CIRJE-F-811

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August 2011

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This Version

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Abstract

In this paper, we study market liberalization in an imperfectly competitive environment in the presence of price effects. For this purpose, we build a three-country model of international trade under monopolistic competition with endogenous prices and wages. The neighboring effect translates how the size effect propagates across countries. When some country increases in size, its nominal wage increases, as well as that in a small and near country, while that in a large and distant country falls. We also show that a preferential trade agreement increases the relative wage, the welfare, and the terms-of-trade in the partner countries, where the integration effect dominates, while it lowers those in the third country.

JEL Classification: F12, F15, R13

Key-words: monopolistic competition, market size effect, preferential trade agreement

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1 Introduction

In imperfect competition models of international trade, the existence of a costlessly traded homogeneous good sector has often been assumed, especially when dealing with multi-country models. This leads to Factor Price Equalization (FPE) across countries, which significantly simplifies the analysis.[1] In that context, markets integrate via the relocation of firms and workers across countries, see Krugman (1980), Baldwin *et al.* (2003), Behrens *et al.* (2007), Venables (1987), or Ossa (2011). By focusing on the consequences of production shifting and of the relocation of industry, that line of research abstracts completely from any price effect present during the liberalization process. In particular, it assumes away terms-of-trade considerations and their impact on welfare. Moreover, in the real world, FPE does not hold, even between developed countries.

In this paper, we address the consequences of market liberalization in a framework dealing with size, neighboring, price, and integration effects. For this purpose, following Venables (1987) and Ossa (2011), among others, we build on Krugman's (1980) new trade theory to construct a three-country model of international trade under monopolistic competition. In contrast to the existing literature, we relax the assumption of FPE by removing the costlessly traded good sector, so that prices and wages are endogenous, and price effects are included into the analysis. As our framework deals with an arbitrary trade cost structure between countries, our results go beyond the analysis of specific examples such as the symmetric or the hub-and-spoke configurations studied by Puga and Venables (1997). Moreover, unlike in Ossa (2011), no trade restriction is placed between countries. Hence, our model deals with general trade patterns and any spatial distribution of resources in terms of location and size.

The aim of this paper is twofold. First, we look at the role of country size on nominal wages and welfare. Second, we analyze the impact of Preferential Trade Agreements (PTAs) on wages and welfare in the partner countries and the left-out country.

The first set of results relate to size and neighboring effects. When some country increases in size, its nominal wage increases, as well as that in a small and near country, while that in a large and distant country falls. This result extends the size effect emphasized by Krugman (1980) in the case of two countries by introducing a neighboring effect, which translates how the size effect propagates across countries. The market potential increases more in a neighboring

[1] FPE is a direct consequence of costless trade in the constant returns sector. Davis (1998) shows how costly trade in both the constant and the increasing returns sectors substantially alters the equilibrium outcome.

country than in a distant one. In terms of welfare, all countries gain from the increase in size of some country because world production and consumption end up increasing.

The second set of results relate to the consequences of PTAs. When some countries engage in a PTA, the integration effect induces relative prices including wages to increase in the integrating area. By raising the export price in the partner countries, the effect of a PTA is to improve the terms-of-trade in the integrating area, while lowering that in the excluded country. While a PTA is beneficial to the partner countries in utility terms, it is always detrimental to the third country because the latter one does not benefit from the integration effect and is exposed to the negative price effect: it has to import goods produced at higher prices in the integrating area.

The terms-of-trade gain provides a strong incentive for countries to engage in bilateral trade agreements. This result is similar to that obtained in other new trade models in the presence of a freely traded homogeneous good, see the trade policy implications derived by Puga and Venables (1997), as well as the model by Ossa (2011) where the third country trades with one partner country only. However, in general, when trade is not restricted so that each country trades with any other one, a PTA may hurt some partner country in terms of welfare under FPE. This has been shown to happen when the hub effect is large enough, see the example provided by Behrens *et al.* (2007). In that case, although the two countries engaging in the PTA have a better market access and attract firms overall, firms in the smaller partner country move to the larger one (Baldwin and Robert-Nicoud, 2000), which reduces the welfare in the smaller country concluding the PTA. Here, in contrast to the models relying on FPE, for the partner countries, the integration effect always dominates the price effect irrespective of country size and of the spatial distribution of resources across countries, so that welfare always increases in the integrating area.

Under FPE, falling trade costs between countries concluding the PTA is accompanied by the relocation of firms to the PTA partners, while it does not lead to terms-of-trade movements. The relocation of firms from the third country to the PTA partners implies a worse access of the third country to the manufacturing varieties. In contrast, in this paper, we provide another rationale for the lower utility in the third country. Falling trade costs between PTA partners raises prices and wages in the integrating area relative to the price level in the third country. As a result, consumers in the left-out country suffer a terms-of-trade loss because they have to import varieties produced at higher prices in the partner countries, which lowers their welfare.

The remainder of the paper is organized as follows. The three-country model of international trade under imperfect competition is introduced in Section 2. Section 3 presents some preliminary results of the model. In Section 4, we analyze the role of country size and the impact of a PTA on wages and welfare. Section 5 concludes.

2 The model

The economy consists of three countries and a manufacturing sector producing a differentiated good. The mass of immobile workers in country i is denoted by L_i.

The utility of an individual in country i is given by Dixit-Stiglitz preferences

$$U_i = \left(\sum_{j=1}^{3} \int_{\omega \in \Omega_j} q_{ji}(\omega)^{\frac{\sigma-1}{\sigma}} \mathrm{d}\omega \right)^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q_{ji}(\omega)$ is the amount of variety ω produced in country j and consumed in country i, Ω_j is the set of varieties produced in country j, and $\sigma > 1$ is the elasticity of substitution between any two varieties. The budget constraint of a worker in country i earning a wage w_i is

$$\sum_j \int_{\omega \in \Omega_j} p_{ji}(\omega) q_{ji}(\omega) \mathrm{d}\omega = w_i \tag{2}$$

where $p_{ji}(\omega)$ is the delivery price of variety ω produced in country j and consumed in country i.

In order to simplify the notation, we drop the variety label ω from now on. The maximization of utility (1) subject to the budget constraint (2) yields the following worker's demand in country i for a variety produced in country j:

$$q_{ji} = \frac{p_{ji}^{-\sigma}}{P_i^{1-\sigma}} w_i \tag{3}$$

with the price index P_i in country i given by

$$P_i = \left(\sum_k n_k p_{ki}^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \tag{4}$$

where n_k is the number of firms located in country k.

Assuming iceberg trade costs, $\tau_{ij} > 1$ units of variety have to be shipped from country i for one unit to reach country $j(\neq i)$. We also assume that these trade costs are symmetric $\tau_{ij} = \tau_{ji}$ and $\tau_{ii} = 1$.

The production technology requires a fixed and a constant marginal labor requirements, labeled F and c respectively.[2] In order to satisfy the demand $q_{ij}L_j$ in country j, each firm in

[2] Because immobile labor is the only production factor, the equilibrium number of firms in each country turns out to be constant. As a result, there is no production relocation effect à la Krugman (1980).

country i has to produce $\tau_{ij} q_{ij} L_j$ units. Thus, the profit of a firm in country i is given by

$$\pi_i = \left(\sum_j p_{ij} q_{ij} L_j \right) - w_i \left(F + c \sum_j \tau_{ij} q_{ij} L_j \right) \tag{5}$$

By plugging the worker´s demand (3) into expression (5), profit maximization with respect to prices yields

$$p_{ij} = \frac{\sigma c \tau_{ij}}{\sigma - 1} w_i \tag{6}$$

By assuming free entry and exit of manufacturing firms, profit (5) is zero. Given that $p_{ij} = p_{ii}\tau_{ij}$, we have

$$(p_{ii} - c w_i) \sum_j \tau_{ij} q_{ij} L_j = w_i F \tag{7}$$

Because labor inputs are given by the second bracketed terms in expression (5), the labor market clearing condition is

$$n_i \left(F + c \sum_j \tau_{ij} q_{ij} L_j \right) = L_i \tag{8}$$

Using relations (6), (7), and (8), the equilibrium number of firms is proportional to the number of workers as follows:

$$n_i = \frac{L_i}{\sigma F} \tag{9}$$

By substituting relations (6) and (9) into the profit expression (5), we have

$$\sum_j \frac{\tau_{ij}^{1-\sigma} L_j w_j}{\sum_k \tau_{kj}^{1-\sigma} L_k w_k^{1-\sigma}} = w_i^{\sigma}, \qquad \text{for } i = 1, 2, 3 \tag{10}$$

Wages w_i are determined by these three equilibrium conditions. By Walras law, one of these conditions is redundant, so that labor in some country can serve as numéraire.

The equilibrium utility in country i is given by

$$U_i^* = \frac{w_i}{P_i} = \frac{w_i}{\frac{\sigma c}{\sigma - 1} \left(\frac{1}{\sigma F} \sum_k \tau_{ki}^{1-\sigma} L_k w_k^{1-\sigma} \right)^{\frac{1}{1-\sigma}}} \tag{11}$$

where wages are evaluated at equilibrium (10).

3 Preliminary results

First of all, as stated in previous Section already, we assume that shipping a manufacturing variety from one country to another is costly, so that we exclude the case of costless trade $\tau_{ij} = 1$, $i \neq j$.

Assumption 1 *For any distinct* $i, j, k \in \{1, 2, 3\}$, $\tau_{ij} > 1$.

Assumption 1 implies costly international trade and excludes perfect integration between countries. This assumption is in no way restrictive given that otherwise, the number of countries would reduce to two or less. While τ_{ij} represents the trade cost between countries i and j, the product $\tau_{ik}\tau_{jk}$ corresponds to the trade cost between these countries via country k. In the following Lemma, we show that direct trade between countries is less costly than trade via a third country for at least two pairs of countries.

Lemma 1 *For any distinct* $i, j, k \in \{1, 2, 3\}$, *at least two of the following three triangle inequalities* $\tau_{ij} < \tau_{ik}\tau_{jk}$ *hold.*

Proof. Suppose, on the contrary, that two of the three triangle inequalities are violated, so that $\tau_{ik} \geq \tau_{ij}\tau_{jk}$ and $\tau_{ij} \geq \tau_{ik}\tau_{jk}$. Then, using them leads to

$$\tau_{ik} \geq \tau_{ij}\tau_{jk} \geq \tau_{ik}\tau_{jk}^2$$

This implies

$$1 \geq \tau_{jk}^2$$

which contradicts the assumption $\tau_{jk} > 1$. ■

>From Lemma 1, two cases may arise:

$$\begin{array}{lrl} (i) & \tau_{ik} < & \tau_{ij}\tau_{jk},\ \tau_{ij} < \tau_{ik}\tau_{jk},\ \text{and } \tau_{jk} < \tau_{ij}\tau_{ik} \\ (ii) & \tau_{ik} \geq & \tau_{ij}\tau_{jk},\ \tau_{ij} < \tau_{ik}\tau_{jk},\ \text{and } \tau_{jk} < \tau_{ij}\tau_{ik} \end{array}$$

In case (i), direct trade is less costly than trade via a third country for any pair of countries, and the triangle inequality always holds. In case (ii), direct trade is more costly than trade via a third country for one pair of countries (i, k), and the triangle inequality is violated for that pair of countries. In this latter case, we will assume that firms transport goods from country i to country k via country j rather than directly so that $\tau_{ik} = \tau_{ij}\tau_{jk}$. For example, if tariffs between countries i and k are very high, then firms will avoid direct trade, and ship goods via the third country j in order to reduce trade costs. Note that case (ii) is more likely to occur in international trade than in interregional trade because within a country trade costs increase in the geographical distance. In general, we assume $\tau_{ik} \equiv \min\{\tau_{ik}, \tau_{ij}\tau_{jk}\}$. This condition can be rewritten in terms of the freeness of trade $\phi_{ij} \equiv \tau_{ij}^{1-\sigma} \in (0, 1]$ between countries i and j in the following way.

Assumption 2 *For any distinct* $i, j, k \in \{1, 2, 3\}$, $\phi_{ik} \equiv \max\{\phi_{ik}, \phi_{ij}\phi_{jk}\}$.

$>$From Assumptions 1 and 2, we set

$$\phi_{23} \equiv \max\{\phi_{23}, \phi_{12}\phi_{13}\},\ \phi_{12} > \phi_{13}\phi_{23},\ \text{and}\ \phi_{13} > \phi_{12}\phi_{23} \tag{12}$$

from now on without loss of generality.

As indicated in previous Section already, w_1 can be normalized to one by Walras law. So as to look at how wages are determined in equilibrium, we rewrite the wage equations (10) in the following way:

$$e_i \equiv L_{i+1} w_1^{\sigma-1} f_1 - L_1 w_{i+1}^{\sigma-1} f_{i+1} = 0, \qquad i = 1, 2 \tag{13}$$

where

$$f_i \equiv (1 - \phi_{jk}^2) w_i^\sigma + (\phi_{ik}\phi_{jk} - \phi_{ij}) w_j^\sigma + (\phi_{ij}\phi_{jk} - \phi_{ik}) w_k^\sigma,\ \text{ for distinct } i, j, k \in \{1, 2, 3\}$$

We now show that admissible wages belong to the triangle (Δ) defined by the lines ($f_i = 0$, $i = 1, 2, 3$) in the plane (w_2^σ, w_3^σ). As line ($f_i = 0$) corresponds to $L_i = 0$, $(f_j = 0) \cap (f_k = 0)$ corresponds to $L_i = 1$ for distinct i, j, k. Because of relation (12), the slope of line ($f_1 = 0$) is negative, that of line ($f_2 = 0$) is positive, and that of line ($f_3 = 0$) is nonnegative.

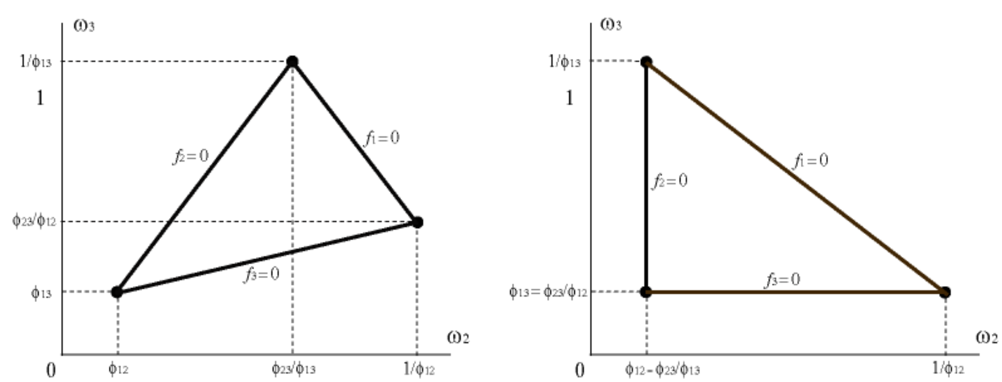

Figure 1a: The case of $\phi_{23} > \phi_{12}\phi_{13}$ Figure 1b: The case of $\phi_{23} = \phi_{12}\phi_{13}$

Based on the foregoing, we start showing a set of Lemmas.

Lemma 2 $\Phi \equiv 1 - \phi_{12}^2 - \phi_{13}^2 - \phi_{23}^2 + 2\phi_{12}\phi_{13}\phi_{23} > 0$.

Proof. Function Φ can be rewritten as

$$(\phi_{12} - \phi_{13}\phi_{23})(1 - \phi_{12}) + (\phi_{13} - \phi_{12}\phi_{23})(1 - \phi_{13}) + (\phi_{23} - \phi_{12}\phi_{13})(1 - \phi_{23})$$
$$+(1 - \phi_{12})(1 - \phi_{13})(1 - \phi_{23})$$

By using the trade cost Assumption 2, $\Phi > 0$. ∎

Because of the above Lemma, the slope of $(f_2 = 0)$ is steeper than that of $(f_3 = 0)$. This is because $(\phi_{13}^2 - 1)/(\phi_{12}\phi_{13} - \phi_{23}) > (\phi_{12}\phi_{13} - \phi_{23})/(\phi_{12}^2 - 1) \iff \Phi > 0$ if $\phi_{23} > \phi_{12}\phi_{13}$. Otherwise, $(f_2 = 0)$ is vertical and $(f_3 = 0)$ is horizontal. Triangle (Δ) can then be represented as illustrated in Figures 1a and 1b, respectively. Both scenarios are consistent with the trade cost Assumption 2. Because $f_i > 0$ inside triangle (Δ), $\forall i$, admissible wages (w_2^σ, w_3^σ) belong to (Δ).

In what follows, because $L_i > 0$, $\forall i$, our analysis focuses on wages belonging to the interior of triangle (Δ). Otherwise the number of countries would reduce to two or less. The following Lemmas involve the study of curves Γ_1 and Γ_2 in the plane (w_2^σ, w_3^σ) defined respectively by relations (13), $e_1 = 0$ and $e_2 = 0$.

Lemma 3 *Inside triangle* (Δ),

(i) along curve Γ_1, $\mathrm{d}w_3/\mathrm{d}w_2 \gtreqless 0$ *if* $w_2^\sigma \lesseqgtr \phi_{23}/\phi_{13}$, *and*

(ii) along curve Γ_2, $\mathrm{d}w_3/\mathrm{d}w_2 \gtreqless 0$ *if* $w_3^\sigma \lesseqgtr \phi_{23}/\phi_{12}$.

The proof is contained in Appendix 1.

Using Lemma 3, we are ready to prove the existence and uniqueness of equilibrium as follows.

Lemma 4 *There exists a unique wage equilibrium* (w_2^*, w_3^*).

Proof. (i) Consider the curve Γ_1 defined by $e_1 = 0$. The point $(\phi_{23}/\phi_{13}, \phi_{13}^{-1})$ belongs to Γ_1. >From Lemma 3, the slope of Γ_1 is $\mathrm{d}w_3/\mathrm{d}w_2 \gtreqless 0$ if $w_2^\sigma \lesseqgtr \phi_{23}/\phi_{13}$ meaning that along Γ_1, w_3^σ is a monotone function of w_2^σ which increases until reaching $(\phi_{23}/\phi_{13}, \phi_{13}^{-1})$ when $w_2^\sigma < \phi_{23}/\phi_{13}$ (resp. decreases from $(\phi_{23}/\phi_{13}, \phi_{13}^{-1})$ when $w_2^\sigma > \phi_{23}/\phi_{13}$).

By evaluating e_1 as given by (13) along the sides $(f_1 = 0)$ and $(f_2 = 0)$ of triangle (Δ), we get that e_1 is respectively negative and positive. This implies that the curve Γ_1 separates the sides ($f_1 = 0$) and ($f_2 = 0$) of triangle (Δ) except in ($\phi_{23}/\phi_{13}, \phi_{13}^{-1}$), and therefore intersects side ($f_3 = 0$).

(ii) Consider now the curve Γ_2 defined by $e_2 = 0$. A symmetric argument to that developed in (i) can be applied so as to show that Γ_2 is a monotone curve going through ($\phi_{12}^{-1}, \phi_{23}\phi_{12}^{-1}$) in the plane ($w_2^g, w_3^g$) which intersects the side ($f_2 = 0$) of triangle (Δ).

(iii) Because the curves Γ_1 and Γ_2 obtained in (i) and (ii) are monotone and join a side of triangle (Δ) to its opposite corner, they intersect once in a point interior to (Δ). This proves the existence and uniqueness of equilibrium. ■

Hence, wages (w_2, w_3) are uniquely determined by solving Equations (10). In what follows, E denotes the Jacobian matrix of (e_1, e_2)

$$E = \begin{pmatrix} \frac{\partial e_1}{\partial w_2} & \frac{\partial e_1}{\partial w_3} \\ \frac{\partial e_2}{\partial w_2} & \frac{\partial e_2}{\partial w_3} \end{pmatrix}$$

The determinant of E can be signed as follows.

Lemma 5 $\det(E) > 0$.

Proof. As of the proof of Lemma 4, the curves Γ_1 and Γ_2 cannot be parallel at their intersection point. This means that the slopes $-\frac{\partial e_1}{\partial w_2}/\frac{\partial e_1}{\partial w_3}$ and $-\frac{\partial e_2}{\partial w_2}/\frac{\partial e_2}{\partial w_3}$ are not equal, and the gradients ($\frac{\partial e_1}{\partial w_2}, \frac{\partial e_1}{\partial w_3}$) and ($\frac{\partial e_2}{\partial w_2}, \frac{\partial e_2}{\partial w_3}$) are not colinear. Hence, $\det(E) \neq 0$. By a continuity argument, $\det(E)$ is signed inside triangle (Δ). If it were not, it would necessarily equate zero in some point of (Δ), which cannot happen. ■

The above Lemmas will be used to determine the general equilibrium impacts of changes in exogenous parameters on endogenous variables.

4 Comparative statics

First, we examine the effect of an exogenous increase in country size on nominal wages and utilities. We will then study the consequences of a PTA by considering an exogenous increase in the freeness of trade between the countries concluding the PTA.

4.1 Size effect

Proposition 1 *For any distinct* $i, j, k \in \{1, 2, 3\}$,

(i) $\mathrm{d}(w_j/w_i)/\mathrm{d}L_j > 0$, *and*

(ii) $\mathrm{d}\left(w_k/w_i\right)/\mathrm{d}L_j \gtreqless 0$ *if* $w_k \lesseqgtr \left(\phi_{jk}/\phi_{ij}\right)^{1/\sigma}$.

The proof is contained in Appendix 2.

Proposition 1(i) implies that the larger a country, the higher its nominal wage. This result corresponds to the size effect emphasized by Krugman (1980, p. 954) in the case of two countries. When the size of the local market increases, local firms face lower transportation costs. In equilibrium, that competitive advantage is offset by higher local wages. The implication of Proposition 1(ii) may be as follows. We expect from Proposition 1(i) that $w_k < \left(\phi_{jk}/\phi_{ij}\right)^{1/\sigma}$ if country k is sufficiently small and the freeness of trade between countries j and k sufficiently high. This implies that *an exogenous increase in the size of some country tends to benefit a small and near country in terms of nominal wage, but harms a large and distant country with which trade is infrequent.* This *neighboring effect* translates how the size effect propagates across countries: an increase in the size of some country raises the market potential more in a neighboring country than in a distant country. Because a smaller country is more sensitive to outside changes, the impact of the neighboring effect is stronger for a small country than for a large one.

We have shown that relative wages in neighboring countries may rise or fall when some country increases in size. So as to examine the impact on welfare, we rely on the following monotonic transformation of the indirect utility function:

$$\widehat{U}_i^* \equiv \sigma F \left(\frac{\sigma c}{\sigma - 1} U_i^*\right)^{\sigma-1} = \sum_k \phi_{ki} L_k \left(\frac{w_i}{w_k}\right)^{\sigma-1} \tag{14}$$

This is a sufficient statistic of the welfare in country i because we focus on the impacts of changes in country size and trade costs on the indirect utility.

Proposition 2 $\mathrm{d}U_i/\mathrm{d}L_j > 0$, $\forall i$ *and* j.

The proof is contained in Appendix 3.

An increase in the local labor force is beneficial to all countries because the world production and demand end up increasing. While the increase in the manufacturing workers raises the number of local varieties and the quantity produced, the increase in demand raises the wages of local workers, which in turn raises the prices of local goods. Though the impact on wages in other countries may be positive or negative (depending on the sign of the neighboring effect), Proposition 2 shows that overall, the local country as well as neighboring countries gain in terms of welfare.

4.2 The impact of a PTA

In this Section, we consider the scenario where countries j and k engage in a PTA and we study market integration by investigating the impact of an increase in the freeness of trade between the PTA partners on nominal wages and welfare. By neglecting the source of potential tariff revenues for partner countries, our approach follows Venables (1987), Behrens *et al.* (2007), and Ossa (2011).

Proposition 3 *For any distinct* $i, j, k \in \{1, 2, 3\}$,

$\mathrm{d}\left(w_j/w_i\right)/\mathrm{d}\phi_{jk} > 0$ *and* $\mathrm{d}\left(w_k/w_i\right)/\mathrm{d}\phi_{jk} > 0$.

The proof is contained in Appendix 4.

Proposition 3 states that a PTA between two countries via a reduction in their trade cost increases their wages relative to that available in the third country. The *integration effect* due to a better market access between the PTA partners induces the price index in the integrating area to fall and local consumption to rise. However, because supply is fixed, the *price effect* leads prices and wages in the integrating area to rise so as to restore equilibrium.

Because the export price is proportional to the wage in the export country (see expression (6)), a PTA raises the export price in the partner countries, and therefore improves the terms-of-trade of the integrating area, while lowering that in the excluded country. The implication on welfare is derived in the following Proposition.

Proposition 4 *For any distinct* $i, j, k \in \{1, 2, 3\}$,

(i) $\mathrm{d}U_j/\mathrm{d}\phi_{jk} > 0$ *and* $\mathrm{d}U_k/\mathrm{d}\phi_{jk} > 0$.

(ii) $\mathrm{d}U_i/\mathrm{d}\phi_{jk} < 0$.

The proof is contained in Appendix 5.

Proposition 4(i) is intuitive: the terms-of-trade gain provides a strong incentive for countries to engage in bilateral trade agreements. This result is similar to that obtained in other new trade models in the presence of a freely traded homogeneous good, see the trade policy implications derived in the symmetric and hub-and-spoke configurations by Puga and Venables (1997), as well as the model by Ossa (2011) where the third country trades with one partner country only. However, in general, when trade is not restricted so that each country trades with any other one, a PTA may hurt some partner country in terms of welfare under FPE. This has been shown to happen when the hub effect is large enough, see the example with $\mathrm{d}U_j/\mathrm{d}\phi_{jk} < 0$ provided by

Behrens *et al.* (2007, p. 637). In that case, although the two countries engaging in a PTA have a better access and attract firms overall, firms in the smaller country move to the larger one (Baldwin and Robert-Nicoud, 2000), which reduces the welfare in the smaller country concluding the PTA. Here, in contrast to the models relying on FPE, for the PTA partners, the integration effect always dominates the price effect irrespective of country size and of the spatial distribution of countries, so that welfare always increases in the integrating area.

Proposition 4(ii) is an important finding of this paper. It states that *while a PTA is beneficial to the PTA partners, it is always detrimental to the third country.* This result is in agreement with Puga and Venables (1997), Behrens *et al.* (2007, Proposition 3), and Ossa (2011, Proposition 3). However, here, we provide another rationale for the lower utility in the third country. Under FPE, falling trade costs between countries concluding the PTA is accompanied by the relocation of firms to the PTA partners, while it does not lead to terms-of-trade movements given FPE in the presence of a freely traded homogeneous good. That is, the relocation of firms from the third country to the PTA partners implies a worse access of the third country to the manufacturing varieties. In contrast, in this paper, falling trade costs ϕ_{jk} between PTA partners raises prices and wages in the integrating area, countries j and k, relative to the price level in the third country i (see Proposition 3). Rewriting the utility in the excluded country i (11) as

$$U_i^* = \left[\frac{1}{\sigma F} \left(L_i + \phi_{ij} L_j \left(\frac{w_i}{w_j} \right)^{\sigma-1} + \phi_{ik} L_k \left(\frac{w_i}{w_k} \right)^{\sigma-1} \right) \right]^{\frac{1}{\sigma-1}}$$

shows that it does not involve the freeness of trade ϕ_{jk} between the partner countries. Stated differently, a PTA affects the utility U_i^* in the third country only via relative wages and reduces welfare in that country. This is because the relative wage increase in the partner countries raises production costs and product prices (see relation (6)) in the integrating area relative to those in the third country. As a result, consumers in the third country do not benefit from the integration effect and are exposed to the negative price effect. They suffer a terms-of-trade loss because they have to import the varieties produced at higher prices in the partner countries, which lowers their welfare.

5 Conclusion

In this paper, we have built a three-country model of international trade under monopolistic competition. In contrast to the existing literature which relies on FPE across countries, we have integrated price effects into the analysis. We have then used our model to address several market integration issues.

We have determined the role of country size on nominal wages and welfare. When some country increases in size, its nominal wage increases, as well as that in a small and near country, while that in a large and distant country falls. The size effect, emphasized by Krugman in the case of two countries, propagates across countries, giving rise to a neighboring effect. The market potential increases more in a neighboring country than in a distant one. We have also determined the impact of a PTA on prices, wages and welfare in the participating countries and the left-out country. A PTA increases the relative wage, the welfare, and the terms-of-trade in the integrating area, while it lowers those in the third country.

Appendix 1: Proof of Lemma 3

(i) Along the curve Γ_1, the derivative $\mathrm{d}w_3/\mathrm{d}w_2$ is computed by applying the implicit function theorem to relation (13) and plugging L_2 as given by (13) into the expression as follows

$$\begin{aligned}\left.\frac{\mathrm{d}w_3}{\mathrm{d}w_2}\right|_{\Gamma_1} &= -\frac{\partial e_1/\partial w_2}{\partial e_1/\partial w_3} = -\frac{L_2 w_1^{\sigma-1} f_{12} - (\sigma-1) L_1 w_2^{\sigma-2} f_2 - L_1 w_2^{\sigma-1} f_{22}}{L_2 w_1^{\sigma-1} f_{13} - L_1 w_2^{\sigma-1} f_{23}} \\ &= -\frac{f_2 f_{12} - (\sigma-1) f_1 f_2/w_2 - f_1 f_{22}}{f_2 f_{13} - f_1 f_{23}} \\ &= \frac{w_3^{1-\sigma} g_1(w_2^\sigma, w_3^\sigma)}{\sigma \Phi w_2 (\phi_{13} w_2^\sigma - \phi_{23})} \qquad (15)\end{aligned}$$

where $f_{ij} \equiv \partial f_i / \partial w_j$ and

$$\begin{aligned} & g_1(w_2^\sigma, w_3^\sigma) \equiv \\ & (\sigma-1)(\phi_{12} - \phi_{13}\phi_{23})(-1+\phi_{23}^2) - (\sigma-1)(-2\phi_{12}\phi_{13} + \phi_{23} + \phi_{12}^2\phi_{23} + \phi_{13}^2\phi_{23} - \phi_{23}^3) w_3^\sigma \\ & +[-1 - \phi_{12}^2 + \phi_{13}^2 + 2\phi_{12}\phi_{13}\phi_{23} + \phi_{23}^2 - 2\phi_{13}^2\phi_{23}^2 + 2\sigma(-1+\phi_{13}^2)(-1+\phi_{23}^2)] w_2^\sigma \\ & +[(2\sigma-1)\phi_{13}^3 + 2(\sigma-1)\phi_{12}\phi_{23} - 2\sigma\phi_{12}\phi_{13}^2\phi_{23} + \phi_{13}(1 - 2\sigma + \phi_{12}^2 + \phi_{23}^2)] w_2^\sigma w_3^\sigma \\ & +(\sigma-1)(\phi_{13}^2 - 1)(\phi_{12} - \phi_{13}\phi_{23}) w_2^{2\sigma} + (\sigma-1)[\phi_{13}\phi_{23} + \phi_{12}^2\phi_{13}\phi_{23} - \phi_{12}(\phi_{13}^2 + \phi_{23}^2)] w_3^{2\sigma} \end{aligned}$$

We consider the numerator of (15) and show that $g_1(w_2^\sigma, w_3^\sigma) > 0$ in the interior of triangle (Δ).

By evaluating the expression $g_1(w_2^\sigma, w_3^\sigma)$ at the three corners of (Δ), we obtain $g_1(\phi_{12}, \phi_{13}) = \sigma\phi_{12}(1-\phi_{13}^2)\,\Phi > 0$, $g_1(\phi_{23}/\phi_{13}, \phi_{13}^{-1}) = 0$, and $g_1(\phi_{12}^{-1}, \phi_{23}/\phi_{12}) = \sigma(\phi_{12} - \phi_{13}\phi_{23})\Phi/\phi_{12}^2 > 0$. We show that the curve Γ_3 defined by the expression $g_1(w_2^\sigma, w_3^\sigma) = 0$ does not intersect triangle (Δ), except at corner $(\phi_{23}/\phi_{13}, \phi_{13}^{-1})$. By substituting the expression of line $(f_1 = 0)$ into $g_1(w_2^\sigma, w_3^\sigma) = 0$, we have that line $(f_1 = 0)$ intersects the curve Γ_3 in two points (w_2^σ, w_3^σ) with w_3^σ satisfying

$(w_3^\sigma)_1 = \phi_{13}^{-1}$ and $(w_3^\sigma)_2 = (1-\phi_{23}^2)/(\phi_{13}-\phi_{12}\phi_{23})$. Because $(w_3^\sigma)_2 - (w_3^\sigma)_1 = \phi_{23}(\phi_{12}-\phi_{13}\phi_{23})/[\phi_{13}(\phi_{13}-\phi_{12}\phi_{23})] > 0$, the curve Γ_3 intersects triangle (Δ) at corner $(\phi_{23}\phi_{13}^{-1}, \phi_{13}^{-1})$ only. By substituting the expression of line $(f_2 = 0)$ into $g_1(w_2^\sigma, w_3^\sigma) = 0$, we have that line $(f_2 = 0)$ intersects the curve in two points (w_2^σ, w_3^σ) with w_3^σ satisfying $(w_3^\sigma)_1 = \phi_{13}^{-1}$ and $(w_3^\sigma)_2 = (\phi_{12} - \phi_{13}\phi_{23})/(\phi_{12}\phi_{13} - \phi_{23}) < 0$ and $\lim_{\phi_{23}\to\phi_{12}\phi_{13}} (w_3^\sigma)_2 = -\infty$. This second intersection point is outside (Δ). By substituting the expression of line $(f_3 = 0)$ into $g_1(w_2^\sigma, w_3^\sigma) = 0$, we obtain that line $(f_3 = 0)$ intersects the curve Γ_3 in (w_2^σ, w_3^σ) with w_3^σ satisfying the following quadratic equation

$$
\begin{aligned}
g_2(w_3^\sigma) \equiv\ & (-\sigma\phi_{12}\phi_{13}^2 - \phi_{13}\phi_{23} + 2\sigma\phi_{13}\phi_{23} + \phi_{12}^2\phi_{13}\phi_{23} + \phi_{13}^3\phi_{23} - \sigma\phi_{13}^3\phi_{23} - \sigma\phi_{12}\phi_{23}^2 \\
& -2\phi_{12}\phi_{13}^2\phi_{23}^2 + 2\sigma\phi_{12}\phi_{13}^2\phi_{23}^2 + \phi_{13}\phi_{23}^3 - \sigma\phi_{13}\phi_{23}^3) + (\phi_{12}\phi_{13} - \phi_{12}^3\phi_{13} - \phi_{12}\phi_{13}^3 \\
& +2\sigma\phi_{12}\phi_{13}^3 + \phi_{23} - 2\sigma\phi_{23} - \phi_{12}^2\phi_{23} + 2\sigma\phi_{12}^2\phi_{23} - \phi_{13}^2\phi_{23} + 2\phi_{12}^2\phi_{13}^2\phi_{23} \\
& -3\sigma\phi_{12}^2\phi_{13}^2\phi_{23} + \phi_{12}\phi_{13}\phi_{23}^2 - \phi_{23}^3 + \sigma\phi_{23}^3)w_3^\sigma + (-\phi_{12} + \sigma\phi_{12} + \phi_{12}^3 - \sigma\phi_{12}^3 + \phi_{12}\phi_{13}^2 \\
& -2\sigma\phi_{12}\phi_{13}^2 + \sigma\phi_{12}^3\phi_{13}^2 + \sigma\phi_{13}\phi_{23} - 2\phi_{12}^2\phi_{13}\phi_{23} + \sigma\phi_{12}^2\phi_{13}\phi_{23} + \phi_{12}\phi_{23}^2 - \sigma\phi_{12}\phi_{23}^2)w_3^{2\sigma} \\
=\ & 0
\end{aligned}
$$

Because the coefficient of $w_3^{2\sigma}$ can be rewritten as $(\sigma-1)(1-\phi_{12}^2)(\phi_{12}-\phi_{13}\phi_{23}) + (\phi_{23}-\phi_{12}\phi_{13})[\sigma\phi_{13}(1-\phi_{12}^2) + (\sigma-1)(\phi_{13}-\phi_{12}\phi_{23})] > 0$, and $g_2(\phi_{13}) = -\sigma\phi_{12}(1-\phi_{13}^2)(\phi_{23}-\phi_{12}\phi_{13})^2 \leq 0$ and $g_2(\phi_{23}/\phi_{12}) = -\sigma(\phi_{12}-\phi_{13}\phi_{23})(\phi_{23}-\phi_{12}\phi_{13})^2/\phi_{12}^2 \leq 0$, the quadratic expression $g_2(w_3^\sigma)$ is signed along the side $(f_3 = 0)$ of triangle (Δ) except in the corners, and the curve Γ_3 does not intersect along that side of (Δ) except in the corners.

This means that the curve Γ_3 intersects (Δ) in $(\phi_{23}\phi_{13}^{-1}, \phi_{13}^{-1})$ and eventually in the other corners when $\phi_{23} = \phi_{12}\phi_{13}$. Therefore, the expression $g_1(w_2^\sigma, w_3^\sigma)$ is strictly positive inside triangle (Δ).

Finally, given the denominator of (15), $\mathrm{d}w_3/\mathrm{d}w_2 \gtrless 0$ if $w_2^\sigma \lessgtr \phi_{23}/\phi_{13}$.

(ii) Along the curve Γ_2, the derivative $\mathrm{d}w_3/\mathrm{d}w_2$ is also obtained by applying the implicit function theorem to relation (13) as follows

$$
\begin{aligned}
\left.\frac{\mathrm{d}w_3}{\mathrm{d}w_2}\right|_{\Gamma_2} &= -\frac{\partial e_2/\partial w_2}{\partial e_2/\partial w_3} = -\frac{f_3 f_{12} - f_1 f_{32}}{f_3 f_{13} - (\sigma-1)\, f_1 f_3/w_3 - f_1 f_{33}} \\
&= \frac{\sigma\Phi w_2^{1-\sigma} w_3(\phi_{23} - \phi_{12}w_3^\sigma)}{g_3(w_2^\sigma, w_3^\sigma)}
\end{aligned}
\tag{16}
$$

where $f_{ij} \equiv \partial f_i / \partial w_j$ and expression $g_3(w_2^\sigma, w_3^\sigma)$ is given by

$$g_3(w_2^\sigma, w_3^\sigma) \equiv -(\sigma-1)\left(1-\phi_{12}^2\right)(\phi_{13}-\phi_{12}\phi_{23})\, w_3^{2\sigma}$$
$$\left[\phi_{12}-2\phi_{13}\phi_{23}+\phi_{12}\phi_{13}^2+\phi_{12}\phi_{23}^2-\phi_{12}^3-2\sigma\left(1-\phi_{12}^2\right)(\phi_{12}-\phi_{13}\phi_{23})\right] w_2^\sigma w_3^\sigma$$
$$+\left[2\sigma\left(1-\phi_{12}^2\right)\left(1-\phi_{23}^2\right)-1+\phi_{12}^2-\phi_{13}^2+\phi_{23}^2+2\phi_{12}\phi_{13}\phi_{23}-2\phi_{12}^2\phi_{23}^2\right] w_3^\sigma$$
$$-(\sigma-1)\left[\phi_{13}-\phi_{12}\phi_{23}+(\phi_{23}-\phi_{12}\phi_{13})\, w_2^\sigma\right]\left[1-\phi_{23}^2-(\phi_{12}-\phi_{13}\phi_{23})\, w_2^\sigma\right]$$

We consider the denominator of (16) and show that $g_3(w_2^\sigma, w_3^\sigma) > 0$ in the interior of triangle (Δ). For this purpose, we evaluate g_3 along the three sides of triangle (Δ).

First, solving $f_1(w_2^\sigma, w_3^\sigma) = 0$ for w_3^σ and plugging the solution into $g_3(w_2^\sigma, w_3^\sigma)$ lead to

$$g_3(w_2^\sigma, w_3^\sigma)|_{f_1=0} = \frac{\sigma\Phi\left(1-\phi_{12}w_2^\sigma\right)}{\phi_{13}-\phi_{12}\phi_{23}} g_4(w_2^\sigma)$$

where

$$\begin{aligned} g_4(w_2^\sigma) &\equiv 1-\phi_{23}^2-(\phi_{12}-\phi_{13}\phi_{23})\, w_2^\sigma \\ &\geq 1-\phi_{23}^2-(\phi_{12}-\phi_{13}\phi_{23})/\phi_{12} \\ &= (\phi_{13}-\phi_{12}\phi_{23})\,\phi_{23}/\phi_{12} > 0 \end{aligned}$$

Hence, $g_3(w_2^\sigma, w_3^\sigma)|_{f_1=0} \geq 0$.

Second, solving $f_2(w_2^\sigma, w_3^\sigma) = 0$ for w_3^σ and plugging the solution into $g_3(w_2^\sigma, w_3^\sigma)$ lead to

$$g_3(w_2^\sigma, w_3^\sigma)|_{f_2=0} = \frac{\sigma\Phi}{(\phi_{23}-\phi_{12}\phi_{13})^2} g_5(w_2^\sigma)$$

where $g_5(w_2^\sigma)$ is given by

$$g_5(w_2^\sigma) \equiv g_6(\phi_{12})\, w_2^{2\sigma} + [\phi_{12}^3(\phi_{13}-2\sigma\phi_{13})+\phi_{12}^2\phi_{23}\left(3\sigma\phi_{13}^2-2\phi_{13}^2+1\right)$$
$$-(2\sigma-1)\left(\phi_{13}^2-1\right)\phi_{23}-(\sigma-1)\phi_{23}^3+\phi_{12}\phi_{13}\left(\phi_{13}^2-\phi_{23}^2-1\right)]w_2^\sigma$$
$$+\phi_{12}\phi_{23}\left(\sigma\phi_{12}^2-2\sigma-\phi_{12}^2-\phi_{13}^2+1\right)+\phi_{13}\phi_{23}^2\left(\sigma-2(\sigma-1)\phi_{12}^2\right)+\sigma\phi_{12}^2\phi_{13}+(\sigma-1)\phi_{12}{\phi_{23}}^3$$

with

$$g_6(\phi_{12}) \equiv \left(2\sigma-\sigma\phi_{13}^2-1\right)\phi_{13}\phi_{12}^2-\left(\sigma+\sigma\phi_{13}^2-2\phi_{13}^2\right)\phi_{23}\phi_{12}-(\sigma-1)\phi_{13}\left(1-\phi_{13}^2-\phi_{23}^2\right)$$

While g_5 is quadratic in w_2^σ, g_6 is quadratic and convex in ϕ_{12}. Therefore, so as to show that g_5 is concave in w_2^σ, we show that $g_6(\phi_{12})$ is negative by evaluating it at the three corners of (Δ). By plugging $\phi_{12} = \phi_{13}\phi_{23}$, ϕ_{13}/ϕ_{23}, ϕ_{23}/ϕ_{13} into $g_6(\phi_{12})$, we have respectively

$$\begin{aligned} g_6(\phi_{13}\phi_{23}) &= -\phi_{13}\left(1-\phi_{13}^2\right)\left[\sigma\left(1-\phi_{13}^2\phi_{23}^2\right)-1+\phi_{23}^2\right] \\ &< -\phi_{13}\left(1-\phi_{13}^2\right)\left[\left(1-\phi_{13}^2\phi_{23}^2\right)-1+\phi_{23}^2\right] \\ &= -\phi_{13}\left(1-\phi_{13}^2\right)^2\phi_{23}^2 < 0 \end{aligned}$$

$$\begin{aligned} g_6\left(\phi_{13}/\phi_{23}\right) &= -\frac{\phi_{13}\left(\phi_{23}^2-\phi_{13}^2\right)}{\phi_{23}^2}\left[\sigma\left(2-\phi_{13}^2-\phi_{23}^2\right)-1+\phi_{23}^2\right] \\ &< -\frac{\phi_{13}\left(\phi_{23}^2-\phi_{13}^2\right)}{\phi_{23}^2}\left[\left(2-\phi_{13}^2-\phi_{23}^2\right)-1+\phi_{23}^2\right] \\ &= -\frac{\phi_{13}\left(\phi_{23}^2-\phi_{13}^2\right)}{\phi_{23}^2}\left(1-\phi_{13}^2\right)<0 \end{aligned}$$

and

$$g_6\left(\phi_{23}/\phi_{13}\right)=-\frac{(\sigma-1)\,\phi_{13}\left(\phi_{13}^2-\phi_{23}^2\right)}{\phi_{13}}<0.$$

As a consequence, $g_6\left(\phi_{12}\right)<0$ always holds, and $g_5(w_2^\sigma)$ is concave in w_2^σ. Furthermore, the values of $g_5(w_2^\sigma)$ at the two vertices, where $w_2^\sigma=\phi_{12}$, ϕ_{23}/ϕ_{13}, are given by

$$\begin{aligned} g_5(\phi_{12}) &= \sigma\left(1-\phi_{12}\right)^2\left(\phi_{23}-\phi_{12}\phi_{13}\right)^2\phi_{13}>0 \\ g_5(\phi_{23}/\phi_{13}) &= \frac{\sigma\left(\phi_{13}-\phi_{12}\phi_{23}\right)^2\left(\phi_{23}-\phi_{12}\phi_{13}\right)^2}{\phi_{13}^2}>0 \end{aligned}$$

Hence, $g_5(w_2^\sigma)>0$ and $g_3(w_2^\sigma,w_3^\sigma)|_{f_2=0}\geq 0$.

Third, solving $f_3(w_2^\sigma,w_3^\sigma)=0$ for w_3^σ and plugging the solution into $g_3(w_2^\sigma,w_3^\sigma)$ lead to

$$g_3(w_2^\sigma,w_3^\sigma)|_{f_3=0}=\frac{\sigma\Phi\left(1-\phi_{12}w_2^\sigma\right)}{\left(1-\phi_{12}^2\right)}\left[\left(\phi_{13}-\phi_{12}\phi_{23}\right)+\left(\phi_{23}-\phi_{12}\phi_{13}\right)w_2^\sigma\right]>0$$

Hence, g_3 is positive along the three sides of triangle (Δ) except at the vertex $(1/\phi_{12},\phi_{23}/\phi_{12})$. By a continuity argument, g_3 is positive inside (Δ).

Finally, given the numerator of (16), $\mathrm{d}w_3/\mathrm{d}w_2\gtreqless 0$ if $w_3^\sigma\lesseqgtr\phi_{23}/\phi_{12}$.

Appendix 2: Proof of Proposition 1

We show the Proposition for $(i,j,k)=(1,2,3)$. The other results can be obtained simply by reindexing country numbers.

(i) By applying the implicit function theorem to relations (13) and plugging L_2 and L_3 as given by (13) into the expression, we have

$$\frac{\mathrm{d}\left(w_2/w_1\right)}{\mathrm{d}L_2}=\frac{\mathrm{d}w_2}{\mathrm{d}L_2}=-\frac{\frac{\partial e_1}{\partial L_2}\frac{\partial e_2}{\partial w_3}}{\det(E)}=\frac{L_1w_3^{\sigma-2}}{\det(E)}g_3(w_2^\sigma,w_3^\sigma)>0 \tag{17}$$

where $g_3(w_2^\sigma,w_3^\sigma)$ has been defined and shown to be positive inside triangle (Δ) in the proof of Lemma 3 in Appendix 1.

Given Lemma 5, $\det(E)>0$, and therefore $\mathrm{d}\left(w_2/w_1\right)/\mathrm{d}L_2>0$.

(ii) Also by applying the implicit function theorem, we readily get

$$\frac{\mathrm{d}w_3}{\mathrm{d}L_2}=\frac{L_1w_3^{\sigma-1}}{\det(E)}\left(f_3f_{12}-f_1f_{32}\right)=\frac{\sigma\Phi L_1w_2^{\sigma-1}w_3^{\sigma-1}}{\det(E)}(\phi_{23}-\phi_{12}w_3^\sigma) \tag{18}$$

Given Lemma 5, $\mathrm{d}w_3/\mathrm{d}L_2 \gtreqqless 0$ if $w_3^\sigma \lesseqgtr \phi_{23}/\phi_{12}$.

Appendix 3: Proof of Proposition 2

We show the Proposition for $j = 2$. The other results can be obtained in a similar way.

(i) By deriving the utility $\widehat{U}_1^*$ in country 1 as given by (14) with respect to L_2, we get

$$\frac{\mathrm{d}\widehat{U}_1^*}{\mathrm{d}L_2} = \frac{\partial \widehat{U}_1^*}{\partial L_2} + \frac{\partial \widehat{U}_1^*}{\partial w_2}\frac{\mathrm{d}w_2}{\mathrm{d}L_2} + \frac{\partial \widehat{U}_1^*}{\partial w_3}\frac{\mathrm{d}w_3}{\mathrm{d}L_2}$$

where $\mathrm{d}w_2/\mathrm{d}L_2$ and $\mathrm{d}w_3/\mathrm{d}L_2$ are given respectively by expressions (17) and (18) in Appendix 2. Substituting L_2 and L_3 as given by relation (13) into the above expression yields

$$\frac{\mathrm{d}\widehat{U}_1^*}{\mathrm{d}L_2} = \frac{L_1^2 w_2^{\sigma-1} w_3^{\sigma-2}}{\det(E)\ f_1^3}\ g_7(w_2^\sigma, w_3^\sigma)$$

where

$$\begin{aligned}
&g_7(w_2^\sigma, w_3^\sigma) \equiv \\
&(\sigma-1)\left(\phi_{12}-\phi_{13}\phi_{23}\right)\left(\phi_{13}-\phi_{12}\phi_{23}\right)+(\sigma-1)\left(\phi_{12}-\phi_{13}\phi_{23}\right)\left(\phi_{23}-\phi_{12}\phi_{13}\right)w_2^\sigma - \\
&\left[(3\sigma-1)\,\phi_{12}^2\phi_{13}\phi_{23}+\phi_{12}\left(2\sigma-1-\sigma\phi_{13}^2-\sigma\phi_{23}^2\right)-(2\sigma-1)\,\phi_{12}^3-(\sigma-1)\,\phi_{13}\phi_{23}\right]w_3^\sigma
\end{aligned}$$

We compute the values of $g_7(w_2^\sigma, w_3^\sigma)$ at the three vertices of triangle (Δ) as

$$\begin{aligned}
g_7(\phi_{12}, \phi_{13}) &= \sigma\phi_{12}\phi_{13}\Phi > 0 \\
g_7(1/\phi_{12}, \phi_{23}/\phi_{12}) &= \sigma\phi_{23}\Phi > 0 \\
g_7(\phi_{23}/\phi_{13}, 1/\phi_{13}) &= \frac{\Phi}{\phi_{13}}\left[\phi_{13}\phi_{23}-\phi_{12}+\sigma\left(2\phi_{12}-\phi_{13}\phi_{23}\right)\right] \\
&> \frac{\Phi}{\phi_{13}}\left[\phi_{13}\phi_{23}-\phi_{12}+\left(2\phi_{12}-\phi_{13}\phi_{23}\right)\right] = \frac{\Phi}{\phi_{13}}\phi_{12} > 0
\end{aligned}$$

all of which are positive. Because $g_7(w_2^\sigma, w_3^\sigma)$ is linear in w_2^σ and w_3^σ, solving $f_i(w_2^\sigma, w_3^\sigma) = 0$ along each side of triangle (Δ) for w_3^σ and plugging the solution into $g_7(w_2^\sigma, w_3^\sigma)$ leads to another linear expression in w_2^σ. This implies that $g_7(w_2^\sigma, w_3^\sigma)$ is positive along each side of (Δ). By a continuity argument, $g_7(w_2^\sigma, w_3^\sigma)$ is positive inside (Δ), which implies that $\mathrm{d}\widehat{U}_1^*/\mathrm{d}L_2 > 0$.

(ii) The sufficient statistic of welfare $\widehat{U}_2^*$ in country 2 is given by expression (14)

$$\widehat{U}_2^* = \phi_{12}L_1\left(\frac{w_2}{w_1}\right)^{\sigma-1} + L_2 + \phi_{23}L_3\left(\frac{w_2}{w_3}\right)^{\sigma-1} \tag{19}$$

The first term of (19) is increasing in L_2 as resulting from Proposition 1(i). Obviously, the second term is also increasing in L_2. Thus, so as to prove that $\mathrm{d}\widehat{U}_2^*/\mathrm{d}L_2 > 0$, it is sufficient to show that w_2/w_3 is increasing in L_2.

$$\frac{\mathrm{d}\left(w_2/w_3\right)}{\mathrm{d}L_2} = \frac{\frac{\mathrm{d}w_2}{\mathrm{d}L_2}w_3 - \frac{\mathrm{d}w_3}{\mathrm{d}L_2}w_2}{w_3^2} = \frac{w_3^{\sigma-3}L_1}{\det(E)\ f_1^2}\ g_8(w_2^\sigma, w_3^\sigma)$$

where

$$g_8 \equiv (w_2^\sigma, w_3^\sigma)(\sigma-1)(\phi_{23}-\phi_{12}\phi_{13})(\phi_{12}-\phi_{13}\phi_{23})w_2^{2\sigma} - (\sigma-1)\left(1-\phi_{12}^2\right)(\phi_{13}-\phi_{12}\phi_{23})w_3^{2\sigma}$$
$$+(\sigma-1)\left[\phi_{12}^3-\phi_{12}\left(\phi_{13}^2+\phi_{23}^2+1\right)+2\phi_{13}\phi_{23}\right]w_2^\sigma w_3^\sigma$$
$$+\left(-2\sigma\phi_{12}\phi_{13}\phi_{23}^2+2\sigma\phi_{12}\phi_{13}+2\sigma\phi_{23}^2-2\sigma\phi_{23}+\phi_{12}^2\phi_{23}-2\phi_{12}\phi_{13}+\phi_{13}^2\phi_{23}-\phi_{23}^3+\phi_{23}\right)w_2^\sigma$$
$$+\left[2\sigma\left(1-\phi_{12}^2\right)\left(1-\phi_{23}^2\right)+\phi_{12}^2+\phi_{23}(2\phi_{12}(\phi_{13}-\phi_{12}\phi_{23})+\phi_{23})-\phi_{13}^2-1\right]w_3^\sigma$$
$$-(\sigma-1)\left(1-\phi_{23}^2\right)(\phi_{13}-\phi_{12}\phi_{23})$$

We compute the values of $g_8(w_2^\sigma, w_3^\sigma)$ at the three vertices of triangle (Δ) as

$$\begin{aligned} g_8(\phi_{12},\phi_{13}) &= \sigma(\phi_{13}-\phi_{12}\phi_{23})\Phi > 0 \\ g_8(1/\phi_{12},\phi_{23}/\phi_{12}) &= 0 \\ g_8(\phi_{23}/\phi_{13},1/\phi_{13}) &= -\frac{\sigma\left(1-\phi_{23}^2\right)\Phi}{\phi_{13}} > 0 \end{aligned}$$

all of which are nonnegative.

First, solving $f_1(w_2^\sigma, w_3^\sigma)=0$ for w_3^σ and plugging the solution into $g_8(w_2^\sigma, w_3^\sigma)$ yields

$$g_8(w_2^\sigma, w_3^\sigma)|_{f_1=0} = \frac{\sigma\left(1-\phi_{23}^2\right)\Phi\left(1-\phi_{12}w_2^\sigma\right)}{\phi_{13}-\phi_{12}\phi_{23}} > 0$$

Second, solving $f_2(w_2^\sigma, w_3^\sigma)=0$ for w_3^σ and plugging the solution into $g_8(w_2^\sigma, w_3^\sigma)$ yields

$$g_8(w_2^\sigma, w_3^\sigma)|_{f_2=0} = \frac{\Phi}{(\phi_{23}-\phi_{12}\phi_{13})^2}\; g_9(w_2^\sigma)$$

where $g_9(w_2^\sigma)$

$$g_9(w_2^\sigma) \equiv -(\sigma-1)\phi_{13}\Phi w_2^{2\sigma} + [\phi_{12}^3(\phi_{13}-\sigma\phi_{13})+\phi_{12}^2\phi_{23}\left((\sigma-2)\phi_{13}^2-\sigma+1\right)$$
$$+\phi_{12}\phi_{13}\left((3\sigma-1)\phi_{23}^2+\phi_{13}^2-1\right)-(2\sigma-1)\phi_{23}\left(\phi_{13}^2+\phi_{23}^2-1\right)]w_2^\sigma$$
$$+\phi_{12}\phi_{23}\left(\sigma\left(\phi_{12}^2-2\right)-{\phi_{12}}^2-{\phi_{13}}^2+1\right)+\phi_{13}\phi_{23}^2\left(\sigma-2(\sigma-1)\phi_{12}^2\right)+\sigma\phi_{12}^2\phi_{13}+(\sigma-1)\phi_{12}\phi_{23}^3$$

Since $g_9(w_2^\sigma)$ is concave in w_2^σ and

$$\begin{aligned} g_9(\phi_{12}) &= \sigma(\phi_{13}-\phi_{12}\phi_{23})(\phi_{23}-\phi_{12}\phi_{13})^2 > 0 \\ g_9(\phi_{23}/\phi_{13}) &= \frac{\sigma(1-\phi_{23}^2)(\phi_{23}-\phi_{12}\phi_{13})^2}{\phi_{13}} > 0 \end{aligned}$$

$g_9(w_2^\sigma)$ and $g_8(w_2^\sigma, w_3^\sigma)$ are positive along the side $(f_2=0)$ of triangle (Δ).

Third, solving $f_3(w_2^\sigma, w_3^\sigma)=0$ for w_3^σ and plugging the solution into $g_8(w_2^\sigma, w_3^\sigma)$ yields

$$g_8(w_2^\sigma, w_3^\sigma)|_{f_3=0} = \frac{\sigma(\phi_{13}-\phi_{12}\phi_{23})\Phi\left(1-\phi_{12}w_2^\sigma\right)}{1-\phi_{12}^2} > 0.$$

By a continuity argument, $g_8(w_2^\sigma, w_3^\sigma)$ is positive inside triangle (Δ), which implies that w_2/w_3 is increasing in L_2, and therefore $\mathrm{d}\widehat{U}_2^*/\mathrm{d}L_2 > 0$.

(iii) By symmetry, a proof similar to (i) applies for country 3, and $\mathrm{d}\widehat{U}_3^*/\mathrm{d}L_2$.

Appendix 4: Proof of Proposition 3

We show the Proposition for $(i, j, k) = (1, 2, 3)$. The other results can be obtained simply by reindexing country numbers.

The expression $\mathrm{d}w_2/\mathrm{d}\phi_{23}$ is computed by applying the implicit function theorem to relation (13) and plugging L_2 and L_3 as given by (13) into the expression

$$\frac{\mathrm{d}w_2}{\mathrm{d}\phi_{23}} = \frac{w_2^{\sigma-1} w_3^{\sigma-2} L_1^2\ f_3(w_2^\sigma, w_3^\sigma)}{\det(E)\ f_1^3}\ g_{10}(w_2^\sigma, w_3^\sigma) \tag{20}$$

where

$$\begin{aligned} g_{10}(w_2^\sigma, w_3^\sigma) \equiv{}& (1-\sigma)(\phi_{13} - 2\phi_{12}\phi_{23} + \phi_{13}\phi_{23}^2) + 2(\sigma-1)(\phi_{13}^2 - 1)\phi_{23} w_2^\sigma \\ &+(-1 + \phi_{12}^2 - 2\sigma(-1+\phi_{12}^2) - \phi_{13}^2 + 2\phi_{12}\phi_{13}\phi_{23} - \phi_{23}^2) w_3^\sigma \\ &+(\sigma-1)\phi_{13}(1-\phi_{13}^2) w_2^{2\sigma} + (\sigma-1)(-1+\phi_{12}^2)\phi_{13} w_3^{2\sigma} \end{aligned}$$

Given that $f_i > 0$ and $\det(E) > 0$ by Lemma 5, we need to show that expression g_{10} is signed in the interior of triangle (Δ). We show that the curve Γ_4 defined by $g_{10}(w_2^\sigma, w_3^\sigma) = 0$ does not intersect triangle (Δ).

First, by substituting the expression of line $(f_1 = 0)$ into $g_{10}(w_2^\sigma, w_3^\sigma)$, we have that line $(f_1 = 0)$ intersects the curve Γ_4 with w_2^σ satisfying the following quadratic equation

$$\begin{aligned} &g_{11}(w_2^\sigma) \equiv (\sigma-1)\left(\phi_{12}^2 - \phi_{13}^2\right)\phi_{13} w_2^{2\sigma} + \left(2\sigma - 1 - \phi_{23}^2\right)\phi_{12}\phi_{23} - \phi_{13}\left((\sigma-2)\,\phi_{23}^2 + \sigma\right) \\ &+[(2\sigma-3)\phi_{13}^2\phi_{23} - (2\sigma-1)\,\phi_{12}^2\phi_{23} + \phi_{12}\phi_{13}\left(1+\phi_{23}^2\right)] w_2^\sigma = 0 \end{aligned} \tag{21}$$

By evaluating function g_{11} at the 2 corners of side $(f_1 = 0)$, we get $g_{11}(\phi_{23}/\phi_{13}) = -\sigma\Phi/\phi_{13} < 0$ and $g_{11}(\phi_{12}^{-1}) = -((\sigma-1)\phi_{13} + \phi_{12}\phi_{23})\Phi/\phi_{12}^2 < 0$.

(i) If $\phi_{12} \geq \phi_{13}$, then g_{11} is convex, and thus signed along side $(f_1 = 0)$, implying no intersection point of Γ_4 with side $(f_1 = 0)$.

(ii) If $\phi_{12} < \phi_{13}$, then g_{11} is concave. Solving $\partial g_{11}/\partial w_2^\sigma = 0$ with respect to w_2^σ and plugging the solution into g_{11} leads to

$$\begin{aligned} g_{11} &= \frac{\left(\phi_{13} - \phi_{12}\phi_{23}\right)^2}{4\,(\sigma-1)\left(\phi_{13}^2 - \phi_{12}^2\right)\phi_{13}} A_1 \\ A_1 &\equiv (2\sigma-1)^2\,\phi_{12}^2 - \left(4\sigma^2 - 4\sigma + \phi_{23}^2\right)\phi_{13}^2 - 2\phi_{12}\phi_{23}\phi_{13} \end{aligned} \tag{22}$$

Therefore, if $A_1 < 0$, then $g_{11} < 0$ holds on side $(f_1 = 0)$. Next, we get

$$\left.\frac{\partial g_{11}}{\partial w_2^\sigma}\right|_{w_2^\sigma=1/\phi_{12}} = \frac{(\phi_{13} - \phi_{12}\phi_{23})}{\phi_{12}} A_2 \tag{23}$$
$$A_2 \equiv (2\sigma - 1)^2 \phi_{12}^2 - 2(\sigma - 1)\phi_{13}^2 - \phi_{12}\phi_{23}\phi_{13}$$

Therefore, if $A_2 > 0$, then the curve $g_{11} = 0$ does not intersect $(f_1 = 0)$ for all $w_2^\sigma \in [\phi_{23}/\phi_{13}, 1/\phi_{12}]$. It can be readily shown that $A_1 < A_2$. If $A_1 < A_2 \leq 0$, then from expression (22), the curve $g_{11} = 0$ does not intersect $(f_1 = 0)$ for all $w_2^\sigma \in [\phi_{23}/\phi_{13}, 1/\phi_{12}]$. On the other hand, if $A_2 > 0$, then the slope (23) is positive, and g_{11} is signed for all $w_2^\sigma \in [\phi_{23}/\phi_{13}, 1/\phi_{12}]$ too. Hence, the curve Γ_4 does not intersect $(f_1 = 0)$ inside triangle (Δ).

Second, by substituting the expression of line $(f_2 = 0)$ into $g_{10}(w_2^\sigma, w_3^\sigma)$, we have that line $(f_2 = 0)$ intersects the curve Γ_4 at w_2^σ satisfying

$$g_{12}(w_2^\sigma) \equiv \sigma\phi_{12}^2\phi_{13} - 2\sigma\phi_{12}\phi_{23} - \phi_{12}(-1 + \phi_{13}^2)\phi_{23} + \sigma\phi_{13}\phi_{23}^2 - (\sigma - 1)\phi_{13}(1 - \phi_{13}^2)w_2^{2\sigma}$$
$$+(\phi_{12}\phi_{13}(-1 + \phi_{13}^2) + 2\sigma\phi_{23} - 2\sigma\phi_{13}^2\phi_{23} + (-1 + \phi_{13}^2)\phi_{23})w_2^\sigma = 0$$

Given that $g_{12}(w_2^\sigma)$ is concave, $g_{12}(\phi_{12}) = \sigma\phi_{13}\Phi > 0$, and $g_{12}(\phi_{23}/\phi_{13}) = \sigma\Phi/\phi_{13} > 0$, the quadratic expression $g_{12}(w_2^\sigma)$ is signed along side $(f_2 = 0)$, and thus the curve Γ_4 does not intersect $(f_2 = 0)$ inside triangle (Δ). It can easily be shown that $g_{10} = 0$ always intersect $(f_2 = 0)$ outside (Δ).

Third, by substituting the expression of line $(f_3 = 0)$ into $g_{10}(w_2^\sigma, w_3^\sigma)$, we have that line $(f_3 = 0)$ intersects the curve Γ_4 at w_2^σ satisfying

$$g_{13}(w_2^\sigma) \equiv \sigma\phi_{13} - \phi_{13}\phi_{23} + (\phi_{12}\phi_{13} - 2\sigma\phi_{12}\phi_{13} + \phi_{23})w_2^\sigma + (\sigma - 1)\phi_{13}w_2^{2\sigma} = 0$$

Note that g_{13} is convex in w_2^σ, $g_{13}(\phi_{12}) = \sigma\phi_{13}\Phi > 0$, and $g_{13}(\phi_{12}^{-1}) = ((\sigma - 1)\phi_{13} + \phi_{12}\phi_{23})\Phi/\phi_{12}^2 > 0$. Because the minimum of g_{13} is reached at $w_2^{\sigma^*} = (\phi_{12}\phi_{13} - 2\sigma\phi_{12}\phi_{13} + \phi_{23})/(2\phi_{13} - 2\sigma\phi_{13})$, with $\phi_{12} - w_2^{\sigma^*} = (\phi_{23} - \phi_{12}\phi_{13})/(2\phi_{13}(\sigma - 1)) \geq 0$, the quadratic expression g_{13} remains positive in the interval $[\phi_{12}, \phi_{12}^{-1}]$, and the curve Γ_4 does not intersect side $(f_3 = 0)$ along triangle (Δ).

Therefore, we have shown that the curve Γ_4 does not intersect any side of triangle (Δ). By a continuity argument, $g_{10}(w_2^\sigma, w_3^\sigma)$ is positive inside triangle (Δ). Given expression (20), we have $\mathrm{d}w_2/\mathrm{d}\phi_{23} = \mathrm{d}(w_2/w_1)/\mathrm{d}\phi_{23} > 0$.

Similarly, it can be shown that $\mathrm{d}(w_3/w_1)/\mathrm{d}\phi_{23} > 0$.

Appendix 5: Proof of Proposition 4

We show the Proposition for $(i, j, k) = (1, 2, 3)$. The other results can be obtained simply by reindexing country numbers.

(i) By differentiating $\widehat{U}_1^*$ as given by (14) with respect to ϕ_{23}, we get

$$\begin{aligned} \frac{\mathrm{d}\widehat{U}_1^*}{\mathrm{d}\phi_{23}} &= \frac{\partial \widehat{U}_1^*}{\partial w_2}\frac{\partial w_2}{\partial \phi_{23}} + \frac{\partial \widehat{U}_1^*}{\partial w_3}\frac{\partial w_3}{\partial \phi_{23}} + \frac{\partial \widehat{U}_1^*}{\partial \phi_{23}} \\ &= \frac{(\sigma-1)L_1^3 w_2^{\sigma-2} w_3^{\sigma-2}\ f_2\ f_3}{\det(E)\ f_1^4} g_{14}(w_2^\sigma, w_3^\sigma) \end{aligned}$$

where

$$\begin{aligned} &g_{14}(w_2^\sigma, w_3^\sigma) \equiv 2(\sigma-1)(\phi_{12}\phi_{23}-\phi_{13})(\phi_{12}-\phi_{13}\phi_{23}) \\ &+[2\sigma\phi_{12}\phi_{13}^2\phi_{23} - (2\sigma-1)\phi_{13}^3 - 2(\sigma-1)\phi_{12}\phi_{23} - \phi_{13}(1-2\sigma+\phi_{12}^2+\phi_{23}^2)]w_2^\sigma \\ &+[\phi_{12}((2\sigma-1)(1-\phi_{12}^2)-\phi_{13}^2) - 2(\sigma-1-\sigma\phi_{12}^2)\phi_{13}\phi_{23} - \phi_{12}\phi_{23}^2]w_3^\sigma \end{aligned}$$

In order to show that the linear expression $g_{14}(w_2^\sigma, w_3^\sigma)$ is signed, we show that the line defined by $g_{14}(w_2^\sigma, w_3^\sigma) = 0$ in the plane (w_2^σ, w_3^σ) does not intersect triangle (Δ). Given Lemma 2, the evaluation of g_{14} at the three corners of triangle (Δ) leads to $g_{14}(\phi_{12}, \phi_{13}) = -2\sigma\phi_{12}\phi_{13}\Phi < 0$, $g_{14}(\phi_{23}\phi_{13}^{-1}, \phi_{13}^{-1}) = -[(2\sigma-1)\phi_{12} + \phi_{13}\phi_{23}]\Phi/\phi_{13} < 0$, and $g_{14}(\phi_{12}^{-1}, \phi_{23}\phi_{12}^{-1}) = -[(2\sigma-1)\phi_{13} + \phi_{12}\phi_{23}]\Phi/\phi_{12} < 0$. This means that $g_{14}(w_2^\sigma, w_3^\sigma)$ is strictly negative inside triangle (Δ). By using Lemma 5, we have that $\mathrm{d}\widehat{U}_1^*/\mathrm{d}\phi_{23} < 0$.

(ii) Differentiating $\widehat{U}_2^*$ as given by (14) with respect to ϕ_{23} leads to

$$\frac{\mathrm{d}\widehat{U}_2^*}{\mathrm{d}\phi_{23}} = \frac{L_1^3 w_2^{\sigma-1} w_3^{\sigma-2}\ f_3}{\det(E)\ f_1^4} g_{15}(w_2^\sigma, w_3^\sigma)$$

where

$$
\begin{aligned}
&g_{15}(w_2^\sigma, w_3^\sigma) \equiv \\
&2(\sigma-1)^2\left(1-\phi_{13}^2\right)(\phi_{12}-\phi_{13}\phi_{23})(\phi_{23}-\phi_{12}\phi_{13})w_2^{2\sigma}+(\sigma-1)\{\phi_{12}^3\left(2\sigma-1-2(\sigma-1)\phi_{13}^2\right) \\
&+\phi_{12}\left[1-2\phi_{13}^4+\phi_{13}^2+3\phi_{23}^2-2\sigma\left(1-\phi_{13}^2\right)\left(\phi_{13}^2+\phi_{23}^2+1\right)\right]+\phi_{13}\phi_{23}\left[(4\sigma-3)\left(1-\phi_{13}^2\right)+\phi_{23}^2\right] \\
&-3\phi_{12}^2\phi_{13}\phi_{23}\}w_2^\sigma w_3^\sigma-(\sigma-1)(\phi_{13}-\phi_{12}\phi_{23})[2\sigma\left(1-\phi_{12}^2\right)\left(1-\phi_{13}^2\right)+\phi_{12}^2\left(1-2\phi_{13}^2\right) \\
&-1+\phi_{13}^2-\phi_{23}^2]w_3^{2\sigma}+\{(\sigma-1)\phi_{12}\phi_{13}\left[4\sigma\left(1-\phi_{13}^2\right)\left(1-\phi_{23}^2\right)-2\phi_{13}^2\phi_{23}^2+3\phi_{13}^2-\phi_{23}^2-3\right] \\
&-(\sigma-1)\phi_{23}\left[4\sigma\left(1-\phi_{13}^2\right)\left(1-\phi_{23}^2\right)+\phi_{13}^4-3\phi_{13}^2\phi_{23}^2+\phi_{13}^2+2\phi_{23}^2-2\right]+\phi_{12}^3(\phi_{13}-\sigma\phi_{13}) \\
&+(\sigma-1)\phi_{12}^2\left(\phi_{13}^2+2\right)\phi_{23}\}w_2^\sigma+\{2\sigma^2[\phi_{12}^3\left(\phi_{12}-3\phi_{13}\phi_{23}\right)+\phi_{12}\left(3\phi_{12}-\phi_{13}\phi_{23}\right)\left(\phi_{13}^2+\phi_{23}^2-1\right) \\
&+2\left(1-\phi_{13}^2\right)\left(1-\phi_{23}^2\right)]\}+\sigma[3\phi_{12}^3\phi_{13}\phi_{23}-\phi_{12}^4+\phi_{12}^2\left(6\phi_{13}^2\phi_{23}^2-7\phi_{23}^2-7\phi_{13}^2+5\right) \\
&+\phi_{12}\phi_{13}\phi_{23}\left(-3\phi_{13}^2-3\phi_{23}^2+7\right)+2\left(\phi_{13}^4-2\phi_{13}^2\phi_{23}^2+\phi_{13}^2+\phi_{23}^4+\phi_{23}^2-2\right)]-\phi_{12}^3\phi_{13}\phi_{23} \\
&+\phi 12^2\left[\phi_{13}^2\left(3-2\phi_{23}^2\right)+3\phi_{23}^2-1\right]+\phi_{12}\phi_{13}\phi_{23}\left(\phi_{13}^2+\phi_{23}^2-5\right)-\left(\phi_{13}^2-\phi_{23}^2\right)^2+1]\}w_3^\sigma \\
&+(\sigma-1)\{\phi_{13}\left[(6\sigma-1)\phi_{12}^2\phi_{23}^2+(2\sigma-1)\phi_{23}^4+1-2\sigma+\phi_{12}^2\right] \\
&+2\phi_{12}\phi_{23}\left[2\sigma-\sigma\phi_{12}^2-2\sigma\phi_{23}^2+\phi_{23}^2-1\right]-2\sigma\phi_{12}\phi_{13}^2\phi_{23}\left(\phi_{23}^2+2\right)+\phi_{13}^3\left(2\sigma+\phi_{13}^2-1\right)\}
\end{aligned}
$$

By evaluating expression $g_{15}(w_2^\sigma, w_3^\sigma)$ at the three corners of triangle (Δ), we obtain $g_{15}(\phi_{12}, \phi_{13}) = (2\sigma-1)\phi_{13}\Phi^2 > 0$, $g_{15}(\phi_{23}\phi_{13}^{-1}, \phi_{13}^{-1}) = \sigma(2\sigma-1)\Phi^2/\phi_{13} > 0$, and $g_{15}(\phi_{12}^{-1}, \phi_{23}\phi_{12}^{-1}) = \sigma\phi_{23}\Phi^2/\phi_{12} > 0$. We want to show that the curve Γ_5 defined by $g_{15}(w_2^\sigma, w_3^\sigma) = 0$ does not intersect (Δ).

First, by substituting the expression of line $(f_1 = 0)$ into $g_{15} = 0$, we have that line $(f_1 = 0)$ intersects the curve Γ_5 at $w_2^\sigma = (2\sigma - 1 - \phi_{23}^2)/[(2\sigma - 1)\phi_{12} - \phi_{13}\phi_{23}] > 1/\phi_{12}$, which means that the intersection point is outside (Δ).

Second, by plugging the expression of line $(f_2 = 0)$ into $g_{15} = 0$, we obtain that $(f_2 = 0)$ intersects the curve Γ_5 in (w_2^σ, w_3^σ) with w_3^σ satisfying the following quadratic equation

$$g_{16}(w_3^\sigma) \equiv -(\sigma-1)\phi_{13}w_3^{2\sigma} + \left(2\sigma - 1 - \phi_{13}^2\right)w_3^\sigma - (\sigma-1)\phi_{13} = 0$$

Because the coefficient of $w_3^{2\sigma}$ is negative, and $g_{16}(\phi_{13})$ and $g_{16}(\phi_{13}^{-1})$ are positive, the quadratic expression $g_{16}(w_3^\sigma)$ is positive along the side $(f_2 = 0)$ of (Δ), and the curve Γ_5 does not intersect that side of (Δ).

Third, by substituting the expression of line $(f_3 = 0)$ into $g_{15} = 0$, we obtain that $(f_3 = 0)$ intersects the curve Γ_5 in (w_2^σ, w_3^σ) with w_3^σ satisfying the following quadratic equation

$$g_{17}(w_3^\sigma) \equiv -(\sigma-1)\phi_{12}w_3^{2\sigma} + \left[(2\sigma-1)(\phi_{23}+\phi_{12}\phi_{13}) - 2\sigma^2\phi_{12}\phi_{13}\right]w_3^\sigma + (\sigma-1)(2\sigma-1)\phi_{13}\phi_{23} = 0$$

Because the coefficient of $w_3^{2\sigma}$ is negative, and $g_{17}(\phi_{13})$ and $g_{17}(\phi_{23}\phi_{12}^{-1})$ are positive, the quadratic expression $g_{17}(w_3^\sigma)$ is positive along the side ($f_3 = 0$) of (Δ), and the curve Γ_5 does not intersect that side of (Δ).

Hence, the curve Γ_5 intersects no side of triangle (Δ) and expression $g_{15}(w_2^\sigma, w_3^\sigma)$ is positive inside (Δ). By using Lemma 5, we have that $\mathrm{d}\widehat{U}_2^*/\mathrm{d}\phi_{23} > 0$.

(iii) A proof similar to (ii) applies, so that $\mathrm{d}\widehat{U}_3^*/\mathrm{d}\phi_{23} > 0$.

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