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Bayesian Estimation of Demand Functions under Block Rate Pricing

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Abstract

This article proposes a Bayesian estimation method of demand functions under block rate pricing, focusing on increasing one, where we first considered the separability condition explicitly which has been ignored in the previous literature. Under this pricing structure, price changes when consumption exceeds a certain threshold and the consumer faces a utility maximization problem subject to a piecewise-linear budget constraint. Solving this maximization problem leads to a statistical model that includes many inequalities, such as the so-called separability condition. Because of them, it is virtually impractical to numerically maximize the likelihood function. Thus, taking a hierarchical Bayesian approach, we implement a Markov chain Monte Carlo simulation to properly estimate the demand function. We find, however, that the convergence of the distribution of simulated samples to the posterior distribution is slow, requiring an additional scale transformation step for parameters to the Gibbs sampler. These proposed methods are applied to estimate the Japanese residential water demand function.

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JEL classification: C11, C24, Q25.

1 Introduction

Block rate pricing is a nonlinear pricing system often applied in public utilities, such as water. In contrast to other goods and services offered at a single price, consumers under block rate pricing face several prices corresponding to their level of consumption. Income tax also has this pricing structure because the marginal tax rate changes according to total income. General microeconomic theory suggests that efficient allocation is achieved by setting a good's unit price equal to its production cost per unit, which is called marginal cost pricing. At a practical level, several market failures exist, which makes this marginal cost pricing inapplicable. In such cases, block rate pricing is often selected by regulators.

To derive the demand function under block rate pricing, we adopt a discrete/continuous choice approach, which Burtless and Hausman (1978) first used to analyze taxation's effect on labor supply (see also Hanemann (1984); Hausman (1985); Moffitt (1986)). Model specifications of this kind are commonly used to evaluate tax policy or examine consumer behavior under block rate pricing structures, such as labor supply (Burtless and Hausman, 1978), expenditure with food stamps (Moffitt, 1989), car ownership and use (de Jong, 1990), electricity demand (Herriges and King, 1994; Reiss and White, 2005), and water demand (Hewitt and Hanemann, 1995; Olmstead, Hanemann, and Stavins, 2007).

While this approach is based on the consumer's maximization problem, a corresponding statistical model includes many inequality constraints. It is virtually impractical to numerically maximize the likelihood function evaluating them. Further, as Moffitt (1986) pointed out, there is not only a computational burden but also nondifferentiability of the likelihood function. Thus, previous studies estimated the demand function in a simplified manner: all consumers face the same two-block rate pricing. The exception seems to be Olmstead et al. (2007), who consider multiple-block (the number of blocks varies from 2 to 4) rate pricing by the maximum likelihood method. Their method, however, ignores the so-called separability condition, which becomes important as the number of blocks increases. In Japan, consumers usually face more than two blocks (five to eleven for water and three to four for electricity), which requires them to consider multiple-block rate pricing.

Therefore, this article, taking a hierarchical Bayesian approach, implements a Markov chain Monte Carlo (MCMC) simulation to estimate the demand function, (see Chib (2001) for the MCMC methodology, and Chib and Greenberg (1996) for its use in econometrics), and incorporates two practical characteristics compared to the previous studies. First, we allow the number of blocks to be more than two. Then, the discrete/continuous approach derives the demand function as a multinomial generalization of the type V Tobit model (see Chapter 10 of Amemiya (1985) for Tobit classifications, and Chib (1992), which is a pioneering work of the Bayesian approach in Tobit modeling). Second, the separability condition is explicitly considered, which guarantees that consumer preference is divided disjointly by blocks. This condition has been ignored in previous studies, yet plays a critical role especially when facing multiple-block rate pricing.

We find, however, that the distribution of samples obtained from the Gibbs sampler converges very slowly to the posterior distribution. To improve sampling inefficiency, we introduce an additional scale transformation step for parameters to the Gibbs sampler based on the generalized Gibbs step (GGS) by Liu and Sabatti (2000).

The rest of this article is organized as follows. In Section 2, we describe the discrete/continuous choice approach and the demand function under block rate pricing. Section 3 explains the statistical model, derives its likelihood function and joint posterior distribution, and accounts for the separability condition. With this posterior distribution, this section presents the MCMC algorithm and corresponding generalized Gibbs step. Section 4 carries out a simulation study and reveals several properties of our algorithms. After data description, Section 5 applies our proposed method to estimate the residential water demand function in Japan using microdata, and the price and income elasticities are also estimated to investigate the sensitivity of the demands in detail. Some remarks in Section 6 conclude this article.

2 Demand Function

First, we explain the model settings following the discussion of Moffitt (1986). There are two goods: a good under block rate pricing and all other goods. Suppose that the consumer's demand for a good, Y, is subject to K-block rate pricing, and that its demand is strictly positive, $Y > 0$. Let Y_a and I be the expenditure for other goods except Y and total income, respectively. The price system of Y is described as follows. There are K prices, P_k ($k = 1, \ldots, K$), in relation to K blocks. These prices are fixed and considered to be given constants throughout this article. In practice, price often changes monotonically, such as $P_k < P_{k+1}$ or $P_k > P_{k+1}$ for $k = 1, \ldots, K-1$. This article focuses on the price system where price increases monotonically, that is, $P_k < P_{k+1}$ ($k = 1, \ldots, K-1$), which is called increasing block rate pricing. Japanese residential water demand data in Section 5 offer an example of increasing block rate pricing. Let $\bar{Y}_k$ denote the upper limit, or threshold, of the k-th block ($k = 0, \ldots, K$), where we set $\bar{Y}_0 \equiv 0$ and $\bar{Y}_K \equiv \infty$ for convenience. In addition to marginal prices and thresholds, there is a fixed cost FC that is independent of the consumption Y. At a practical level, this fixed cost represents a minimum access charge. The threshold values, $\bar{Y}_k$, and a fixed cost FC are given fixed constants.

Let $U(Y, Y_a)$ be the well-defined utility function. Then, the consumer's utility maximization problem is given by:

$$V = \max_{Y, Y_a} U(Y, Y_a) \quad \text{subject to } c(Y) + Y_a \leq I, \tag{1}$$

where $c(Y) = FC + P_k(Y - \bar{Y}_{k-1}) + \sum_{j=1}^{k-1} P_j(\bar{Y}_j - \bar{Y}_{j-1})$, if $\bar{Y}_{k-1} \leq Y < \bar{Y}_k$ for $k = 1, \ldots, K$. Figure 1 illustrates a budget constraint and indifference curve under three-block increasing block rate pricing, where the second block is optimal with its optimal demand Y_{opt} and level of indirect utility, V. The budget constraint of this form is called a piecewise-linear budget constraint because it becomes linear given

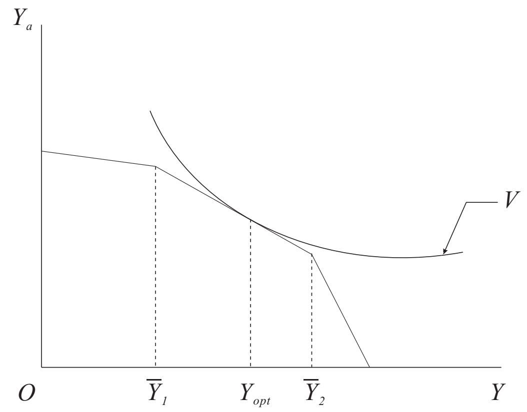


Figure 1: Utility maximization problem: three-block case.

the choice of a block.

Finally, the demand function is derived. Before its derivation, we need to define K conditional utility maximization problems. For $k = 1, \ldots, K$, the k-th conditional problem is given by:

$$\max_{Y, Y_a} U(Y, Y_a) \quad \text{subject to } P_k Y + Y_a \leq Q_k, \quad \text{where } Q_k = I - FC - \sum_{j=1}^{k-1} (P_j - P_{j+1}) \bar{Y}_j, \tag{2}$$

and Q_k is an augmented income (also referred to as virtual income). Under Problem (2), the consumer can maximize utility as if facing a single price P_k and virtual income Q_k. Let conditional demand Y_k be the solution to this conditional utility maximization problem, and we have the demand function under increasing block rate pricing:

$$Y = \begin{cases} Y_k, & \text{if } \bar{Y}_{k-1} < Y_k < \bar{Y}_k \text{ and } k = 1, \ldots, K, \\ \bar{Y}_k, & \text{if } Y_{k+1} \leq \bar{Y}_k \leq Y_k \text{ and } k = 1, \ldots, K-1. \end{cases} \tag{3}$$

In the study of the demand function under block rate pricing, there are several functional forms such as linear, quadratic, and log-linear functions for the conditional demand, Y_k in Eq.(3); the log-linear conditional demand model is one of the most popular models used in previous studies (see Hewitt and Hanemann (1995) for example). Thus, this article focuses on the log-linear model for conditional demand, but our proposed estimation method would apply to other models in a similar manner. The log-linear model is given by $\ln Y_k = \beta_1 \ln P_k + \beta_2 \ln Q_k$, where the parameters β_1 and β_2 represent the price and income elasticities conditional on the block choice, respectively. For simplicity, let y, y_k, $\bar{y}_k$, p_k, and q_k denote logarithm of the demand, Y, k-th conditional demand, Y_k, k-th threshold, $\bar{Y}_k$, k-th marginal price, P_k, and k-th virtual income, Q_k, respectively. Then, we have Eqs (3) with log-linear model as:

$$y = \begin{cases} y_k, & \text{if } \bar{y}_{k-1} < y_k < \bar{y}_k \text{ and } k = 1, \ldots, K, \\ \bar{y}_k, & \text{if } y_{k+1} \le \bar{y}_k \le y_k \text{ and } k = 1, \ldots, K-1, \end{cases} \tag{4}$$

$$y_k = \beta_1 p_k + \beta_2 q_k \equiv \boldsymbol{x}_k' \boldsymbol{\beta}, \tag{5}$$

where $\boldsymbol{x}_k = (p_k, q_k)'$, $\boldsymbol{\beta} = (\beta_1, \beta_2)'$, $\bar{y}_0 \equiv -\infty$ and $\bar{y}_K \equiv \infty$.

3 Bayesian Analysis of Demand Functions under Block Rate Pricing

3.1 Statistical Model

From this section, we append the subscript i to the i-th consumer's variables ($i = 1, \ldots, n$), and the superscript $*$ to latent variables. For examples, $y_i, \bar{y}_{ik}, p_{ik}, q_{ik}, K_i$ are observed variables, while w_i^*, s_i^* are unobserved that will be explained in the following paragraph. We notice that y_{ik}, the k-th log conditional demand, is unobserved, having no superscript $*$ to avoid tedious notation. Because we construct a statistical model which assumes different block rate pricing for different observation, variables for block rate pricing are also marked by the subscript i.

Our statistical model, which is a multinomial extension of Moffitt (1986), is described as follows (see also Hewitt and Hanemann (1995)). First, we introduce two unobserved random variables into the demand function of the i-th consumer: heterogeneity, w_i^*, and state variable, s_i^*. Heterogeneity is a stochastic term that models consumers' characteristics. The w_i^* is assumed to follow the linear model:

$$w_i^* = \boldsymbol{z}_i'\boldsymbol{\delta} + v_i, \quad v_i \sim \text{i.i.d. } N(0, \sigma_v^2), \tag{6}$$

where $\boldsymbol{z}_i$ and $\boldsymbol{\delta}$ are $d \times 1$ vectors of explanatory variables for heterogeneity and corresponding parameters, respectively, and v_i is an independently and identically distributed disturbance term with a normal distribution of mean 0 and variance σ_v^2. We assume that heterogeneity for the i-th observation, w_i^*, is additive to log conditional demand y_{ik}. Then, the log conditional demand with heterogeneity, y_i^*, for the i-th consumer is given by:

$$y_i^* = \begin{cases} y_{ik} + w_i^*, & \text{if } \bar{y}_{i,k-1} < y_{ik} + w_i^* < \bar{y}_{ik} \text{ and } k = 1, \ldots, K_i, \\ \bar{y}_{ik}, & \text{if } y_{i,k+1} + w_i^* \leq \bar{y}_{ik} \leq y_{ik} + w_i^* \text{ and } k = 1, \ldots, K_i - 1, \end{cases} \tag{7}$$

where $y_{ik} = \boldsymbol{x}_{ik}'\boldsymbol{\beta}$ and $\boldsymbol{x}_{ik} = (p_{ik}, q_{ik})'$.

Another latent variable is the state variable, s_i^*. There are $2K_i - 1$ potential outcomes in the demand function Eq. (7): K_i conditional demands with heterogeneity $(y_{ik} + w_i^*)$ and $K_i - 1$ threshold demands $(\bar{y}_{ik})$. The state variable s_i^* is an unobserved discrete random variable taking values from 1 to $2K_i - 1$ and indicates which outcome the i-th observation selects: if s_i^* is odd, observation i chooses conditional demand with heterogeneity, and if s_i^* is even, it selects kink point as its demand. More precisely:

$$s_i^* = \begin{cases} 2k - 1, & \text{if } y_i^* = y_{ik} + w_i^* \text{ and } k = 1, \ldots, K_i, \\ 2k, & \text{if } y_i^* = \bar{y}_{ik} \text{ and } k = 1, \ldots, K_i - 1. \end{cases} \tag{8}$$

It is straightforward from Eq. (7) that the condition whether y_i^* is $y_{ik} + w_i^*$ or $\bar{y}_{ik}$ is equivalent to the interval condition for heterogeneity.

$$\bar{y}_{i,k-1} < y_{ik} + w_i^* < \bar{y}_{ik} \iff w_i^* \in R_{i,2k-1} = (\bar{y}_{i,k-1} - y_{ik}, \bar{y}_{ik} - y_{ik}) \iff s_i^* = 2k - 1, \tag{9}$$

$$y_{i,k+1} + w_i^* \leq \bar{y}_{ik} \leq y_{ik} + w_i^* \Longleftrightarrow w_i^* \in R_{i,2k} = (\bar{y}_{ik} - y_{ik}, \bar{y}_{ik} - y_{i,k+1}) \Longleftrightarrow s_i^* = 2k. \tag{10}$$

Further, we assume that the conditional demand y_i for the i-th consumer is observed with a disturbance:

$$y_i = y_i^* + u_i, \quad u_i \sim \text{i.i.d. } N(0, \sigma_u^2), \quad i = 1, \ldots, n, \tag{11}$$

where u_i implies optimization and specification error as well as measurement error.

Finally, the statistical model for the demand function under increasing block rate pricing is given by following equations:

$$y_{ik} = \boldsymbol{x}_{ik}'\boldsymbol{\beta}, \quad \boldsymbol{x}_{ik} = (p_{ik}, q_{ik})', \quad k = 1, \ldots, K_i, \tag{12}$$

$$w_i^* = \boldsymbol{z}_i'\boldsymbol{\delta} + v_i, \quad v_i \sim \text{i.i.d. } N(0, \sigma_v^2), \tag{13}$$

$$s_i^* = \begin{cases} 2k-1, & \text{if } w_i^* \in R_{i,2k-1} \text{ and } k = 1, \ldots, K_i, \\ 2k, & \text{if } w_i^* \in R_{i,2k} \quad \text{and } k = 1, \ldots, K_i - 1, \end{cases} \tag{14}$$

$$y_i^* = \begin{cases} y_{ik} + w_i^*, & \text{if } s_i^* = 2k-1 \text{ and } k = 1, \ldots, K_i, \\ \bar{y}_{ik}, & \text{if } s_i^* = 2k \text{ and } k = 1, \ldots, K_i - 1, \end{cases} \tag{15}$$

$$y_i = y_i^* + u_i, \quad u_i \sim \text{i.i.d. } N(0, \sigma_u^2). \tag{16}$$

Error terms, measurement error u_i and error for heterogeneity v_i, are assumed to be mutually independent conditional on the block choice s_i^* because they represent different sources of error. This model is a multinomial extension of the Type V Tobit model (see Section 10.10 in Amemiya (1985) for the Type V Tobit model).

Remark 1. Suppose $K_i = 2$. Then, Eqs (14) and (15) reduce to:

$$s_i^* = \begin{cases} 1, & \text{if } w_i^* \in R_{i1} = (-\infty, \bar{y}_{i1} - y_{i1}), \\ 2, & \text{if } w_i^* \in R_{i2} = (\bar{y}_{i1} - y_{i1}, \bar{y}_{i1} - y_{i2}), \\ 3, & \text{if } w_i^* \in R_{i3} = (\bar{y}_{i1} - y_{i2}, \infty), \end{cases} \quad y_i^* = \begin{cases} y_{i1} + w_i^*, & \text{if } s_i^* = 1, \\ \bar{y}_{i1}, & \text{if } s_i^* = 2, \\ y_{i2} + w_i^*, & \text{if } s_i^* = 3. \end{cases} \tag{17}$$

Remark 2. There might be consumers whose first block is zero marginal price, $P_{i1} = 0$. They are assumed to consume more than or equal to the first threshold, $\bar{y}_{i1}$, as suggested by economic theory, which leads to $s_i^* = 2, \ldots, K_i$ and $R_{i2} = (-\infty, \bar{y}_{i1} - y_{i2})$.

3.2 Likelihood Function

The augmented likelihood function for observation i is derived by multiplying two densities. First, we derive the joint density of unobserved variables, s_i^* and w_i^*. These variables are modeled through Eqs (13) and (14). Thus:

$$\begin{aligned} f\left(s_i^*, w_i^* \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_v^2\right) &= f\left(w_i^* \mid \boldsymbol{\delta}, \sigma_v^2\right) f\left(s_i^* \mid w_i^*, \boldsymbol{\beta}\right) \\ &\propto \sigma_v^{-1} \exp\left[-\frac{1}{2\sigma_v^2}\left(w_i^* - \boldsymbol{z}_i'\boldsymbol{\delta}\right)^2\right] I\left(w_i^* \in R_{is_i^*}\right) \prod_{k=1}^{K_i-1} I\left(\boldsymbol{x}_{i,k+1}'\boldsymbol{\beta} \le \boldsymbol{x}_{ik}'\boldsymbol{\beta}\right), \end{aligned} \tag{18}$$

where $I(A)$ is an indicator function taking value 1 if A is true and 0 otherwise. The last truncation term, $\prod_{k=1}^{K_i-1} I(\boldsymbol{x}_{i,k+1}'\boldsymbol{\beta} \le \boldsymbol{x}_{ik}'\boldsymbol{\beta})$, is the separability condition, which is explicitly considered in this article. The role of the separability condition is explained in the next subsection.

Then, after the unobserved variables are determined by Eq. (18), the conditional density of y_i is derived through Eqs (15) and (16), and given by:

$$f\left(y_i \mid s_i^*, w_i^*, \boldsymbol{\beta}, \sigma_u^2\right) \propto \begin{cases} \sigma_u^{-1} \exp\left[-\frac{1}{2\sigma_u^2}\left(y_i - \boldsymbol{x}_{ik}'\boldsymbol{\beta} - w_i^*\right)^2\right], & \text{if } s_i^* = 2k-1 \text{ and } k = 1, \ldots, K_i, \\ \sigma_u^{-1} \exp\left[-\frac{1}{2\sigma_u^2}(y_i - \bar{y}_{ik})^2\right], & \text{if } s_i^* = 2k \text{ and } k = 1, \ldots, K_i - 1, \end{cases} \tag{19}$$

$$= \sigma_u^{-1} \exp\left[-\frac{1}{2\sigma_u^2}\left(y_i - y_i^*\right)^2\right]. \tag{20}$$

Finally, multiplying these two densities (18) and (20), we obtain the augmented likelihood function for observation i:

$$\begin{aligned} f\left(y_i, s_i^*, w_i^* \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2\right) &= f\left(y_i \mid s_i^*, w_i^*, \boldsymbol{\beta}, \sigma_u^2\right) f\left(s_i^*, w_i^* \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_v^2\right) \\ &\propto \sigma_u^{-1}\sigma_v^{-1} \exp\left[-\frac{1}{2}\left\{\sigma_u^{-2}\left(y_i - y_i^*\right)^2 + \sigma_v^{-2}\left(w_i^* - \boldsymbol{z}_i'\boldsymbol{\delta}\right)^2\right\}\right] \\ &\quad \times I\left(w_i^* \in R_{is_i^*}\right) \prod_{k=1}^{K_i-1} I\left(\boldsymbol{x}_{i,k+1}'\boldsymbol{\beta} \le \boldsymbol{x}_{ik}'\boldsymbol{\beta}\right). \end{aligned} \tag{21}$$

3.3 Separability Condition

We briefly describe the role of the separability condition in this model. The separability condition is a condition that disjointly creates heterogeneity intervals. It guarantees that the upper and lower limit for intervals in (10) would not be upside down. Under multiple-block rate pricing, it is given by:

$$y_{i,k+1} \le y_{ik}, \left(\Longleftrightarrow \boldsymbol{x}_{i,k+1}'\boldsymbol{\beta} \le \boldsymbol{x}_{ik}'\boldsymbol{\beta}\right) \quad \text{for } k = 1, \ldots, K_i - 1 \text{ and } i = 1, \ldots, n. \tag{22}$$

Let us illustrate the maximization of augmented likelihood under two-block increasing block rate pricing assuming there is only one observation. Then, condition (22) reduces to be $y_{i2} \le y_{i1}$, which is the only condition. Without this condition, the interval for kink point demand is allowed to be upside down and, hence, there could be the case that $\bar{y}_1 - y_{i2} < w_i^* < \bar{y}_1 - y_{i1}$ (see R_{i2} of Eq. 17). Such a situation leads to ambiguity in the state variable, $s_i^* = 1$ or 3. Therefore, estimation without the separability condition causes disagreement with the model.

Once the separability condition is adopted, it is difficult to take care of many inequality conditions using the maximum likelihood method. The separability condition is one of reasons we take the

Bayesian approach.

3.4 Posterior Distribution and Gibbs Sampler

First, we assume proper prior distributions for the model parameters $(\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2)$. For these parameters, we assume normal distributions for $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ conditional on σ_u^2 and σ_v^2, and inverse gamma distributions for σ_u^2 and σ_v^2.

$$\boldsymbol{\beta} \mid \sigma_u^2 \sim N_2\left(\boldsymbol{\mu}_{\boldsymbol{\beta},0}, \sigma_u^2 \boldsymbol{\Sigma}_{\boldsymbol{\beta},0}\right), \boldsymbol{\delta} \mid \sigma_v^2 \sim N_d\left(\boldsymbol{\mu}_{\boldsymbol{\delta},0}, \sigma_v^2 \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}\right), \sigma_u^2 \sim IG\left(\frac{n_{u,0}}{2}, \frac{S_{u,0}}{2}\right), \sigma_v^2 \sim IG\left(\frac{n_{v,0}}{2}, \frac{S_{v,0}}{2}\right), \quad (23)$$

where $\boldsymbol{\mu}_{\boldsymbol{\beta},0}$, is a 2×1 known vector, $\boldsymbol{\Sigma}_{\boldsymbol{\beta},0} = \text{diag}(\sigma^2_{\beta_1,0}, \sigma^2_{\beta_2,0})$ is a 2×2 known diagonal matrix with diagonal elements $(\sigma^2_{\beta_1,0}, \sigma^2_{\beta_2,0})$, $\boldsymbol{\mu}_{\boldsymbol{\delta},0}$ is a $d \times 1$ known vector, $\boldsymbol{\Sigma}_{\boldsymbol{\delta},0}$ is a known $d \times d$ covariance matrix, and $n_{u,0} > 0$, $S_{u,0} > 0$, $n_{v,0} > 0$, $S_{v,0} > 0$ are some known constants. In this article, subscript on the normal distribution indicates its dimension.

Then, the posterior distribution for the statistical model (12)-(16) is obtained by multiplying the augmented likelihood function (21) over all observations with prior distribution $\pi\left(\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2\right)$:

$$\begin{aligned}
\pi\left(\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2, \boldsymbol{s}^*, \boldsymbol{w}^* \mid \boldsymbol{y}\right) &\propto \pi\left(\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2\right) \\
&\times \sigma_u^{-n} \sigma_v^{-n} \exp\left[-\frac{1}{2}\left\{\sigma_u^{-2} (\boldsymbol{y} - \boldsymbol{y}^*)' (\boldsymbol{y} - \boldsymbol{y}^*) + \sigma_v^{-2} (\boldsymbol{w}^* - \boldsymbol{Z}\boldsymbol{\delta})' (\boldsymbol{w}^* - \boldsymbol{Z}\boldsymbol{\delta})\right\}\right] \\
&\times \prod_{i=1}^{n} \left\{ I(w_i^* \in R_{i s_i^*}) \prod_{k=1}^{K_i - 1} I\left(y_{i,k+1} \leq y_{ik}\right) \right\}, \quad (24)
\end{aligned}$$

where $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)'$, $\boldsymbol{y}^* = (y_1^*, y_2^*, \ldots, y_n^*)'$, $\boldsymbol{s}^* = (s_1^*, s_2^*, \ldots s_n^*)'$, $\boldsymbol{w}^* = (w_1^*, w_2^*, \ldots w_n^*)'$ and $\boldsymbol{Z} = (\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_n)'$.

Under log-linear conditional demand and above priors, the full conditional posterior distributions are all standard distributions, which are provided in Appendix A.1. Then, we implement a standard Gibbs sampler to draw samples from the posterior distribution (24), which is summarized in the following seven steps:

Algorithm 1.1: MCMC algorithm for model (12)-(16)

Step 1. Initialize $\boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$ and σ_v^2.

Step 2. Generate β_1 given $\beta_2, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$.

Step 3. Generate β_2 given $\beta_1, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$.

Step 4. Generate $\left(\sigma_v^2, \boldsymbol{\delta}\right)$ given $\boldsymbol{w}^*$.

Step 5. Generate $\left(s_i^*, w_i^*\right)$ given $\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2$ for $i = 1, \ldots, n$.

Step 6. Generate σ_u^2 given $\boldsymbol{\beta}, \boldsymbol{s}^*, \boldsymbol{w}^*$.

Step 7. Go to Step 2.

A blocking technique is used to sample (s_i^*, w_i^*) in order to isolate the relationship in which w_i^* determines s_i^*, while blocking in $(\sigma_v^2, \boldsymbol{\delta})$ is to accelerate the convergence of MCMC draws.

3.5 Convergence Acceleration

As we shall see in Sections 4 and 5, the obtained samples of parameters are sometimes highly autocorrelated so that their convergence to the posterior distribution is slow. This subsection introduces a generalized Gibbs step proposed by Liu and Sabatti (2000) to improve sampling inefficiency. While its simple implementation, the GGS improves sampling efficiency to some extent in the estimation of the discrete/continuous choice model.

The main idea of the GGS is to add one more sampling step for a transformation group keeping the transition kernel of MCMC invariant, such that we can obtain acceleration effects similar to those of reparametrization or blocking (see Section 2 of Liu and Sabatti (2000) and Section 8.3 of Liu (2001) for a general definition of the GGS).

In our case, we apply the GGS to all parameters $\boldsymbol{\zeta} = (\boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{w}^*, \sigma_u, \sigma_v)$ so as to implement the one-step Metropolis-Hastings (MH) algorithm described below, and take a scale transformation group, that is, $\Gamma = \{g > 0 : g(\boldsymbol{\zeta}) = g\boldsymbol{\zeta}\}$. Then, the full conditional distribution of $\tilde{g} \equiv g^{-1}$ is derived as Eq. (A.36) in

Appendix A.2. This full conditional distribution is a nonstandard distribution, so that the MH algorithm is adopted to draw a sample of $\tilde{g}$. Starting from the initial value $\tilde{g} = 1$, we draw a candidate $\tilde{g}'$, which follows the truncated normal distribution with mean $\mu_{\tilde{g}}$, variance $\sigma^2_{\tilde{g}}$ and truncation interval $R_{\tilde{g}}$:

$$TN_{R_{\tilde{g}}}\left(\mu_{\tilde{g}}, \sigma^2_{\tilde{g}}\right), \tag{25}$$

where $\mu_{\tilde{g}} = a_2/a_1$ and $\sigma^2_{\tilde{g}} = a_1^{-1}$ (see Eqs (A.37), (A.38), and (A.40) in Appendix A.2 for definitions of a_1, a_2, and $R_{\tilde{g}}$). The candidate is accepted with probability: $\alpha(\tilde{g}, \tilde{g}') = \min[1, (\tilde{g}'/\tilde{g})^{a_0-1}]$, where $a_0 = n + n_{u,0} + n_{v,0}$.

It is usually the case to repeat the MH step in order to obtain a sample from the conditional posterior distribution of $\tilde{g}$. As we have proved in Appendix A.3, however, it suffices to draw a sample only once using the initial value $\tilde{g} = 1$. Therefore, the GGS is implemented by replacing Step 7 of Algorithm 1.1 described in the previous subsection:

Algorithm 1.2: Generalized Gibbs step for model (12)-(16)

Step 7. Generate $\tilde{g}$ given $\boldsymbol{\beta}, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u, \sigma_v$.

(a) Generate $\tilde{g}' \sim TN_{R_{\tilde{g}}}(\mu_{\tilde{g}}, \sigma^2_{\tilde{g}})$ and $u \sim U(0,1)$ where $U(0,1)$ denotes a uniform distribution on interval $(0,1)$.

(b) Accept a candidate $\tilde{g}'$ if $u \leq \alpha(1, \tilde{g}')$. If rejected, set $\tilde{g} = 1$.

Step 8. Transform parameters $(\boldsymbol{\beta}, \boldsymbol{\delta}, \boldsymbol{w}^*, \sigma_u, \sigma_v)$ by multiplying parameters by $g = \tilde{g}^{-1}$.The state variable $\boldsymbol{s}^*$ is also updated by this new $\boldsymbol{w}^*$.

Step 9. Go to Step 2.

4 Illustration Using Simulated Data

This section illustrates our Bayesian estimation of the statistical model (12)-(16) with simulated data. We consider two-block increasing block rate pricing with 100 observations. The marginal price for the first block is generated using $\left|N\left(2, 0.4^2\right)\right|$, which is the absolute value of a random number following a normal distribution with mean 2 and variance 0.4^2. The absolute value is taken to guarantee a positive marginal price. The second block's marginal price is similarly generated by adding $\left|N\left(0.7, 0.2^2\right)\right|$ to the first block's price. There is one threshold in this price system, which is set equal to 2. As for other variables, income is generated by $\left|N\left(3, 0.3^2\right)\right|$ and fixed cost is equal to 0. We consider only one explanatory variable other than the constant term for heterogeneity following $N(2.5, 1)$. Thus, $\boldsymbol{\delta} = (\delta_0, \delta_1)'$.

The true parameter values are $(\beta_1, \beta_2, \delta_0, \delta_1, \sigma_u, \sigma_v) = (-0.6, 0.3, 0.1, 0.1, 0.3, 0.1)$. The regression parameter for price is set to be more elastic than that for income because of the evidence often reported in previous studies on the water demand function (see Table 2 of Hewitt and Hanemann (1995)). The prior distributions are:

$$\boldsymbol{\beta} \mid \sigma_u^2 \sim N_2\left(\mathbf{0}, 10^2\sigma_u^2\boldsymbol{I}\right), \boldsymbol{\delta} \mid \sigma_v^2 \sim N_2\left(\mathbf{0}, 10^2\sigma_v^2\boldsymbol{I}\right), \sigma_u^2 \sim IG\left(10^{-2}, 10^{-2}\right), \sigma_v^2 \sim IG\left(10^{-2}, 10^{-2}\right). \quad (26)$$

The mean and variance for the precision parameters σ_u^{-2} and σ_v^{-2} are 1 and 10^2, respectively. In hierarchical modeling, it is often pointed out that flat or improper prior distributions for variance parameters may lead to (almost) improper posterior distributions (see, for example, Section 5.3 of Gelman, Carlin, Stern, and Rubin (1995)), which makes Bayesian inference unreliable. Thus, we use relatively tight proper prior distributions for σ_u^2 and σ_v^2 to avoid (almost) improper posterior distributions.

We draw MCMC samples by the Gibbs sampler, Algorithm 1.1, and find its sample autocorrelations are very high. Thus, we apply the generalized Gibbs step, Algorithm 1.2, to accelerate convergence of samples to their posterior distribution. After deleting 3×10^4 samples, we draw 10^5 samples by applying these accelerations to make Bayesian inference. The GGS results are reported in Table 1. Because other results obtained by the Gibbs sampler is very similar to those obtained by the GGS, we omit them.

Table 1 reports true values, posterior means, posterior standard deviations, 95% credible intervals,

Table 1: Estimation summary with simulated data by GGS

Parameter	*True*	*Mean*	*SD*	*95% interval*		*INEF GGS / GS* *	*CD* **
β_1	$-.6$	$-.71$	.31	[−1.33	− .11]	345 / 400	.123
β_2	.3	.15	.25	[− .42	.60]	731 / 1113	.836
δ_0 (constant)	.1	.45	.27	[.072	1.20]	1034 / 1972	.304
δ_1	.1	.070	.031	[.011	.13]	25 / 16	.051
σ_u (measurement error)	.3	.26	.062	[.10	.35]	152 / 84	.314
σ_v (heterogeneity)	.1	.17	.078	[.059	.33]	144 / 99	.370

* "INEF GGS / GS" denotes the inefficiency factors using the Gibbs sampler with Generalized Gibbs step and the standard Gibbs sampler

** "CD" denotes the convergence diagnostic.

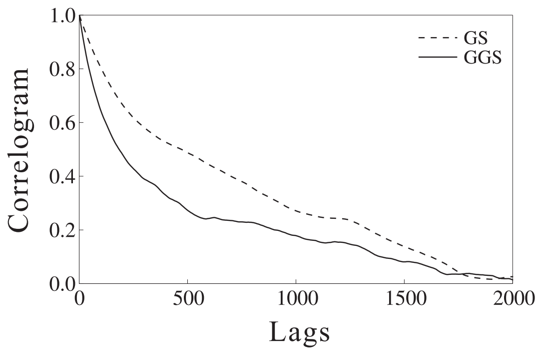


Figure 2: Sample autocorrelation functions for β_2.

estimated inefficiency factors, and the convergence diagnostic, that is, the two-sided p-value of the test for convergence. The inefficiency factor is defined as $1 + 2\sum_{j=1}^{\infty} \rho(j)$, where $\rho(j)$ is the sample autocorrelation at lag j. It is calculated by using the spectral density (see Section 3.2 of Chib (2001) for details). It is interpreted as a ratio of the variance of the sample mean from the Markov chain to the variance of uncorrelated draws. If the inefficiency factor is close to one, the sampling method is almost as efficient as an independent draw. The greater the inefficiency factor becomes, the more samples we should take to reach convergence. The convergence diagnostic, on the other hand, is the test statistic with a null of convergence, proposed in Section 3.2 of Geweke (1992). We use the first 10% and the last 50% of samples to calculate this test statistic as suggested by Geweke (1992).

In Table 1, we found smaller inefficiency factors by GGS than GS. We compare these two samplers in terms of their sample autocorrelation functions. Figure 2 shows sample autocorrelation function of β_2 for GS and GGS results, where autocorrelation decays more quickly in GGS than in GS. Thus, it is concluded that the GGS is effective for β_2 in improving its sample convergence.

There are two findings regarding this simulation. First is the role of the kink point. The state change helps us to separately estimate measurement error σ_u and heterogeneity error σ_v. When kink point is chosen, however, other information, such as regression coefficients $\boldsymbol{\beta}$ and coefficients for heterogeneity, $\boldsymbol{\delta}$, is lost, which causes the sampling of our model to become inefficient.

Second, initial values are important for efficient sampling, which is also a difficult task in the maximum likelihood method (see Section 4 of Moffitt (1986)). The full conditional distributions, especially for $\boldsymbol{\beta}$, are truncated by so many inequality constraints that samples cannot move freely in their state space. Thus, when initial values are far from true values, it takes a considerable amount of time for the distribution of the MCMC samples to converge to the posterior distribution. Because we do not know much about the true parameter values in empirical analysis, testing several initial values might be effective when sampling appears to be inefficient.

5 Estimation of the Japanese Residential Water Demand Function

5.1 Data Description

We use the household-level dataset gathered by ourselves through an online questionnaire on the Internet. The data are monthly and cover randomly-chosen 1250 households in Tokyo and Chiba prefectures. All household faces increasing block rate pricing whose number of blocks varies from five to eleven. The dependent variable is the amount of water calculated from each payment using corresponding price tables. The explanatory variables to be used for empirical analysis are summarized in Table 2 and their summary statistics are found in Table 3 and Figure 3.

Table 2: Variables used in the water demand function

Variable	*Coefficient*	*Description*
year		June 2006
num. of obs.		365
price	β_1	water+sewer (¥10^3/m^3)
virtual income	β_2	monthly income augmented by price (¥10^3)
variables for w_i^*	δ_0	constant
	δ_1	number of members in household
	δ_2	number of rooms in house/apartment
	δ_3	total floor space of house/apartment (50m^2)

Table 3: Summary statistics of variables used in the water demand function

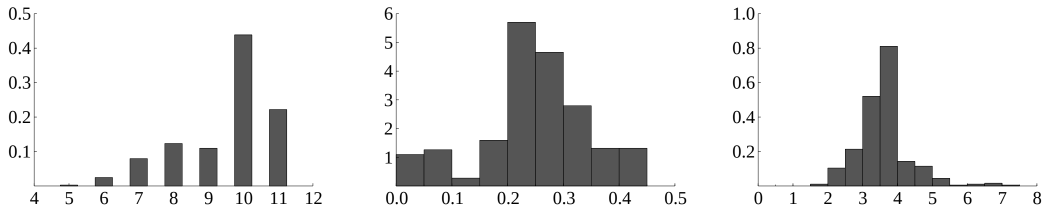
Variable	*Unit*	*Mean*	*SD*	*Min.*	*Max.*
amount of water	$\log m^3$	3.53	.51	.098	4.87
monthly income	¥10^3	1145.00	566.11	166.67	4666.70
number of members in household	person	3.05	1.22	1	7
number of rooms in house/apartment	room	4.29	1.10	1	8
total floor space of house/apartment	50m^2	1.66	.70	.24	4.60

The number of observations is reduced because of their missing or inappropriate answers. Observations are also omitted for technical reasons, as listed below.

- Consumption within the zero marginal price block is observed.
- Living in cities that have discontinuous parts in their price system.
- Living in cities that changed rate tables in June 2006.
- Using a well for water use because of its special charge system.

Observations linked to any of these five reasons are omitted.

Regarding the income variable, it is a sensitive issue to ask households their exact income level. In our research, the household annual income is recorded in one of eight intervals: 0-2, 2-4, 4-6, 6-8, 8-10, 10-12, 12-15, over 15 million yen. Then, we use the median of the interval divided by 12 as a proxy for monthly income for most households. Households whose incomes are over 15 million yen



(a) Number of blocks (number). (b) Price (¥10^3/m^3). (c) Fixed cost (¥10^3).

Figure 3: Histograms of the number of blocks, price, and fixed cost.

are asked to answer their approximate annual income, and we divide it by 12 as monthly income for such households.

5.2 Empirical Result

Initially, the following prior distributions are assumed for parameters of the demand function:

$$\boldsymbol{\beta} \mid \sigma_u^2 \sim N_2\left(\mathbf{0}, 10\sigma_u^2 \boldsymbol{I}\right), \boldsymbol{\delta} \mid \sigma_v^2 \sim N_4\left(\mathbf{0}, 10\sigma_v^2 \boldsymbol{I}\right), \sigma_u^2 \sim IG(0.1, 0.1), \sigma_v^2 \sim IG(0.1, 0.1). \tag{27}$$

Because the Gibbs sampler (Algorithm 1.1) in Subsection 3.4 is very slow to converge to the posterior distribution, we accelerate the convergence of the MCMC samples using the GGS described in Algorithm 1.2 of Subsection 3.5. The initial 16×10^5 samples are discarded and the subsequent 4×10^6 samples are recorded. The recorded samples are reduced to 10^4 samples by picking up every 400-th value. These estimation results are shown in Table 4.

At first, to check the plausibility of our proposed model, we carried out the numerical posterior predictive checks (PPCs) based on these results (see, e.g., Chapter 6 of Gelman, Carlin, Stern, and Rubin (2003)). Seven test quantities, the first and third quartile, mean, median, standard deviation, minimum, and maximum, are chosen to conduct PPCs, and the results are found in Figure 4. The density plots

Table 4: Water demand function

Parameter	*Mean*	*SD*	*95% interval*	*INEF GGS / GS* *	*CD* **
β_1 (price)	−1.09	.22	[−1.52 − .67]	242.55 / 370.44	.201
β_2 (income)	.067	.044	[− .028 .14]	272.18 / 583.76	.025
δ_0 (constant)	.23	.51	[− .89 1.12]	312.61 / 500.76	.055
δ_1 (num. of members)	.23	.039	[.16 .31]	54.90 / 63.16	.983
δ_2 (num. of rooms)	.14	.049	[.038 .23]	7.65 / 9.31	.198
δ_3 (floor space)	.041	.077	[− .11 .20]	7.07 / 7.47	.987
σ_u (measurement error)	.42	.018	[.38 .45]	8.91 / 19.41	.531
σ_v (heterogeneity)	.20	.038	[.14 .28]	17.39 / 10.60	.021

* "INEF GGS / GS" denotes the inefficiency factors using the Gibbs sampler with Generalized Gibbs step and the standard Gibbs sampler

** "CD" denotes the convergence diagnostic.

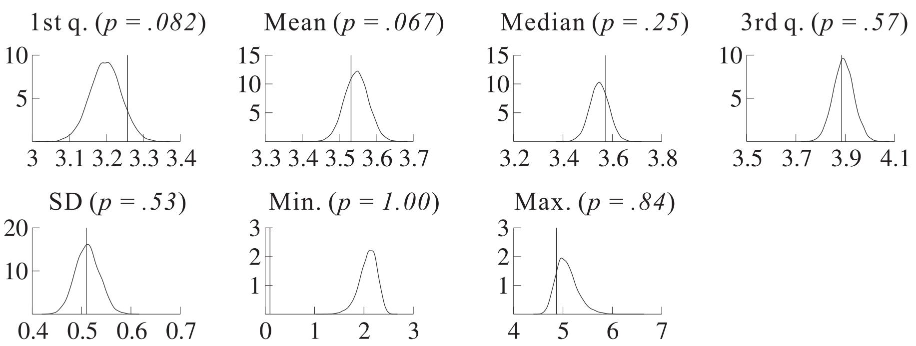


Figure 4: Posterior predictive checks.

represent those of test quantities based on the replicated data from the predictive distribution and the vertical lines denote the values of test quantities based on the observed data. We also calculated the posterior predictive p-values, which are shown in parentheses. All density plots and p-values, except those for the minimum, indicate that the discrete/continuous choice model would be plausible to our Japanese residential water demand data. The small p-value for the minimum indicates that we may need to improve our model for small consumptions (five smallest consumptions are 0.098, 1.79, 1.86, 1.92, and 1.95 $\log \mathrm{m}^3$). As stated in Remark 2 of Subsection 3.1, for simplicity, we excluded households that consume within the zero marginal price block so that all consumptions are above this block. The PPC result for the minimum would be improved when we consider the model with these small amount consumers, which would be a future work.

Next, we analyze the GGS results. The posterior mean of the price elasticity β_1 is estimated to be negative, -1.09. Because its 95% credible interval does not include 0, the probability of $\beta_1 < 0$ is greater than 0.95. This is consistent with what we expect from the economic theory. On the other hand, the posterior mean of the income elasticity β_2 may not differ from 0, because its 95% credible interval includes 0.

Among the independent variables that are expected to explain individual heterogeneity, the number of members in household and number of rooms in house/apartment show positive effects on residential water demand because $\Pr(\delta_j > 0 \mid \boldsymbol{y}) > .95$ $(j = 1, 2)$. Further, the former has larger marginal effect on the demand than the latter, that is, one person increase in household has larger effect for water demand than one room extension to house/apartment does. In contrast, total floor space of house/apartment (δ_3) has no effect on water demand in terms of its 95% credible interval.

We compare these parameter estimates with those of previous studies on water demand. Hewitt and Hanemann (1995) used the microdata from Denton, Texas, introduced the discrete/continuous choice model as their underlying statistical model, and estimated the water demand function under block rate pricing by the maximum likelihood method. Because of its complex form of the likelihood function, their analysis simply focuses on households under two-block increasing block rate pricing. They reported that the price and income parameters are -1.8989 and 0.1782, respectively. While they are larger in absolute values than ours, Hewitt and Hanemann (1995)'s estimates show a similar pattern to ours, that is, the larger price and smaller income elasticities in their absolute values.

Olmstead et al. (2007) also applied the discrete/continuous choice model to estimate the water demand function. They used the microdata from the United States and Canada facing different price schedules, that is, two-block and four-block increasing block rate pricings, and the uniform price system. They estimated that the price and income parameters for households under block rate pricing are

−0.6411 and 0.1959, respectively.

Dalhuisen, Florax, de Groot, and Nijkamp (2003) analyzed 64 studies on water demand and presented the meta analysis on price and income elasticities. They showed that the price and income elasticity are dispersed with means −0.41 and 0.43, and standard deviations 0.86 and 0.79, respectively. Their estimates are somewhat similar when we take their large standard deviations into consideration.

5.3 Predictive Analysis

At the end of this section, we conduct a posterior predictive analysis on water demand when the block rate price schedule is changed to the uniform pricing. We consider two types of uniform pricing, that is, the same uniform price for all households and the different uniform prices for each households. For the former, let the unit price be ¥100/m^3, ¥250/m^3, or ¥500/m^3, setting the fixed cost to ¥3,500 for every unit price cases. These unit prices are inexpensive, almost as high as, or expensive ones for the majority of households compared to the present increasing block rate pricing, and fixed cost is set close to the present one for most households (see Figures 3(b) and 3(c)). For the latter, on the other hand, we pick up the price of the block where the consumption is actually made as the single price of this suppositional uniform price system and the fixed cost remains the same with the present schedule. To analyze the effect of such price schedule changes, we generate samples of predictive demand using the Gibbs with GGS samples, and draw boxplots of predictive distributions for each household found in Figure 5.

In this figure, solid lines and each boxplots represent plots of actual log demands and boxplots of predictive distribution for each households, respectively. The water consumptions of households are arranged in ascending order, and the number of household is reduced to 60 by picking up every 6-th household. Each box, upper and lower whiskers denote the range between the first and third quartiles, 95-th and 5-th percentiles, respectively.

First three figures in Figure 5 reveals that most households consume more water as the price be-

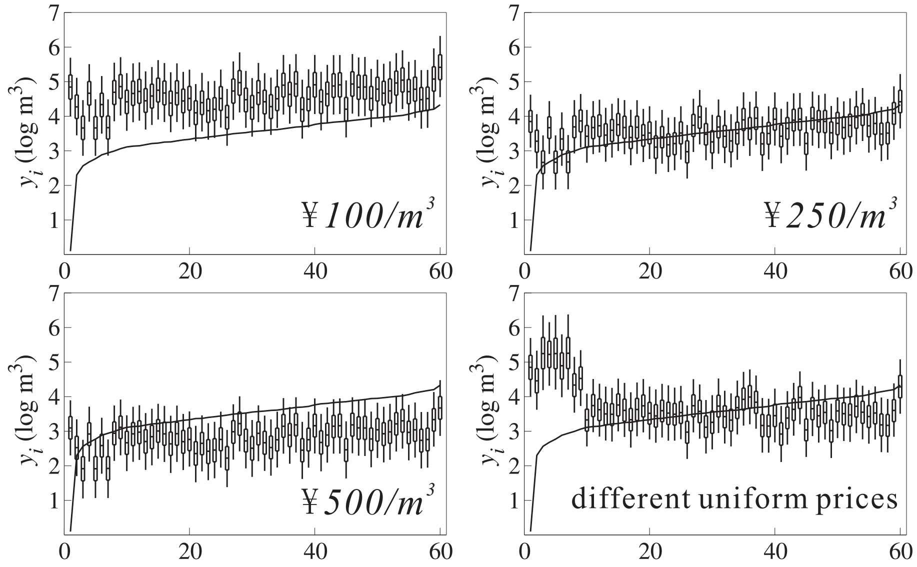


Figure 5: Effect of uniform pricings.

comes less inexpensive, which is expected from the negatively estimated price elasticity. The lower right figure of Figure 5, on the other hand, suggests that households who consume less tend to choose a suboptimal block. From the microeconomic theory view point, the price system change of this kind has no effect on the consumption as far as a underlying preference satisfies regular assumptions. Our statistical modeling, however, introduces the measurement error. Due to the measurement error, households would choose a suboptimal block, and such a suboptimal choice is partly captured by this suppositional price system.

6 Concluding Remarks

This article proposed a Bayesian estimation method for demand functions under block rate pricing and conducted empirical analysis with Japanese residential water demand data. Furthermore, the separability condition is explicitly considered to obtain appropriate estimates. Our method is useful for analyzing the demand for water services as well as for other goods or services facing block rate pricing, including taxation. Furthermore, it would be possible to apply our method to examine consumer's choice over multiple product categories and brands (Song and Chintagunta, 2007) and consumer's selection of calling plans for wireless services (Iyengar, 2004).

Future research may be conducted on several related issues. First, the supply structure needs to be considered to apply our method to other goods under block rate pricing. Water companies are regional monopolists and obliged to supply as much as consumers require. Thus, we excluded firm competition. Other suppliers, such as telecommunication services and deregulated electricity services, face no such obligation and compete with each other. To analyze the demand of these services, it is necessary to consider the supply structure explicitly in our model. Disequilibrium models are a framework that can handle such a market structure. See Kunitomo and Sato (1996) and Maddala (1983) for a discussion of disequilibrium models.

Second, as pointed out in the previous section, there are households who consume less than the zero marginal price block. The discrete/continuous choice model proposed by this article assumes to exclude such a behavior. Another structural approach to these consumers is necessary as a future work.

Thirdly, substitution among electricity, gas, and other fuels needs to be considered. It is possible for the block rate pricing model proposed here to be extended to a multivariate setting in a natural way. Furthermore, Japanese gas services are provided under decreasing block rate pricing. Thus, a subsequent study will examine the energy demand function under a mixture of increasing and decreasing block rate pricing.

Finally, an improved convergence acceleration method needs to be proposed. Although the generalized Gibbs step improved sampling efficiency, the regression coefficients, $\boldsymbol{\beta}$, still show high sample autocorrelations. Further improvement of convergence acceleration is a subject of future research.

Appendices

A.1 Full Conditional Distributions for the Statistical Model (12)-(16)

This section provides the full conditional distributions for the statistical model (12)-(16) following the standard Gibbs sampler's steps (see Algorithm 1.1 in Subsection 3.4). Let $\mathcal{A}$ denote a set of observations who do not select the threshold or kink point as their demand, that is, $\mathcal{A} = \{i|s_i^*$ is odd and equal to $2k_i - 1\}$. Furthermore, without loss of generality, we assume that $p_{i1}, q_{i1}, \bar{y}_{i1}$ are strictly positive, that is, $p_{i1}, q_{i1}, \bar{y}_{i1} > 0$. This can be accomplished by adjusting the unit of measurement of price and income. When $P_{i1} = 0$ ($p_{i1} = -\infty$), we assume $s_i^* \geq 2$ and let $p_{i2} > 0$.

Step 2. Generate β_1 *given* $(\beta_2, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2)$. The full conditional distribution for β_1 is the truncated normal distribution $TN_{R_1}(\mu_1, \sigma_u^2 \sigma_1^2)$, where:

$$
\begin{aligned}
\mu_1 &= \sigma_1^2 \left[\sigma_{\beta_1,0}^{-2} \mu_{\beta_1,0} + \sum_{i \in \mathcal{A}} p_{ik_i} \left(y_i - \beta_2 q_{ik_i} - w_i^* \right) \right], \quad \sigma_1^{-2} = \sigma_{\beta_1,0}^{-2} + \sum_{i \in \mathcal{A}} (p_{ik_i})^2, \\
R_1 &= \left(\max_i (-\infty, BL_i), \min_{i,k} \left(BU_i, -\beta_2 \frac{q_{i,k+1} - q_{ik}}{p_{i,k+1} - p_{ik}} \right) \right),
\end{aligned}
\tag{A.28}
$$

and $\mu_{\beta_1,0}$ is a prior mean of β_1. The BL_i and BU_i are the lower and upper bounds of the interval B_i such that:

$$
B_i = \begin{cases} \left(\frac{\bar{y}_{i,k-1} - \beta_2 q_{ik} - w_i^*}{p_{ik}}, \frac{\bar{y}_{i,k} - \beta_2 q_{ik} - w_i^*}{p_{ik}} \right), & \text{if } s_i^* = 2k - 1, \\ \left(\frac{\bar{y}_{i,k} - \beta_2 q_{ik} - w_i^*}{p_{ik}}, \frac{\bar{y}_{i,k} - \beta_2 q_{i,k+1} - w_i^*}{p_{i,k+1}} \right), & \text{if } s_i^* = 2k. \end{cases}
\tag{A.29}
$$

These B_is are constructed from intervals $R_{is_i^*}$ defined in (9) and (10) of Subsection 3.1. To sample from the truncated normal distributions, we use the inverse cumulative distribution function sampling method (see Section 1.3 of Gamerman (1997)).

Step 3. Generate β_2 *given* $\beta_1, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_u^2$ As in Step 2, the full conditional distribution for β_2 is the truncated normal distribution $TN_{R_2}(\mu_2, \sigma_u^2 \sigma_2^2)$, where:

$$\mu_2 = \sigma_2^2 \left[\sigma_{\beta_2,0}^{-2} \mu_{\beta_2,0} + \sum_{i \in \mathcal{A}} q_{ik_i} \left(y_i - \beta_1 p_{ik_i} - w_i^* \right) \right], \quad \sigma_2^{-2} = \sigma_{\beta_2,0}^{-2} + \sum_{i \in \mathcal{A}} (q_{ik_i})^2,$$
$$R_2 = \left(\max_i \left(-\infty, BL_i^\dagger \right), \min_{i,k} \left(BU_i^\dagger, -\beta_1 \frac{p_{i,k+1} - p_{ik}}{q_{i,k+1} - q_{ik}} \right) \right), \tag{A.30}$$

and $\mu_{\beta_2,0}$ is a prior mean of β_2. The $BL_i^\dagger$ and $BU_i^\dagger$ are the lower and upper bounds of the interval $B_i^\dagger$ such that:

$$B_i^\dagger = \begin{cases} \left(\frac{\bar{y}_{i,k-1} - \beta_1 p_{ik} - w_i^*}{q_{ik}}, \frac{\bar{y}_{i,k} - \beta_1 p_{ik} - w_i^*}{q_{ik}} \right), & \text{if } s_i^* = 2k - 1, \\ \left(\frac{\bar{y}_{i,k} - \beta_1 p_{ik} - w_i^*}{q_{ik}}, \frac{\bar{y}_{i,k} - \beta_1 p_{i,k+1} - w_i^*}{q_{i,k+1}} \right), & \text{if } s_i^* = 2k. \end{cases} \tag{A.31}$$

Step 4. Generate $(\sigma_v^2, \boldsymbol{\delta})$ *given* $\boldsymbol{w}^*$. Because a blocking technique is applied in this step, $\boldsymbol{\delta}$ is integrated over the full conditional distribution of $(\boldsymbol{\delta}, \sigma_v^2)$ to obtain the full conditional of σ_v^2. Thus, sample σ_v^2 from the inverse gamma distribution, $IG(\frac{n_{v,1}}{2}, \frac{S_{v,1}}{2})$, and $\boldsymbol{\delta}$ from the multivariate normal distribution, $N_d(\boldsymbol{\mu}_{\boldsymbol{\delta},1}, \sigma_v^2 \boldsymbol{\Sigma}_{\boldsymbol{\delta},1})$, where $n_{v,1} = n_{v,0} + n$,

$$S_{v,1} = S_{v,0} + \boldsymbol{\mu}_{\boldsymbol{\delta},0}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0} + \boldsymbol{w}^{*\prime} \boldsymbol{w}^* - \boldsymbol{\mu}_{\boldsymbol{\delta},1}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},1}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},1},$$
$$\boldsymbol{\mu}_{\boldsymbol{\delta},1} = \boldsymbol{\Sigma}_{\boldsymbol{\delta},1} \left(\boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0} + \boldsymbol{Z}' \boldsymbol{w}^* \right), \; \boldsymbol{\Sigma}_{\boldsymbol{\delta},1}^{-1} = \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} + \boldsymbol{Z}'\boldsymbol{Z}. \tag{A.32}$$

Step 5. Generate (s_i^*, w_i^*) *given* $\boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2$ *for* $i = 1, \ldots, n$. We again apply a blocking technique in drawing samples of (s_i^*, w_i^*). Then, the conditional posterior distribution of s_i^* is discrete such that:

$$Pr\left(s_i^* = s \mid \boldsymbol{\beta}, \boldsymbol{\delta}, \sigma_u^2, \sigma_v^2 \right) \propto \tau_s \left[\Phi\left\{ \tau_s^{-1} (RU_{is} - \theta_{is}) \right\} - \Phi\left\{ \tau_s^{-1} (RL_{is} - \theta_{is}) \right\} \right] \exp\left(-\frac{m_{is}}{2} \right), \tag{A.33}$$

for $s = 1, \ldots, 2K_i - 1$ where $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution, RU_{is} and RL_{is} denote the upper and lower limit of the interval for heterogeneity, R_{is}, given by Eqs (9) and (10), and $(m_{is}, \theta_{is}, \tau_s^2)$ are defined as follows:

$$\left(m_{is}, \theta_{is}, \tau_s^2\right) = \begin{cases} \left(\dfrac{\sigma_u^{-2}\sigma_v^{-2}\left(y_i - y_{ik} - \boldsymbol{z}_i'\boldsymbol{\delta}\right)^2}{\sigma_u^{-2} + \sigma_v^{-2}}, \ \dfrac{\sigma_u^{-2}(y_i - y_{ik}) + \sigma_v^{-2}\boldsymbol{z}_i'\boldsymbol{\delta}}{\sigma_u^{-2} + \sigma_v^{-2}}, \ \{\sigma_u^{-2} + \sigma_v^{-2}\}^{-1} \right), \\ \qquad\qquad\qquad\qquad \text{if } s = 2k-1, \\ \left(\sigma_u^{-2}(y_i - \bar{y}_{ik})^2, \ \boldsymbol{z}_i'\boldsymbol{\delta}, \ \sigma_v^2 \right), \quad \text{if } s = 2k. \end{cases} \tag{A.34}$$

Given $s_i^* = s$, we generate w_i^* from $TN_{R_{is}}(\theta_{is}, \tau_s^2)$.

Step 6. Generate σ_u^2 given $\boldsymbol{\beta}, \boldsymbol{s}^*, \boldsymbol{w}^*$. It is straightforward to show that the full conditional posterior distribution of σ_u^2 is the inverse gamma distribution $IG(\frac{n_{u,1}}{2}, \frac{S_{u,1}}{2})$, where $n_{u,1} = n_{u,0} + 2 + n$ and

$$S_{u,1} = S_{u,0} + \left(\boldsymbol{\beta} - \boldsymbol{\mu}_{\boldsymbol{\beta},0}\right)' \Sigma_{\boldsymbol{\beta},0}^{-1} \left(\boldsymbol{\beta} - \boldsymbol{\mu}_{\boldsymbol{\beta},0}\right) + (\boldsymbol{y} - \boldsymbol{y}^*)'(\boldsymbol{y} - \boldsymbol{y}^*). \tag{A.35}$$

A.2 Full Conditional Distribution of $\tilde{g}$

We assume that $\bar{y}_{i1}$ is strictly positive. Then, the full conditional distribution of $\tilde{g}$ $(= g^{-1})$ is derived as follows. First, plug parameters multiplied by g $(= \tilde{g}^{-1})$ into the posterior distribution (24). Because the number of parameters to be accelerated is $4 + d + n$, the Jacobian of this transformation is $g^{-(4+d+n)}$. Transforming g to $\tilde{g}$, the conditional probability density of $\tilde{g}$ is given by:

$$\pi(\tilde{g} \mid \boldsymbol{\beta}, \boldsymbol{s}^*, \boldsymbol{w}^*, \sigma_v, \sigma_u) \propto \tilde{g}^{a_0} \exp\left[-\frac{1}{2} \left\{ a_1 \tilde{g}^2 - 2a_2 \tilde{g} \right\} \right] I\left(\tilde{g} \in R_{\tilde{g}}\right) L(d\tilde{g}), \tag{A.36}$$

where $L(d\tilde{g}) = \tilde{g}^{-1} d\tilde{g}$ is the left-Haar measure, $a_0 = n + n_{u,0} + n_{v,0}$, and

$$a_1 = \sigma_u^{-2}\left(S_{u,0} + \boldsymbol{\mu}_{\boldsymbol{\beta},0}' \boldsymbol{\Sigma}_{\boldsymbol{\beta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\beta},0} + \sum_{i=1}^{n} a_{4i}^2 \right) + \sigma_v^{-2}\left(S_{v,0} + \boldsymbol{\mu}_{\boldsymbol{\delta},0}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0} \right), \tag{A.37}$$

$$a_2 = \sigma_u^{-2}\left(\boldsymbol{\beta}' \boldsymbol{\Sigma}_{\boldsymbol{\beta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\beta},0} + \sum_{i=1}^{n} a_{3i} a_{4i} \right) + \sigma_v^{-2} \boldsymbol{\delta}' \boldsymbol{\Sigma}_{\boldsymbol{\delta},0}^{-1} \boldsymbol{\mu}_{\boldsymbol{\delta},0}, \tag{A.38}$$

$$(a_{3i}, a_{4i}) = \begin{cases} \left(y_{ik} + w_i^*,\ y_i\right), & \text{if } s_i^* = 2k-1, \\ (0,\ y_i - \bar{y}_{ik}), & \text{if } s_i^* = 2k, \end{cases} \tag{A.39}$$

$$R_{\tilde{g}} = \left(\max_i \left(0, BL_i^*\right), \min_i \left(BU_i^*, \infty\right)\right). \tag{A.40}$$

The BL_i^* and BU_i^* are the lower and upper bounds of the interval B_i^*, which is given by:

$$B_i^* = \begin{cases} \left(\frac{\beta_1 p_{ik} + \beta_2 q_{ik} + w_i^*}{\bar{y}_{ik}}, \frac{\beta_1 p_{ik} + \beta_2 q_{ik} + w_i^*}{\bar{y}_{i,k-1}}\right), & \text{if } s_i^* = 2k-1, \\ \left(\frac{\beta_1 p_{i,k+1} + \beta_2 q_{i,k+1} + w_i^*}{\bar{y}_{ik}}, \frac{\beta_1 p_{ik} + \beta_2 q_{ik} + w_i^*}{\bar{y}_{ik}}\right), & \text{if } s_i^* = 2k. \end{cases} \tag{A.41}$$

A.3 Proof for the One-step MH Algorithm

We prove that it suffices to implement a one-step MH algorithm using the initial value $\tilde{g} = 1$ in our GGS. By Theorem 2 of Liu and Sabatti (2000), it is adequate to prove that for all $\tilde{g}, \tilde{g}', \tilde{g}_0 \in \tilde{\Gamma} = \{\tilde{g} > 0 : \tilde{g}(x) = \tilde{g}^{-1} x\}$,

$$T_{\boldsymbol{\zeta}}(\tilde{g}, \tilde{g}')\, L(d\tilde{g}') = T_{\tilde{g}_0^{-1}\boldsymbol{\zeta}}(\tilde{g}\tilde{g}_0, \tilde{g}'\tilde{g}_0)\, L(d\tilde{g}'), \tag{A.42}$$

where $T_{\boldsymbol{\zeta}}(\tilde{g}, \tilde{g}')\, L(dg')$ is the transition kernel of our Markov chain.

Let $q_{\boldsymbol{\zeta}}(\tilde{g}')$ denote our proposal density. Then, the transition kernel becomes $T_{\boldsymbol{\zeta}}(\tilde{g}, \tilde{g}') = q_{\boldsymbol{\zeta}}(\tilde{g}')\alpha(\tilde{g}, \tilde{g}')\tilde{g}'$, where the last $\tilde{g}'$ is the adjustment term for the left-Haar measure. It is obvious that the acceptance probability $\alpha(\tilde{g}, \tilde{g}')$ is invariant to the scale transformation of $\tilde{g}_0$. Moreover, we have:

$$q_{\boldsymbol{\zeta}}(\tilde{g}')\tilde{g}' = \sigma_{\tilde{g}'}^{-1} \phi\left(\frac{\tilde{g}' - \mu_{\tilde{g}'}}{\sigma_{\tilde{g}'}}\right) I\left(\tilde{g}' \in R_{\tilde{g}}\right)\tilde{g}', \tag{A.43}$$

$$q_{\tilde{g}_0^{-1}\boldsymbol{\zeta}}(\tilde{g}'\tilde{g}_0)\tilde{g}'\tilde{g}_0 = \left(\tilde{g}_0\sigma_{\tilde{g}'}\right)^{-1} \phi\left(\frac{\tilde{g}'\tilde{g}_0 - \tilde{g}_0\mu_{\tilde{g}'}}{\tilde{g}_0\sigma_{\tilde{g}'}}\right) I\left(\tilde{g}'\tilde{g}_0 \in \tilde{g}_0 R_{\tilde{g}}\right)\tilde{g}'\tilde{g}_0 = q_{\boldsymbol{\zeta}}(\tilde{g}')\tilde{g}', \tag{A.44}$$

where $\phi(\cdot)$ is the density function of the standard normal distribution. Thus, the transition kernel of the Markov chain is invariant to transformation $\tilde{g}$, which completes the proof.

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