CIRJE-F-800

Multi-period Contract Problems with Verifiable and Unverifiable Outputs

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April 2011

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April 12, 2011

Abstract

Labour contracts tend to be more complicated than one simple short or long-term contract which is the basis of previous studies. Combinations of different length contracts become essential when principals expect to maximize not only verifiable outputs but also observable but unverifiable outputs, e.g., leadership. This paper is the first to develop a theoretical model of multi-period contracts that combine short-, mid-, and long-term contracts. We show that combinations of different length contracts vary by the relative importance of verifiable and unverifiable outputs and relative efficiency of investments in human capital made for each output. We also determine thresholds where the principal switches from offering one type of contract to the other.

Keywords: Different Length Contracts; Unverifiable Outputs; Unverifiable Investments; Unverifiable Ability

JEL Codes: D86; J41; J31

1 Introduction

Some labour contracts guarantee employees a certain period of employment, while some guarantee permanent employment. These contracts may be used independently, but often they are combined for labor agreements. That is, an employer may offer an employee a short-term contract first, but she may offer permanent employment later. It is widely known that combinations of different length contract are used in practice. However, in previous models labour contracts tend to be depicted as either short or long-term contracts. (For example, Fudenberg, Holmstrom and Milgrom [1990] and Dutta and Reichel-

*We would like to thank Hideshi Itoh, Mary Milne, and Hiroshi Osano for helpful comments.

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stein [2003].) Moreover, most models tend to incorporate only verifiable outputs when a lot of outputs that are expected to be realized are observable but unverifiable. (For example, Mirrlees[1976], Harris and Raviv [1979], Holmstrom [1979], and Grossman and Hart [1983].) This paper examines how a multi-period optimal contract is designed as a combination of different length contracts when verifiable and unverifiable outputs exist.[1]

Suppose there are T contractible periods. We define a contract which guarantees employment for all T periods as a long-term contract, employment for one period as a short-term contract, and anything between one and T periods to be a mid-term contract. Such differences in the length of contracts exist in practice, even though they may not be referred to as short-, mid-, or long-term contracts. As an example of offering contracts of different lengths in a multi-period relationship, a firm (principal) may first offer a short-term contract to a newly hired employee (agent), but after observing the employee's performance represented by his skills and efforts, leadership, and an ability to communicate and interact effectively with a variety of co-workers, she may offer him a longer contract[2]. An example of repeating mid-term contracts is given by a professional soccer team (principal) which may offer the player (agent) a short term contract in the initial stage but after observing his performance represented by the number of goals he has kicked, team work skills, and his popularity, she may repeat mid-term contracts[3]. An example of switching from a mid or a long-term contract to a short-term contract is a firm (principal) offering a short-term directorship to her high-ranking employee (agent) who is currently on his mid- or long-term contract.

As the above examples show, one agent may be offered several different contracts by the same principal during his lifetime. We show that this happens when the principal's utility level is determined by both verifiable and unverifiable outputs the agent produces.[4]

[1] There are some contracts that are made to be short considering the principal's financial state. However, such exogenous factors are beyond the scope of this paper.

[2] For simplicity, we use "she" for the principal, "he" for the agent, unless otherwise noted.

[3] In the case of Ichiro, a famous baseball player, Seattle Mariners first offered him a short-term contract, but nowadays she repeats offering a mid-term contracts which extend one year by one year. This is because Mariners valued his teamwork skill and popularity at the very beginning as well as the number of hits he can make. It can be considered that Ichiro's popularity has now saturated that Mariners are offering mid-term contracts.

[4] More precisely, we assume that the principal wishes to maximize the addition of verifiable and unverifiable

[5] In short, principals deliberately need to use different length contracts sequentially to induce the efforts of agents to achieve optimal effort levels for producing both verifiable and unverifiable outputs.

We indicate that long-term contracts can always replicate short-term contracts when there are only verifiable outputs, and therefore principals are always better off or equivalent in offering long-term contracts than short-term contracts.[6] However, some principals may wish to have both types of outputs. Then they may want to induce both types of efforts at the same time, or may want to induce one before the other.[7] Then, different combinations of verifiable and unverifiable outputs affect the efforts of the agent to obtain the skills that produce each type of outputs.

Bernheim and Whinston [1998] study that if there are some unverifiable actions and if agents' actions are taken sequentially, there are cases in which an efficient outcome is obtained only by an incomplete contract of verifiable actions. More precisely, restricting the second mover's (verifiable) action space in the contract, the shape of the best-response function can be modified so that the efficient outcome is obtained as a Nash equilibrium. Although our paper incorporates both verifiable and unverifiable outputs and wages for verifiable outputs are sometimes unspecified in the contract, the logic is quite different from that of Bernheim and Whinston. In our contracts, the wages for future verifiable outputs are not specified in order to induce incentives for investments in the current period. Our logic has nothing to do with the modification of best-response functions.

outputs in section three onwards.

[5] If the principal is concerned with only verifiable outputs, the principal only needs to offer the agent a long-term contract with some clauses, e.g., if the agent's work does not meet the principal's expectation level α, within duration β, the agent is fired, or if the agent produces more than α, he is retained. This can occur only when outputs (in this case, α) are verifiable.

[6] In Dutta and Reichelstein [2003], the principal sometimes chooses a short-term contract, even when outputs are verifiable. This is because in their model, the optimal short-term contract requires firing the incumbent agent and hiring a new one in the second period. It is clear that this scenario is ruled out under long-term contracting with the same incumbent agent.

[7] If an architect runs her own company and if she wishes to have an apprentice, her first concern may be whether or not he could work in team with her. (His unverifiable output must be realized before his verifiable outputs.) This is because agents who are engaged in work which requires special skills must have a certain qualification which verifies his talent. There is a difference in the level of talent for each individuals with qualification, and certainly the verfiable outputs represented by his qualified skills must be realized but this can happen in the later stages.

This paper is also related to the study of Fudenberg, Holmstrom, and Milgrom [1990]. They discuss the environment in which an efficient long-term contract can be implemented as a sequence of one-period short-term contracts. However, they do not investigate when a multi-period optimal contract is implemented as a combination of different length contracts. Our paper is the first study which shows that the optimal contract in the multi-period principal-agent relationship consists of several contracts of different lengths.

The structure of this paper is as follows. Next section provides more technical examples of the use of several contracts of different lengths. Section three presents a benchmark model with two-period cases. Section four and five analyze the main model of the three-period and five-period cases. Section six discusses some concluding remarks.

2 A Variety of Contracts

In this section, we present a technical explanation of the use of several contracts of different lengths. There is an employer (principal) and an employee (agent). The employee makes two types of investments (efforts) related to human capital, and produces two types of outputs corresponding to the investments. Both types of investments are observable but not verifiable. The first type of output x is observable and verifiable (contractible), and the second type of output y is observable but not verifiable (non-contractible). Examples of x are the batting average of a baseball player or an amount of sales a salesman makes. The employee makes an effort I_c (makes investments in human capital) to obtain a skill for producing x. The principal can write a wage which depends on x. On the other hand, examples of y are the popularity of a baseball player or the leadership of a high ranking employee. The employee also makes an effort I_n (makes investment in human capital) to obtain a skill for producing y. However, because y may be observable but unverifiable, the wage cannot reflect y. We suppose that both skills are firm-specific to some extent and thus the investments are relationship-specific. We also suppose that a market of workers without a firm-specific skill is competitive and that an employee who has a firm-specific skill has bargaining power. Therefore, when the employer hires an employee who has no firm-specific skill, she posts a take-it-or-leave-it wage offer to him, but when she hires an employee who has a firm-specific skill, she negotiates his wage with him. For simplicity, we adopt Nash bargaining.

Below, we explain why a principal and an agent sign a variety of contracts depending on the parameters such as relative efficiency of investments. We first consider a two-period case. In the two-period case, the employer has the following choices of contracts to offer: a short-term contract or a long-term contract. In the short-term contract, they sign a contract on the first period wage at the beginning of the first period, and they bargain over the second period wage at the beginning of the second period. The investments in the human capital to improve the agent's skills are made during the first period. In the long-term contract, they sign the contract on both the first and second periods wages at the beginning of the first period.

In the long-term contract, at the beginning of the first period, the principal can write a fixed amount of second period wage depending on the output x the agent is going to produce in the second period. (For example, "If $0 < x \leq 1$, w_a will be paid, if $1 < x \leq 2$, w_b (where, $w_a < w_b$) will be paid, and so on.") However, she cannot write the second period wage to reflect the amount of y the agent is going to produce in the second period, as y may be observable but unverifiable. Then, it is clear that the long-term contract deprives an employee of making I_n, an effort to improve his skill to produce y. The benefit of the long-term contract is that the employer can motivate the employee to make a great deal of I_c than it can under short-term contracts. (We later show in section three that long-term contracts can achieve the first-best level of x.)

In the short-term contract, the bargaining position of the employee at the beginning of the second period depends on his skill to produce y as well as on his skill to produce x. Therefore, the employee has an incentive to make $I_{n,}$ which is also beneficial for the employer. In this case, the wage for the second period can be divided into two components: a *fixed* compensation corresponding to the unverifiable output y and a *variable* compensation corresponding to the verifiable output x.

Thus, this example of two-period case shows that the employer chooses a type of contract depending on the relative efficiency of investments. In other words, if the principal values x relatively more than y, and if the principal expects the investment the agent makes for x is efficient, she prefers the long-term contract to the short-term contract. Otherwise, she prefers the short-term contract.

In a three-period case, a more complicated combination of contracts are chosen depending on the parameters. First, suppose that I_n is relatively efficient and easily saturated. Then the employer chooses a short-term contract for the first period wage in order to induce I_n, and after the investments have been made she contracts wages for the second and third periods on a mid-term contract at the beginning of the second period. That is, at the beginning of the second period, the skill to produce y has already saturated (and hence no more I_n is needed) and thus the employer's concern for the following periods would be to induce I_c from the employee. Second, suppose that the employee should accumulate the skill to produce x in order to obtain the skill to produce y; for example, the employee needs experience in sales in order to obtain leadership skills. Then the employer decides to write the first and second periods wages on a mid-term contract at the beginning of the first period to induce I_c. Then, she chooses to write the third period wage on a short-term contract at the beginning of the third period in order to induce I_n. In this scenario, the employer first wishes to motivate the employee to induce I_c in order to accumulate employee's skill to produce x. After that, she wishes the employee to induce I_n in order to obtain the skill to produce y.

In the T-period case, a furthermore complicated combination of contracts are obtained as an equilibrium. For example, suppose that a skill to produce y depreciates with some depreciation rate. Then the employer wishes her employee to occasionally make efforts I_n to keep the skill to produce y at the certain level. Then, the employer repeats mid-term contracts. Suppose that every two periods the employer and employee bargain over wages. Then every two periods the employee has an incentive to invest I_n which makes up for depreciation.

3 Two-Period Model: The Benchmark

Before presenting the main models – three-period models and five-period models in Sections 4 and 5 – we investigate a two-period model in this section. The logic developed in this section will be used in the main models. There is a principal and an agent. We assume that both of them are risk neutral. There are two types of outputs: an observable and contractible output $x \geq 0$ and an observable but non-contractible output $y \geq 0$. There are two contractible output levels, x^H and x^L, where $x^H > x^L > 0$. The probabilities of x^H and x^L are denoted by $P^H \in [0,1]$ and $P^L = 1 - P^H$. There are two non-contractible output levels, θy^H and θy^L, where $y^H > y^L > 0$. Note that $\theta \geq 0$ is a parameter introduced for later use. The probabilities of y^H and y^L are denoted by $Q^H \in [0,1]$ and $Q^L = 1 - Q^H$. In the first period, the agent makes two types of investments, $I_c \geq 0$ and $I_n \geq 0$. We assume that both I_c and I_n are observable but non-contractible, and that P^H and Q^H in the second period are functions of I_c and I_n, denoted by $P^H(I_c)$ and $Q^H(I_n)$, respectively. As will be formally stated in Assumptions 1 and 2, we assume that the random variables x and y are stochastically independent and that $P^H = Q^H = 0$ in the first period. That is, we assume that the investments in the human capital made in the first period will increase the skills in the second periods onwards. The agent incurs disutilities in making investments, denoted by $D_c(I_c)$ and $D_n(I_n)$. Let $\delta \in (0,1)$ be the discount factor. The wages for each period are paid out at the end of each period, or after the realization of outputs in each period. Since only x is contractible, the wage depends only on the realization of x: the wages in the cases of x^H and x^L are denoted by w^H and w^L, respectively. w^i, $i = H, L$, in period t is denoted by $w_t^i, t = 1, 2$. Note that because of the risk-neutrality, w_2^i needs not depend on the realization of an output in the first period. We first investigate the model without the limited liability constraint, and later show that almost the same results can be obtained with the constraint.

Throughout this section we make the following three assumptions. The first assumption on D_c, D_n, P^H, and Q^H are standard.

Assumption 1 1. $\frac{dD_i}{dI_i} > 0$, $\frac{d^2D_i}{dI_i^2} > 0$, $D_i(0) = 0$, and $\frac{d^2D_i(0)}{dI_i^2} = 0$, $i = c, n$.

2. $\frac{dP^H}{dI_c} > 0$ and $\frac{d^2P^H}{dI_c^2} < 0$.

3. $\frac{dQ^H}{dI_c} > 0$ and $\frac{d^2Q^H}{dI_c^2} < 0$.

4. The random variables x and y are stochastically independent.

For simplicity, we make the following assumption.

Assumption 2 In the first period, the probabilities of x^H and y^H are zero.

By the above assumption, the principal need not determine w_1^H in the first period.

We suppose that the market of workers without firm-specific skills are competitive. We also assume that the agent obtains some firm-specific skills in the first period and hence he has bargaining power to negotiate his wage at the beginning of the second period.[8] Therefore, when the principal hires an agent without firm-specific skills, she posts a take-it-or-leave-it wage offer. After the agent has obtained the skills the principal and the agent bargain over wages at the beginning of the second period. For simplicity, we adopt Nash bargaining with the threat point set at $(0, 0)$. That is, we assume that their bargaining powers are the same and that if they lose a partner, they cannot find a new one, i.e., they can access the labor market just once and their reservation utilities are zero. It is worthwhile noting that we can obtain similar results even if they have different bargaining power or their reservation utilities are non-zero in the second period. Note that in the discussion of renegotiation-proofness in the following theorems, we consider the bargaining, where the status quo is the wage contract signed in the previous periods.

Assumption 3 When a contract is signed at the beginning of the first period, the principal posts a take-it-or-leave-it wage offer. When a contract is signed at the beginning of the second period, they Nash bargain over wages with the threat point held at $(0, 0)$.

In this section, we consider two types of wage contracts: a short-term contract and a long-term contract. In the short-term contract, the wages are determined at the beginning of each period and paid out at the end of each period. In the long-term contract, the wages for both periods are determined at the beginning of the first period but paid out at the

[8] Alternatively, we assume that the agent who made investments in the first period, that is the agent with $I_c > 0$ or $I_n > 0$, has bargaining power. We could assume that bargaining power is only given to the agent with $I_c > 0$ or $I_n > 0$, but we can obtain the same result even when we give bargaining power to the agent who did not make any investments.

end of each period. As will be shown later, the equilibrium contracts are renegotiation-proof. We also discuss limited liability constraint in Remark 1 at the end of this section, and demonstrate that almost the same results can be obtained when the constraint is imposed.

3.1 Short-Term Contract

Under the short-term contract, the principal and the agent sign the contract on the first period wage at the beginning of the first period, and they bargain over the second period wage at the beginning of the second period. The agent can make investments in human capital during the first period. By Assumption 3, the contracting problem of the short-term contract in the first period is a take-it-or-leave-it offer on the first period wage, subject to the individual rationality constraint and the incentive compatibility constraint on investments:

$$\max_{w_1^L, I_c, I_n} x^L - w_1^L + \theta y^L + \delta V_2^p(I_c, I_n) \tag{1}$$

$$\text{s.t. } w_1^L - D_c(I_c) - D_n(I_n) + \delta V_2^a(I_c, I_n) \geq u, \tag{2}$$

$$\begin{aligned} & w_1^L - D_c(I_c) - D_n(I_n) + \delta V_2^a(I_c, I_n) \\ & \geq w_1^L - D_c(I_c') - D_n(I_n') + \delta V_2^a(I_c', I_n'), \ ^\forall I_c', I_n', \end{aligned} \tag{3}$$

where u is the reservation utility determined in the competitive market, and $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$ are the principal's value and the agent's value when the investments are I_c and I_n. (2) and (3) are the the individual rationality constraint and the incentive compatibility constraint, respectively. Note that $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$ are determined by the backward induction given below.

The agent has bargaining power at the beginning of the second period. Applying Assumption 3, the principal and the agent Nash bargain over wages: for a given (I_c, I_n),

$$\max_{w_2^H, w_2^L} \left\{ \sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right\} \left\{ \sum_{j=H,L} P^j(I_c) w_2^j \right\},$$

where $g(I_n, \theta) = \sum_{i=H,L} Q^i(I_n)\theta y^i$. Since both players are risk neutral, their utilities are the same in the Nash bargaining solution and is equal to a half of the total utility, i.e., their utilities are

$$\frac{1}{2}\left\{\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right\},$$

and it is clearly equal to $V_2^p(I_c, I_n)$ and $V_2^a(I_c, I_n)$.

Thus in the first period, the agent chooses I_c and I_n satisfying the incentive compatibility constraint:

$$\max w_1^L - D_c(I_c) - D_n(I_n) + \frac{1}{2}\delta\left\{\sum_{j=H,L} P^j(I_c)x^j + g(I_n,\theta)\right\}. \tag{4}$$

The first-order condition yields

$$\frac{dD_c(I_c)}{dI_c} = \frac{1}{2}\delta\frac{dP^H(I_c)}{dI_c}(x^H - x^L), \tag{5}$$

and

$$\frac{dD_n(I_n)}{dI_n} = \frac{1}{2}\delta\frac{\partial g(I_n,\theta)}{\partial I_n}.$$

Note that by Assumption 1 the second-order condition is satisfied. Let the solutions of the above equation be I_c^* and I_n^*. On the other hand, by the individual rationality constraint, the principal clearly sets

$$w_1^L = D_c(I_c^*) + D_n(I_n^*) - \delta V_2^a(I_c^*, I_n^*) + u. \tag{6}$$

Then the principal's utility is obtained as follows:

$$\begin{aligned} x^L - w_1^L + \theta y^L + \delta V_2^p(I_c^*, I_n^*) &= x^L + \theta y^L - D_c(I_c^*) - D_n(I_n^*) + 2\delta V_2^p(I_c^*, I_n^*) - u \\ &= x^L + \theta y^L - D_c(I_c^*) - D_n(I_n^*) \\ &\quad +\delta\left\{\sum_{j=H,L} P^j(I_c^*)x^j + g(I_n^*,\theta)\right\} - u \end{aligned} \tag{7}$$

3.1.1 Long-Term Contract

Under the long-term contract, the principal and the agent sign the contract on the first and second periods wages at the beginning of the first period. The agent can make investments during the first period. By Assumption 3, the contracting problem is a take-it-or-leave-it offer on the first and second periods wages, subject to the individual rationality constraint and the incentive compatibility constraint on investments:

$$\max_{w_1^L, I_c, I_n, w_2^H, w_2^L} x^L - w_1^L + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c)(x^j - w_2^j) + g(I_n, \theta) \right) \tag{8}$$

$$\text{s.t.} \qquad w_1^L - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c) w_2^j \geq u, \tag{9}$$

$$\begin{aligned} & w_1^L - D_c(I_c) - D_n(I_n) + \delta \sum_{j=H,L} P^j(I_c) w_2^j \\ \geq \; & w_1^L - D_c(I_c') - D_n(I_n') + \delta \sum_{j=H,L} P^j(I_c') w_2^j, \ {}^\forall I_c', I_n'. \end{aligned} \tag{10}$$

The principal's utility is (8), and the agent's utility is the left-hand side of (9). (9) and (10) are expressions satisfying individual rationality and incentive compatibility of the agent.

By (10), $I_n^{**} = 0$ is chosen. Since both the principal and the agent are risk neutral, the joint utility

$$x^L + \theta y^L - D_c(I_c) + \delta \left(\sum_{j=H,L} P^j(I_c) x^j + g(0, \theta) \right) \tag{11}$$

is maximized with respect to I_c. Indeed, setting $w_2^j = x^j - r, j = H, L$, where r is the principal's utility in period two, (10) yields the following first-order condition for maximizing (11):

$$\frac{dD_c(I_c)}{dI_c} = \delta \frac{dP^H(I_c)}{dI_c}(x^H - x^L). \tag{12}$$

Let I_c^{**} be the solution. Then by the individual rationality constraint,

$$w_1^L = D_c(I_c^{**}) - \delta \sum_{j=H,L} P^j(I_c^{**}) w_2^j + u. \tag{13}$$

Then the principal's utility is expressed as follows:

$$\begin{aligned} & x^L - w_1^L + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c^{**})(x^j - w_2^j) + g(0, \theta) \right) \\ & = x^L - D_c(I_c^{**}) + \theta y^L + \delta \left(\sum_{j=H,L} P^j(I_c^{**}) x^j + g(0, \theta) \right) - u. \end{aligned} \tag{14}$$

3.1.2 Comparison of two types of contracts

First, comparing (5) and (12), the agent makes more investment I_c under the long-term contract than under the short-term contract, i.e., $I_c^* < I_c^{**}$.

When $\theta = 0$, the principal prefers the long-term contract to the short-term contract, i.e., (14) is larger than (7). Indeed, when $\theta = 0$, $I_n^* = 0$ is chosen even in the short-term contract and thus

$$(14) - (7) = -D_c(I_c^{**}) + \delta \sum_{j=H,L} P^j(I_c^{**})x^j - \left(-D_c(I_c^*) + \delta \sum_{j=H,L} P^j(I_c^*)x^j \right) > 0.$$

The last inequality follows from (12), i.e., I_c^* satisfies the first-order condition for maximizing $-D_c(I_c) + \delta \sum_{j=H,L} P^j(I_c)x^j$.

In order to investigate the effect of θ on the choice of contracts, we only need to investigate

$$-D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta)$$

in (4), since I_c^* does not depend on θ.

Let

$$\kappa(\theta) = \max_{I_n} -D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta)$$

and

$$h(\theta) = \arg\ \max_{I_n} -D_n(I_n) + \frac{1}{2}\delta g(I_n, \theta).$$

Then by envelope theorem

$$\kappa'(\theta) = \frac{1}{2}\delta \frac{\partial g(h(\theta), \theta)}{\partial \theta}.$$

Therefore κ is a strictly increasing function of θ and κ goes to $+\infty$ as θ goes to $+\infty$, since $\frac{\partial g(h(\theta),\theta)}{\partial \theta} = \sum_{i=H,L} Q^i(h(\theta))y^i \geq y^L > 0$. This implies that the principal's utility under the short-term contract (7) also goes to $+\infty$ as θ goes to $+\infty$. Since when $\theta = 0$ the principal strictly prefers the long-term contract to the short-term contract, then there exists a $\bar{\theta} > 0$ such that the principal prefers the long-term contract to the short-term contract for $\theta \in [0, \bar{\theta})$, and prefers the short-term contract to the long-term contract for $\theta \in (\bar{\theta}, \infty)$.

It is clear that the long-term contract in this section is renegotiation-proof. Indeed, because of the risk neutrality of the principal and the agent, any renegotiation on the wages does not induce Pareto improvement. Therefore, it is obvious that there is no need to discuss renegotiation-proofness when short-term contracts are repeated.

Theorem 1 1. *The investment for the contractible output x is larger under the long-term contract than under the short-term contract, i.e., $I_c^{**} > I_c^*$.*

2. *There exists a $\bar{\theta} > 0$ such that the principal prefers the long-term contract to the short-term contract at the beginning of the first period for $\theta \in [0, \bar{\theta})$, and prefers the short-term contract to the long-term contract for $\theta \in (\bar{\theta}, \infty)$, i.e., (7) is smaller than (14) for $\theta \in [0, \bar{\theta})$ and (7) is larger than (14) for $\theta \in (\bar{\theta}, \infty)$. Moreover, the equilibria are renegotiation-proof.*

Remark 1 Below, we discuss limited liability constraint. In the case of short-term contracts, we can set $w_2^H = w_2^L = V_2^a(I_c^*, I_n^*)$. Then by (6)

$$w_1^L + \delta w_2^i = D_c(I_c^*) + D_n(I_n^*) + u \geq 0, i = H, L$$

holds. Thus the limited liability constraint is always satisfied. In the case of long-term contracts, recall that $w_2^H = x^H - r$ and $w_2^L = x^L - r$, where r is the principal's utility in period two. Then by (13),

$$\delta r = w_1^L - D_c(I_c^{**}) + \delta \sum_{j=H,L} P^j(I_c^{**}) x^j - u.$$

Thus

$$w_1^L + \delta w_2^H > w_1^L + \delta w_2^L = \delta x^L + D_c(I_c^{**}) - \delta \sum_{j=H,L} P^j(I_c^{**}) x^j + u.$$

Since I_c^{**} does not depend on u, the RHS is positive for sufficiently large u. Thus the limited liability constraint is not binding for sufficiently large u. In the case that u is not sufficiently large, the limited liability constraint is binding in the long-term contract, and thus the total utility is smaller than the case without the constraint. Even in this case, when $\theta = 0$, the long-term contract is clearly better than the short-term contract.

Thus Theorem 1.2 still holds with smaller $\bar{\theta}$, since in the case of short-term contract the principal obtains the same gain as in the case of sufficiently large u and in the case of long-term contract the principal obtains smaller gain.

4 Three-period Model

4.1 Model

In this section, we assume that the principal and the agent live for three periods. In this case, some complicated combinations of contracts are chosen depending on the parameters. For example, in some cases, the principal chooses to contract the wage for the first period on a short-term contract, and contract the wages for both the second and third periods on a mid-term contract at the beginning of the second period. In some other cases, the principal may contract the wages for both the first and second periods on a mid-term contract, and contract the wage of the third period on a short-term contract at the beginning of the third period.

In order to investigate the three-period model, we introduce abilities (skills) and slightly change the notations. That is, we assume that the abilities increase by investments. Let $P^H(\alpha_c) \in [0,1]$ be the probability that x^H occurs when an ability corresponding to a contractible output is $\alpha_c \in [0,\infty)$. Let $P^L(\alpha_c) = 1 - P^H(\alpha_c)$. Let $Q^H(\alpha_n) \in [0,1]$ be the probability that y^H occurs when an ability corresponding to a non-contractible output is $\alpha_n \in [0,\infty)$. Let $Q^L(\alpha_n) = 1 - Q^H(\alpha_n)$. For a given parameter $\theta \geq 0$, $g(\alpha_n,\theta) = \sum_{i=H,L} Q^i(\alpha_n)\theta y^i$ denotes the expected value of non-contractible output. Let $f_c : R_+^2 \to R_+$ and $f_n : R_+^2 \to R_+$ be the transition functions of abilities. That is, for $i = c, n$, $\alpha_i' = f_i(I_i, \alpha_i)$ means that when the ability in the current period is α_i and the investment is I_i, the next period ability denoted by α_i', is $f_i(I_i, \alpha_i)$.

Throughout this section, we make the following assumptions. The first assumption is standard:

Assumption 4 1. $\frac{dD_i}{dI_i} > 0$, $\frac{d^2 D_i}{dI_i^2} > 0$, $D_i(0) = 0$, and $\frac{d^2 D_i(0)}{dI_i^2} = 0$, $i = c, n$.

2. $\frac{dP^H}{d\alpha_c} > 0$ and $\frac{d^2 P^H}{d\alpha_c^2} < 0$.

3. $\frac{dQ^H}{d\alpha_n} > 0$ and $\frac{d^2Q^H}{d\alpha_n^2} < 0$.

4. The random variables x and y are stochastically independent.

For simplicity, we make the following assumption:

Assumption 5 In the first period, the abilities α_c and α_n are zero, and $P^H(0) = Q^H(0) = 0$.

As in the previous section, we make the following assumption on the bargaining:

Assumption 6 When a contract is signed at the beginning of the first period, the principal posts a take-it-or-leave-it wage offer. When a contract is signed at the beginning of the second period or the third period, they Nash bargain over the wages with the threat point $(0, 0)$.

Note that in the discussion of renegotiation-proofness in the following theorems, we consider bargaining, where the status quo is the wage contract signed in the previous periods.

4.2 Equilibria

Below, we show that depending on θ, f_c, f_n, D_c, D_n, P^H, and Q^H, four types of equilibria can be obtained:

1. On the equilibrium path, the wages for each period are determined at the beginning of each period. (The short-short-short-term equilibrium contract)

2. On the equilibrium path, the wages for all periods are determined at the beginning of the first period. (The long-term equilibrium contract)

3. On the equilibrium path, the wage for the first period is determined at the beginning of the first period, and the wages for the second and third periods are determined at the beginning of the second period. (The short-mid-term equilibrium contract)

4. On the equilibrium path, the wages for the first and second periods are determined at the beginning of the first period, and the wage for the third period is determined at the beginning of the third period. (The mid-short-term equilibrium contract)

The rigorous definitions of the above equilibrium contracts are obtained by backward induction as in section three. That is, whenever the players sign a contract, they choose the one that is maximizing the discounted sum of the total utility. This is because in the second and third periods, each player obtains a half of the discounted sum, while in the first period, the principal obtains all gains. We define equilibrium contracts according to the lengths of contracts on equilibrium path. The rigorous definitions are rather complicated, and thus are given in the Appendix.

Remark 2 One might think that other types of equilibrium contracts could exist; namely, the wages for the first and third periods are determined at the beginning of the first period, and the wage for the second period is determined by Nash bargaining at the beginning of the second period. However, the outcome of this contract can be replicated by a short-mid-term contract.

4.3 Long-Term Contract and Short-Short-Short-Term Contract

If θ is sufficiently small, then it is clearly better to induce an incentive for I_c and thus a long-term contract is chosen. Note that this holds for any f_c, f_n, D_c, D_n, P^H, and Q^H satisfying the above assumptions. On the other hand, if θ is sufficiently large, then under an additional condition it is always better to induce an incentive for I_n and thus a short-short-short-term contract is chosen. The additional condition is that the investment I_n is sufficiently costly and the cost function is sufficiently convex. Then α_n is not saturated in all periods and the principal always wishes to induce an incentive for I_n. For simplicity, in this subsection we suppose $Q^H(\alpha_n) = \alpha_n$, $f_n(I_n, \alpha_n) = \min\{I_n + \alpha_n, 1\}$, and $D_n(I_n) = bI_n^2$, where $b > 0$ is a parameter. Note that f_c, D_c, and P^H can be any functions satisfying the above assumptions.

Theorem 2 *1. There exists a* $\underline{\theta} > 0$ *such that* $\forall\theta \leq \underline{\theta}$*, the equilibrium contract is of long-term. Moreover, it is renegotiation-proof.*

2. Suppose $b > \frac{1}{4}\theta(2\delta+\delta^2)(y^H - y^L)$*. Then there exists a* $\bar{\theta} > 0$ *such that* $\forall\theta \geq \bar{\theta}$*, the equilibrium contract is of short-short-short-term. (Note that it is clearly renegotiation-proof.)*

Proof[9]**:**

In (i)-(iv) below, we focus on each contract and obtain its total equilibrium utility from non-contractible output. We use backward induction, if necessary. Then in (v), we show that if θ is sufficiently small, then the equilibrium contract is a long-term, and otherwise it is a short-short-short-term. In (vi), we discuss renegotiation-proofness.

(i) First, we focus on a long-term contract. The agent chooses $I_{n1} = I_{n2} = 0$ and thus $g(\alpha_n, \theta) = \theta y^L$ holds for all periods. The total utility obtained from the non-contractible output, denoted L_n, is clearly $(1 + \delta + \delta^2)\theta y^L$.

(ii) Next, we focus on a short-short-short-term contract. The wages for the second and third periods are determined by Nash bargaining at the beginning of each period. Below, we only consider the utilities obtained from the non-contractible outputs. In the third period, the agent obtains a half of the total utility, i.e., $\frac{1}{2}\sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i$. Thus in the second period the agent chooses I_{n2} satisfying the incentive compatibility constraint:

$$\max_{I_{n2}} \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI_{n2}^2,$$

where $\alpha_{n3} = \min\{\alpha_{n2} + I_{n2}, 1\}$. Below, suppose that the optimal α_{n2} and α_{n3} are less than 1, i.e., the optimal I_{n1} and I_{n2} are determined by the first-order condition. Then

$$I_{n2}^* = \tfrac{1}{4b}\delta\theta(y^H - y^L).$$

Note that I_{n2}^* does not depend on α_{n2}. In the second period the agent obtains a half of

[9] We show the proof here and not in the Appendix, because the logic used in this proof is used in section 4.4 onwards.

the total utility

$$\frac{1}{2}\left(\sum_{i=H,L} Q^i(\alpha_{n2})\theta y^i - b(I_{n2}^*)^2\right) + \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n2} + I_{n2}^*)\theta y^i,$$

where $\alpha_{n2} = I_{n1}$. Thus in the first period the agent chooses I_{n1} satisfying the incentive compatibility constraint:

$$\max_{I_{n1}} \frac{1}{2}\left(\sum_{i=H,L} Q^i(I_{n1})\theta y^i - b(I_{n2}^*)^2\right) + \frac{1}{2}\delta \sum_{i=H,L} Q^i(I_{n1} + I_{n2}^*)\theta y^i - bI_{n1}^2.$$

Then the optimal I_{n1} is obtained as follows:

$$I_{n1}^* = \tfrac{1}{4b}\theta(\delta + \delta^2)(y^H - y^L).$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds, thus $\alpha_{n2}^* = I_{n1}^*$ and $\alpha_{n3}^* = I_{n1}^* + I_{n2}^*$ are indeed less than one because $I_{n1}^* + I_{n2}^* = \frac{1}{4b}\theta(2\delta + \delta^2)(y^H - y^L)$.

Then $S_n(\theta)$, the total utility obtained from the non-contractible output, is obtained as follows:

$$\begin{aligned} S_n(\theta) &= \theta y^L - b(I_{n1}^*)^2 + \delta\left(I_{n1}^*\theta y^H + (1 - I_{n1}^*)\theta y^L - b(I_{n2}^*)^2\right) \\ &\quad + \delta^2\left((I_{n1}^* + I_{n2}^*)\theta y^H + (1 - I_{n1}^* - I_{n2}^*)\theta y^L\right) \\ &= (1 + \delta + \delta^2)\theta y^L + \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta^2 + 3\delta + 1). \end{aligned}$$

(iii) Next, we focus on a mid-short-term contract. The agent's utility in the third period is clearly $\frac{1}{2}\sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i$. The wage for the second period is determined by a take-it-or-leave-it offer at the beginning of the first period. Thus the agent is only interested in α_{n3}, since the wage for the second period does not depend on α_{n2}. That is, the agent solves the following problem with respect to I_{n1} and I_{n2} in the first period:

$$\max_{I_{n1}, I_{n2}} \frac{1}{2}\delta^2 \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI_{n1}^2 - \delta b I_{n2}^2,$$

where $\alpha_{n1} = \min\{I_{n1}, 1\}$ and $\alpha_{n2} = \min\{\alpha_{n1}+I_{n2}, 1\}$. Suppose the optimal $\alpha_{n3} = I^*_{n1}+I^*_{n2}$ is less than one. Then I^*_{n1} and I^*_{n2} are obtained as follows:

$$\begin{aligned} I^*_{n1} &= \tfrac{1}{4b}\delta^2\theta(y^H - y^L) \\ I^*_{n2} &= \tfrac{1}{4b}\delta\theta(y^H - y^L). \end{aligned}$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds, thus $I^*_{n1} + I^*_{n2}$ is indeed less than one. Then, the total utility obtained from the non-contractible output denoted $MS_n(\theta)$ is obtained as follows:

$$MS_n(\theta) = (1 + \delta + \delta^2)\theta y^L + \frac{1}{16b}(y^H - y^L)^2\delta^3\theta^2(3\delta + 7).$$

(iv) Finally, we focus on a short-mid-term contract. The wage for the third period is determined at the beginning of the second period. Thus the agent chooses $I_{n2} = 0$ in the second period. Therefore, the agent solves the following problem with respect to I_{n1} in the first period:

$$\max_{I_{n1}} \frac{1}{2}\delta \sum_{i=H,L} Q^i(\alpha_{n2})\theta y^i + \frac{1}{2}\delta^2 \sum_{i=H,L} Q^i(\alpha_{n3})\theta y^i - bI^2_{n1},$$

where $\alpha_{n2} = \min\{I_{n1}, 1\}$ and $\alpha_{n3} = \alpha_{n2}$. Suppose the optimal $\alpha_{n3} = I^*_{n1}$ is less than one. Then it is obtained as follows:

$$I^*_{n1} = \tfrac{1}{4b}\theta(\delta + \delta^2)(y^H - y^L).$$

By the premise of the theorem, $b > \frac{1}{4}\theta(2\delta + \delta^2)(y^H - y^L)$ holds, thus I^*_{n1} is indeed less than one. Then $SM_n(\theta)$, the total utility obtained from the non-contractible output, is obtained as follows:

$$SM_n(\theta) = (1 + \delta + \delta^2)\theta y^L + \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta + 1)^2.$$

Below in (v), we compare the total utilities obtained both from the contractible and non-contractible outputs, and show that if θ is sufficiently small, then the equilibrium contract is of long-term, and if θ is sufficiently large, it is of short-short-short-term.

(v) In the case of long-term contract the total utilities obtained from the contractible output, denoted L_c, is the first-best as shown in the previous section. In the cases of the short-short-short-term contract, the mid-short-term contract, and the short-mid-term contract, the total utilities obtained from the contractible output, denoted S_c, MS_c, and SM_c, are obtained as in Section 3. As shown in Section 3, in the cases of short-short-short-term and short-mid-term contracts, the agent chooses I_{c1} so as to maximize a half of the second and third periods utilities obtained from the contractible output, and in the case of mid-short-term contract, the agent chooses I_{c2} so as to maximize a half of the third period utility obtained from the contractible output. Thus in these cases I_{c1} and/or I_{c2} are different from the first-best levels. Thus

$$S_c < L_c, SM_c < L_c, \text{ and } MS_c < L_c$$

hold for all $\theta > 0$. Since $S_n(0) = L_n = SM_n(0) = MS_n(0)$, there exists $\underline{\theta} > 0$ such that $^\forall\theta \leq \underline{\theta}$,

$$S_c + S_n(\theta) < L_c + L_n, SM_c + SM_n(\theta) < L_c + L_n, MS_c + MS_n(\theta) < L_c + L_n.$$

That is, $L_c + L_n$ is the largest. Thus the long-term contract is chosen for all $\theta \leq \underline{\theta}$. Suppose the contrary. Then the equilibrium contract derived from backward induction is one of short-short-short, mid-short-, or short-mid-term contracts, and the equilibrium total utility is larger than $L_c + L_n$. This contradicts the above inequalities. Note that the renegotiation-proofness will be discussed later.

On the other hand,

$$\begin{aligned} S_n(\theta) - L_n &= \frac{3}{16b}(y^H - y^L)^2\delta^2\theta^2(\delta^2 + 3\delta + 1) > 0, \\ S_n(\theta) - MS_n(\theta) &= \frac{1}{16b}(y^H - y^L)^2\delta^2\theta^2(2\delta + 3) > 0, \\ S_n(\theta) - SM_n(\theta) &= \frac{3}{16b}(y^H - y^L)^2\delta^3\theta^2 > 0 \end{aligned}$$

hold. Since $S_n(\theta) - L_n$, $S_n(\theta) - MS_n(\theta)$, and $S_n(\theta) - SM_n(\theta)$ are strictly increasing functions of θ and go to $+\infty$ as θ goes to $+\infty$, then there exists a $\bar{\theta} > 0$ such that $^\forall\theta \geq \bar{\theta}$,

$$L_c + L_n < S_c + S_n(\theta), SM_c + SM_n(\theta) < S_c + S_n(\theta), MS_c + MS_n(\theta) < S_c + S_n(\theta).$$

That is, $S_c+S_n(\theta)$ is the largest. Thus the short-short-short-term contract is chosen for all $^\forall\theta \geq \bar{\theta}$. Suppose the contrary. Then the equilibrium contract derived from the backward induction is one of long-, mid-short-, or short-mid-term contracts, and the equilibrium total utility is larger than $S_c + S_n(\theta)$. This contradicts the above inequalities.

(vi) Finally, we show that the above long-term equilibrium contract is renegotiation-proof. There is no need to discuss a renegotiation at the beginning of the third period, since the argument on renegotiation-proofness for the case of two-period model applies. Below, we investigate a renegotiation at the beginning of the second period. Suppose they choose a mid-term contract; that is, they sign a contract on the wages for the second and third periods. Suppose it is not renegotiation-proof; that is, their utilities resulting from this renegotiation are Pareto superior to those of the initial long-term contract at the beginning of the second period. Note that the threat point of the bargaining depends only on the wages signed in the first period. Moreover, $I_{n2} = 0$ and I_{c2} is the first-best level given the first period investment. Given this, suppose the agent chooses I_{c1} and I_{n1} maximizing the discounted sum of her expected utility. Then their utilities are even Pareto superior to those of the long-term contract at the beginning of the first period, since the outputs in the first period are always x^L and y^L. The investments in the case with the renegotiation are clearly equal to those of the short-mid-term contract. However, it has been shown that the total utility is larger under the long-term contract than under the short-mid-term contract $^\forall\theta \leq \underline{\theta}$. This is a contradiction. Suppose in the renegotiation at the beginning of the second period they choose a short-term contract; that is, they sign a contract on the wage for the second period only. Then a similar argument as the above applies and a contradiction can be obtained. Thus the long-term contract is renegotiation-proof if $\theta \leq \underline{\theta}$.

Clearly, there is no need to discuss the renegotiation-proofness of the short-short-short-term contract.

∎

4.4 Short-Mid-Term Contract

In the three-period case, more complicated combinations of contracts are chosen depending on f_n, D_n and Q^H. Suppose that I_n is relatively efficient and easily saturated. Then the principal chooses a short-term contract for the first period wage in order to induce I_n. After the agent makes an investment for I_n in the first period, she chooses a mid-term contract for the second and third periods wages at the beginning of the second period. That is, after the saturation of α_n, the principal wishes to induce I_c. In order to illustrate this point, in this subsection we suppose $f_n(I_n, \alpha_n) = I_n + \alpha_n$, $D_n(I_n) = aI_n$, where $a > 0$ is a parameter, and

$$Q^H(\alpha_n) = \begin{cases} b\alpha_n & \text{if } 0 \leq \alpha_n \leq \bar{\alpha}_n \\ 1 & \text{if } \bar{\alpha}_n \leq \alpha_n, \end{cases}$$

where $b > 0$ and $\bar{\alpha}_n = \frac{1}{b}$. Note that f_c, D_c, and P^H can be any functions satisfying the above assumptions.

Theorem 3 *Suppose $a < \frac{1}{2}\delta\theta b(y^H - y^L)$. Then there exists a $\bar{\theta} > 0$ such that, for all $\theta \geq \bar{\theta}$, the equilibrium contract is of short-mid-term. Moreover, it is renegotiation-proof.*

Proof:

As in the proof of Theorem 2, L_c, L_n, $S_c, S_n(\theta), MS_c, MS_n(\theta), SM_c$, and $SM_n(\theta)$ are similarly obtained.

Suppose a short-mid-term contract is chosen. By $a < \frac{1}{2}\delta\theta b(y^H - y^L)$, the marginal cost of I_n is strictly smaller than the marginal utility so that the agent chooses $I_{n1} = \bar{\alpha}_n$ in the first period. Thus,

$$SM_n(\theta) = \theta y^L + \delta\theta y^H + \delta^2\theta y^H - \frac{a}{b}$$

holds. Note that $S_n(\theta) = SM_n(\theta)$ clearly holds.

Then suppose a mid-term contract on the first and second periods wages is signed at the beginning of the first period. Since the marginal cost of I_n is strictly smaller than the marginal utility, then the agent chooses $I_{n1} = 0$ and $I_{n2} = \bar{\alpha}_n$ due to the discount factor. Thus

$$MS_n(\theta) = \theta y^L + \delta\theta y^L + \delta^2\theta y^H - \frac{\delta a}{b}.$$

For sufficiently large θ,

$$SM_n(\theta) - MS_n(\theta) = \delta\theta(y^H - y^L) - \frac{a}{b} + \frac{\delta a}{b} > 0$$

holds. Since $SM_n(\theta) - MS_n(\theta)$ and $SM_n(\theta)$ are strictly increasing and $SM_n(\theta) - MS_n(\theta) \to +\infty$ and $SM_n(\theta) \to +\infty$ as $\theta \to +\infty$,

$$L_c + L_n < SM_c + SM_n(\theta), \text{ and } MS_c + MS_n(\theta) < SM_c + SM_n(\theta)$$

hold for sufficiently large θ. Moreover, as shown in the previous section $S_c < SM_c$ holds and thus

$$S_c + S_n(\theta) < SM_c + SM_n(\theta).$$

Using the same argument as in the proof of Theorem 2, the short-mid-term contract is an equilibrium contract.

Finally, as shown in the previous section, the above equilibrium is clearly renegotiation-proof, since we should only consider the renegotiation in the third period and the argument on renegotiation-proofness in the case of two-period model applies. ∎

4.5 Mid-Short-Term Contract

Suppose that θ is sufficiently large and the agent should accumulate the first type ability (α_c) in order to obtain the second type ability (α_n); for example, the agent needs experience in sales in order to obtain leadership skills. Then the principal writes the first and second periods wages on a mid-term contract in order to induce I_c, and writes the third period wage on a short-term contract in order to induce I_n. (This is after the investment has been made.) In this subsection, we suppose $P^H(\alpha_c) = \alpha_c$, $f_c(I_c, \alpha_c) = \min\{I_c + \alpha_c, 1\}$, $D_c(I_c) = I_c^2$, $Q^H(\alpha_n) = \alpha_n$, $f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, and $D_n(I_n) = I_n^2$. Note that the transition function f_n depends not only on I_n and α_n but also on α_c. More precisely, if α_c is small, then the investment I_n is not efficient, since, in $f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, I_n is multiplied by α_c.

Theorem 4 [10] *There exists a $\bar{\theta}$ such that $^\forall\theta \geq \bar{\theta}$, the equilibrium contract is of mid-short-term. Moreover, it is renegotiation-proof.*

Proof:
By $f_n(I_n, \alpha_n, \alpha_c) = \min\{\alpha_c I_n + \alpha_n, 1\}$, α_n in the second period is zero even if $I_n > 0$ in the first period, because $\alpha_c = 0$ at the beginning of the first period. Thus the principal does not choose a short-term contract at the beginning of the first period; either a long-term contract or a mid-short-term contract is chosen. If a mid-short-term contract is chosen, then Q^H in the third period is $\alpha_{n3} = \min\{\alpha_{c2} I_{n2}, 1\}$. Note that the optimal α_{c2} is positive and an increasing function of θ in these contracts, since I_{c1} has a positive effect on the expected value of non-contractible output. Thus the agent maximizes

$$\delta(\alpha_{n3}\theta y^H + (1 - \alpha_{n3})\theta y^L) - I_{n2}^2$$

with respect to I_{n2}. Suppose the optimal α_{n3} is less than 1, then the optimal I_{n2} is equal to $\frac{1}{2}\delta\theta\alpha_{c2}(y^H - y^L)$. Therefore the following total utility from non-contractible output is obtained:

$$(1 + \delta + \delta^2)\theta y^L + \delta^2\theta^2(y^H - y^L)^2\alpha_{c2}(\frac{1}{2} - \frac{1}{4}\alpha_{c2}). \tag{15}$$

Suppose the optimal α_{n3} is equal to 1, then the agent chooses $I_{n2}^* = \frac{1}{\alpha_{c2}}$. Therefore the following total utility from non-contractible output is obtained:

$$(1 + \delta + \delta^2)\theta y^L + \delta^2\theta y^H - \frac{1}{\alpha_{c2}^2}. \tag{16}$$

If the principal chooses a long-term contract, then $I_{n1}^* = I_{n2}^* = 0$ and the total utility from non-contractible output is $(1 + \delta + \delta^2)\theta y^L$. Since the optimal α_{c2} is an increasing function of θ, then (15) and (16) go to $+\infty$ as $\theta \to +\infty$. Using the same arguments as in the proof of Theorem 2, there exists a $\bar{\theta}$ such that $^\forall\theta \geq \bar{\theta}$, the mid-short-term contract is an equilibrium contract.

Finally, we show that the above equilibria are renegotiation-proof. There is no need to discuss a renegotiation at the beginning of the third period, since the argument on renegotiation-proofness for the case of two-period model applies. Below, we investigate a renegotiation at the beginning of the second period. Suppose they choose a mid-term

[10] We can also show that a mid-short-term contract is an equilibrium if I_c is relatively efficient and easily saturated. That is, the principal chooses a mid-term contract on the first and second period wages in order to induce I_c, and after the investments he chooses a short-term contract on the third period wages in order to induce I_n.

contract; that is, they sign a contract on the wages for the second and third periods. Suppose it is not renegotiation-proof; that is, their utilities resulting from this renegotiation are Pareto superior to those of the initial mid-short-term contract at the beginning of the second period. Note that the threat point of the bargaining depends only on the wages signed at the beginning of the first period. Moreover, $I_{n2} = 0$ and I_{c2} are the first-best level given the investments made during the first period. Given this, suppose the agent chooses I_{c1} and I_{n1} maximizing the discounted sum of her expected utility. Then their utilities are even Pareto superior to those of the mid-short-term contract at the beginning of the first period, since the outputs in the first period is always x^L and y^L. The investments in the case with the renegotiation are clearly equal to those of the short-mid-term contract. However, it has been shown that the total utility is larger under the mid-short-term contract than under the short-mid-term contract $^{\forall}\theta \geq \bar{\theta}$. This is a contradiction. Suppose in the renegotiation at the beginning of the second period they choose a short-term contract; that is, they sign a contract on the wage for the second period only. Then a similar argument as the above applies and a contradiction arises. Thus the mid-short-term contract is renegotiation-proof $^{\forall}\theta \geq \bar{\theta}$. ∎

5 Five-Period Model

In this section, we assume that the principal and the agent live for five periods. In this case, much more complicated combinations of contracts could be chosen. For example, suppose that α_n is depreciates. Then the principal wishes to occasionally induce an incentive for I_n. For example, suppose that every two periods the principal and the agent bargain over wages. Then every two periods the agent has an incentive to invest for α_n which makes up for depreciation.

We assume that α_n is a function of investments in previous two periods; namely, $\alpha_{nt} = I_{n,t-2} + I_{n,t-1}$, i.e., the investments before period $t-2$ are totally depreciated. Moreover, we suppose $D_n(I_n) = I_n$, and

$$Q^H(\alpha_n) = \begin{cases} \alpha_n & \text{if } 0 \leq \alpha_n \leq 1 \\ 1 & \text{if } 1 \leq \alpha_n. \end{cases}$$

We adopt the same environment and the assumptions as in the three-period model besides the length of life and the arguments of f_n. Note that f_c, D_c, and P^H can be any functions satisfying the above assumptions.

Although even though many types of equilibrium contracts could exist in this model, we focus on the following contract.

> The one-two-two-term contract (which is short-mid-mid-term contract under five-period model): on the equilibrium path, the wage for the first period is determined at the beginning of the first period, the wages for the second and third periods are determined at the beginning of the second period, and the rest of the wages are determined at the beginning of the fourth period.

Note that the other combinations, such as the two-two-one-term (mid-mid-short-term) contract, are similarly defined.

Theorem 5 *Suppose $x^H - x^L \leq 2$, $y^H - y^L \geq 2$, and $\theta \geq 5$. Then there exists a $\bar{\delta} \in (0,1)$ such that the equilibrium contract is a one-two-two-term contract for $\delta \in (\bar{\delta}, 1]$.*

Proof:

Suppose $\delta = 1$. In the one-two-two-term contract, the agent clearly chooses $I_{n1} = I_{n3} = 1$ and $I_{n2} = I_{n4} = 0$, since in periods two and four the principal and the agent bargain over the wages, and in periods one and three the marginal cost of I_n is one and the marginal utility is $\frac{1}{2}(\delta + \delta^2)\theta(y^H - y^L) = \theta(y^H - y^L) \geq 10$ for $I_n < 1$. Thus $\alpha_n = 1$ holds in periods two, three, four, and five, and the total utility obtained from I_n is the first-best amount they can obtain, i.e.,

$$(\delta + \delta^2 + \delta^3 + \delta^4)\theta y^H - I_{n1} - \delta^2 I_{n3} = 4\theta y^H - I_{n1} - I_{n3} \geq 38.$$

Other than a one-two-two-term contract, logically there are three other combinations of contracts under five period principal-agent relationship: (i) combinations which involve a contract that covers at least three periods, e.g., a one-three-one-term contract, (ii) combinations which involve at most one contract that covers two periods, e.g., a one-one-two-one-term contract, and (iii) combinations which include two contracts which cover two periods, e.g., a two-two-one-term contract. In the case of (i), since there exists a period in which $\alpha_n = 0$, they lose at least $\theta(y^H - y^L) - 1 \geq 9$ in the total non-contractible utility and obtain at most $4(x^H - x^L) \leq 8$ in the total contractible utility, where 4 is the number of periods in which α_c could be increased. In the case of (ii), although the total utility obtained from I_n is the maximum amount, the total utility obtained from I_c is smaller than the case of the one-two-two-term contract. This is because combinations which fall in the category of (ii) involve more bargaining periods than the one-two-two-term contract. In the case of (iii), a two-two-one-period contract and a two-one-two-period contract are the only possibilities. In both cases, since $I_{n1} = 0$ in the first period , they lose at least $\theta(y^H - y^L) - 1 \geq 9$ in the total non-contractible utility and obtain at most $4(x^H - x^L) \leq 8$ in the total contractible utility, where 4 is the number of periods in which α_c could be increased. Therefore, using the same arguments as in the proof of Theorem 2, the one-two-two-term contract is an equilibrium contract.

Note that the above arguments apply to any $\delta > 0$ close to one. ∎

6 Concluding Remarks

In this paper, we investigated multi-period contracting problems with verifiable and unverifiable outputs. Confining our attention to simple wage contracts, we found that combined multi-period contracts arise as equilibrium contracts. One may wonder that some sophisticated contracts as discussed below may be more efficient. Rather than neglecting such contracts with the argument that parties are not sufficiently rational to use them in practice, we consider how such contracts can be applied in our setting.

It is known that, giving the residual control right to an investing party mitigates the hold-up problem to some extent. We consider the two-period model in Section 3 and suppose that the agent has the residual control right: if the bargaining in the second period breaks down, the agent can produce by himself and sells the output in the market. The verifiable output levels in the market are denoted by $\bar{x}^H$ and $\bar{x}^L$. Since $\bar{x}^H$ and $\bar{x}^L$

are smaller than those of x^H and x^L, we assume that, for $i = H, L$, $\bar{x}^i = \alpha x^i$ for some $\alpha \in (0,1)$. Similarly, we assume that, for $i = H, L$, $\bar{y}^i = \gamma y^i$ for some $\gamma \in (0,1)$. Therefore, the following Nash product would be maximized in the second period:

$$\max \left(\sum_{i=H,L} P^i(I_c)(x^i - w^i) + g(I_n, \theta) \right) \left(\sum_{i=H,L} P^i(I_c) w^i - \sum_{i=H,L} P^i(I_c)\bar{x}^i - \bar{g}(I_n, \theta) \right),$$

where $\bar{g}(I_n, \theta) = \sum_{i=H,L} Q^i(I_n)\theta \bar{y}^i$ Thus the agent would maximize the following:

$$\begin{aligned} &\max \frac{1}{2} \left(\sum_{i=H,L} P^i(I_c)x^i + g(I_n, \theta) - \sum_{i=H,L} P^i(I_c)\bar{x}^i - \bar{g}(I_n, \theta) \right) \\ &+ \sum_{i=H,L} P^i(I_c)\bar{x}^i + \bar{g}(I_n, \theta) - D_c(I_c) - D_n(I_n). \end{aligned}$$

The first-order condition is then:

$$\begin{aligned} \frac{dD_c}{dI_c} &= \frac{1}{2} \frac{dP^H(I_c)}{dI_c}(x^H - x^L + \bar{x}^H - \bar{x}^L) = \frac{1}{2} \frac{dP^H(I_c)}{dI_c}(1+\alpha)(x^H - x^L) \\ \frac{dD_n}{dI_n} &= \frac{1}{2} \left(\frac{\partial g(I_n, \theta)}{\partial I_n} + \frac{\partial \bar{g}(I_n, \theta)}{\partial I_n} \right) = \frac{1}{2} \frac{dQ^H(I_n)}{dI_n}(1+\gamma)(y^H - y^L). \end{aligned}$$

Thus the principal is more likely to choose a short-term contract and the total utility would become larger than the case without the residual control right, since the agent has an incentive to invest more. However, I_c is less than the first-best levels. Thus Theorem 1 clearly holds with a smaller $\bar{\theta}$.

Edlin and Reichelstein [1996] show that in a certain environment, where the threat point of a renegotiation is a function of investments, an appropriately chosen initial contract can provide the right incentive for investment and can lead the first-best output. As shown in the proof of renegotiation-proofness of the long-term contract in the three-period model (Theorem 2), the investments do not affect the threat point of the renegotiation in our case. Thus the initial contracting has no value, as verified in Che and Hausch [1999].

It is known that the "shoot the liar mechanism" can attain the first-best output provided that the parties can commit themselves not to renegotiate a contract. (See, for example, Moore and Repullo [1988].) In the mechanism, both parties report their observations of investments to a third party. If their reports match, then the production takes place and the wage corresponding to the investment is paid, and otherwise they are penalized by the third party. Moore and Repullo show that any outputs can be implemented as a subgame-perfect equilibrium by using a sequential type of this mechanism. However, such an equilibrium is clearly not renegotiation-proof. Since we assumed that the parties can NOT commit themselves not to renegotiate a contract, such a mechanism is not available in our case.

Maskin and Tirole [1999b] show that an "option to sell contract" can induce the right incentive to invest in some environments. If we apply this to our case, then the story might be as follows: the principal has a right to sell her property to the agent, and she exercises the option if the investment is not efficient. However, such a contract is not feasible in our case under the limited liability constraint. Note that, as shown in Remark 1, all the theorems still hold with slight modifications even if we impose the constraint.

Finally, in the case with risk averse parties and unverifiable outputs, Maskin and Tirole [1999a] argue that as long as the parties can foresee the probabilities of their possible payoffs, then even with renegotiation the parties can attain the same utilities as when full description is possible in the contract. This argument clearly does not apply to our case, since both parties are risk neutral.

A Appendix

In the three-period model, the equilibrium contract is obtained by the backward induction as below. Since the players are risk neutral, the wages need not depend on the realization of outputs in the previous period. For example, the wage for the third period in the case of x^H, denoted w_3^H, does not depend on the realization of the output in the second period.

A.1 The Third Period

Suppose that the abilities at the beginning of the third period are α_c and α_n. If the wage for the third period is not determined yet at the beginning of the period the contracting problem is expressed as follows:

$$\max_{w_3^i} \left(\sum_{i=H,L} P^i(\alpha_c) \cdot (x^i - w_3^i) + g(\alpha_n, \theta) \right) \left(\sum_{i=H,L} P^i(\alpha_c) w_3^i \right) \tag{17}$$

The values of principal and agent when the optimum value is plugged in can be written as $V_3^p(\alpha_c, \alpha_n)$ and $V_3^a(\alpha_c, \alpha_n)$, respectively. Both of them are clearly equal to

$$\frac{1}{2} \left(\sum_{i=H,L} P^i(\alpha_c) x^i + g(\alpha_n, \theta) \right).$$

A.2 The Second Period

Suppose that the abilities at the beginning of the second period are α_c and α_n. If the wage for the second period is not determined yet at the beginning of the second period, the contracting problem for the short-term second period is expressed as follows.

$$\begin{aligned} Max_{w_2^H, w_2^L, I_{c2}, I_{n2}} \quad & \left(\sum_{i=H,L} P^i(\alpha_c) \cdot (x^i - w_2^i) + g(\alpha_n, \theta) + \delta V_3^p(f_c(I_{c2}, \alpha_c), f_n(I_{n2}, \alpha_n)) \right) \times \\ & \left(u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta V_3^a(f_c(I_{c2}, \alpha_c), f_n(I_{n2}, \alpha_n)) \right) \\ \text{s.t.} \quad & u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta V_2^a(f_c(I_{c2}, \alpha_c), f_n(I_{n2}, \alpha_n)) \\ & \geq u_2^a(w_2^H, w_2^L, I'_{c2}, I'_{n2}) + \delta V_2^a(f_c(I'_{c2}, \alpha_c), f_n(I'_{n2}, \alpha_n)), \ {}^\forall I'_{c2}, \ I'_{n2}, \end{aligned}$$

where

$$u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) = \sum_{i=H,L} P^i(\alpha_c) w_2^i - D_n(I_{n2}) - D_c(I_{c2}).$$

The principal's and agent's values when the optimum value is plugged in are written as $\hat{V}_2^p(\alpha_c, \alpha_n)$ and $\hat{V}_2^a(\alpha_c, \alpha_n)$, respectively, and they are clearly equal.

If a mid-term contract is chosen at the second period, then the bargaining problem is as follows:

$$\begin{aligned} & Max_{w_2^H, w_2^L, w_3^H, w_3^L, I_{c2}, I_{c3}} \left(u_2^p(w_2^H, w_2^L) + \delta u_3^p(w_3^H, w_3^L, I_{c2}, I_{n2}) \right) \\ & \left(u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta u_3^a(I_{c2}, w_3^H, w_3^L) \right) \end{aligned}$$

$$\begin{aligned} \text{s.t.} \ & u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta u_3^a(I_{c2}, w_3^H, w_3^L) \\ & \geq u_2^a(w_2^H, w_2^L, I'_{c2}, I'^a_{n2}(I'^H_{c2}, w_3^L), \ {}^\forall I'_{c2}, I'_{n2}, \end{aligned} \tag{18}$$

where

$$\begin{aligned} u_2^p(w_2^H, w_2^L) &= \sum_{i=H,L} P^i(\alpha_c)(x^i - w_2^i) + g(\alpha_n, \theta), \\ u_3^p(w_3^H, w_3^L, I_{c2}, I_{n2}) &= \sum_{j=H,L} P^j(f_c(I_{c2}, \alpha_c))(x^j - w_3^j) + g(f_n(I_{n2}, \alpha_n), \theta) \\ u_3^a(I_{c2}, w_3^H, w_3^L) &= \sum_{j=H,L} P^j(f_c(I_{c2}, \alpha_c))w_3^j - D_c(I_{c2}). \end{aligned}$$

The principal's and agent's values when the optimum value is plugged in are written as $\tilde{V}_2^p(\alpha_c, \alpha_n)$ and $\tilde{V}_2^a(\alpha_c, \alpha_n)$, respectively, and they are equal. Clearly,

$$V_2^p(\alpha_c, \alpha_n) = \max\{\hat{V}_2^p(\alpha_c, \alpha_n), \tilde{V}_2^p(\alpha_c, \alpha_n)\}.$$

Since $\hat{V}_2^p = \hat{V}_2^a$ and $\tilde{V}_2^p = \tilde{V}_2^a$, if $V_2^p(\alpha_c, \alpha_n) = \hat{V}_2^p(\alpha_c, \alpha_n)$, then $V_2^a(\alpha_c, \alpha_n) = \hat{V}_2^a(\alpha_c, \alpha_n)$, and otherwise $V_2^a(\alpha_c, \alpha_n) = \tilde{V}_2^a(\alpha_c, \alpha_n)$.

A.3 The First Period

The contracting problem for the short-term first period is expressed as follows.

$$\begin{aligned} Max_{w_1^L, I_{c1}, I_{n1}} \quad & x^L - w_1^L + \theta y^L + \delta V_2^p(f_c(I_{c1}, 0), f_n(I_{n1}, 0)) \\ \text{s.t.} \quad & w^L - D_c(I_{c1}) - D_n(I_n1) + \delta V_2^a(f_c(I_{c1}, 0), f_n(I_{n1}, 0)) \geq u \\ & w_1^L - D_c(I_{c1}) - D_n(I_{n1}) + \delta V_2^a(f_c(I_{c1}, 0), f_n(I_{n2}, 0)) \\ & \geq w_1^L - D_c(I'_{c1}) - D_n(I'_{n1}) + \delta V_2^a(f_c(I'_{c1}, 0), f_n(I'_{n1}, 0)), \ ^\forall I'_{c1}, \ I'_{n1}. \end{aligned}$$

The principal's and agent's utilities when the optimum value is plugged in can be written as $\hat{V}_1^p(0,0)$ and $\hat{V}_1^a(0,0)$, respectively.

If the wages for the first and second periods are determined in the contract at the first period, the bargaining problem is as follows:

$$\begin{aligned} Max \quad & x^L - w_1^L + \theta y^L + \delta u_2^p(w_2^H, w_2^L, I_{c1}, I_{c2}) + \delta^2 V_3^p(\alpha_c(I_{c1}, I_{c2}), \alpha_n(I_{n1}, I_{n2})) \\ \text{s.t.} \quad & u_1^a(w_1^L, I_{c1}, I_{n1}) + \delta u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta^2 V_3^a(\alpha_c(I_{c1}, I_{c2}), \alpha_n(I_{n1}, I_{n2})) \geq u \\ & u_1^a(w_1^L, I_{c1}, I_{n1}) + \delta u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) + \delta^2 V_3^a(\alpha_c(I_{c1}, I_{c2}), \alpha_n(I_{n1}, I_{n2})) \\ & \geq u_1^a(w_1^L, I'_{c1}, I'^a_{n1}(w_2^H, w_2^L, I'_{c2}, I'^2_{n2} V_3^a(\alpha_c(I'_{c1}, I'_{c2}), \alpha_n(I'_{n1}, I'_{n2})), \ ^\forall I'_{c1}, I'_{c2}, I'_{n1}, I'_{n2}, \end{aligned}$$

where $\alpha_c(I_{c1}, I_{c2}) = f_c(I_{c2}, f_c(I_{c1}, 0))$, $\alpha_n(I_{n1}, I_{n2}) = f_n(I_{n2}, f_c(I_{n1}, 0))$,

$$\begin{aligned} u_2^p(w_2^H, w_2^L, I_{c1}, I_{c2}) &= \sum_{j=H,L} P^j(f_c(I_{c1}, 0))(x^j - w_2^j) + g(f_n(I_{n1}, 0), \theta), \\ u_1^a(w_1^L, I_{c1}, I_{n1}) &= w_1^L - D_n(I_{n1}) - D_c(I_{c1}), \\ u_2^a(w_2^H, w_2^L, I_{c2}, I_{n2}) &= \sum_{j=H,L} P^j(f_c(I_{c1}, 0))w^j - D_c(I_{c2}) - D_n(I_{n2}) \end{aligned}$$

The principal's and agent's utilities when the optimum value is plugged in can be written as $\tilde{V}_1^p(0,0)$ and $\tilde{V}_1^a(0,0)$, respectively.

If the wages for all periods are determined in the contract at the first period, then the bargaining problem is as follows:

$$\begin{array}{rl} Max & x^L - w_1^L + \theta y^L + \delta u_2^p(w_2^H, w_2^L, I_{c1}, I_{n1}) + \delta^2 u_3^p(w_3^H, w_3^L, I_{c1}, I_{n1}, I_{c2}, I_{n2}) \\ \text{s.t.} & u_1^a(w_1^L, I_{c1}, I_{n1}) + \delta u_2^a(w_2^H, w_2^L, I_{c1}, I_{c2}, I_{n2}) + \delta^2 u_3^a(w_3^H, w_3^L, I_{c1}, I_{c2}) \geq u \\ & u_1^a(w_1^L, I_{c1}, I_{n1}) + \delta u_2^a(w_2^H, w_2^L, I_{c1}, I_{c2}, I_{n2}) + \delta^2 u_3^a(w_3^H, w_3^L, I_{c1}, I_{c2}) \\ & \geq u_1^a(w_1^L, I'_{c1}, I'^{a}_{n1}(w_2^H, w_2^L, I'_{c1}, I'_{c2}, I'^{2}_{n2}u_3^a(w_3^H, w_3^L, I'_{c1}, I'_{c2}), \ {}^{\forall}I'_{ct}, I'_{nt}, t = 1, 2. \end{array}$$

where

$$\begin{aligned} u_2^p(w_2^H, w_2^L, I_{c1}, I_{n1}) &= \sum_{j=H,L} P^j(f_c(I_{c1}, 0))(x_2^j - w_2^j) + g(f_n(I_{n1}, 0), \theta), \\ u_3^p(w_3^H, w_3^L, I_{c1}, I_{n1}, I_{c2}, I_{n2}) &= \sum_{k=H,L} P^k(f_c(f_c(I_{c1}, 0)))(x^k - w_3^k) + g(f_n(I_{n2}, f_n(I_{n1}, 0)), \theta), \\ u_1^a(w_1^L, I_{c1}, I_{n1}) &= w_1^L - D_c(I_{c1}) - D_n(I_{n1}), \\ u_2^a(w_2^H, w_2^L, I_{c1}, I_{c2}, I_{n2}) &= \sum_{j=H,L} P^j(f_c(I_{c1}, 0))w_2^j - D_c(I_{c2}) - D_n(I_{n2}), \\ u_3^a(w_3^H, w_3^L, I_{c1}, I_{c2}) &= \sum_{k=H,L} P^k(f_c(I_{c2}, f_c(I_{c1}, 0)))w_3^k \end{aligned}$$

The principal's and agent's values when the optimum value is plugged in can are written as $\check{V}_1^p(0,0)$ and $\check{V}_1^a(0,0)$, respectively. In the first period, the principal posts a take-it-or-leave-it wage offer, choosing the largest one among $\{\hat{V}_1^p(0,0), \tilde{V}_1^p(0,0), \check{V}_1^p(0,0)\}$, i.e.,

$$V_1^p(0,0) = \max\{\hat{V}_1^p(0,0), \tilde{V}_1^p(0,0), \check{V}_1^p(0,0)\}.$$

If $V_1^p(0,0) = \hat{V}_1^p(0,0)$, then $V_1^a(0,0) = \hat{V}_1^a(0,0)$, if $V_1^p(0,0) = \tilde{V}_1^p(0,0)$, then $V_1^a(0,0) = \tilde{V}_1^a(0,0)$, and otherwise $V_1^a(0,0) = \check{V}_1^a(0,0)$.

There are four types of equilibrium contracts:

1. The short-short-short-term equilibrium contract: On the equilibrium path, if wages for each period are determined at the beginning of each period in an equilibrium contract, it is called a short-short-short-term equilibrium contract, i.e., the case that $V_1^p(0,0) = \hat{V}_1^p(0,0)$ and $V_2^p(\alpha_c, \alpha_n) = \hat{V}_1^p(\alpha_c, \alpha_n)$.

2. The long-term equilibrium contract: On the equilibrium path, if wages for all periods are determined at the beginning of the first period, it is called a long-term equilibrium contract, i.e., the case that $V_1^p(0,0) = \check{V}_1^p(0,0)$.

3. The short-mid-term equilibrium contract: On the equilibrium path, if the wage for the first period is determined at the beginning of the first period, and the rest of wages are determined in the second period, it called a short-mid-term equilibrium contract, i.e., the case that $V_1^p(0,0) = \hat{V}_1^p(0,0)$ and $V_2^p(\alpha_c, \alpha_n) = \tilde{V}_2^p(\alpha_c, \alpha_n)$.

4. The mid-short-term equilibrium contract: On the equilibrium path, if the wages for the first and second periods are determined at the beginning of the first period, and the rest of the wages are determined at the beginning of the third period, it is called a mid-short-term equilibrium contract, i.e., the case that $V_1^p(0,0) = \tilde{V}_1^p(0,0)$.

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