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The Role of Uncertainty in the Term Structure of Interest Rates: A Macro-Finance Perspective

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Abstract

Using a macroeconomic perspective, we examine the effect of uncertainty arising from policy-shock volatility on yield-curve dynamics. Many macro-finance models assume that policy shocks are homoskedastic, while observed policy shock processes are significantly time varying and persistent. We allow for this key feature by constructing a no-arbitrage GARCH affine term structure model, in which monetary policy uncertainty is modeled as the conditional volatility of the error term in a Taylor rule. We find that monetary policy uncertainty increases the medium- and longer-term spreads in a model that incorporates macroeconomic dynamics.

JEL Classification: C13, C32, E43, E44, E52

Keywords: GARCH, Estimation, Term Structure of Interest Rates, Financial markets and the macroeconomy, Monetary policy

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1 Introduction

The growing macro-finance literature has yet to examine the links between yield curves and the volatility of factors that explain yield curves. In particular, prevailing macro-finance no-arbitrage affine term structure models (ATSMs) are mostly homoskedastic. A body of empirical evidence, however, indicates that homoskedasticity is disputable (e.g., Brenner, Harjes, and Kroner 1996). Moreover, time-varying volatility per se may have important macroeconomic implications: if the short-term interest rate follows a monetary policy rule such as a Taylor rule, then its conditional volatility captures monetary policy uncertainty as perceived by market participants. In line with this widely acknowledged idea, some authors (Rudebusch 2002, Rudebusch, Swanson, and Wu, 2006) have suggested investigating the role of uncertainty factors in explaining yield curve dynamics. However, little formal analysis has followed. In order to fill this gap, this paper examines the role of uncertainty arising from the heteroskedastic policy shock process in accounting for yield curve dynamics.

In general, policy uncertainty may at times be large and long-lived, while at other times relatively small and short-lived. At a time of unusual distress—for example the Volcker shock in the early 1980s, 9/11 in 2001, and the Lehman shock in 2008—the Fed undertook extraordinary action deviating from any known simple policy rule. As a result, uncertainty in the federal funds (FF) and other financial markets has increased. On the other hand, there are indications that FF market volatility has declined since the Federal Open Market Committee (FOMC) began publicly announcing the target FF rate in 1995 (Favero and Mosca, 2001). In a somewhat similar vein, in 2004, the FOMC explicitly signaled that its future course of monetary policy would be less volatile and more predictable for market participants.[1]

On these grounds, it may be more reasonable to assume that the policy shock process consists of large occasional shocks. Once the size of deviation changes, it lasts for a reasonably long period of time, as uncertainty in financial markets, once present, cannot easily be eliminated. One way to accommodate this type of shock process is to apply a generalized autoregressive conditional heteroskedasticity (GARCH) process that allows

[1] For example, the FOMC made explicit policy commitments with statements such as, "Policy accommodation can be maintained for a considerable period" (August 2003) and "Accommodative monetary policy stance will be removed at a measured pace" (June 2004).

for serial correlation in the conditional volatility.[2] To this end we construct a discrete-time macro-finance GARCH term structure model.[3] Specifically, we extend Heston and Nandi's (2003) multivariate GARCH "ATSM"[4] with a richer macro structure. The main difference between Heston and Nandi's (2003) model and other GARCH term-structure models is that the yield equation in their model can be written as an affine function of state variables. This allows for greater tractability and generates a closed-form solution for term rates with any maturity as well as option pricing.

With the existing macro-finance ATSMs having performed broadly successfully,[5] we take Ang and Piazzesi (2003) as a point of departure and generalize their model in three directions. First, we allow the short-term rate to follow a GARCH-type process with the conditional volatility of the error term following an autoregressive moving average process. Second, we allow the dynamics of macro variables[6] to depend on the lagged short rate as well as their own lagged variables, in a spirit similar to Ang, Piazzesi, and Wei (2006) and Hőrdahl, Tristani, and Vestin (2006). Thus, the policy interest rate can directly influence future macro variables, and vice versa. Third, to enhance the link between financial econometrics and macroeconomics, we include no latent variables, which are commonly used in many term structure models to improve empirical performance,

[2] Previously developed "pure finance" ATSMs (e.g., Dai and Singleton 2000) are compatible with stochastic volatility, and they typically assume a square-root process for factor heterosckedasticity—for example, in a single-factor ATSM, where the short rate is the only factor explaining yield curves, the factor variance is the level of short rate itself. However, the square-root models tend to overstate the sensitivity of volatility to levels (Brenner *et al.*, 1996), and to date no consensus has been reached on how one should model the short-rate volatility.

[3] Evidence of time-varying conditional volatility can be provided by single-equation GARCH estimation. A regression of the FF rate on a constant, its first lag, 12-month inflation, 12-month change in unemployment (in percent), where the conditional variance of the FF rate follows the autoregressive moving average process, generates statistically significant GARCH and ARCH terms.

[4] "ATSM" in the sense that model-implied yields can be expressed as an affine function of state variables. Because the continuous version of the GARCH equation reduces to an ordinary differential equation rather than an affine diffusion process, our model lies outside the continuous ATSM framework formally defined by Dai and Singleton (2000).

[5] For example, Ang and Piazzesi (2003), using a discrete-time version of the affine class introduced by Duffie and Kan (1996), found that macro factors explain up to 85 perceent of movements in the short and middle parts of yield curves, and around 40 percent at the long end.

[6] In the baseline model, we assume homoskedasity for the dynamics of inflation and real activity. We can extend our model to allow heteroskedasticity for the macro dynamics, though such heteroskedasticity is less evidently confirmed when the sample period is short.

because they alone cannot outfit any macroeconomic interpretations. We show that the inclusion of economically interpretable conditional volatility can significantly improve the empirical fit of the ATSMs, effectively replacing uninterpretable latent factors.

The model-implied conditional volatility is significantly time varying and persistent—it soared in the early 1980s and tapered off during the period of the "Great Moderation." The gradual decline halted in the early 2000s, when the Fed undertook expansionary policy deviating from the Taylor rule (Taylor 2009), but resumed its decline after the FOMC began making explicit policy announcements.

Our model-estimated results indicate that the conditional volatility of the short rate—monetary policy uncertainty—plays a significant role in determining the shape of yield curves in the presence of the Taylor rule and endogenous macro dynamics. The uncertainty factor increases term spreads by lifting the middle and longer-end parts of the yield curves. In addition, we focus on a new aspect of policy shock process—policy shock volatility—in explaining yield curves, whereas the existing literature focuses on the policy shock itself, assuming that policy shocks are i.i.d. normal, presumably for tractability. For example, Evans and Marshall (2001), using VARs with yields of various maturities and macro variables, find that positive monetary policy shocks would bear-flatten a yield curve.

To exemplify how our model performs on real data, we set forth a case study, highlighting the so-called Greenspan conundrum period of 2004-06, on the grounds that monetary policy uncertainty declined during this period (for example, see Figure 1). Our model with the estimated parameters replicates well the actual bear-flattening of the yield curve.[7] It also suggests that the greater predictability in monetary policy in this period partially reined in the risk premiums. Meanwhile, it offsets the upward pressures from the rising short rate and the expanding economic activity.

The paper is organized as follows. The next section describes our macro-finance GARCH term-structure model. Section 3 sets out our estimation strategy, and Section 4 discusses estimated results and a case study on the conundrum period of 2004-06

[7] In the run-up to the 2008 global financial crisis, US yield curves continued to bearflatten, despite the consecutive hikes in the FF rate and the expanding economic activity.

during which monetary policy uncertainty declined. Section 5 concludes.

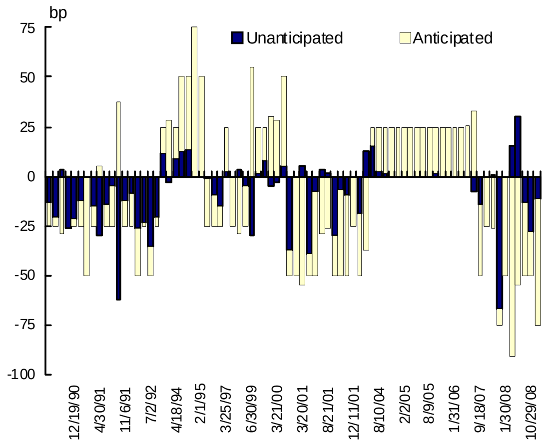

Figure 1. Monetary policy uncertainty (in basis points). Following the methodology of Kuttner (2001), this figure reports recent developments in monetary policy uncertainty; unanticipated policy changes are estimated by differences between the spot-month futures rates before and after each FOMC meeting; anticipated changes are the actual minus the estimated unanticipated changes). During the tightening period of 2004—2007, as can be seen from the figure, the interest rate hikes were mostly well anticipated by investors.

2 The Model

The basic setup of our model essentially builds on the prevailing discrete macro-finance no-arbitrage term structure model, where the stochastic process of the short-term interest rate is driven by a Taylor-type (1993) monetary policy rule. With no-arbitrage bond pricing restrictions, term rates for any maturity can be expressed as an affine function of factors such as the short rate and macro variables.

2.1 Short-term rate and macro-variable dynamics

We employ a few variants of the standard Taylor rule that includes the lagged short-term rate and expected inflation rate (rather than the concurrent inflation rate). This specification including the expected inflation may be labeled a forward-looking version of the Taylor rule as proposed by Clarida *et al.* (2000). The baseline dynamics of short-term and macro variables are given by

$$r_{t+1} = \underset{1\times1}{\mu_0} + \underset{1\times1}{\mu_1} r_t + \underset{1\times2}{\boldsymbol{\mu}_2} X_{t+1} + \sqrt{h_{t+1}} z_{t+1} \tag{1}$$

$$X_{t+1} = \underset{2\times1}{\boldsymbol{\delta}_0} + \underset{2\times1}{\boldsymbol{\delta}_1} r_t + \underset{2\times2}{\Phi} X_t + \underset{2\times2}{\Sigma} \varepsilon_{t+1} \tag{2}$$

$$h_{t+1} = \underset{1\times1}{\beta_0} + \underset{1\times2}{\boldsymbol{\beta}_1} \mathbf{h}_t + \underset{1\times1}{\alpha} z_t^2 \tag{3}$$

$$X_t = [\pi_t \; y_t]', \; \mathbf{h}_t = [h_t \; h_{t-1}]', \tag{4}$$

where r_t denotes the short-term rate (FF rate). X_t is a 2×1 macro-variable vector of inflation (π), and real activity (y) measures following an autoregressive (AR) process. Σ is an upper triangular matrix, while h_t is the conditional variance of the short-term rate. A scalar random shock z and a 2×1 random shock vector ε are assumed to be independent and jointly normal.

We take Ang and Piazzesi (2003) as a point of departure and generalize their model in two directions. First, we allow the short-term rate to follow a GARCH-type process with the conditional volatility of the error term following an autoregressive moving average process given by equation (3). Note that h_{t+1} is included in the information set in period t by (3). The $\sqrt{h_{t+1}} z_{t+1}$ term in the short-rate equation (1) could be interpreted as discretionary changes in the FF rate deviated from the Taylor rule. In some preceding macro-finance models as well as in broader monetary policy-related works, the "policy shock" is broadly assumed to be a random shock following i.i.d. normal distribution on account of tractability rather than empirical plausibility. As discussed in the previous section, empirical evidence supports that the policy shock has time-varying (conditional) variance as opposed to the homoskedasticity frequently assumed in most of the early macro-finance studies.

Second, we allow the dynamics of macro variables to depend on the lagged short rate as well as their own lagged variables, in a spirit similar to Ang, Piazzesi, and Wei (2006) and Hőrdahl, Tristani, and Vestin (2006). Thus, the policy interest rate can directly influence future macro variables. In the next model-estimation section, we will explain that the inclusion of the lagged short rate requires us to modify the Ang and Piazzesi-type specification of the system of equations.

Third, our model has no latent variables, which are commonly used in term structure models to explain the yield curve dynamics, because they alone cannot provide any macroeconomic interpretations. Instead, we treat the conditional volatility of the short rate as an additional factor that explains the yield curves. We then jointly estimate this unobservable variable via maximum likelihood estimation.

Substituting (2) into (1), we obtain

$$\begin{aligned} r_{t+1} &= \mu_0 + \mu_1 r_t + \boldsymbol{\mu}_2 X_{t+1} + \sqrt{h_{t+1}} z_{t+1} \\ &= \mu_0 + \mu_1 r_t + \boldsymbol{\mu}_2 \left(\boldsymbol{\delta}_0 + \boldsymbol{\delta}_1 r_t + \Phi X_t + \boldsymbol{\Sigma} \varepsilon_{t+1} \right) + \sqrt{h_{t+1}} z_{t+1} \\ &= \underbrace{(\mu_0 + \boldsymbol{\mu}_2 \boldsymbol{\delta}_0)}_{\overline{\mu}_0} + \underbrace{(\mu_1 + \boldsymbol{\mu}_2 \boldsymbol{\delta}_1)}_{\overline{\mu}_1} r_t + \underbrace{(\boldsymbol{\mu}_2 \Phi)}_{\overline{\mu}_2} X_t + \sqrt{h_{t+1}} z_{t+1} + \mu_2 \boldsymbol{\Sigma} \varepsilon_{t+1} \end{aligned} \tag{5}$$

$$\text{where } \bar{\mu}_0 = \mu_0 + \boldsymbol{\mu}_2 \boldsymbol{\delta}_0,\ \bar{\mu}_1 = \mu_1 + \boldsymbol{\mu}_2 \boldsymbol{\delta}_1,\ \bar{\mu}_2 = \boldsymbol{\mu}_2 \Phi. \tag{6}$$

The above short-term rate and macro-variable dynamics can be rewritten in a more concise form:

$$\begin{pmatrix} r_{t+1} \\ X_{t+1} \end{pmatrix} = \begin{pmatrix} \overline{\mu}_0 \\ \delta_0 \end{pmatrix} + \begin{pmatrix} \overline{\mu}_1 \\ \delta_1 \end{pmatrix} r_t + \begin{pmatrix} \overline{\mu}_2 \\ \Phi \end{pmatrix} X_t + \underbrace{\begin{pmatrix} \sqrt{h_{t+1}} & \boldsymbol{\mu}_2 \Sigma \\ 0 & \Sigma \end{pmatrix}}_{\equiv \Sigma_{t+1}} \underbrace{\begin{pmatrix} z_{t+1} \\ \varepsilon_{t+1} \end{pmatrix}}_{\equiv e_{t+1}}$$

$$h_{t+1} = \beta_0 + \boldsymbol{\beta}_1 \mathbf{h}_t + \alpha z_t^2.$$

2.2 Pricing kernel and the price of risk

We define a time-dependent 1×3 price of risk vector Ω_t and assume that the price of risk takes a certain affine form in state variables, as handled in many existing affine term structure models.

$$\begin{aligned} \Omega_t' &\equiv (\omega_{r,t}\ \omega_{\pi,t}\ \omega_{y,t}) \\ & \underbrace{\begin{bmatrix} \omega_{0r} \\ \omega_{0\pi} \\ \omega_{0y} \end{bmatrix}}_{\Omega_0} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ \omega_{21} & \omega_{22} & \omega_{23} \\ \omega_{31} & \omega_{32} & \omega_{33} \end{bmatrix}}_{\Omega_1} \begin{bmatrix} r_t \\ \pi_t \\ y_t \end{bmatrix}. \end{aligned} \tag{7}$$

$$\omega_0 \equiv [\omega_{0\pi}\ \omega_{0y}]'\ \omega_1 \equiv [\omega_{21}\ \omega_{31}]'\ \ \tilde{\Omega}_1 \equiv \begin{bmatrix} \omega_{22} & \omega_{23} \\ \omega_{32} & \omega_{33} \end{bmatrix},$$

where Ω_0 is a 3×1 constant vector, and Ω_1 is a 3×3 constant matrix where we impose some zero restrictions.[8] Note that with the zero restriction, $\omega_{r,t} = \omega_{0r}$.

[8] The first row in Ω_js must be zero, as this is a critical condition to ensure that the model lies within the affine framework (in the sense that yield equations can be written as a linear function of factors).

Now suppose that the pricing kernel (m)[9] is given by

$$m_{t+1} \equiv \exp(-r_t + \Omega_t \Sigma_{t+1} e_{t+1} - \frac{1}{2}\Omega_t \Sigma_{t+1} \Sigma_{t+1}' \Omega_t').$$

Then the log price of n-period bond follows the following affine form (see Appendix B for the derivation):

$$\begin{aligned}
p_t^n &= \exp(\bar{A}_n + \bar{B}_n r_t + \bar{\mathbf{C}}_n \mathbf{h}_{t+1} + \bar{\mathbf{D}}_n X_t), \\
\text{where } \bar{A}_{n+1} &= \bar{A}_n + \bar{B}_n \overline{\mu}_0 + \bar{\mathbf{C}}_n s_1 \beta_0 + \bar{\mathbf{D}}_n \delta_0 + \frac{1}{2} H_n \Sigma \Sigma H_n' \qquad (8) \\
&\quad -\frac{1}{2} \log(1 - 2\bar{\mathbf{C}}_n s_1 \alpha) + \omega_{0r} \boldsymbol{\mu}_2 \Sigma \Sigma' H_n' + H_n \Sigma \Sigma' \omega_0 \\
\bar{B}_{n+1} &= \bar{B}_n \mu_1 + \bar{\mathbf{D}}_n \delta_1 + H_n \Sigma \Sigma' \omega_1 - 1 \qquad (9) \\
\bar{\mathbf{C}}_{n+1} &= \bar{\mathbf{C}}_n \left(s_1 \beta_1 + S\right) + \bar{B}_n \omega_r s_1' + \frac{1}{2} \bar{B}_n^2 s_1' \qquad (10) \\
\bar{\mathbf{D}}_{n+1} &= \bar{\mathbf{D}}_n \Phi + \bar{B}_n \overline{\mu}_2 + H_n \Sigma \Sigma' \tilde{\Omega}_1 \qquad (11) \\
\overline{\mu}_1 &= \mu_1 + \boldsymbol{\mu}_2 \delta_1 \,, \ \overline{\mu}_2 = \boldsymbol{\mu}_2 \Phi, \ H_n = \bar{B}_n \mu_2 + \mathbf{D}_n, \ s_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \ S = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}.
\end{aligned}$$

Note that according to basic asset pricing theory, the n-period bond yield is given by

$$y_t^n = A_n + B_n r_t + \mathbf{C}_n \mathbf{h}_{t+1} + \mathbf{D}_n X_t,$$

where $A_n = -\bar{A}_n/n,\ B_n = -\bar{B}_n/n,\ \mathbf{C}_n = -\bar{\mathbf{C}}_n/n,\ \mathbf{D}_n = -\bar{\mathbf{D}}_n/n.$

3 Model Estimation

For our estimation, we use monthly data on interest rates and macro variables that capture inflation and real activity from July 1954 to December 2006.[10] We assume that the policy reaction function remains fully stable throughout the period.[11] The summary

[9] For the pricing kernel expressed in terms of risk-neutral probabilities, see Appendix A.

[10] Our sample period starts from July 1954 because the FF rate data are available from that month.

[11] We have also estimated the model with a shorter sample period from January 1988 to December 2006, i.e., the period that covers Alan Greenspan's tenure as Fed chairman. The main results did not change, although the convergence of maximum likelihood estimators became less smooth.

statistics and data sources are provided in Appendix C.

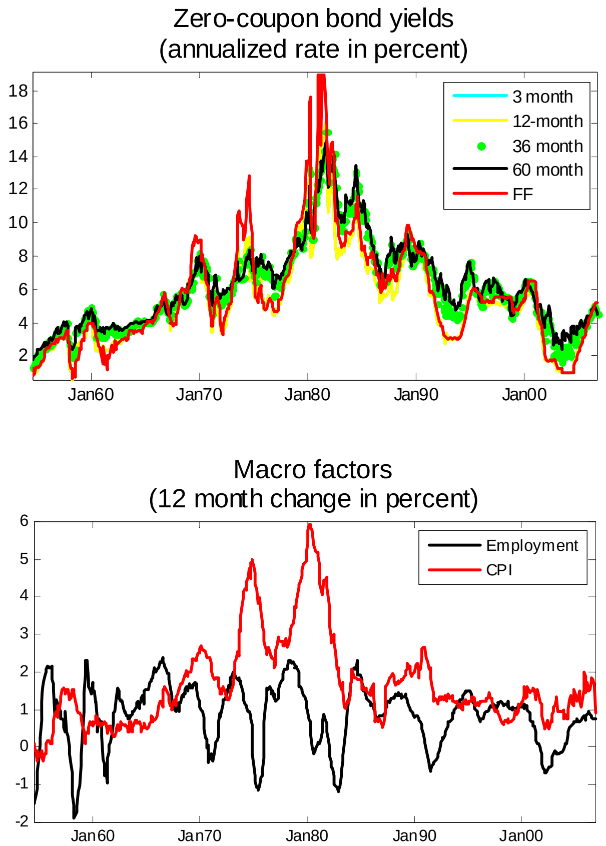

Figure 2. Bond yields and macro principal components. The top panel plots the monthly FF rate and zero-coupon bond yields of maturity 3 months, 12 months, 36 months, and 60 months at an annualized rate in percent. The bottom panel plots employment and CPI in year-on-year percentage change, representing real activity and inflation, respectively. The sample period is July 1954 to December 2006.

We use the FF rates for the short rate and zero-coupon bond yields of 3-, 12-, 36-, and 60-month maturities (Figure 2, top panel); the FF rates are obtained from the Fed. The bond yields are from the CRSP US Treasury Database (the Fama-Bliss Discount Bond Files for 12-, 36-, and 60-month data and from the Risk-Free Rate Files for 3-month data). All bond yields are continuously compounded and expressed at annualized rates in percentages. Regarding inflation and real activity measures, we use the consumer price index (CPI) and employment data (Figure 2, bottom panel). These macro variables are expressed in the year-on-year difference in logs of the original series.

As explained in the previous section, our model dynamics consist of macro and yield dynamics. The macro dynamics are summarized by equation (2) and the yield dynamics are given by

$$R_t = A + Br_t + \mathbf{C}\mathbf{h}_{t+1} + \mathbf{D}X_t + \Sigma^m \varepsilon^m_{t+1},$$

where $R_t = \left[r^3_t, r^{12}_t, r^{36}_t, r^{60}_t\right]'$ is a 4×1 vector of bond yields with maturities corresponding to the superscript numbers (in months). The yield dynamics are an affine function of the state variables with the coefficient vectors of $A, B, \mathbf{C}$, and $\mathbf{D}$ corresponding to (i) the constant term, (ii) the short-rate term, (iii) the conditional variance term, and (iv) the macro-variable term, respectively. These vectors are time-invariant 4×1 vectors with maturities corresponding to the subscript numbers (i.e., $A = [A_3, A_{12}, A_{36}, A_{60}]'$ $B = [B_3, B_{12}, B_{36}, B_{60}]'$ $\mathbf{C} = [C_3, C_{12}, C_{36}, C_{60}]'$ $\mathbf{D} = [D_3, D_{12}, D_{36}, D_{60}]'$). Their elements are derived from the recursive equations; in other words, the model implicitly imposes cross-equation restrictions reducing the number of parameters to be estimated. Measurement errors ε^m are assumed to have constant variance and Σ^m is a diagonal matrix.

We can summarize the system of equations to be estimated as follows:

$$\underbrace{\begin{pmatrix} r_{t+1} \\ X_{t+1} \\ R_t \end{pmatrix}}_{\equiv Y_{t+1}} = \begin{pmatrix} \overline{\mu}_0 \\ \delta_0 \\ A \end{pmatrix} + \begin{pmatrix} \overline{\mu}_1 \\ \delta_1 \\ B \end{pmatrix} r_t + \begin{pmatrix} 0 \\ 0 \\ \mathbf{C} \end{pmatrix} \mathbf{h}_{t+1} \tag{12}$$

$$+ \begin{pmatrix} \overline{\mu}_2 \\ \Phi \\ \mathbf{D} \end{pmatrix} X_t + \begin{pmatrix} \sqrt{h_{t+1}} & \mu_2\Sigma & 0 \\ 0 & \Sigma & 0 \\ 0 & 0 & \Sigma^m \end{pmatrix} \begin{pmatrix} z_{t+1} \\ \varepsilon_{t+1} \\ \varepsilon^m_{t+1} \end{pmatrix}$$

$$\begin{aligned} h_{t+1} &= \lambda + \boldsymbol{\beta}_1(\mathbf{h}_t - \lambda) + \alpha(z^2_t - 1) \\ \mathbf{h}_t &= [h_t \;\; h_{t-1}],\; \boldsymbol{\beta}_1 = [\beta_{11} \; \beta_{12}], \end{aligned} \tag{13}$$

where z, ε, and ε^m are jointly normal and independent. Note that because ε^m_{t+1} is the vector of the measurement errors, it is independent of the current and past $Y's$ (i.e., $Y_t, Y_{t-1}, ...$), even though it is observable in period t. We set the lag of X_t at one and that of $\mathbf{h}_t$ at two.[12] λ is the unconditional variance of the short rate given by $(\alpha+\beta_0)/(1-\beta_{11}-\beta_{12})$. We estimate this system using the maximum likelihood method (for details, see the Appendix D). A cursory glance at the model-implied yields (Figure

[12] We tried other lag lengths, but the corresponding coefficients were insignificant.

3) indicates a good fit to the data. The parameter estimates of our model are reported in Table 1.

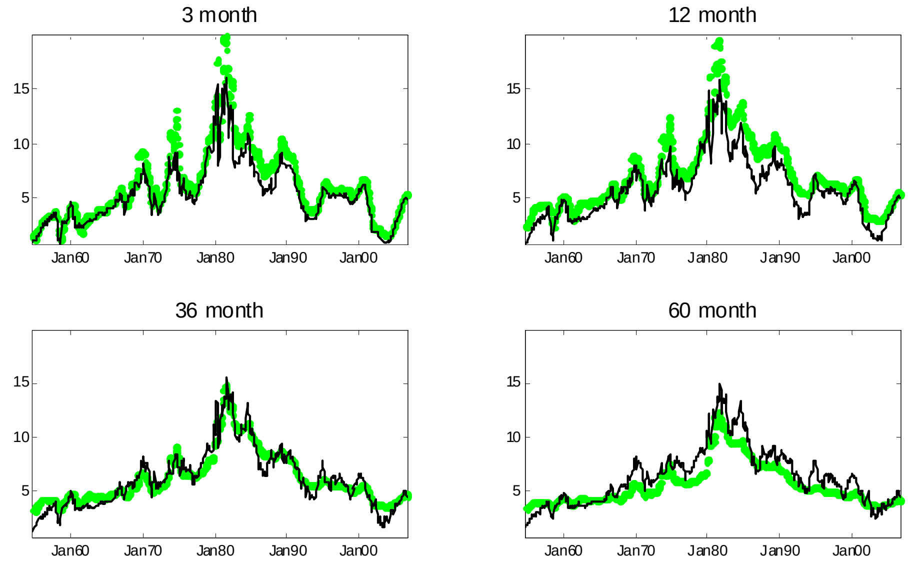

Figure 3: Model-implied yields (in annualized rate in percent). These figures plot model-implied yields for the indicated maturities in annualized rate in percent. The dotted-lines show one-period-ahead in-sample forecasting, and the solid lines show the actual data.

Short-rate dynamics

$\bar{\mu}_0$	$\bar{\mu}_1$	$\bar{\mu}_2$	
0.156	0.897	0.107	0.134
0.016	*0.003*	*0.000*	*0.005*
λ	α	β_1	
5.130E-02	6.000E-04	0.732	0.256
4.5E-03	*3.3E-05*	*0.046*	*0.045*

Dynamics of macro variables

δ_0	δ_1	Φ		Σ	
0.009	-8.00E-04	0.987	0.045	0.129	-0.0028
1.090E-02	*0.0021*	*0.0059*	*0.0061*	*0.0034*	*0.0042*
0.196	-0.004	-0.0514	0.9298	---	0.137
0.0109	*0.0020*	*0.0061*	*0.0071*	---	*0.0001*

Table 1. Estimated coefficients. This table reports estimated coefficients in our macro-finance GARCH term-structure model. Numbers in italic indicate standard errors.

Prices of risk

	Ω_0	$\tilde{\Omega}_1$ Inflation	$\tilde{\Omega}_1$ Real activity
Inflation	1.984	-3.300	-7.296
	0.902	*0.591*	*0.981*
Real activity	6.17E-05	1.61E-04	9.87E-05
	1.0E-04	*1.0E-04*	*1.0E-04*
FF	7.6227		
	0.0262		

Measurement error

3 months	12 months	36 months	60 months
1.785	3.020	0.676	1.404
0.1078	*0.2184*	*0.0338*	*0.1173*

Table 1 (continued). we set insignificant prices-o- risk parameters to zero.

The estimated dynamics of real activity and inflation are robust to different model specifications: they are comparable to on those based on a multivariate GARCH model (Appendix E). The estimated Taylor rule coefficients are statistically significant, and their signs and magnitudes are in line with those previously estimated in the macro-finance literature (e.g., Ang and Piazzesi, 2003). The GARCH and ARCH coefficients in the GARCH equation (3) are statistically significant as well.

4 Estimated Results

4.1 Estimation summary

The key results are as follows. First, our model-implied conditional volatility is considerably time varying and persistent. Figure 4 reports the dynamics of conditional variance[13] and shows that the model-implied conditional standard deviation increased notably in the wake of the Volcker shock in the early 1980s (left panel) and tapered off during the "Great Moderation." The gradual decline halted in the early 2000s when the Fed undertook expansionary policy deviating from the Taylor rule (Taylor 2009) but resumed its decline when the FOMC made explicit policy announcements with statements such as, "policy accommodation can be maintained for a considerable period" (August

[13] This GARCH process is stationary, as the absolute values of the corresponding polynomial roots are all greater than one.

2003) and "accommodative monetary policy stance will be removed at a measured pace" (June 2004) (right panel).

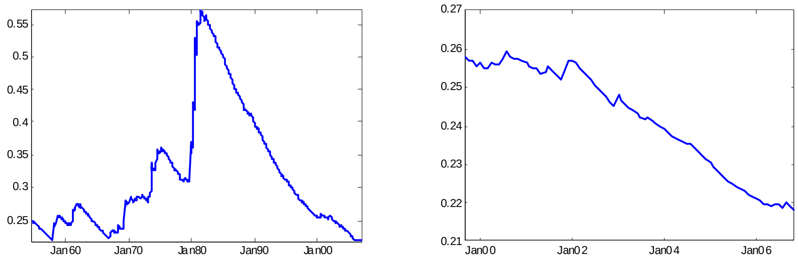

Figure 4: Model-implied conditional standard deviation of the short rate (at an annualized rate in percent). The left panel shows the dynamics of the conditional standard deviation of the short rate for the entire sample period. The right panel enlarges the dynamics in recent years.

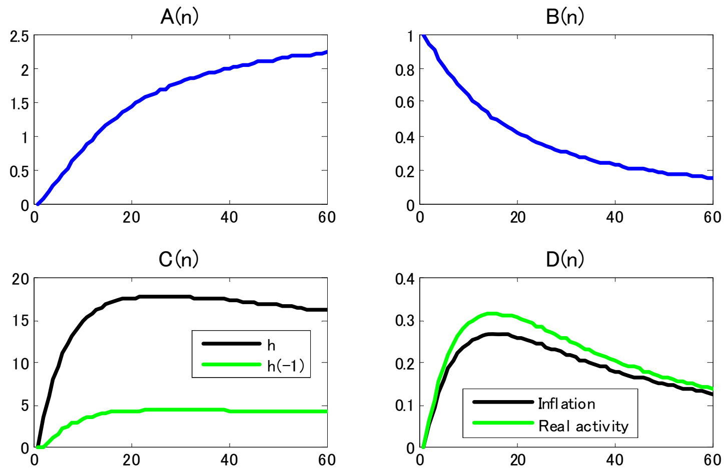

Figure 5. Factor weights against maturity. This figure plots the coefficients of the yield equation against maturity. A(n), B(n), C(n), and D(n) correspond to the constant term, the short-rate term, the conditional-variance term, and macro-variable term, respectively.

Second, our results confirm that the conditional volatility of the short rate plays a significant role in determining yield curves in the presence of endogenous macro dynamics. Figure 5 shows how the yield-equation coefficients change against maturity. The upward-sloping of A_n represents the shape of average yield curves, while the downward slope of B_n implies that an increase in the short rate has a more positive impact on the shorter-end of yield curves, thereby reducing term spreads. The shape of $\mathbf{C}_n$ implies that the conditional volatility increases the term spreads by lifting the middle parts and longer-end of yield curves. The curves of $\mathbf{D}_n$ appear similar to the corresponding dynamics in the existing macro-finance literature, and capture the positive impact of macro variables on yield curves.

Third, the model broadly supports the practitioner's view that the term premium is negatively associated with economic expansion. Simple correlation between the term premium and employment changes is -0.12. We define the term premium as the n-period yield term premium (TP^n), i.e., $TP^n_t = R^n_t - \frac{1}{n}\sum_{j=0}^{n-1} E_t(r_{t+j})$, where R^n_t is the n-period bond yield, and $\frac{1}{n}\sum_{j=0}^{n-1} E_t(r_{t+j})$ is the average of expected future short rates or the yields under the expectations hypothesis. We can calculate R^n_t from the affine yield equation and $\frac{1}{n}\sum_{j=0}^{n-1} E_t(r_{t+j})$ from the short-rate dynamics.[14]

Fourth, in the absence of heteroskedasticity, the model performance deteriorates considerably. Note that we can obtain the homoskedastic version of the model by simply setting the coefficients of the ARCH term (α) and GARCH term (β_1) in the GARCH equation equal to zero and re-maximizing the log-likelihood function. Clearly, this homoskedastic model with no other latent variables turns out to be overly inflexible to provide a reasonable fit to the data, notably at the longer-end of yield curves as shown in Figure 6.

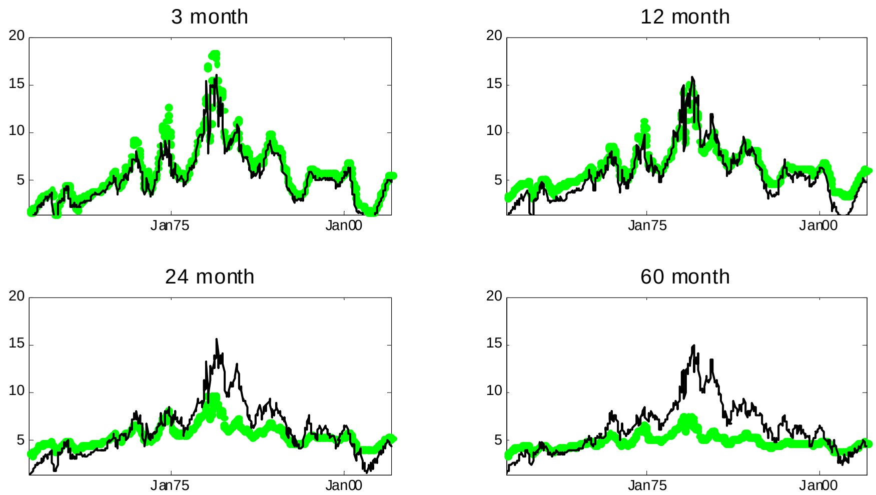

Figure 6: Model-implied yields without heteroskedasticity. With no other latent variables, the model has a poor and unreasonable fit to the data, notably at the longer-end of yield curves.

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[14] We can calculate $\frac{1}{n}\sum_{j=0}^{n-1} E_t(r_{t+j})$ with the following equation: $\frac{1}{n}\sum_{j=0}^{n-1} E_t(r_{t+j}) = \frac{1}{n}r_t + [1\ 0\ 0]\left[(1-G_1)^{-1}(1-G_1^j)G_0 + G_1^j q_t\right]$, where $q_t = [r_t\ X_t]'$ and G_1 is a 3×3 matrix in which the first row is given by $\overline{\mu}_1, \overline{\mu}_2$, and the second and third rows together are given by δ_1 and Φ as implied by the model estimates.

4.2 A case study: Around the time of the conundrum period

In the runup to the 2008 global financial crisis, US yield curves continued to bear-flatten, despite the consecutive hikes in the FF rate and expanding economic activity. This development, labeled a "conundrum" by then-Fed Chairman Alan Greenspan, poses a challenge to the existing macro-finance models, because they tend to perform poorly in explaining this period unless the term premiums fell beyond the range predicted by these models.

In our paper, on the other hand, the model-implied yield curves (Figure 7) successfully generate the continued bear-flattening of yield curves between 2004-06. To facilitate understanding of the mechanism behind this bear-flattening, Figure 8 reports factor dynamics around this period: they are characterized by a decline in conditional variance while the short rate was rising and economic activity was expanding. Keeping in mind the factor weights discussed in the previous paragraph, we originally conjectured that it must have been the volatility channel that put downward pressure on the longer rate during this period. The contribution of each term to the model-implied yields, however, only *partially* confirms this conjecture (Figure 9, bottom left panel), as there was a significant decline in model residuals with respect to longer-maturity yield equations, particularly in 2002 (Figure 10). This suggests that there are still unexplained factors accounting for the conundrum. In particular, a demand shift caused by the increased demand for the long-maturity bonds by foreign central banks and institutions might be an important underlying factor.

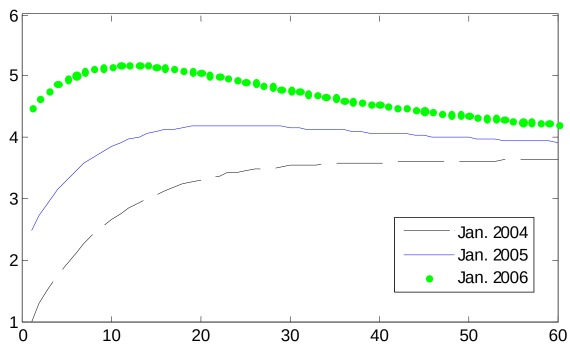

Figure 7. Model-implied yield curves (at an annualized rate in percent) The implied yield curves continued to bear-flatten during the low-yield period.

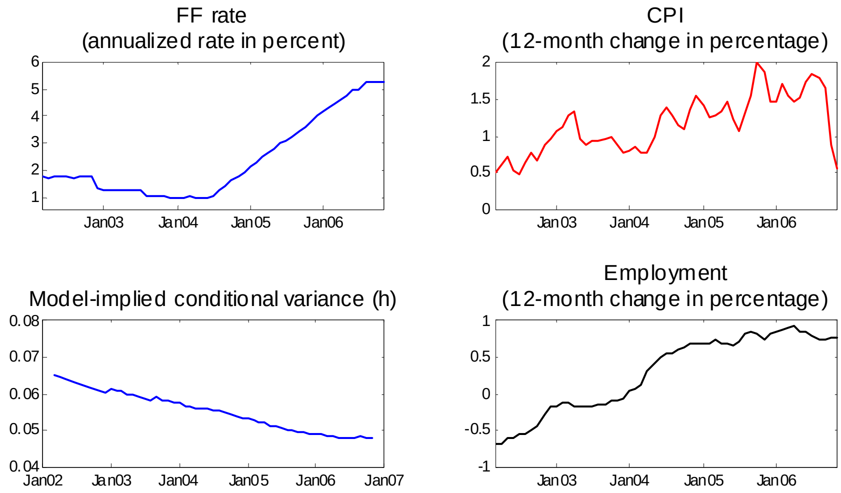

Figure 8. Factor dynamics around the conundrum. These figures plot the dynamics of state variables (i.e., the short rate, the conditional volatility of the short rate, and macro variables between January 2002 and December 2006.

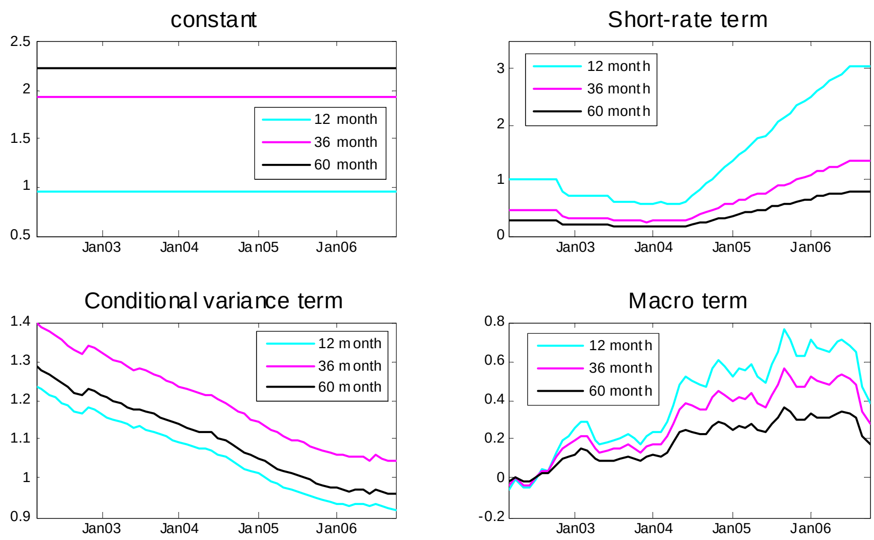

Figure 9. Contributions to the model-implied yields (in annualized rate in percent). These figures demonstrate the contribution to the model-implied yields by each term in the yield equation. Note that the sum of each factor contribution is equal to the model-implied yields.

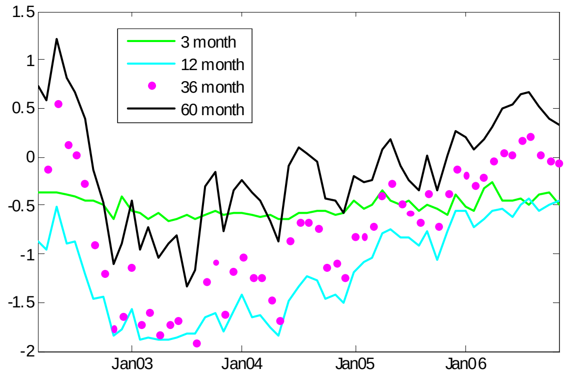

Figure 10. Model residuals for the yield equations (at an annualized rate in percent). The model residuals of longer-maturity yield equations dropped in 2002 and remained more negative than the shorter-maturity counterparts.

5 Conclusion

We analyzed a new aspect of monetary policy effects—the role of the policy shock volatility or policy uncertainty—rather than the policy shock itself (i.e., its level or the first moment, in contrast to our focus; the second moment), in accounting for yield curve dynamics. Our estimation results confirmed that the newly included uncertainty factor improved the empirical performance of our ATSM remarkably, greatly reducing the unexplained portion or residuals. Furthermore, the results indicate that the time-varying and persistent policy shocks increase term spreads as they lift the middle-part or longer-end of the yield curves. There may be, however, other factors not yet included that could further reveal the unexplained portion of term premium dynamics or model residuals. For example, at a time of unusual distress, if the Fed were to undertake extraordinary policy actions, investors might lose their risk appetite, collectively switching to treasury bonds or other risk-free assets. This sort of "flight to quality" driven by a demand shift could fully offset the upward pressure on the interest rates arising from the elevated uncertainty as discussed in this paper. Looking ahead, the impact of demand-side shifts (i.e., investors' preference) on yield curves could be stressed more in the future research, particularly focusing on the crisis experience.

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A Pricing kernel and the Risk-Neutral Measure

Assume the existence of an equivalent martingale measure (or risk-neutral measure) Q, such that the price of any asset p_t with no dividends at time $t+1$ satisfies

$$p_t = E_t^Q\left(\exp(-r_t)p_{t+1}\right) \simeq E_t^Q\left(\frac{p_{t+1}}{1+r_t}\right),$$

where expectation is taken under the measure Q and $-log(1+r_t) = log(1+r_t)^{-1} \simeq -r_t$. Let the Radon-Nikodym derivative, which converts the risk-neutral measure to the data-generating measure exploiting the Girsanov theorem, be denoted by ζ_{t+1}. Then, for any random variable Z_{t+1}, we have

$$E_t^Q Z_{t+1} = E_t\left(\frac{\zeta_{t+1}}{\zeta_t} Z_{t+1}\right). \tag{14}$$

Condition 1 *Assume ζ_{t+1} follows the process described as,*

$$\begin{aligned}
\zeta_{t+1} &= \zeta_t \exp\left(\Omega\Sigma_{t+1}e_{t+1} - \frac{1}{2}\Omega\Sigma_{t+1}\Sigma'_{t+1}\Omega'\right) \\
E_t\Sigma_{t+1} &= \Sigma_{t+1},
\end{aligned}$$

where e_t is a vector of random variables that jointly follows $N(0,1)$ distribution and Σ_{t+1} denotes a lower or upper triangular standard deviation matrix. Σ_{t+1} can vary depending on t while it needs to be known at period t.

Under the condition, we define the pricing kernel m_{t+1} as,

$$m_{t+1} \equiv \exp(-r_t) \times \frac{\zeta_{t+1}}{\zeta_t}.$$

Using the kernel, the price of an asset without any dividend can be written as,

$$\begin{aligned}
p_t &= E_t\left(m_{t+1}p_{t+1}\right) \\
&= E_t\left[\exp(-r_t) \times \left(\frac{\zeta_{t+1}}{\zeta_t}\right) \times p_{t+1}\right] = \exp(-r_t)E^Q\left(p_{t+1}\right).
\end{aligned}$$

This clarifies the relationship between the pricing kernel and the risk-neutral measure. As shown here, the pricing kernel effectively adjusts the measure in addition to the discount effect arising from $\exp(-r_t)$.

B Recursive Bond Prices

We can confirm that the n-period bond pricing formula in

$$\begin{aligned}
p_t^{n+1} &= E_t\left(m_{t+1}p_{t+1}^n\right) \\
&= E_t\left[\begin{array}{c} \exp(-r_t + \Omega_t\Sigma_{t+1}e_{t+1} - \frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma'_{t+1}\Omega'_t) \\ \times\exp(\bar{A}_n + \bar{B}_n r_{t+1} + \bar{\mathbf{C}}_n\mathbf{h}_{t+2} + \bar{\mathbf{D}}_n X_{t+1}) \end{array}\right] \\
&= \exp(-r_t + \bar{A}_n - \frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma'_{t+1}\Omega'_t) \\
&\quad \times E_t\left[\exp(\Omega_t\Sigma_{t+1}e_{t+1} + \bar{B}_n r_{t+1} + \bar{\mathbf{C}}_n\mathbf{h}_{t+2} + \bar{\mathbf{D}}_n X_{t+1})\right].
\end{aligned}$$

Plugging in the dynamics of X_{t+1}, r_{t+1}, and h_{t+2} into the above gives

$$
\begin{aligned}
p_t^{n+1} &= \exp(-r_t + \bar{A}_n - \frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma'_{t+1}\Omega'_t) \\
&\quad \times E_t\left[\exp\left(\begin{array}{c} \Omega_t\Sigma_{t+1}e_{t+1} + \bar{B}_n\left(\overline{\mu}_0 + \overline{\mu}_1 r_t + \overline{\mu}_2 X_t + \sqrt{h_{t+1}}z_{t+1} + \boldsymbol{\mu}_2\boldsymbol{\Sigma}\varepsilon_{t+1}\right) \\ +\bar{\mathbf{C}}_n\mathbf{h}_{t+2} + \bar{\mathbf{D}}_n\left(\boldsymbol{\delta}_0 + \boldsymbol{\delta}_1 r_t + \Phi X_t + \Sigma\varepsilon_{t+1}\right) \end{array}\right)\right] \\
&= \exp\left(\begin{array}{c} -r_t + \bar{A}_n - \frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma'_{t+1}\Omega'_t + \bar{B}_n\left(\overline{\mu}_0 + \overline{\mu}_1 r_t + \overline{\mu}_2 X_t\right) \\ +\bar{\mathbf{D}}_n\left(\boldsymbol{\delta}_0 + \boldsymbol{\delta}_1 r_t + \Phi X_t\right) \end{array}\right) \\
&\quad \times E_t\left[\exp\left(\begin{array}{c} \Omega_t\Sigma_{t+1}e_{t+1} + \bar{B}_n\left(\sqrt{h_{t+1}}z_{t+1} + \boldsymbol{\mu}_2\boldsymbol{\Sigma}\varepsilon_{t+1}\right) \\ +\bar{\mathbf{C}}_n\mathbf{h}_{t+2} + \bar{\mathbf{D}}_n\left(\Sigma\varepsilon_{t+1}\right) \end{array}\right)\right].
\end{aligned}
$$

At this point, we can spell out the $\bar{\mathbf{C}}_n(.)$ and $\mathbf{h}_{t+2}(.)$ terms in the above as:

$$
\begin{aligned}
\bar{\mathbf{C}}_n\mathbf{h}_{t+2} &= \bar{\mathbf{C}}_n\begin{bmatrix} \beta_0 + \boldsymbol{\beta}_1\mathbf{h}_{t+1} + \alpha z_{t+1}^2 \\ h_{t+1} \end{bmatrix} \\
&= \beta_0\bar{\mathbf{C}}_n\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \bar{\mathbf{C}}_n\begin{bmatrix} 1 \\ 0 \end{bmatrix}\boldsymbol{\beta}_1\mathbf{h}_{t+1} + \bar{\mathbf{C}}_n\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}\mathbf{h}_{t+1} + \bar{\mathbf{C}}_n\begin{bmatrix} 1 \\ 0 \end{bmatrix}\alpha z_{t+1}^2 \\
&= \beta_0\bar{\mathbf{C}}_n s_1 + \bar{\mathbf{C}}_n s_1\boldsymbol{\beta}_1\mathbf{h}_{t+1} + \bar{\mathbf{C}}_n S\mathbf{h}_{t+1} + \bar{\mathbf{C}}_n s_1\alpha z_{t+1}^2 \\
\text{where } s_1 &\equiv \begin{bmatrix} 1 \\ 0 \end{bmatrix},\ S \equiv \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix},
\end{aligned}
$$

where s_1 and S are the selection vector and matrix, respectively. In the expectations operator, rearranging the terms leaves:

$$
\left[\exp\left(\begin{array}{c} \Omega_t\Sigma_{t+1}e_{t+1} \\ +\bar{B}_n\left(\sqrt{h_{t+1}}z_{t+1} + \mu_2\boldsymbol{\Sigma}\varepsilon_{t+1}\right) \\ +\bar{\mathbf{C}}_n\mathbf{h}_{t+2} \\ +\bar{\mathbf{D}}_n\left(\Sigma\varepsilon_{t+1}\right) \end{array}\right)\right] = \left[\exp\left(\begin{array}{c} \Omega_t\Sigma_{t+1}e_{t+1} \\ +\bar{B}_n\sqrt{h_{t+1}}z_{t+1} \\ +\left(\bar{B}_n\mu_2 + \bar{\mathbf{D}}_n\right)\boldsymbol{\Sigma}\varepsilon_{t+1} \\ +\beta_0\bar{\mathbf{C}}_n s_1 \\ +\left(\bar{\mathbf{C}}_n s_1\boldsymbol{\beta}_1 + \bar{\mathbf{C}}_n S\right)\mathbf{h}_{t+1} \\ +\bar{\mathbf{C}}_n s_1\alpha z_{t+1}^2 \end{array}\right)\right].
$$

Putting this back into the bond pricing formula leaves

$$
\begin{aligned}
p_t^{n+1} &= E_t\left(m_{t+1}p_{t+1}^n\right) \\
&= \exp\begin{pmatrix} -r_t+\bar{A}_n-\frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma_{t+1}'\Omega_t'+\bar{B}_n\left(\overline{\mu}_0+\overline{\mu}_1 r_t+\overline{\mu}_2 X_t\right) \\ +\bar{\mathbf{D}}_n\left(\boldsymbol{\delta}_0+\boldsymbol{\delta}_1 r_t+\Phi X_t\right) \\ +\beta_0\bar{\mathbf{C}}_n s_1+\left(\bar{\mathbf{C}}_n s_1\boldsymbol{\beta}_1+\bar{\mathbf{C}}_n S\right)\mathbf{h}_{t+1} \end{pmatrix} \\
&\quad \times E_t\left(\exp\begin{pmatrix} \Omega_t\Sigma_{t+1}e_{t+1} \\ +\bar{B}_n\sqrt{h_{t+1}}z_{t+1}+\underbrace{\left(\bar{B}_n\mu_2+\bar{\mathbf{D}}_n\right)}_{\equiv H_n}\boldsymbol{\Sigma}\varepsilon_{t+1} \\ +\bar{\mathbf{C}}_n s_1\alpha z_{t+1}^2 \end{pmatrix}\right) \\
&= \exp\begin{pmatrix} -r_t+\bar{A}_n-\frac{1}{2}\Omega_t\Sigma_{t+1}\Sigma_{t+1}'\Omega_t'+\bar{B}_n\left(\overline{\mu}_0+\overline{\mu}_1 r_t+\overline{\mu}_2 X_t\right) \\ +\bar{\mathbf{D}}_n\left(\boldsymbol{\delta}_0+\boldsymbol{\delta}_1 r_t+\Phi X_t\right) \\ +\beta_0\bar{\mathbf{C}}_n s_1+\left(\bar{\mathbf{C}}_n s_1\boldsymbol{\beta}_1+\bar{\mathbf{C}}_n S\right)\mathbf{h}_{t+1} \end{pmatrix} \\
&\quad \times E_t\left(\exp\left(\left[\Omega_t\Sigma_{t+1}+\underbrace{\left(\bar{B}_n\sqrt{h_{t+1}}\ \ H_n\Sigma\right)}_{\equiv J_n}\right]e_{t+1}+\bar{\mathbf{C}}_n s_1\alpha z_{t+1}^2\right)\right).
\end{aligned}
$$

Now with the aid of proposition used in Heston and Nandi (2003), i.e., $E_t\exp\left(az_{t+1}\right)=\exp(a^2 1/2)$, and $E_t\exp\left[k\left(z_{t+1}-a\right)^2\right]=\exp\left(\frac{ka^2}{1-2k}-\frac{1}{2}\log\left(1-2k\right)\right)$, where z is i.i.d standard normal, all $t+1$ variables $\left(z_{t+1},\varepsilon_{t+1},z_{t+1}^2\right)$ can be taken out from the expectations operators:

$$
\begin{aligned}
E_t\left[\exp\left(\left[\Omega_t\Sigma_{t+1}+J_n\right]e_{t+1}\right)\right] &= \exp\left[\frac{1}{2}\left(\Omega_t\Sigma_{t+1}\Sigma_{t+1}'\Omega_t'+J_nJ_n'+2\Omega_t\Sigma_{t+1}J_n'\right)\right] \\
&= \exp\left[\frac{1}{2}\begin{pmatrix} \Omega_t\Sigma_{t+1}\Sigma_{t+1}'\Omega_t'+\bar{B}_n^2 s_1\mathbf{h}_{t+1}+H_n\Sigma\Sigma'H_n' \\ +2\begin{pmatrix} \omega_{0r}\bar{B}_n s_1\mathbf{h}_{t+1}+\omega_{0r}\mu_2\Sigma\Sigma'H_n' \\ +H_n\Sigma\Sigma'\omega_0+H_n\Sigma\Sigma'\omega_1 r_t \\ +H_n\Sigma\Sigma'\tilde{\Omega}_1 X_t \end{pmatrix} \end{pmatrix}\right] \\
E_t\left[\bar{\mathbf{C}}_n s_1\alpha z_{t+1}^2\right] &= \exp\left[-\frac{1}{2}\log\left(1-2\bar{\mathbf{C}}_n s_1\alpha\right)\right].
\end{aligned}
$$

The bond price equation can finally be rewritten as

$$
\begin{aligned}
p_t^{n+1} &= E_t\left(m_{t+1}p_{t+1}^n\right) \\
&= \exp\left(\begin{array}{c}
+\bar{A}_n+\bar{B}_n\overline{\mu}_0+\beta_0\bar{\mathbf{C}}_n s_1+\bar{\mathbf{D}}_n\boldsymbol{\delta}_0 \\
+\frac{1}{2}H_n\Sigma\Sigma' H_n'-\frac{1}{2}\log\left(1-2\bar{\mathbf{C}}_n s_1\alpha\right) \\
+\omega_{0r}\mu_2\Sigma\Sigma' H_n'+H_n\Sigma\Sigma'\omega_0 \\
\\
+\left(\bar{B}_n\overline{\mu}_1+\bar{\mathbf{D}}_n\boldsymbol{\delta}_1-1+H_n\Sigma\Sigma'\omega_1\right)r_t \\
\\
+\left(\bar{\mathbf{C}}_n s_1\boldsymbol{\beta}_1+\bar{\mathbf{C}}_n S+\frac{1}{2}\bar{B}_n^2 s_1+\omega_{0r}\bar{B}_n s_1\right)\mathbf{h}_{t+1} \\
\\
+\left(\bar{B}_n\overline{\mu}_2+\bar{\mathbf{D}}_n\Phi+H_n\Sigma\Sigma'\tilde{\Omega}_1\right)X_t
\end{array}\right),
\end{aligned}
$$

corresponding to equations (8) - (11).

C Data

Table AC-1. Summary Statistics of the Data

	Central moments				Autocorrelations		
	Mean	Stdev	Skew	Kurt	Lag 1	Lag 2	Lag 3
FF rate	5.703	3.3544	1.2273	5.0431	0.9865	0.9627	0.9387
3-month	5.303	2.8486	1.087	4.6614	0.9839	0.9634	0.9447
12-month	5.698	2.8388	0.8936	3.9897	0.9849	0.9658	0.949
36-month	6.076	2.705	0.8848	3.7083	0.989	0.9752	0.9631
60-month	6.282	2.6353	0.8752	3.5146	0.9905	0.9795	0.9698
CPI	1.666	1.200	1.384	4.796	0.993	0.982	0.969
Employment	0.834	0.846	-0.676	3.150	0.983	0.951	0.901

Note: Normal distribution has skewness of zero and kurtosis of 3.

Table AC-2. Data sources

Variable	Source
Federal funds rate	Fed
zero coupon bond yields (3, 12, 36, 60 month) 1/	CRSP US Treasury Database
Consumer Price Index	Bureau of Labor Statistics
Employment	Bureau of Labor Statistics

1/ CRSP currently does not provide zero-coupon bond yield data longer than five years.

D The Log-Likelihood Function

We estimate the model dynamics (equation (12)) by numerically maximizing the following log-likelihood function:

$$L(\theta) = -\frac{1}{2}\sum_{t=1}^{T} \log(\det(H_t)) - \frac{1}{2}\sum_{t=1}^{T} u_t' H_t^{-1} u_t,$$

where θ is the vector of parameters to be estimated;

$$\theta = \left[\delta_0, \delta_1, \Phi, \lambda, \beta_1, \alpha, \Sigma, \overline{\mu}_0, \overline{\mu}_1, \overline{\mu}_2, \Sigma^m, \Omega_0, \omega_1, \tilde{\Omega}_1\right].$$

H is the covariance-variance matrix,

$$H_t = \begin{bmatrix} h_t + \mu_2\Sigma(\mu_2\Sigma)' & \mu_2\Sigma\Sigma' & 0 \\ \Sigma(\mu_2\Sigma)' & \Sigma\Sigma & 0 \\ 0 & 0 & \Sigma^m\Sigma^{m\prime} \end{bmatrix},$$

where the initial value of h is calculated by the sum of squared residuals of the short-rate dynamics based on the low-inflation period of the 1950s, and u is the error term in the modeldefined by

$$u_t = Y_t - A_Y - B_Y r_{t-1} - \mathbf{C}_Y \mathbf{h}_t - \mathbf{D}_Y X_{t-1},$$

where

$$Y_t = \begin{pmatrix} r_t \\ X_t \\ R_{t-1} \end{pmatrix}, \; A_Y = \begin{pmatrix} \overline{\mu}_0 \\ \delta_0 \\ A \end{pmatrix}, \; B_Y = \begin{pmatrix} \overline{\mu}_1 \\ \delta_1 \\ B \end{pmatrix}, \; \mathbf{C}_Y = \begin{pmatrix} 0 \\ 0 \\ \mathbf{C} \end{pmatrix}, \; \mathbf{D}_Y = \begin{pmatrix} \overline{\mu}_2 \\ \Phi \\ \mathbf{D} \end{pmatrix}.$$

E Estimating Macro Dynamics Without the Term Structure of Interest Rates

To see if our estimated parameters for macro dynamics lie within a reasonable range, we estimate the macro dynamics given by (2) and report the estimated results. The only difference between (2) and our macro-finance GARCH ATSM is that the former excludes the term structure.

The log-likelihood function is given by

$$L(\tilde{\theta}) = -\frac{1}{2}\sum_{t=1}^{T}\log(\det(\tilde{H}_t)) - \frac{1}{2}\sum_{t=1}^{T}\varepsilon_t'\tilde{H}_t^{-1}\varepsilon_t,$$

where θ is the vector of parameters to be estimated;

$$\tilde{\theta} = [\delta_0, \delta_1, \Phi, \beta_0, \beta_1, \alpha, \Sigma, \overline{\mu}_0, \overline{\mu}_1, \overline{\mu}_2, \mu_2\Sigma].$$

H is the covariance-variance matrix

$$\tilde{H}_t = \begin{bmatrix} h_t + \mu_2\Sigma(\mu_2\Sigma)' & \mu_2\Sigma\Sigma' \\ \Sigma(\mu_2\Sigma)' & \Sigma\Sigma' \end{bmatrix},$$

and ε is the error term in the model defined by:

$$\varepsilon_{t+1} = \begin{pmatrix} r_{t+1} \\ X_{t+1} \end{pmatrix} - \begin{pmatrix} \overline{\mu}_0 \\ \delta_0 \end{pmatrix} - \begin{pmatrix} \overline{\mu}_1 \\ \delta_1 \end{pmatrix} r_t - \begin{pmatrix} \overline{\mu}_2 \\ \Phi \end{pmatrix} X_t.$$

The estimation results are reported in Table AE.

Table AE. Estimation Results: A Multivariate GARCH Model

Short-rate dynamics

$\overline{\mu}_0$	$\overline{\mu}_1$	$\overline{\mu}_2$	
0.028	0.962	0.032	0.128
0.023	*0.007*	*0.016*	*0.016*

λ	α	β_1	
2.390E-02	6.360E-02	0.764	-0.003
7.4E-02	*1.4E-02*	*0.029*	*0.065*

Dynamics of macro variables

δ_0	δ_1	Φ		Σ	
-0.028	-7.00E-04	0.994	0.052	0.136	0.0002
1.160E-02	*0.0026*	*0.0073*	*0.0067*	*0.0039*	*0.0061*
0.077	-4.00E-04	*-0.0303*	*0.9739*	---	0.151
0.0130	*0.0029*	*0.0081*	*0.0075*	---	*0.0044*