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Hotelling Meets Weber*

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Abstract

In this paper, spatial competition between two sellers in a market (Hotelling, 1929) and total transportation costs minimization (Weber, 1909) are combined, and equilibrium and optimum locations of firms are analyzed along with the consequent policy implications. We reveal that when the output prices are fixed and equal, then both firms agglomerate at the market center, irrespective of the distribution of inputs. Further, we also show that the middle point of firm locations in Hotelling's model is identical to the Weber's point. Finally, it is indicated that the location of firms in Hotelling's model is far from the socially optimal location.

Keywords: Hotelling, Weber, location competition, social welfare.

JEL Classification: D43, L13

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1 Introduction

In their respective economic theories, Weber (1909) considered the location of a monopolist's manufacturing firm, whereas Hotelling (1929) considered the location of duopolists' retail firms. It is interesting to imagine what would have happened if they met early in the early 20th century.

Weber examined the least shipping cost location of a manufacturing firm with respect to the locations of input materials and consumer demand. In relation to this, Weber established the Weber triangle or the locational triangle. In this triangle, there is one product and two sources of raw material and the main interest of a monopolist is to minimize the transportation costs of two inputs to produce one output.

On the other hand, Hotelling dealt with interfirm competition between duopolists in location and price of an identical good. Each duopolist attempts to maximize their profit by selling an identical good to consumers who are equally distributed over a line segment. Hotelling ignored the input materials and focused only on the output market, while Weber took both inputs and output into account. Weber (1909) is more realistic in terms of the fact that manufacturing firms use inputs in producing a final product. However, Hotelling (1929) is more realistic in terms of the fact that there is competition between firms.

The purpose of this paper is to combine the two classical location theories. Specifically, we consider a situation in which two firms use input materials that involve transportation costs a la Weber (1909), and compete in location and price a la Hotelling (1929). This is a generalization of Weber's transportation cost minimization and Hotelling's profit maximization. In this paper, it is shown that Weber's firm location coincides with the midpoint of Hotelling's firm locations. The coincidence implies the similarity between these two classical location theories, the establishment of which is the main objective of this study.

To the best of our knowledge, there is no existing literature that has attempted such a combination except Hansen et al. (1986) and Birkin and Wilson (1986a, b). Hansen et al. (1986) indicated that the Hotelling's duopolists and the Weber's monopolist will always choose to be located at the place of median demand when consumers are distributed at vertices of a symmetric network. However, their model does not involve

input transportation costs and price competition. On the other hand, in this paper, firms consider the transportation of not only the output but also inputs and compete in terms of not only location but also price. Birkin and Wilson (1986a, b) made an ambitious attempt to unify location models including Weber (1909) and Hotelling (1929). However, they did not consider strategic behavior of firms and focused more on geography rather than economics.

The remainder of this paper is organized as follows. Section 2 considers Weber's location problem for an arbitrary number of input materials. Section 3 studies a location game and a location-then-price game in Hotelling's spatial competition (Anderson, et al. 1992). Section 4 discusses the social welfare by analyzing the first-best and second-best optima. The final section presents the concluding remarks.

2 Weber's monopolist location

A Weberian monopolist establishes a firm (= plant) at location $x \in \mathbb{R}$, uses n raw materials (= inputs) to produce final products, and distributes them to all consumers. Suppose that inputs are distributed discretely and consumers are distributed continuously on a line segment. Specifically, input i ($= 1, 2, \dots, n$) is located at $m_i \in \mathbb{R}$ and consumers are uniformly distributed over a unit line segment $[0, 1]$, as illustrated in Figure 1. Each consumer buys only one unit of the final product from the monopolist. The objective of the monopolist is to minimize the sum of the input and output transportation costs by choosing his plant location.

It is assumed that the transportation costs of inputs and outputs are proportional to the square of the distance,¹ and the unit transportation rates of inputs and outputs are given by t and 1, respectively. Further, assume that producing one unit of the output requires (r_1, r_2, \dots, r_n) units of inputs. Without loss of generality, all input prices are

¹We consider quadratic transportation costs rather than linear ones throughout the paper. This is because the existence of price equilibrium is guaranteed only when the output transportation costs are quadratic (d'Aspremont, Gabszewicz, and Thisse, 1979). Otherwise, we cannot obtain SPNE for any pair of firm locations in Hotelling's spatial competition in Section 3.2. In reality, most transportation costs are often concave in terms of distance. However, when the time costs of transportation are included, the total transportation costs can become convex in distance.

zero; however, distance-related transportation costs are incurred. Then, the total cost is expressed as

$$\begin{aligned} \min_x \text{TC}(x) &= \sum_{i=1}^n t_i(m_i - x)^2 + \int_0^1 (x - z)^2 dz \\ &= S_2 - 2Sx + Tx^2 + x^2 - x + \frac{1}{3}, \end{aligned}$$

where x is the monopolist's location, $t_i \equiv tr_i$, $T \equiv t \sum_{i=1}^n r_i$, $S \equiv t \sum_{i=1}^n r_i m_i$, and $S_2 \equiv t \sum_{i=1}^n r_i m_i^2$.

Solving the first-order condition yields the following optimal location:

$$x^W = \frac{2S + 1}{2(T + 1)}, \tag{1}$$

which is the so-called *Weber's point*. The second-order conditions are obviously satisfied.

When the input transportation rate t is rather low, Weber's point approaches the median point of the consumer distribution, i.e., the *market center*:

$$\lim_{t \rightarrow 0} x^W = \frac{1}{2}.$$

When the input transportation rate reaches infinity, Weber's point is equal to the *center of gravity* of the input distribution:

$$\lim_{t \rightarrow \infty} x^W = \frac{S}{T} = \frac{\sum_i t_i m_i}{\sum_i t_i}.$$

This is nothing less than the principle of median location (Alonso, 1975) because the median location of the inputs and output are always somewhere between the market center and the center of gravity.

3 Hotelling's spatial competition

3.1 Location competition with inputs

In the original Hotelling's (1929) spatial competition, two retail firms sell products to consumers without using any input materials. Consumers are assumed to be uniformly distributed over a line segment $[0, 1]$, visit either one of the firms that provides the

lowest full price of a good, and purchase exactly one unit of the product. The full price is the sum of the mill price of a product (= output) and the transportation cost of the product to the firm's location. As done in the previous section, it is assumed that the transportation cost of the output is equal to the square of the distance.

The utility of a consumer who resides at x and purchases one unit of the product from firm k ($= 1, 2$) is given by

$$u_k = y - p_k - (x_k - x)^2, \quad (2)$$

where y is the consumer's income, p_k is the mill price of a product sold by firm k , and x_k is the location of firm k . Further, it is assumed that firms cannot be situated at the same location: $x_1 \neq x_2$. Let \hat{x} be the location of marginal consumers who are indifferent in terms of purchasing from either firm. Because the full prices are equalized at \hat{x} ,

$$p_1 + (\hat{x} - x_1)^2 = p_2 + (\hat{x} - x_2)^2 \quad (3)$$

should hold. In this section, there is no price competition; thus, the mill prices are fixed and equal to \bar{p} . Solving (3) yields the location of the marginal consumers, which is the market boundary between the two firms:

$$\hat{x} = \frac{x_1 + x_2}{2}.$$

In this sequence, we introduce the transportation costs of input materials into the Hotelling's spatial competition model in order to combine Weber (1909) and Hotelling (1929). Firms need to use r_i units of input i located at m_i for $i = 1, 2, \dots, n$ in order to produce one unit of the homogeneous product. Although $n = 2$ in Weber's triangle, n can be an arbitrary number. When $n = 0$, it degenerates to the original Hotelling's spatial competition. In order to ship input i from its location m_i to the firm's location, firms must incur the unit transportation rate t for each input.

Then, the profit of each firm is given by

$$\begin{aligned} \pi_1 &= [\bar{p} - \sum_i t_i (m_i - x_1)^2] \hat{x}, \\ \pi_2 &= [\bar{p} - \sum_i t_i (m_i - x_2)^2] (1 - \hat{x}). \end{aligned}$$

Since location is the only strategic variable, each firm selects its location in order to maximize its profit. We assume that

$$\bar{p} > \max \left\{ \sum_i t_i (m_i - x_1)^2, \sum_i t_i (m_i - x_2)^2 \right\}$$

so that the profit of each firm is nonnegative.

The first-order condition for firm 1 is

$$\begin{aligned}\frac{\partial \pi_1}{\partial x_1} &= \frac{1}{2} \left[\bar{p} - \sum_i t_i (m_i - x_1)^2 \right] + 2 \sum_i t_i (m_i - x_1) \hat{x} \\ &> 2 \sum_i t_i (m_i - x_1) \hat{x} \\ &= 2(S - Tx_1) \hat{x}.\end{aligned}$$

Then, we have

$$\frac{\partial \pi_1}{\partial x_1} > 0 \quad \forall x_1 \leq \frac{S}{T}, \quad (4)$$

where S/T is the center of gravity of inputs. Similarly, for firm 2, we obtain

$$\begin{aligned}\frac{\partial \pi_2}{\partial x_2} &= -\frac{1}{2} \left[\bar{p} - \sum_i t_i (m_i - x_2)^2 \right] + 2 \sum_i t_i (m_i - x_2) (1 - \hat{x}) \\ &< 2 \sum_i t_i (m_i - x_2) (1 - \hat{x}) \\ &= 2(S - Tx_2) (1 - \hat{x}),\end{aligned}$$

hence,

$$\frac{\partial \pi_2}{\partial x_2} \leq 0 \quad \forall x_2 \geq \frac{S}{T}. \quad (5)$$

When $x_1 < x_2$, the following three cases may arise. (i) If $x_1 \leq S/T \leq x_2$, then both firms choose to locate back-to-back at the center of gravity from (4) and (5). (ii) If $x_1 < x_2 \leq S/T$, then firm 1 moves to the right from (4). If firm 2 moves to the left, both firms locate back-to-back somewhere at $x \leq S/T$. If firm 2 moves to the right, it will hit S/T , which is the upper bound from (5); hence, both firms locate back-to-back at S/T . (iii) If $S/T \leq x_1 < x_2$, similar to case (ii) both firms locate back-to-back at S/T .

In any case, both firms locate back-to-back. If the back-to-back locations are $x > 1/2$, then firm 1's profit π_1 is higher than π_2 . This is because firm 1's revenue $\bar{p}\hat{x}$ is higher while the input transportation costs for both firms 1 and 2 are almost the same. As a result, firm 2 has an incentive to jump and locate on the left side and just next to firm 1. Then, their profits are reversed, which causes firm 1 to jump to the left, and so

on. Such a jumping process continues until they reach the market center. If the location is $x < 1/2$, we obtain the same result. Thus, we have the following proposition.

Proposition 1. *Both firms locate at the market center in the Hotelling's location competition with a fixed equal price and an arbitrary number of inputs.*

This proposition indicates that the outcome of the Hotelling's location competition is invariant to the number and location of input materials.

3.2 Location-then-price competition with inputs

In this section, we introduce price competition to the model presented in the previous section. In other words, the two firms simultaneously select their locations x_1 and x_2 in the first stage, and then simultaneously choose their mill prices p_1 and p_2 in the second stage.

Solving (3) yields the market boundary between the two firms:

$$\hat{x} = \frac{p_1 - p_2 + x_1^2 - x_2^2}{2(x_1 - x_2)}. \quad (6)$$

In order to obtain the SPNE, we solve the second-stage price equilibrium first. Each firm maximizes its profit :

$$\begin{aligned} \pi_1 &= [p_1 - \sum_i t_i (m_i - x_1)^2] \hat{x}, \\ \pi_2 &= [p_2 - \sum_i t_i (m_i - x_2)^2] (1 - \hat{x}) \end{aligned} \quad (7)$$

with respect to its price. Solving the first-order conditions yields the following unique equilibrium prices:

$$\begin{aligned} p_1^H &= \frac{1}{3} [(x_2 - x_1)(2 + x_1 + x_2) + \sum_i t_i [2(m_i - x_1)^2 + (m_i - x_2)^2]], \\ p_2^H &= \frac{1}{3} [(x_2 - x_1)(4 - x_1 - x_2) + \sum_i t_i [(m_i - x_1)^2 + 2(m_i - x_2)^2]]. \end{aligned} \quad (8)$$

The second-order conditions are also satisfied. Including the prices (8) into the profits (7) and manipulating, we obtain profits as functions of locations x_1 and x_2 :

$$\begin{aligned} \pi_1 &= 2(x_2 - x_1)\hat{x}^2, \\ \pi_2 &= 2(x_2 - x_1)(1 - \hat{x})^2. \end{aligned} \quad (9)$$

Next, we solve the first-stage location equilibrium. Differentiating (9) with respect to x_1 and x_2 , we get the first-order conditions. Solving them yields the unique SPNE equilibrium locations pair:

$$(x_1^H, x_2^H) = \left(\frac{S - 1/4}{T + 1}, \frac{S + 5/4}{T + 1} \right). \quad (10)$$

The second-order conditions are also satisfied. The SPNE locations (10) in Hotelling's location-then-price competition differ from $(x_1^{HL}, x_2^{HL}) = (1/2, 1/2)$ in Hotelling's location competition presented in the previous section.²

Further, we define

$$\frac{x_1^H + x_2^H}{2} = \frac{S + 1/2}{T + 1} \quad (11)$$

as *Hotelling's midpoint*. Then, we have

$$\frac{x_1^H + x_2^H}{2} - \frac{1}{2} = \frac{T}{T + 1} \left(\frac{S}{T} - \frac{1}{2} \right). \quad (12)$$

This sign is positive if $S/T > 1/2$. That is, if the center of gravity of input materials is to the right of the market center, then Hotelling's midpoint is also located to the right of the market center. This inequality is true when a majority of the input locations m_i are to the right of the market center.

Plugging (10) into (7), we get

$$p_1^H - p_2^H = \frac{3T}{(T + 1)^2} \left(\frac{S}{T} - \frac{1}{2} \right).$$

Since the sign of this equation is identical to that of (12), it may be stated that if Hotelling's midpoint is also to the right of the market center $1/2$, then firm 1 locates closer to the market center than firm 2, and firm 1 charges a higher price than firm 2.

Substituting (8) and (10) into (6), the location of marginal consumers is computed as

$$\hat{x}^H = \frac{1}{2}.$$

²Hotelling concludes that both firms tend to agglomerate at the market center in location-then-price competition with a linear transportation cost. However, d'Asprement et al. (1979) indicated a flaw in Hotelling's proof and suggested a quadratic transportation cost in order to ensure the existence of SPNE.

That is, the equilibrium market share is always the same for any asymmetric locations of input materials. The higher price of the good sold by the firm located closer to the market center causes the demand to be reduced relative to the demand for the good sold by the other firm. However, since this effect is mitigated by the locational advantage of the closer firm, the market shares of the both firms come out even.

Furthermore, plugging (8) and (10) into (7), we have the same profit for both firms:

$$\pi_1^H = \pi_2^H = \frac{3}{4(T+1)}.$$

In light of the above findings, it may be stated that even in the case of asymmetric configurations of input materials, *although one firm locates closer to the market center and charges a higher price, the market share and the profits of both firms are identical in SPNE*. Although the firm that is located closer to the market center earns the higher revenue due to the higher price with the equal market share, it incurs higher transportation costs of input materials that are located farther. The higher revenue is compensated by the higher input transportation costs, thereby equalizing the profits of the two firms.

Finally, comparing Hotelling's midpoint (11) with Weber's point (1), we immediately obtain coincidence between them. Thus, we have established the following proposition that relates Weber (1909) and Hotelling (1929).

Proposition 2. *The Hotelling's midpoint of duopolists' locations is equal to the Weber's point of monopolist location.*³

The above proposition indicates a similarity between the two classical location theories. *Hotelling's midpoint implies filtering out the interfirm competition effect*; hence, it does not differ from Weber's point of cost minimization. It is evident from this proposition that the two firms under Hotelling's spatial competition always locate symmetrically around Weber's point, irrespective of the locations of the input materials, m_i for $i = 1, 2, \dots, n$.

If the transportation rate t of inputs is sufficiently high relative to the transportation rate of outputs, then the locations of the two Hotelling's firms approach the center of

³This can also be shown in the case of triopoly.

the gravity of input materials, which is also the location of the Weber's monopolist firm. Figure 2 depicts the situation in which one input material is located at a considerable distance from the market with $(t, m_1, r_1) = (5, 5, 1)$.

For example, the input materials of clay deposits are distributed in five counties in Illinois, while firms producing clay products (bricks, tiles, etc.) are more diversified in ten counties in Illinois. We know from Masters et al. (1999, pp. 23-25) that firms tend to locate closer to the final demand of Chicago than the clay deposits. Firms neither locate back-to-back unlike Weber (1909) nor locate far apart unlike d'Aspremont, Gabszewicz, and Thisse (1979). They locate in between, which is consistent with this paper.

4 Welfare considerations

In this section, we first consider the first-best optimum, in which the government is able to control the locations and prices of all firms in order to maximize the social welfare. Thereafter, we investigate the second-best optimum, where the policy variables of the government are limited to the taxes on the transportation of inputs and outputs. For example, they are gasoline taxes when firms and consumers use automobiles in the transportation of input materials and products.

4.1 First-best optimum

The social planner determines the locations of the two firms (= plants), x_1 and x_2 , and the market boundary \hat{x} in order to maximize social welfare. Since utility is quasilinear, the sum of the consumers' expenditure is equal to the sum of the firms' revenues. This implies that maximization of social welfare is equivalent to minimization of the sum of the transportation costs of inputs and outputs. Therefore, the objective function of the social planner is defined by

$$\min_{x_1, x_2, \hat{x}} \text{TC} = \sum_i t_i(m_i - x_1)^2 \hat{x} + \sum_i t_i(m_i - x_2)^2 (1 - \hat{x}) + \int_0^{\hat{x}} (x - z)^2 dz + \int_{\hat{x}}^1 (x - z)^2 dz.$$

Solving the first-order conditions $\partial\text{TC}/\partial x_1 = \partial\text{TC}/\partial x_2 = \partial\text{TC}/\partial \hat{x} = 0$, it is easily verified that there is a unique minimizer given by

$$(x_1^o, x_2^o, \hat{x}^o) = \left(\frac{S + 1/4}{T + 1}, \frac{S + 3/4}{T + 1}, \frac{1}{2} \right).$$

Observe the same properties as Hotelling's price-then-location competition in section 3.2. First, the midpoint of the social optimum locations, $(x_1^o + x_2^o)/2$, coincides with Weber's point. Second, the market boundary of the social optimum locations, \hat{x}^o , also coincides with the market center $1/2$.⁴

Finally, comparing Hotelling's price-then-location competition, we can show that

$$x_1^H < x_1^o \quad \text{and} \quad x_2^o < x_2^H$$

This means that in order to relax price competition, firms in Hotelling's spatial competition tend to further from the socially optimum locations.

The above account may be summarized in the form of the following proposition:

Proposition 3. *The socially optimum midpoint of firm locations is equal to Weber's point. Furthermore, firms tend to locate rather far from the socially optimum locations in Hotelling's spatial competition.*

Consider two polar cases as done in the previous section. When the input transportation rate t becomes zero, the optimal locations of firms are given by

$$\lim_{t \rightarrow 0} (x_1^o, x_2^o) = \left(\frac{1}{4}, \frac{3}{4} \right).$$

This coincides with the social optimum configuration in the absence of input transportation.

On the other hand, when the input transportation rate t reaches infinity, the optimal locations of firms are given by

$$\lim_{t \rightarrow \infty} (x_1^o, x_2^o) = \left(\frac{S}{T}, \frac{S}{T} \right).$$

That is, two firms approach the center of the gravity of input materials, which is the similar to the finding in Section 2. If the input and output transportation rates are finite, then the optimum locations are between the two extreme cases.

⁴Qualitatively similar results hold in the linear transportation cost.

4.2 Second-best optimum

Although the first-best optimum is the most desirable for society as a whole, it may not be possible to achieve it in a market economy. In fact, the social planner cannot determine the locations of two firms in the real world. However, the social planner can impose gasoline taxes. Thus, the second-best optimum may be examined by considering the following three-stage game. In the first stage, the social planner determines the *ad valorem* gasoline tax rates on transporting inputs and outputs, whose rates can be different. In the second stage, the two firms simultaneously decide their locations x_1 and x_2 . In the third stage, they simultaneously choose their product prices p_1 and p_2 . The analysis of the last two stages presented in section 3.2 can be utilized with the addition of the gasoline taxes.

The inclusion of the gasoline tax for outputs, g_τ , leads to the following modification of consumer utility (2) at x visiting firm k

$$u_k = y - p_k - (1 + g_\tau)(x_k - x)^2,$$

and the market boundary between the firms

$$\hat{x} = \frac{p_1 - p_2 + x_1^2 - x_2^2}{2(1 + g_\tau)(x_1 - x_2)}, \quad (13)$$

With the gasoline tax for inputs, g_t , the profits are rewritten as

$$\begin{aligned} \pi_1 &= [p_1 - \sum_i t(1 + g_t)(x_1 - s)^2] \hat{x}, \\ \pi_2 &= [p_2 - \sum_i t(1 + g_t)(x_2 - s)^2] (1 - \hat{x}). \end{aligned} \quad (14)$$

Solving the first-order conditions $\partial\pi_1/\partial p_1 = \partial\pi_2/\partial p_2 = 0$, plugging them into (14), and solving them for the first-order conditions yield the following unique SPNE location:

$$(x_1^s, x_2^s, \hat{x}^s) = \left(\frac{S(1 + g_t) - (1 + g_\tau)/4}{T(1 + g_t) + (1 + g_\tau)}, \frac{S(1 + g_t) + 5(1 + g_\tau)/4}{T(1 + g_t) + (1 + g_\tau)}, \frac{1}{2} \right). \quad (15)$$

The second-order conditions for x_1^* and x_2^* are also satisfied. The market boundary of the second-best optimum locations, \hat{x}^s , also coincides with the market center $1/2$.

The objective of the social planner is to maximize the social welfare, which comprises the consumer surplus, producer surplus, and tax revenues. Since the consumer utility

is quasilinear, it is transferable and can be added. Therefore, the consumer surplus is defined as:

$$\text{CS} \equiv \int_0^{\hat{x}} u_1 dx + \int_{\hat{x}}^1 u_2 dx,$$

where \hat{x} is given by (13). The producer surplus is defined by

$$\text{PS} \equiv \pi_1 + \pi_2.$$

The tax revenue is defined by

$$\text{TR} = g_t \left[\sum_i t_i (x_1 - m_i)^2 \hat{x} + \sum_i t_i (x_2 - m_i)^2 (1 - \hat{x}) \right] + g_\tau \left[\int_0^{\hat{x}} (x_1 - x)^2 dx + \int_{\hat{x}}^1 (x_2 - x)^2 dx \right].$$

Substituting equilibrium prices and (15) into the RHS of CS, PS and TR, the social welfare

$$W \equiv \text{CS} + \text{PS} + \text{TR}$$

is expressed as a function of g_t and g_τ . Solving $\partial W / \partial g_t = \partial W / \partial g_\tau = 0$ yields the two identical equations given by

$$g_\tau^s = \frac{(16S^2 - 16ST + 7T^2)g_t^s - 6T(T + 1)}{16(S - T/2)^2 + 9T^2 + 6T}. \quad (16)$$

This suggests that there is a continuum of optimal combinations of the two tax rates g_t^s and g_τ^s . Stated differently, the second-best optimum is guaranteed as far as (16) is satisfied.⁵ Substituting the optimal tax rates into (15), the second-best locations of firms may be expressed as a function of the primitives S and T .

The firm locations of the first-best, the second-best, and Hotelling's location-then-price competition are illustrated in Figure 3 with $(m_1, r_1, t, y) = (1, 3/4, 1, 1)$. It must be observed that the first-best locations of firms are inside the Hotelling's ones as shown by Proposition 3. Further, it must be noted that it cannot be generally stated whether

⁵For example, we can determine the optimal tax rates $g_t^o = 0$ and

$$g_\tau^o = \frac{-6T(T + 1)}{16(S - T/2)^2 + 9T^2 + 6T} < 0.$$

This implies zero gasoline tax for firms and a gasoline subsidy for consumers.

or not the second-best locations are too far apart in comparison with those of Hotelling's spatial competition.

Finally, comparing the social welfare of the first-best optimum, second-best optimum, and Hotelling's spatial competition, it may be indicated that the social welfare of the first-best is larger than that of the second-best, and that of the second-best is larger than that of Hotelling's spatial competition. From the example presented in Figure 3, they are 0.845, 0.801, and 0.702, respectively.

5 Conclusions

The main objective of this paper is the combination of the two classical location theories: Weber's (1909) total transportation cost minimization in a monopolistic market and Hotelling's (1929) spatial duopoly competition in a duopolistic market under assumptions of discrete input distribution and continuous consumer distribution. First, we showed that Weber's point is located at the center of gravity. Thereafter, solving the SPNE, we showed that Hotelling firms agglomerate at the market center, irrespective of the distribution of inputs with a fixed price of the output. Further, we showed that Weber's point is identical to Hotelling's midpoint with and without price competition. Finally, the analysis revealed that the first- and the second-best locations are different from the equilibrium locations.

References

- [1] W. Alonso (1975) "Location theory," in J. Friedmann and W. Alonso (eds.), *Regional Policy: Readings in Theory and Applications*, Cambridge, (Mass.): MIT Press.
- [2] S.P. Anderson, A. de Palma, J.-F. Thisse (1992) *Discrete Choice Theory of Product Differentiation*, Cambridge, (Mass.): MIT Press.
- [3] M. Birkin, A. G. Wilson (1986a) "Industrial location models 1: a review and an integrating framework," *Environment & Planning A*, 18, 175-205.

- [4] M. Birkin, A. G. Wilson (1986b) “Industrial location models 2: Weber, Palander, Hotelling, and extensions within a new framework,” *Environment & Planning A* 18, 293-306.
- [5] C. d’Aspremont, J. Gabszewicz, J.-F. Thisse (1979) “On Hotelling’s stability in competition,” *Econometrica* 47, 1145-50.
- [6] P. Hansen, J.-F. Thisse, and R.E. Wendell (1986) “Equivalence of solutions to network location problems,” *Mathematics of Operations Research* 11, 672-678.
- [7] H. Hotelling (1929) “Stability in competition,” *Economic Journal* 39, pp.41-57.
- [8] J.M. Masters, V.C. Ipe, L.R. Smith, and M. Falter (1999) *Directory of Illinois Mineral Producers, and Maps of Extraction Sites 1997*, Department of Natural Resources, Illinois State Geological Survey: Illinois Minerals 117.
- [9] A. Weber (1909) *Über den Standort der Industrie*, English translation: *Theory of the Location of Industries*, Chicago: The University of Chicago Press, 1929.

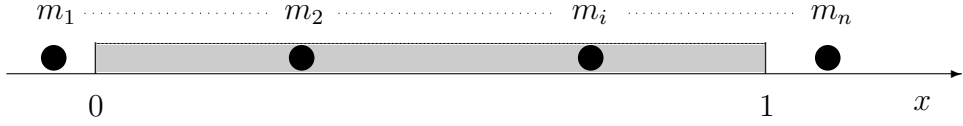


Figure 1: The discrete distribution of input materials and continuous distribution of consumers on a line



Figure 2: The Weber point and the locations of Hotelling's firms

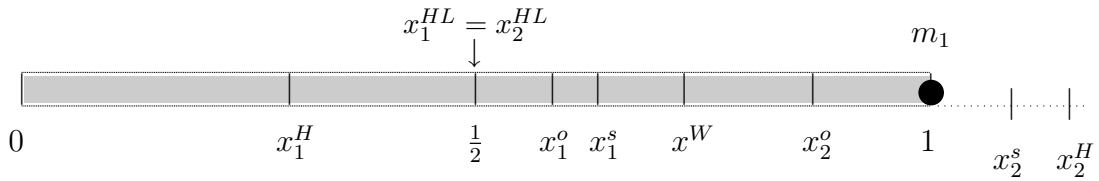


Figure 3: The locations of first-best, second-best, and Hotelling's competition