R_1}\right)^{1-\alpha}}, \quad \lambda_1 = 1 - \lambda_2,$$

where

$$A_j \equiv \alpha|\mathbb{S}_j|E, \quad \tilde{\beta}_j \equiv \Gamma_j \frac{\beta_H}{1+\theta} + (1-\Gamma_j)\beta_L.$$

As for the land market clearing condition, an argument similar to that in the case of the labor market gives land prices as function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$: The demands for land consists of not only those from firms in both skill-intensive and labor-intensive but also those from individuals, i.e., $(1-\alpha)E_j$, where E_j is Region-j income excluding land rents given by

$$E_j = N_{E,j}\int_{\underline{\varphi}}^{\infty} \pi_j(\varphi) g_j^*(\varphi) d\varphi + W_j \lambda_j N_W = \frac{\theta}{1+\theta}\alpha\Gamma_j|\mathbb{S}_j|E - R_j f N_{E,j} + W_j\lambda_j N_W. \tag{17}$$

Noting that the demands from the skill-intensive sector is further divided into those related with variables costs and those related to fixed costs, the market clearing condition is specified by

$$\begin{aligned} R_j &= N_{E,j}\int_{\underline{\varphi}}^{\infty} (1-\beta_H)(1+\theta)^{-1}\tilde{A}_j \varphi^{\frac{1}{\theta}} g_j^*(\varphi) d\varphi + R_j f N_{E,j} + (1-\beta_L)(1-\Gamma_j)\alpha|\mathbb{S}_j|E + (1-\alpha)E_j, \\ &= (1-\beta_H)(1+\theta)^{-1}\alpha\Gamma_j|\mathbb{S}_j|E + R_j f N_{E,j} + (1-\beta_L)(1-\Gamma_j)\alpha|\mathbb{S}_j|E + (1-\alpha)E_j, \end{aligned}$$

where the second equation follows from the definitions of $\tilde{A}_j$ and $\tilde{\varphi}_j$, i.e., (15) and (4). Together with the labor market clearing condition (16) and the equation (17) for the local income E_j, this equation is solved for R_j in order to interpret R_j as a function of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$:

$$R_j = \frac{1}{1-\alpha f N_{E,j}} \eta_j A_j, \quad \text{where} \quad \eta_j \equiv \Gamma_j \frac{1-\alpha\beta_H + (1-\alpha)\theta}{1+\theta} + (1-\Gamma_j)(1-\alpha\beta_L). \tag{18}$$

Given this result, wage rates $\{W_j\}_{j=1}^2$ and the spatial distribution $\{\lambda_j\}_{j=1}^2$ of workers are now functions of three thresholds $(\varphi_1^*, \bar{\varphi}, S_1)$.

C.3 Productivity Thresholds $(\varphi_1^*, \bar{\varphi})$ as Functions of S_1

Now I show that two productivity thresholds $(\varphi_1^*, \bar{\varphi})$ are functions of S_1. For this purpose, two conditions are used: One is for φ_1^* and the other for $\bar{\varphi}$. The first condition states that an individual with productivity φ_1^* is indifferent between becoming a worker and working as an entrepreneur in Region 1, i.e., (6) with $j = 1$. Together with (13)and (15), this reduces to

$$\frac{W_1 + R_1 f}{\sigma^{-1}\alpha\Gamma_1 |\mathbb{S}_1| E/N_{E,1}} \frac{\delta}{\delta - 1/\theta} \frac{1-(\bar{\varphi}/\varphi_1^*)^{-(\delta-\frac{1}{\theta})}}{1-(\bar{\varphi}/\varphi_1^*)^{-\delta}} \varphi_1^* = 1.$$

Further, substituting the labor and land market clearing conditions, (16) and (18), into this equation results in

$$\frac{\delta}{\delta - 1/\theta} \frac{\sigma}{\Gamma_1} \frac{1-(\bar{\varphi}/\varphi_1^*)^{-\left(\delta-\frac{1}{\theta}\right)}}{1-(\bar{\varphi}/\varphi_1^*)^{-\delta}} \left[\frac{\tilde{\beta}_1 N_{E,1}}{(1-\lambda_2)N_W} + \eta_1 \frac{f N_{E,1}}{1-\alpha f N_{E,1}} \right] = 1.$$

Finally, using (9), the first condition is written as follows

$$\begin{aligned} &\frac{\delta}{\delta - 1/\theta} \frac{\sigma}{\Gamma_1} \frac{1-(\varphi_1^*/\bar{\varphi})^{\delta-\frac{1}{\theta}}}{1-(\varphi_1^*/\bar{\varphi})^{\delta}} \\ &\times \left\{ \frac{\tilde{\beta}_1}{1-\lambda_2} \frac{1}{1-\left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta}} + \eta_1 \frac{f}{1-\alpha f \left[1-\left(\frac{\varphi_1^*}{\bar{\varphi}}\right)^{\delta}\right]\left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta}} \right\} \left[1-\left(\frac{\varphi_1^*}{\bar{\varphi}}\right)^{\delta}\right] \left(\frac{\underline{\varphi}}{\varphi_1^*}\right)^{\delta} = 1. \end{aligned} \tag{19}$$

If I define x and y by $x \equiv \varphi_1^*/\bar{\varphi} \in (0,1)$ and $y \equiv \underline{\varphi}/\varphi_1^* \in (0,1)$, respectively, this equation adds an restriction to the relationship between x and y for a given S_1. Note that λ_2 is a function of $(\varphi_1^*, \bar{\varphi}, S_1)$; and that $(\varphi_1^*, \bar{\varphi})$ corresponds to (x, y) equivalently for any given lower bound $\underline{\varphi}$ of productivity.

The second condition is $u(\bar{\varphi}) = 1$, where $\bar{\varphi}$ is assumed to be greater than $\max_j\{\varphi_j^*\}$, or

$$\frac{\sigma^{-1}\tilde{A}_2\bar{\varphi}^{\frac{1}{\theta}} - R_2 f}{\sigma^{-1}\tilde{A}_1\bar{\varphi}^{\frac{1}{\theta}} - R_1 f} = \left(\frac{R_2}{R_1}\right)^{1-\alpha}.$$

After some calculations which use (15), (18), (9), and (10), this equation is restated as follows:

$$\frac{\frac{\delta-1/\theta}{\delta}\frac{\Gamma_2}{\eta_2\sigma}\frac{(xy)^{-\delta}-\alpha f}{f} - 1}{\frac{\delta-1/\theta}{\delta}\frac{1}{x^{1/\theta}-x^{\delta}}\frac{\Gamma_1}{\eta_1\sigma}\frac{1-\alpha f(1-x^{\delta})y^{\delta}}{fy^{\delta}} - 1} = \left[\frac{\frac{1-\alpha f(xy)^{\delta}}{1-\alpha f(1-x^{\delta})y^{\delta}}}{\frac{\eta_2}{\eta_1}\frac{1-S_1}{S_1}}\right]^{\alpha}, \tag{20}$$

which adds another restriction to the relationship between x and y for a given S_1.

Therefore, for a given S_1, there are two unknowns, x and y, and two equations, (19) and (20). This system of equations, if solved, implies that x and y are obtained as functions of S_1. Of course, there might exist multiple solutions for the system, and thus it is more appropriate to state that the system gives x and y as correspondence of S_1. However, in the numerical computation considered in the paper, the system actually gives a unique solution.

C.4 Determination of S_1 through Comparative Advantage

In the above discussion, S_1 is fixed at some point. Stated differently, I considered an interior sorting equilibrium where the spatial distribution of final goods industries is fixed in a particular manner. Thus, finally I discuss how to pin down the value of S_1.

The condition which determines the value of S_1 is the comparative advantage condition, i.e., (8), which states that prices of final goods sector S_1, if posted by two regions, are equalized. Focusing on the case considered in the numerical analysis, i.e., $\beta_H = \beta_L$, this condition is written as follows:

$$\frac{\chi_2(S_1)}{\chi_1(S_1)} = \left(\frac{R_2}{R_1}\right)^{1-\alpha\beta}\left[\frac{N_{E,2}^{\theta}\tilde{\varphi}_2}{N_{E,1}^{\theta}\tilde{\varphi}_1}\right]^{-\gamma(S_1)} = 1.$$

Since all the ratios in braces are functions of S_1 as discussed above, this is a single equation determining the value of S_1. Thus, the computation of an equilibrium can be interpreted as a fixed-point problem with respect to S_1, which nests a system of nonlinear equations for (x, y). Once the value of S_1 which satisfies the above equation is found, values of the other variables are computed. Without loss of generality, the economy-wide income E excluding land rents is normalized to unity.